P. Ammiraju, E. Recami and W. A. Rodrigues Jr. :
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A PHENOMENOLOGICAL AND THEORETICAL ANALYSIS

CHIRONS, GEMINIONS, CENTAUROS, DECAYS INTO PIONS:
A PHENOMENOLOGICAL AND THEORETICAL ANALYSIS[o]

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ABSTRACT

We analyse the data on multiple production of hadrons from very high-energy inter
actions, both at cosmic ray energies and at accelerator energies. We show that these
data are compatible with a discrete mass spectrum of fireballs, which are formed in
the very high-energy collisions and trail the colliding hadrons after the interaction.
Such fireballs seem to possess different decay-modes: either into pions only, or into
baryons only. The decays are statistical and we derive their temperature; for instance,
in the so-called "Chiron-mode" the temperature is about 10 GeV, which shows that the
existence of a limiting temperature of 160 MeV in high-energy collisions is violated.
Moreover, we present a theoretical model for decay-events like Chirons, Centauros
and Geminions in terms of evaporating "strong black-holes". Our analysis seems to
suggest - among the others - that in the considered collisions some "phase transitions"
can take place, associated with the collapse of the colliding matter inside its strong-
-Schwarzshild horizons. The horison radii are in good agreement with experience and,
on their turn, yield the transition temperature through a Hawking-like relation. At
these very high temperatures the emission of heavy objects is expected to be enhanced,
so as it is observed experimentally. Many aspects of the data are reasonably well ex-
plained by our theoretical model (cf. Sect. 4.3).

1. - INTRODUCTION

Extensive data on multiple production of hadrons at extremely high energies ($E_{lab} \sim$ $\sim 1-10^3$ TeV) are now available from Cosmic Ray balloon and emulsion-lead Chambers ($> 10^3$ TeV), as well as from FNAL, ISR and SPS-$p\bar{p}$ collider accelerator experiments. The comparison of both types of data is thus relevant and timely. Although we lack a definite theoretical framework to understand those results, sometimes puzzling so as the Centauro events, even a phenomenological approach, which a future theory would incorporate, would be worthwhile.

This paper is organised as follows. In Section 2, we briefly review the present experimental situation as obtained at Cosmic Ray and accelerator energies. In Section 3, we work out some aspects of a simple "discrete-mass fireball model" to explain the main features of multiplicity and transverse momentum in the "pionization", and of the high multiplicity, high transverse momentum in the "baryonization" of the Centauro, Gemini and Chiron events discovered by the Brazil-Japan Emulsion Chamber Collaboration (herein after referred to as BJECC). In Section 4, we propose a possible theoretical explanation of Chirons, Gemions and Centauros in terms of evaporating "strong black-holes"; this will be attempted within a unified theory of strong and gravitational interactions, by making recourse to the methods of General Relativity applied to elementary particle physics. In Section 5 we present our conclusions.

2. - A BRIEF REVIEW OF THE EXPERIMENTAL DATA FROM COSMIC RAYS AND ACCELERATOR ENERGIES

We shall now recall briefly the main experimental facts about inelastic interactions at high energies, under the following classification:

(i) Transverse momentum distribution,
(ii) Longitudinal momentum distribution,
(iii) Multiplicity - Scaling,
(iv) Centauro, Geminion and Chiron events.

Data on total and elastic scattering cross-sections are left out in this survey for the simple reason that such data are available with accelerator experiments while in Cosmic Rays physics only the inelastic cross-section can be inferred.

2.1. - Transverse momentum distribution

One of the key features of the early experimental data in the secondary particle production processes in high energy interactions is the remarkably stringent limit on the

transverse momentum, p_t, of the secondaries in Cosmic Ray jets, in the tremendous interval of the primary energy $E_1 \sim 10\,GeV - 10^6\,GeV$, with a mean value around 300 MeV/c. Subsequently, at accelerator energies (10-30 GeV), the same limitation of p_t was confirmed, - the mean value of p_t being independent of the type of reaction studied, the observed multiplicity and the incident energy. The differential cross-section for hadron production falls off exponentially as a function of the transverse momentum up to $p_t \simeq 1.0\,GeV/c$ according to the empirical law

$$E\,\frac{d^3\sigma}{dp^3} \;\sim\; e^{-6\,p_t} \,. \tag{2.1}$$

Motivated by the above conclusions, several statistical thermodynamical and bootstrap models were proposed. Prominent among them is the Hagedorn statistical model, which assumes a continuous mass spectrum of hadrons with a universal highest temperature ($\simeq 160\,MeV$) above which no matter can be heated.

This picture changed completely with the pioneer Cosmic Ray mountain experiments of BJECC during 1970-1980's. Lattes et al.[1] classify the multiple production of pions into discrete jet types that are called Mirim, Açu and Guaçu jets, the transverse momentum increasing progressively as is shown in Fig. 1. During the same period, the comissioning of ISR made available higher accelerator energies ($\sqrt{s}$ = 63 GeV) than hitherto available and the data showed the existence of higher average p_t values. With the successful operation of the CERN SPS-$p\bar{p}$ Collider ($\sqrt{s}$ = 540 GeV), which now offers the highest available energy at accelerators, preliminary data on high transverse momenta have been accumulating. These data[2] show (Fig. 2) that the distribution becomes wider, and for increasing values of the transverse momentum ($p_t > 2\,GeV/c$) the observed yield is found to be several orders of magnitude larger than one would expect, were the exponential behaviour to continue. Moreover, the observed effect is energy dependent, in contrast to that observed below 1 GeV/c. This strong deviation from the simple exponential law implies that the mechanism involved in low and high p_t
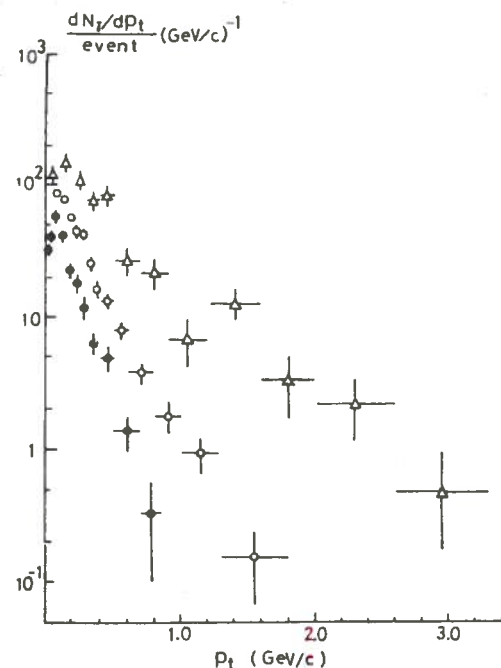

FIG. 1 - Distribution of the gamma ray transverse momentum, p_t, for three types of jets. $\bullet$: Mirim-jet; o: Açu-jet; Δ : Guaçu-jet (Ref. (1)).
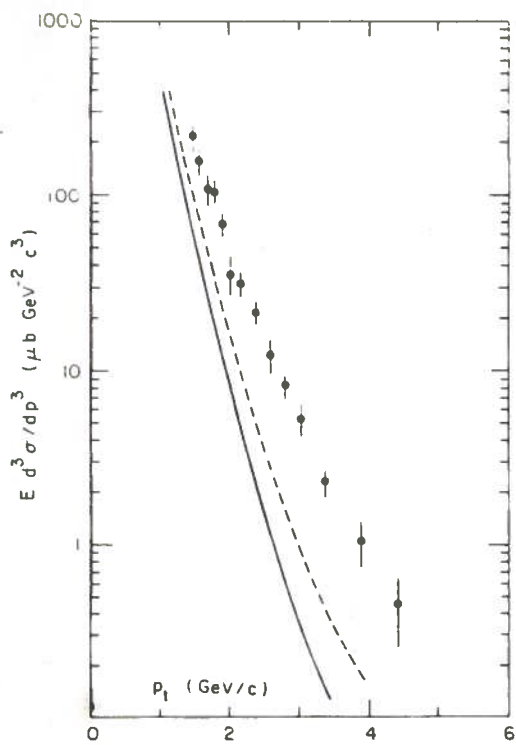
FIG. 2 - $\blacklozenge$: Measured Invariant Cross-Section for $p\bar{p} \rightarrow \pi^0 +$ any thing at $\sqrt{s}$ = 540 GeV; —:idem, at the ISR energy $\sqrt{s}$ = 53 GeV; ---: Extrapolation of data to Collider energies using the form $Ed^3\sigma/dp^3 \sim p_t^{-8}(1 - x_t)^9$; $x_t = 2\,p_t/\sqrt{s}$ (Ref. (2)).

phenomena could be different.

Even more surprising is the fact that BJECC found[1] certain cosmic gamma ray families, which cannot even be explained by the above classification because the observed events have characteristics totally different from the ordinary multiple meson production. They are characterised by high charged hadron multiplicity ($N_{ch} \sim$ $\sim$50-100) and a multiplicity of neutral π^0 mesons which is consistent with zero. Further, these new type of events have average p_t varying from 500 MeV/c to 15 GeV/c. Such extremely high p_t events led to intense speculations regarding the mechanism of their production, suggesting that either there is a new component in the primary Cosmic Radiation (Fe component), or there is a new kind of hadron interaction.

2. 2. - Longitudinal momentum distribution

The severe limitations found for the transverse momentum is in strong contrast to the large range of variations observed for the longitudinal momentum. In the Cosmic Ray experiments of BJECC in 1967, Lattes et al.[1] use the fractional energy of the gamma rays $R_\gamma = E_\gamma/\Sigma E_\gamma$ as a measure of the longitudinal momentum, while it has the common practice among the accelerator workers during 1970's to use the longitudinal momentum via the scaling variable $x = P_\ell^{c.m.} / \frac{1}{2}\sqrt{s} \approx E^{Lab}/E_{inc}$ up to a factor K_γ which is the gamma ray inelasticity. The so called "leading particle effect" or "weak inelasticity" property of the collision process found at accelerator energies is simply a confirmation of the well-known Cosmic Ray result valid up to $E_{inc} \sim 1$ TeV. The fractional gamma ray energy spectra of the BJECC data show that the scaling rule valid in the accelerator region up to $E_{inc} \sim 2$ TeV is not the same as that found in the higher energy region $E_{inc} \sim 7$ to 130 TeV and that this change of scaling rule could be abrupt. Fig. 3 shows[1] the results of the comparison of the 80 C-jet data of BJECC ($\Sigma E_\gamma > 20$ TeV) with the data from 9 C-jets of proton primary, selected under the cri

terion $\Sigma E_\gamma \gtrsim 2$ TeV, of the experiment of Sato et al.[3]. The agreement between the two is quite good in spite of their difference in ΣE_γ value.

FIG. 3 - Distribution of fractional energy of gamma rays. $\bullet$: 80 Chacaltaya C-jets with $\Sigma E_\gamma > 20$ TeV; o: 9 C-jets by proton primary in balloon experiments, with $2 < \Sigma E_\gamma < 5$ TeV (Ref. (3)).
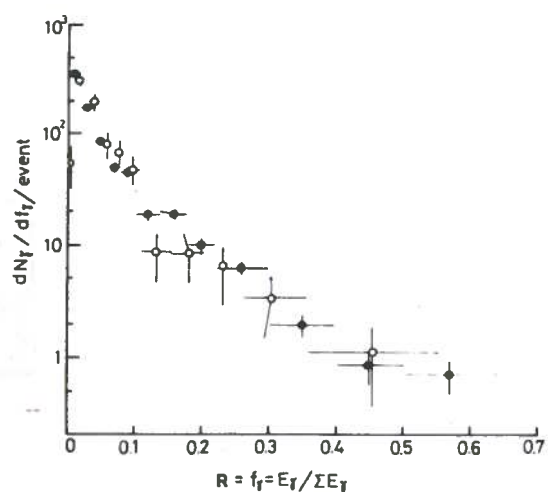
2.3. - Multiplicity - scaling

In Cosmic Ray investigations, it has been noticed for a long time that the average number of secondary particles produced in high energy collisions grows with the energy and the identification of this dependence with $E_{in}^{1/4}$ has been motivated by the statistical model calculations. The more recent data of BJECC point to a slower growth of the multiplicity - perhaps a log s dependence - at least up to $\langle E_{INC} \rangle \sim 130$ TeV of the Chacaltaya C-jets.

At lower accelerator energies (10-30 GeV), one finds relatively low multiplicities of secondaries and their slow increase with energy is in conformity with a log s dependence. This has been the strong motive for the development of the multiperipheral
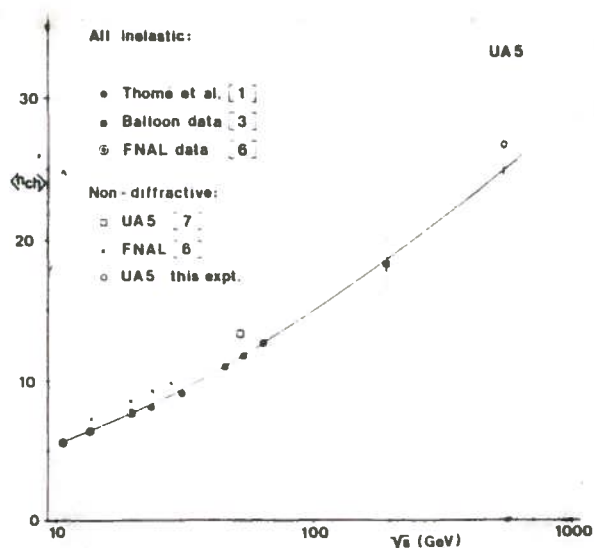

FIG. 4 - Dependence of $\langle n_{ch} \rangle$ upon $\sqrt{s}$ as measured at ISR and lower energies with the fit by Thomé et al.: $\langle n_{ch} \rangle = a + b \log s + c \log^2 s$. The UA5 data point is for sdd excluded (Ref. (4)).

model, suggesting an asymptotic behaviour at high energies. However, at ISR energies a sharper rise of the multiplicity was found and the fit obtained by Thomé et al.[4] to the ISR data has been confirmed by the more recent data of $p\bar{p}$ collider at higher energy. Fig. 4 shows[5] the dependence of $\langle n_{ch} \rangle$ on s for several FNAL, ISR and SPS-$p\bar{p}$ collider energies. The data points fit rather well the parametrization

$$\langle n_{ch} \rangle = a + b \log s + c \log^2 s \qquad (2.2)$$

proposed by Thomé et al.[4]. Assuming that elastic scattering and single diffraction dissociation each contribute 18% to the total cross-section and that the extreme ranges

of the mean charged multiplicity are given by $3 \lesssim \langle n_{ch} \rangle^{sd} \lesssim \frac{1}{2} \langle n_{ch} \rangle^{nd}$ (sd = single diffraction dissociation; nd = non diffraction dissociation), the average inelastic char ged multiplicity is estimated[5] to be in the range $21 \lesssim n_{ch} \lesssim 27$, in reasonable agreement with the prediction $\langle n_{ch} \rangle \approx 25$ of Thomé et al. The fit to the ISR data using a $s^{1/4}$ dependence gives $\langle n_{ch} \rangle \approx 40$ which is clearly ruled out. The $p\bar{p}$ Collider data are in agreement with the general features of the C-jet data, but do not show any significant deviations from extrapolations of the FNAL and ISR data, particularly in the direction of higher multiplicity, as it has been frequently suggested by Cosmic Ray data. But this discrepancy may be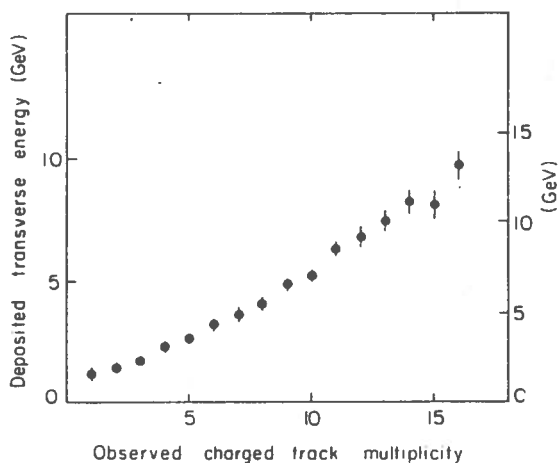 due to different detector thresholds. Fig. 5 shows[6] the deposited transverse energy as a function of the observed charged track multiplicity. Although the average value of the transverse energy per event divided by the charged track multiplicity seen in the UA1 detector does not indicate correlation between transverse energy and multiplicity, this result cannot be meaningfully compared with BJECC C-jet data as the experimental conditions are widely different.

FIG. 5 - Deposited transverse energy as a function of charged track multiplicity (Ref. (5)).

At the SPS-$p\bar{p}$ Collider energies, the UA1 experiment finds[6] that the charged multiplicity distribution, using the coordinates suggested by KNO-scaling, agreed well with the one measured for pp Collisions at the ISR over about the same rapidity interval, the deviation from Poisson distribution indicating the existence of multiparticle correlations (Fig. 6).
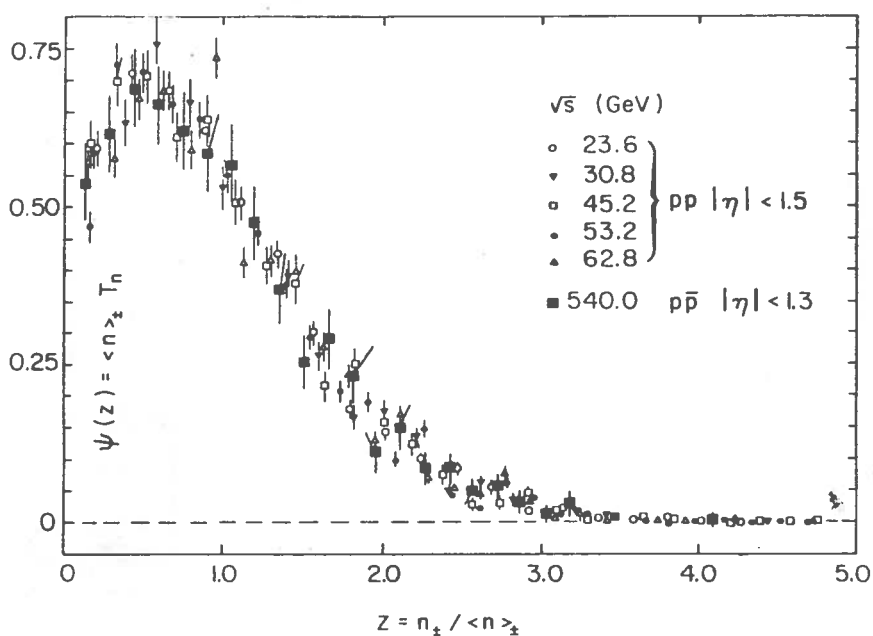

FIG. 6 - The charged track multiplicity $n_{\pm}$ in the fiducial region. T_n is the probability for the observation of n tracks, and $\langle n_{\pm} \rangle = 6.57$ is the number per event (Ref. (6)).

2. 4. - Centauro, Geminion and Chiron events

BJECC found[1] certain peculiar events called Centauros (see Table I) which constitute a significant fraction of the Cosmic Ray families with visible energy 100-1000 TeV. Those authros propose that Centauro is a particular type of nuclear interaction with emission of about a hundred "baryons" (and possibly antibaryons) but without any significant emission of mesons (absence of gamma rays from the neutral pion decays). For two Centauro events, measurement of the particle emission angles have been made. The BJECC detector can separate the electromagnetic shower from hadrons although the latter are detected via secondary cascades produced by them.

TABLE I - Brazil-Japan Emulsion Chamber Collaboration: A summary of the experimental results.

DECAY MODE / TYPE OF CLUSTER	$M^{(0)} \sim 2-3$ GeV/c^2	$M^{(1)} \sim 15-30$ GeV/c^2	$M^{(2)} \sim 100-300$ GeV/c^2
Into Pions (Multipion final state)	DIRECT DECAY $M^{(0)} \to$ Pions $<n>\gamma \sim 22-38$ $<P_T>\gamma \sim 140-150$ MeV/c $\frac{\Delta <n>\gamma}{\Delta y} \sim 2-4$ Nº of events ~ 500 (MIRIM-JET) $T_\pi^{(0)} = 110$ MeV	CASCADE DECAY $M^{(1)} \to M^O <n>_{MO} \to$ Pions $<n>\gamma \sim 22-38$ $<P_T>\gamma \sim 220-250$ MeV/c $\frac{\Delta <n>\gamma}{\Delta y} \sim 6-8$ Nº of events ~ 80 (AÇU-JET) $T_\pi^{(1)} \approx 0.4$ GeV -	CASCADE DECAY $M^{(2)} \to M^O <n>_{MO} \to$ Pions $<n>\gamma \sim 70-100$ $<P_T>\gamma \sim 400-500$ MeV/c $\frac{\Delta <n>\gamma}{\Delta y} \sim 20-30$ Nº of events = 5 or 6 (GUAÇU-JET) $T_\pi^{(2)} \gtrsim 0.4$
"CENTAURO" (Multibaryon final state without Pion emission)		$M^{(1)} \to$ BARYONS $<n>_B \sim 15\pm 2$ $<P_T>_B \sim 1.5$ GeV/c Nº of events = 16 (MINI-CENTAURO) $T_C^{(1)} = 1.5$ GeV	$M^{(2)} \to$ BARYONS $<n>_B \sim 100\pm 20$ $<P_T>_B \sim 1.5$ GeV/c Nº of events = 6 0r 7 (CENTAURO) $T_C^{(2)} = 1.5$ GeV
"CHIRON" (Multibaryon final state without Pion emission); or "GEMINION" (Decay into a pair of baryons)		$M^{(1)} \to$ BARYONS $<n>_B = 2$ $<P_T>_B \sim 10-15$ GeV/c Nº of events = 21 (Castor-Pollux with triangulation) ("GEMINION")	$M^{(2)} \to$ "BARYONS" $<n>_B \sim 22\pm 4$ $<P_T>_B \sim 10-15$ GeV/c Nº of events ~ 2 (with triangulation) but more are seen. ("CHIRON") $T_{Ch}^{(2)} = 10$ GeV

A vigorous search for Centauro-like events has been made with the UA1 (general purpose central detector) and UA2[5] (streamer chamber) at the SPS-$p\bar{p}$ colliding beam machine at CERN. With the UA1 detector, no Centauro-like events have been found[7] in an examination of 48.000 low bias events, using the criteria of large hadronic and low electromagnetic energy content, taking account also of multiplicity

and average transverse momentum. In the UA5 streamer chamber, using similar criteria as UA1, those authors estimate[8] the expected number of charged particles to vary between 30 and 43 with rapidity $\eta > 2$ or $\eta < -2$. However, there was no event with $n_{ch}^{obs}(\eta > 2)$ even remotely comparable to the expected value. If the hadrons occurring in Centauro events are baryons and antibaryons only, a Monte Carlo study of events of this multiplicity with no primary photons gives $n_{\gamma}^{obs}(\eta > 2) = 1 \overset{+}{\underset{-}{}} 1$ owing to secondary hadronic interactions in the vacuum chamber. Experimentally[8], 24 events having $n_{\gamma}^{obs}(\eta > 2) \lesssim 2$ is taken to be consistent with having zero primary photons and are considered as candidates at the level of 1 in 150 of minimum bias events.

No information is available on the search for Geminion or Chiron type of events.

3. - DISCRETE-MASS FIREBALL MODEL.

The experimental evidence strongly suggests that, in the phenomena of multiple production of hadrons at extremely high energies ($E_{lab} \gtrsim 100$ TeV), the interaction is mainly peripheral (since the colliding nucleons survive the collision) and gives rise to the formation of one or more excited fireballs, depending on the energy available (see Fig. 7a and Fig. 7b). Such fireballs trail the colliding nucleons and subsequently de-excite into hadrons. We emphasize that the observed fireballs (see Table I) seem to correspond to a discrete mass spectrum. We make the following assumption: Fireball formation as well as decay are incoherent phenomena.
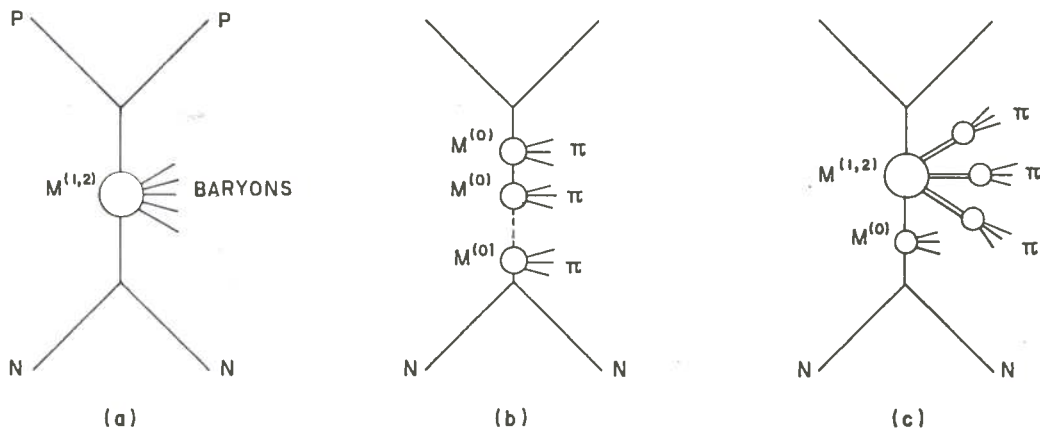
FIG. 7 - (a) "Peripheral" scheme for the decays into hadrons in the discrete mass fireball model (see the text); (b) The multiperipheral mechanism for the decays into pions, below the $M^{(1)}$-fireball production threshold (see the text); (c) Above the threshold, the diagram would include also (multiperipheral) production of the $M^{(1)}$ fireball.

We show in what follows that a discrete mass spectrum of fireballs is consistent with and also a natural explanation for the empirical properties of the energy distri-

butions <u>for pions and gamma rays</u> (and also the multiple production of baryons (Centauro-like jets)) observed at accelerator and Cosmic Ray energies. The analysis also shows that 160 MeV does not seem to be any more a limiting temperature for very high energy collisions[19].

3.1. - Multiple production of pions

As is well known, the variable $x = E_c/E$ where E_c is the energy of a secondary c (produced in the collision process) and E is the total energy of the collision measured in the laboratory system, spans in the limit $E \to \infty$ the so called beam fragmentation region. A variable closely related to x is $R = E_\gamma/\Sigma E_\gamma$, the fractional energy, where ΣE_γ represents the energy released in a hadronic collision in the form of gamma rays (resulting fron the decay $\pi^0 \to 2\gamma$). These gamma rays are observed in x-ray films and nuclear emulsions by BJECC at Chacaltaya.

The spectrum of the energy distribution of gamma rays (Fig. 8)[1] has three branches and each satisfies a similarity property, i.e. the distributions are only a function of the variable R and do not depend on the collision energy E. Such a property of the spectrum is equivalent to the scaling in the beam fragmentation region if the mean inelasticity of the collisions remains constant.
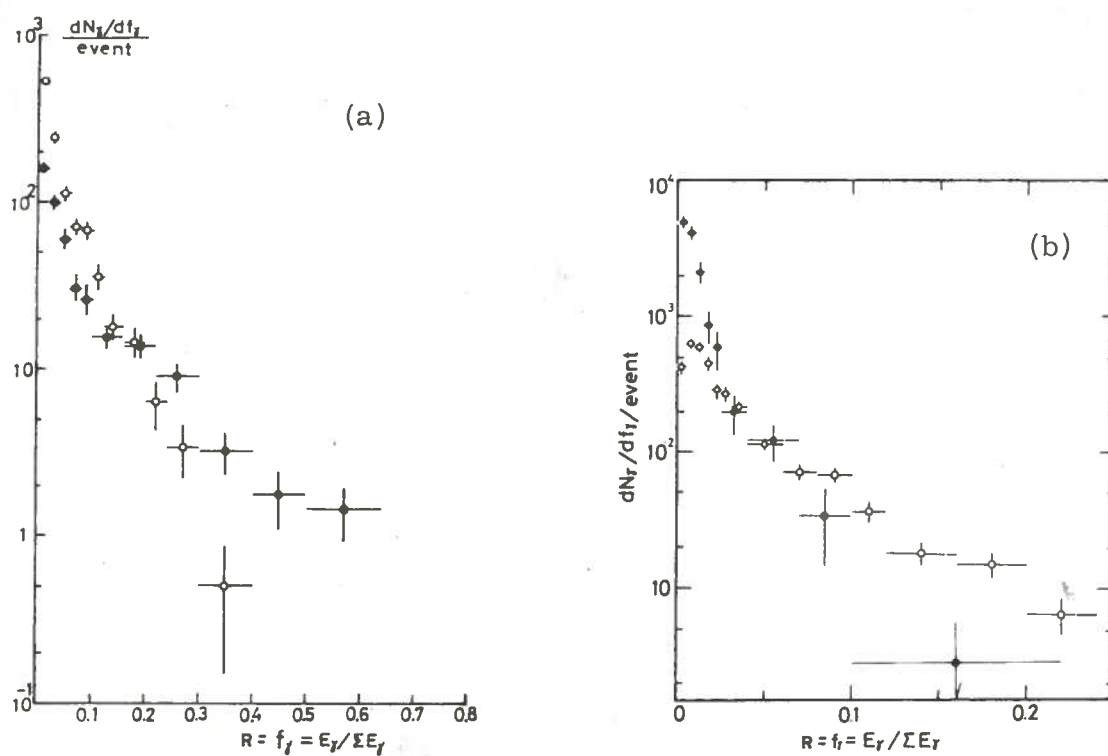

FIG. 8 - Distribution of the fractional energy of gamma rays, $R = f_\gamma$, for three types of jets, Mirim, Açu and Guaçu: (a) ●: Mirim-jets; o: Guaçu-jets; (b) o: Açu-jets; ●: Guaçu-jets (Ref. (1)).

We assume in what follows that in the process of multiple production of pions, one or more intermediate states (fireballs) are produced and then decay into hadrons. For a given intermediate state we assume that the energy distribution of pions result ing from the decay is such that

$$F(E_\pi^*, \theta_\pi^*) \, dE_\pi^* \, d\theta_\pi^* = f(E_\pi^*) \, dE_\pi^* \, \frac{d\Omega^*}{4\pi} \quad . \tag{3.1}$$

The asterisks denote variables measured in the rest system of the fireball (not the centre of momentum of the collision). For the moment we do not specify $f(E_\pi^*)$, but assume that

$$\int_{E_\pi^*(\text{min})}^{E_\pi^*(\text{max})} f(E_\pi^*) \, dE_\pi^* = 1 \quad , \tag{3.2}$$

where $E_\pi^*(\text{min}) = m_\pi$ and $E_\pi^*(\text{max}) = M^{(0)}/2$, quantity m_π being the pion mass and $M^{(0)}$ the mass of the intermediate state, which can in principle be a function of the collision energy: i. e., $M^{(0)} = M^{(0)}(E)$.

In the laboratory frame, the differential energy distribution of pions reads[o]

$$g(E_\pi, \Gamma) = \frac{1}{2} \int_{\overline{E}_\pi^*(\text{min})}^{E_\pi^*(\text{max})} f(E_\pi^*) \, \frac{dE_\pi^*}{\Gamma \beta p_\pi^*} \quad , \tag{3.3}$$

where Γ is the Lorentz factor of the fireball in the laboratory frame, p_π^* is the pion momentum in the fireball rest-frame and

$$\overline{E}_\pi^*(\text{min}) = \Gamma(E_\pi - \beta p_\pi) \quad \text{and} \quad E_\pi^*(\text{max}) = M^{(0)}/2. \tag{3.4}$$

We now investigate the consequences of the hypothesis that the mass of the fireball be independent of E, and also that the mean inelasticity of the collision, K, be a constant when $E \rightarrow \infty$. We write

$$M^{(0)}\Gamma = KE . \tag{3.5}$$

Using approximations valid when $E \gg m$ and $\Gamma \gg 1$, we obtain

$$g(E_\pi, \Gamma) \, dE_\pi = \frac{dE_\pi}{\Gamma} \int_{\frac{E_\pi}{2\Gamma} + \frac{\Gamma m_\pi^2}{2E_\pi}}^{M^{(0)}/2} dE_\pi^* \, \frac{f(E_\pi^*)}{p_\pi^*} + 0(\Gamma^{-2}) . \tag{3.6}$$

(o) We shall adopt natural units ($\hbar = c = 1$) when convenient (and when not differently stated).

Using now eq. (3.5), we can write the differential distribution for pions in terms of the variable $x = E_\pi/E$. We get

$$g(E_\pi, E) \, dE_\pi = a dx \int_{ax + \frac{b}{x}}^{M^{(0)}/2} dE_\pi^* \frac{f(E_\pi^*)}{p_\pi^*} \equiv g(x) \, dx \,, \qquad (3.7)$$

where $a \equiv M^{(o)}/2K$ and $b \equiv m_\pi^2 K/2M^{(0)}$.

Thus, if in the collision only one intermediate state is produced, it holds

$$\frac{1}{\sigma_t} \frac{d\sigma}{dx} = \frac{d}{dx} \langle n(x, E) \rangle = \frac{1}{\langle n_\pi \rangle} g(x). \qquad (3.8)$$

As the variable x spans the beam fragmentation region, we see that eq. (3.8) is equivalent to the scaling of the inclusive distribution in the beam fragmentation region. If the mean inelasticity of the collision remains independent of the energy, we see that the hypothesis that the mass of the fireball is independent of the collision energy is a natural explanation for the distribution of Fig. 9, constructed here for Mirim-jets.

When in the collision several fireballs, all with the same mass $M^{(0)}$, are produced, then $\sigma_t^{-1} d\sigma/dx \propto g(x)$ if the Lorentz factors of the fireballs form a geometrical series[9], i.e. Γ_i/Γ_{i+1} = constant. This corresponds to several fireballs being produced along a multiperipheral chain with constant momentum transfer between the fireballs[9].

If fireballs with different masses are

FIG. 9 - Fractional energy distribution $(1/\sigma_t)(d\sigma^{(\pi^0)}/dR)$. Observe that all data points lie on the same curve. The variable R spans the so-called beam fragmentation region (Ref. (9)).

produced, we break the scaling property. Indeed, in deriving eq. (3.8), we used the hypothesis that the mass of the fireballs is independent of the energy of the collision, and so, if there are intermediate states with a mass different from $M^{(0)}$, say $M^{(1)}$, then also the distribution $d \langle n^{(1)} \rangle/dx$, corresponding to $M^{(1)}$ jets, will not be a function of E, although it will be different from $d \langle n^{(0)} \rangle/dx$. Thus, studying experimentally the energy (or momentum) distribution, one can establish the existence of distinct distributions, if the energy interval under consideration contains contributions from intermediate states of different masses.
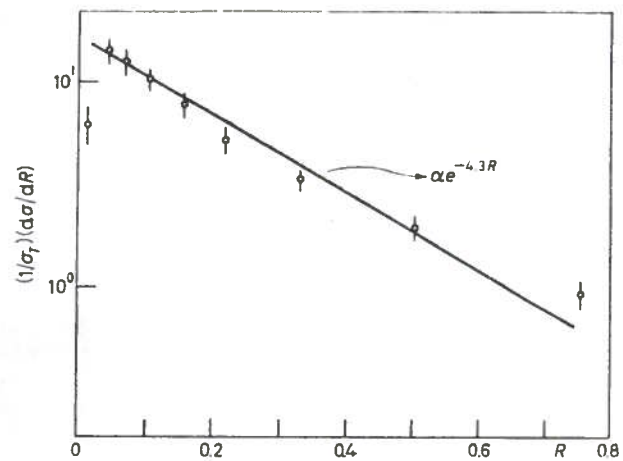
So, the existence of branches in Fig. 8 for the gamma ray energy distribution is compatible with fireballs of <u>distinct</u> masses, and with the fact that these masses do not depend on the collision energy. Fig. 8 tells us, also, that the three distributions satisfy a similarity law, and we now show that this is indeed the case under the hypo_thesis outlined above.

The energy distribution of gamma rays in the fireball rest-system is given by

$$G(\mathscr{E}_\gamma^*) \, d\mathscr{E}_\gamma^* = d\mathscr{E}_\gamma^* \int_{\overline{E_\pi^*}(min)}^{E_\pi^*(max)} \frac{1}{p_\pi^*} \, f(E_\pi^*) \, dE_\pi^* , \tag{3.9}$$

where $\mathscr{E}_\gamma^*$ is the gamma ray energy, $f(E_\pi^*)$ is again the pion energy distribution in the fireball rest-frame, and

$$\overline{E_\pi^*}(min) = \mathscr{E}_\gamma^* + \frac{m_\pi^2}{4\mathscr{E}_\gamma^*} \tag{3.10}$$

is the minimum energy a pion must have in order to produce a gamma ray of energy $\mathscr{E}_\gamma^*$. Also, we have $E_\pi^*(max) = M^{(0)}/2$, quantity $M^{(0)}$ being the mass of the fireball; and eq. (3.9) is valid only in the interval

$$\frac{m_\pi^2}{2M^{(0)}} \leqslant \mathscr{E}_\gamma^* \leqslant M^{(0)}/2 . \tag{3.11}$$

Defining

$$\Phi(M^{(0)}/2, \mathscr{E}_\gamma^* + \frac{m_\pi^2}{4\mathscr{E}_\gamma^*}) = \int_{\mathscr{E}_\gamma^* + \frac{m_\pi^2}{4\mathscr{E}_\gamma^*}}^{M^{(0)}/2} dE_\pi^* \frac{f(E_\pi^*)}{p_\pi^*} \tag{3.12}$$

we obtain, under the hypothesis set in eq. (3.1) (isotropic decay) that:

$$G(\mathscr{E}_\gamma^*, \cos\theta_\gamma^*) \, d\mathscr{E}_\gamma^* \, d(\cos\theta_\gamma^*) = \frac{1}{2} \, \Phi(M^{(0)}/2, \mathscr{E}_\gamma^* + \frac{m_\pi^2}{4\mathscr{E}_\gamma^*}) \, d\mathscr{E}_\gamma^* \, d(\cos\theta_\gamma^*) . \tag{3.13}$$

By using the exact kinematical limits of the appropriate variables, we get in the labo_ratory frame

$$G(\mathscr{E}_\gamma, E) \, d\mathscr{E}_\gamma = \frac{1}{2} \frac{M^{(0)}}{K} \, d\eta \int_{M^{(0)}/2K}^{M^{(0)}/2\eta} \Phi(M^{(0)}/2, \, y\eta + \frac{m_\pi^2}{4y\eta}) \frac{dy}{y} \equiv$$

$$\equiv \frac{1}{2} \frac{M^{(0)}}{K} \, h(\eta) \, d\eta , \tag{3.14}$$

where $y \equiv Eu$, $u \equiv \Gamma(1 - \beta \cos \theta_\gamma)$ and $\eta = \mathscr{E}_\gamma / E$. Since

$$G(\mathscr{E}_\gamma, E) = \frac{1}{\langle N_\gamma \rangle} \frac{d \langle n_\gamma \rangle}{d\mathscr{E}_\gamma},$$ (3.15)

we have

$$\frac{1}{\langle N_\gamma \rangle} \frac{d \langle n_\gamma \rangle}{d\eta} = \frac{1}{2} \frac{M^{(0)}}{K} h(\eta)$$ (3.16)

valid for

$$\frac{m_\pi^2 K}{M^{(0)2}} \leq \eta \leq K,$$ (3.17)

which corresponds to the observable range of the BJECC experiment. Also, the integral spectrum $F(\geq N, E)$, when $m_\pi^2 \Gamma / M^{(0)} \leq W \leq M^{(0)} \Gamma$, satisfies

$$F(\geq W, E) = \int_W^{M^{(0)} \Gamma} G(\mathscr{E}_\gamma, E) d\mathscr{E}_\gamma \equiv H(R);$$ (3.18)

$$H(R) = \frac{1}{2} \frac{M^{(0)}}{K} \int_R^K d\eta \int_{M^{(0)}/2K}^{M^{(0)}/2\eta} \Phi(M^{(0)}/2, y\eta + \frac{m_\pi^2}{4y\eta}) \frac{dy}{y}.$$ (3.19)

Eq. (3.19), thus, agrees with what can be seen in Fig. 8.

3. 2. - Mass and temperature of the fireball

Assuming a Bose-Einstein distribution for the emitted pions in the fireball rest-system, we obtain the p_t^2 distribution as follows

$$\frac{1}{\sigma_t} \frac{d^2 \sigma}{dp_t^2} = N_\pi \left(\frac{\Gamma(3/2)}{\Gamma(1/2)} \right) \sqrt{p_t^2 + m_\pi^2} \sum_{n=1}^{\infty} e^{-n\alpha} K_1 \left(n\beta \sqrt{p_t^2 + m^2} \right),$$ (3.20)

where

K_ν = Bessel-function of the K_ν-type,

α_π = chemical potential of the pion,

β = $1/kT$,

k = Boltzmann constant,

N_π = normalization constant.

The temperature of the fireball is related to the mean-value of p through the equation

$$\langle p(m, T) \rangle \simeq 4 \sqrt{\left(\frac{mT}{2\pi}\right)} K_{5/2}\left(\frac{m}{T}\right)/K_2\left(\frac{m}{T}\right) \simeq 4 \sqrt{\frac{mT}{2\pi}} \tag{3.21}$$

and we have for the mean transverse momentum $\langle p_t \rangle$:

$$\langle p_t(m, T) \rangle = \frac{\pi}{4} \langle p(m, T) \rangle = \sqrt{\frac{\pi m T}{2}} \quad , \tag{3.22}$$

where m is the particle mass taking part in the statistical equilibrium. For Mirim--jets we have $\langle p_t \rangle \simeq 150 \, \text{MeV}/c$ and $T = 105 \, \tex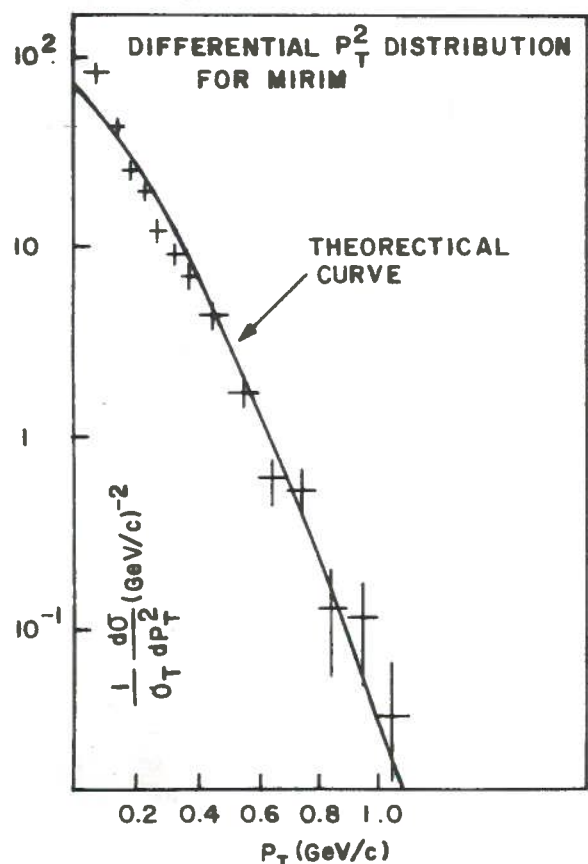t{MeV}$. Fig. 10 shows the p_t distribution for Mirim-jets[10]. The curve has the exponential behaviour $\exp(-7p_t)$ for p_t not very near the origin, and is compatible with what is found in accelerator physics. It is necessary to have in mind here that the transverse momenta in Fig. 10 are measured with respect to the fireball rest-system.

FIG. 10 - Distribution of p_t^2 for π^o mesons produced by the Mirim-jets (Ref. (10)).

Eq. (3.22) shows that the temperature of the fireball is independent of its mass under the above assumption. This yields as a consequence that Açu and Guaçu jets cannot simply be decays of fireballs into pions[9,10,11].

We now estimate the p_t distribution of Açu (or Guaçu) as follows: We suppose that $M^{(1)}$ (or $M^{(2)}$) is a statistical equilibrium of $M^{(0)}$ fireballs with temperature $T^{(1)}$ (or $T^{(2)}$), which subsequently decay into pions. Let $\vec{q}^*$ and E^* be the momentum and energy of a pion produced in the rest-frame of a $M^{(0)}$ fireball with a Bose-Einstein temperature $T^{(0)}$. Let $\vec{p}$ and E be the momentum and energy of the pion in the $M^{(1)}$ (or $M^{(2)}$) rest-frame; and let $\vec{p}^{(0)}$ and $E^{(0)}$ respectively be the momentum and energy of a $M^{(0)}$ fireball in the $M^{(1)}$ (or $M^{(2)}$) rest-frame. Then

$$\begin{cases} \vec{p} = \vec{q}^* + \vec{V}^{(0)} \left\{ \frac{1}{(V^{(0)})^2} \vec{V}^{(0)} \cdot \vec{q}^* (\Gamma^{(0)} - 1) + \Gamma^{(0)} E^* \right\} \\ \\ E = \Gamma^{(0)} (E^* + \vec{V}^{(0)} \cdot \vec{q}^*) \end{cases} \tag{3.23}$$

and

$$\begin{cases} \vec{p}^{(0)} = M^{(0)}\Gamma^{(0)}\vec{V}^{(0)} = M^{(0)}\left[2(E + E^*)/s\right]\vec{\ell} \\ E^{(0)} = M^{(0)}\Gamma^{(0)} = M^{(0)}\left[1 + 2\ell^2/s\right] , \end{cases} \tag{3.24}$$

where $\vec{\ell} = (\vec{p} - \vec{q}^*)$ and $s = (E + E^*)^2 - \vec{\ell}^2$. For simplicity, let us refer in the following to the Açu-jets only. Let $P(\vec{p}, \vec{q}^*)d^3pd^3q^*$ be the probability for a pion produced with momentum $(\vec{q}^*, \vec{q}^* + d\vec{q}^*)$ through the decay of $M^{(0)}$ to have momentum $(\vec{p}, \vec{p} + d\vec{p})$ in the $M^{(1)}$ rest frame. Under the statistical assumptions above, we have

$$P(\vec{p}, \vec{q}^*) = CJ(\vec{p}, \vec{q}^*)W(\vec{p}, \vec{q}^*; T^{(1)}, T^{(0)})d^3pd^3q^* , \tag{3.25}$$

where $T^{(i)}$ (i = 0, 1) is the temperature of the $M^{(i)}$ fireball, C is an appropriate normalization constant, J is the Jacobian from the $M^{(0)}$ rest-system to the $M^{(1)}$ rest-system:

$$J = \frac{8M^{(0)3}}{E}\left(\frac{E + E^*}{s^2}\right)(s + 2\vec{\ell}^2)\left[E(E + E^*) - \vec{p}\cdot\vec{\ell}\right] \tag{3.26}$$

and finally

$$W = \left[\exp(E^{(0)}/kT^{(1)}) - 1\right]^{-1}\left[\exp(E^*/kT^{(0)}) - 1\right]^{-1} =$$

$$= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-nE^{(0)}/kT^{(1)}} \cdot e^{-mE^{(0)}/kT^{(0)}} . \tag{3.27}$$

The inclusive cross-section for pion production is then

$$\frac{d^3\sigma}{dp^3} = C\int_{-1}^{+1}d(\cos\theta)\int_{0}^{M^{(0)}/\sqrt{2}}P(\vec{p}, \vec{q}^*)(\vec{q}^*)^2 dq^* \tag{3.28}$$

where we cut-off the q^* distribution at the value $M^{(0)}/\sqrt{2}$. For $E \gg m_\pi$ we obtain

$$\frac{d^3\sigma}{dp^3} \sim C\left(\frac{\pi}{2x^3}\right)^{1/2}e^{-x}e^{-\ell n}\left[\theta(E - M^{(0)}/2\right] \tag{3.29}$$

where

$$x = \left[M^{(0)}/T^{(1)}(2E/T^{(0)} + M^{(0)}/T^{(1)})\right]^{1/2} \tag{3.30}$$

$$\theta(\mu) = \begin{cases} 1, & \mu > 0 \\ 0, & \mu < 0 . \end{cases}$$

Finally the p_t distribution is

$$\frac{1}{\sigma_t} \frac{d\sigma}{dp_t^2} = \overline{C} \int_0^{M^{(1)}/\sqrt{2}} dP_\ell \int_{-1}^{+1} d(\cos\theta) \int_0^{M^{(0)}/\sqrt{2}} P(\vec{p}, \vec{q}^*)(q^*)^2 dq^*, \qquad (3.31)$$

where we cut-off the p_ℓ distribution in the $M^{(1)}$ rest system at the value $M^{(1)}/\sqrt{2}$, and $\overline{C}$ is an appropriate normalization constant.

We already obtained $kT^{(0)} = 105$ MeV. Under the assumption of isotropic emission we have the following relation between the mass of the fireball and Σp_t:

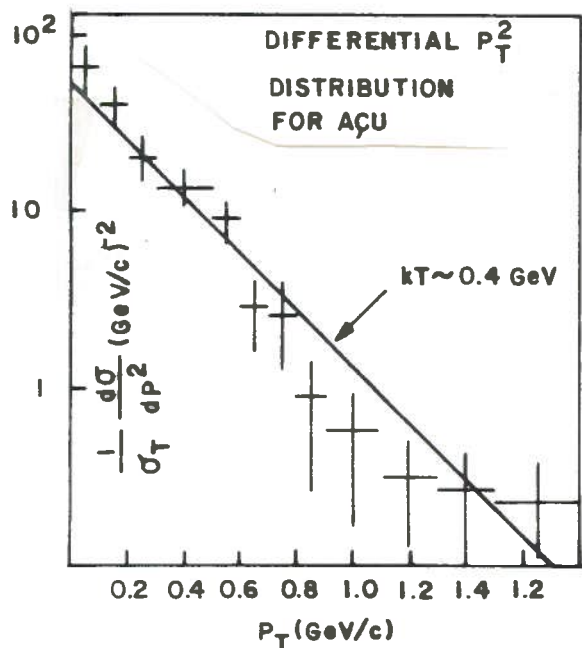$$M = \frac{4}{\pi} \Sigma p_t . \qquad (3.32)$$

FIG. 11 - Distribution of p_t^2 for π^0 mesons produced by the Mirim-jets (Ref. (10)).

This gives $M^{(0)} \sim 2.5$ GeV/c^2, and $M^{(1)} \sim$ ~ 25 GeV/c^2; $M^{(2)} \sim 250$ GeV/c^2. In Fig. 11 we show a comparison of data obtained by BJECC for π^0 mesons coming from Açu--jets. We find $kT^{(1)} \sim 0.4$ GeV. We see also from Table I that $kT^{(2)}$, the temperature of the $M^{(2)}$ fireball (for pion emission), is probably greater than 0.4 GeV, but we do not have yet enough information to estimate the temperature with better precision. Also in Ref. (16) it is shown that data from ISR are consistent with $kT^{(1)} \sim 0.5$-0.6 GeV.

3. 3. - Multiple production of baryons

Table I shows that there are events observed by BJECC where there is direct production of "baryons" without π^0 emission. These are called mini-Centauro, Geminion and Chiron jets. One important point which can be seen from Table I is that these jets have approximately the mass $M^{(1)}$ for mini-Centauro and Geminion events, and the mass $M^{(2)}$ for Centauro and Chiron events.

We are led naturally to suppose that these jets are different decay modes of the same discrete fireballs, produced in the high-energy collisions and able to decay into pions (Açu- and Guaçu-jets$^{(12)}$). One can infer that, above the temperature $kT^{(1)} \sim$ ~ 0.4 GeV and/or a density 3-5 times the nuclear matter density, there exists a kind of phase- transition where pion production is suppressed and we start producing ba-

ryons and/or more exotic objects. These assertions are justified from the phenomeno logical analysis that follows (see also the theory of "strong-black holes" that we develop in Section 4). Others explanations for Centauros and Geminions can be found in Refs. (13, 14, 15). A critical comment on these proposals can be found in Ref. (12). First we note that,for mini-Centauros and Centauros, the energy conservation implies that the mass of the emitted stable baryons is ~ 1 GeV. As these baryons seems to have the same mean-free path in the atmosphere as ordinary baryons, they are com patible with being usual baryons and are similiar in their properties to Cosmic-Ray nucleons, although the real situation is far from clear.

For Geminions and Chiron-jets the situation is as follows. Taking for the Geminion decay products $\langle p_t \rangle \sim 10$ GeV/c, the energy conservation implies m $\simeq 7.5$ GeV/c for the masses of the decay products ($M_{Geminion} \sim 25$ GeV/c^2).

For the Chiron-jets, on the other hand, we have ($M_{Chiron} \sim 250$ GeV/c^2) that the masses of the decay products are in the interval 7.3-13.3 GeV/c^2 for p_t in the inter val 10-15 GeV/c.

Now, supposing that the "baryons" coming from the decay of mini-Centauros, Cen tauros and Chirons are due to a statistical radiation with a Fermi-Dirac statistics, we obtain the following p_t distribution:

$$\frac{1}{\sigma_t} \frac{d\sigma}{dp_t^2} = N_B \left[\frac{\Gamma(3/2)}{\Gamma(1/2)} \right] \sqrt{p_t^2 + m^2} \sum_{n=1}^{\infty} e^{-n\alpha_B} (-1)^n K_1(n\beta \sqrt{p_t^2 + m^2}) \qquad (3.33)$$

where α_B is the Fermi-energy.

The relation eq. (3.21) between the temperature and the mass of the decay products continues to hold also here. This implies that mini-Centauros and Centauros have tem perature kT ~ 1.5 GeV and Chirons have kT $\simeq 10$ GeV ! (See Table I).

It is interesting to note that these very high masses of the stable decay products of Chirons go together with the experimental fact that such "baryons" have an anomalous behavior in the atmosphere with respect to normal baryons[17] (they travel only about 1/3 of the ordinary mean free path).

Also the above analysis led us to classify Geminion and Chiron jets as being of essentially the same nature. These assertions are supported by the theoretical analysis in the following Section 4.

4. - A POSSIBLE THEORETICAL EXPLANATION IN TERMS OF EVAPORATING "STRONG BLACK-HOLES"

4.1. - Introductory remarks

Let us now propose a possible theoretical interpretation of the Chirons[o] and of the Centauros.

We have seen that, in the c.m. of the $M^{(1)}$ and $M^{(2)}$ fireballs:

(i) Their decay-products (either pions, or baryons) can be associated with a generalized Planck distribution[18], i.e. with a thermal-type emission.

(ii) Starting from that fact, it has been moreover possible to realise that the emission temperature involved in the decays of $M^{(1)}$ and $M^{(2)}$ are much higher than the "maximal temperature" (160 MeV) of the ordinary hadron physics. In particular, in the Chiron decay-mode the p_t of the products is so high that the fireball temperature seems to be about 60 times higher than the standard hadronic "maximal temperature" (and about 7 times higher than the temperature implied by the Centauro decay-modes).

This suggests that:

(a) In the Cosmic Ray events we are dealing with, due to the very high energy involved, the initial nucleons create a new type of hadronic-matter.

(b) The new type of hadronic objects should evaporate (isotropically, in their c.m.s.) in a "thermal" way.

(c) We may also infer from experience that such new hadronic objects (in particular the $M^{(1)}$ and $M^{(2)}$ fireballs) seem to be moreover able to suffer different internal phase-transitions. Or, rather, that at least one internal phase-transition seems to intervene between the Centauro and the Chiron decay-modes of $M^{(2)}$.

Let us add that, from the Heisenberg correlation $r \cdot \Delta p_t \gtrsim \hbar/2$, one can roughly evaluate that the Centauro and Chiron decay products come from the "evaporation" of (one or more) objects with radii

$$r \text{ (Centauro)} \gtrsim 0.1 \text{ fm}; \qquad (4.1.a)$$

$$r \text{ (Chiron)} \gtrsim 0.01 \text{ fm}, \qquad (4.1.b)$$

respectively.

(o) By the word Chiron we mean both the Chiron-decay of $M^{(2)}$ and the Geminion-decay of $M^{(1)}$: Cf. Table I.

Our proposal is that such new hadronic (probably mesonic) objects, as well as ordinary hadrons, are associated with the "strong black-hole" solutions of a unified theory of strong and gravitational interactions, and to their possible evaporations[20]. We refer, namely, to the (classical) approach in Refs. (21), even if its results are not very far from the ones of the (quantal) "strong-gravity" approach by Salam et al.[22].

Actually, while a black-body (that absorbs all incoming photons) emits a characteristic spectrum of photons only, and while a gravitational black-hole (that absorbs all incoming particles) should emit characteristic spectra of all possible particles - photons, leptons, hadrons -, a "strong black-hole" (that absorbs all incoming hadrons only) is expected to emit a thermal spectrum only of hadrons. And this does a priori agree with the data we are discussing (cf. Table I). Let us mention, incidentally, that other approaches do not even yield such a preliminary agreement: for instance, Ref. (23) seems to be in contrast with the existence of the Mirin-, Açu-, and Guaçu-jets, which decay into pions.

4. 2. - The theoretical framework

Let us now be more specific. By adopting the geometrical methods of General Relativity and following Ref. (21), let us describe the strong fields surrounding a hadron as a classical tensorial field $s_{\mu\nu}$, to be added to the usual gravitational field $f_{\mu\nu}$. Of course, the strong tensor components $s_{\mu\nu}$ must vanish for distances $r \gg 1$ fm from the considered source-hadron. Hadrons are attributed a "hadronic-charge", or rather a "strong-charge"[o] (or "strong-mass"), which directly deforms the space-time, in analogy to the gravitational mass (or "gravitational charge"), but via the "Strong Universal Constant" $N \equiv \varrho^{-1}G$, where G is the ordinary "Gravitation Universal Constant". The pure number ϱ^{-1} has been shown in Refs. (21) - at least for the most common hadrons - to be given roughly by the ratio S/s between the dimensionless strength S of the strong field and the dimensionless strength s of the gravitational field:

$$\varrho^{-1} \equiv S/s \equiv (Ng^2/\hbar c)/(Gm^2/\hbar c) \simeq 10^{+41} ,$$

where quantities m, g represent mass and strong-mass of one and the same source-hadron, e. g. of a pion[21]. In the case of a pion (or of a nucleon and, approximately, of ordinary hadrons) it is

$$Ng^2 = \varrho^{-1}Gm^2 \simeq 10^{+41} Gm^2;$$

(o) See the following.

so that, if we conventionally choose e. g. to put $g = m$, then:

$$N = \varrho^{-1}G \simeq 10^{+41}G \simeq \frac{hc}{m_\pi^2} \quad . \tag{4.2}$$

In other words, $f_{\mu\nu}$ and $s_{\mu\nu}$ have to be two metric-tensors. The simplest choice is to assume that the total metric tensor is just the sum of the gravitational and strong metric-tensors:

$$g_{\mu\nu} = f_{\mu\nu} + s_{\mu\nu}.$$

Far from a hadron it will be $g_{\mu\nu} = f_{\mu\nu}$ and all particles (with or without strong charge) will feel only the gravitational metric $f_{\mu\nu}$, in agreement with General Relativity. In the micro neighbourhood of a hadron, on the contrary, one shall meet also the strong metric-deformation $s_{\mu\nu}$. The present approach is a bi-metric theory, or rather a "bi-scale" theory[21], since $s_{\mu\nu}$ affects only the objects with hadronic-charge (i. e. with scale factor $\varkappa = \varrho \simeq 10^{-41}$) and not the objects with gravitational charge only (i. e. with scale-factor $\varkappa = 1$).

In the surroundings of hadrons (and in suitable co-ordinates) it is, of course, $f_{\mu\nu} \simeq \eta_{\mu\nu}$, so that:

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu} ; \qquad s_{\mu\nu} \simeq g_{\mu\nu} - \eta_{\mu\nu} \quad . \tag{4.3}$$

In the presence of our two metric-fields, we have to modify the Einstein equations (that, incidentally, we adopted in their form with the "cosmological term", for the reasons explained in Refs. (21)). If we confine ourselves to the motion, in the neighbourhood of a source-hadron - i. e. , of a distribution of strong mass, or strong-charge[o], - of a test-particle endowed both with mass m' and with strong-mass g', the simplest field-equations would be

$$R_{\mu\nu} + \Lambda g_{\mu\nu} + H s_{\mu\nu} = -\frac{8\pi}{c^4}(S_{\mu\nu} + G T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S_\varrho^\varrho - \frac{1}{2} g_{\mu\nu} T_\varrho^\varrho) ,$$

where $S_{\mu\nu}$ is the "strong-matter tensor", denoting the strong-mass distribution of the source-hadron, just as $T_{\mu\nu}$ is the "ordinary matter tensor"; and where the "strong cosmological term" $H s_{\mu\nu}$ takes care of the geometrical properties of $s_{\mu\nu}$ in the neighbourhood of the source-hadron: see Refs. (21). At least for ordinary hadrons it is, as clarified in Refs. (21),

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(o) See the following.

$$ H \equiv \varrho^{-2} \Lambda ; \qquad S_{\mu\nu} \equiv N T_{\mu\nu} ; \qquad N \equiv \varrho^{-1} G . \tag{4.4} $$

For the pion (or nucleon and, roughly, for ordinary hadrons) it is $\varrho^{-1} \approx 10^{+41}$, as we have seen. Quantities Λ and H are the cosmological constant and the "strong cosmological constant" (or "hadronic constant") respectively[21]. As usual, it is taken to be $|\Lambda| \equiv 10^{-56}$ cm^{-2}.

By disregarding the negligible terms, we end up with the equations ($s_{\mu\nu} = 0$ for $r \gg 1$ fm):

$$ R_{\mu\nu} + H s_{\mu\nu} = - \frac{8\pi}{c^4} (S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho}) , \tag{4.5} $$

which are our relevant new field equations, to be associated with eqs. (4.3) and (4.4). Notice explicitly that our modified, "Einstein-type" equations are quite different from Einstein's, even if they have been written down in the simplified form (4.5) to be valid in the neighbourhood of a hadron for a hadronic test-particle. In such form of theirs it is $f_{\mu\nu} \simeq \eta_{\mu\nu}$ and the relevant field is the strong-metric $s_{\mu\nu}$.

Let us now turn to investigate the "Schwarzschild-type" problem for our new field equations (4.5); i.e., let us look for "black-hole-type" solution associated with the strong-gravity field $s_{\mu\nu}$. Therefore, let our hadron be a spherically-symmetric source g of the strong gravity field; and as before let g' be the strong-charge (strong--mass) of the hadronic test particle. Notice incidentally that, to match the usual terminology, as strong field tensor it should be taken not exactly $s_{\mu\nu}$, but the quantity $\Phi_{\mu\nu}$ defined as: $\Phi_{\mu\nu}/g' \equiv \frac{1}{2} c^2 s_{\mu\nu} = \frac{1}{2} c^2 (g_{\mu\nu} - \eta_{\mu\nu})$. Eqs. (4.5) can be easily written also in the form ($s_{\mu\nu} = 0$ for $r \gg 1$ fm):

$$ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - H (g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g^{\alpha\beta} \eta_{\alpha\beta} g_{\mu\nu}) = - \frac{8\pi}{c^4} S_{\mu\nu} , \tag{4.5'} $$

where the last addendum in the l.h.s. has the meaning of "interference term" between the two tensorial metrics $s_{\mu\nu}$ and $f_{\mu\nu} \simeq \eta_{\mu\nu}$. [We can geometrize also the strong--field since the inertia of the test-hadron (in the surroundings of the considered source-hadron) coincides[21] with its strong-mass g', and not with its gravitational mass: $M_I \equiv g'$].

Let us specify the total metric, acting on the test-hadron in the surroundings of the considered source-hadron, by setting[o]:

$$ds^2 = \exp\left[a(r)\right]c^2 dt^2 - \exp\left[b(r)\right]dr^2 - r^2(d\theta^2 + \sin^2\theta \cdot d\varphi^2) , \qquad (4.6)$$

where $a(r)$, $b(r)$ are functions still to be determined.

Before going on, let us recall that the strong black-holes we are looking for are suitable to represent hadrons also for the fact that they result to be surrounded - at the static limit and in the "intermediate region" - by the Yukawa potential, and not by a Newtonian potential (as it would be the case, on the contrary, for black-holes associated with Einstein equations). It is evident that hadrons could not be represented by the strong black-hole solutions of equations obtained from Einstein's merely by substituting N to G. In fact, by linearising eqs. (4.5) in the weak field approximation we get[21] at the static limit for the scalar potential $V \equiv \frac{1}{2} c^2 s_{oo}$ the spherically symme_tric solution $V = - \frac{g}{2} \exp\left[- r \sqrt{2H}\right]$, to be valid for a point-like particle at rest in the origin (and endowed with strong-charge g). By identifying $\sqrt{2H} \equiv m_S c/\hbar$ we find for the field mass - in the case of ordinary source-hadrons - the value $m_S = \hbar \sqrt{2H}/c \simeq$ $\simeq m_\pi$, and therefore

$$V = - \frac{g}{r} \cdot \exp\left[- r m_\pi c/\hbar\right].$$

Let us, first of all, notice also that the gravitational Schwarzchild radius for a pion or nucleon would be

$$R_G = \frac{2Gm}{c^2} \approx 10^{-55} \text{ m.}$$

We then expect the strong Schwarzchild radius for the pion or nucleon to be of the order of 1 fm $(N = \varrho^{-1}G)$[21]:

$$r_S = \frac{2Nm}{c^2} \simeq 10^{40} \times 10^{-55} \text{ m} \approx 1 \text{ fm .}$$

Also in this respect we are a priori allowed to identify hadrons with our strong black-holes (SBH). Notice, by the way, that our horizon-radii r do not necessarily correspond to the hadron radii but rather to an upper value of them; actually, the observed radius could correspond to an average radius (See Refs. (21), where the present ap-

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(o) Since, as we said, under our conditions it is $f_{\mu\nu} \simeq \eta_{\mu\nu}$ and the relevant metric-field remains $s_{\mu\nu}$, for simplicity we might call it the "strong-metric" interval" tout court.

proach is extended by considering the hadron interior as a "micro-universe").

To solve our main problem, however, we have of course to release the weak field condition. The "strong-horizon" radii will be given by the equations

$$\exp\left[-b(r)\right] = 0 \; ; \qquad \exp\left[a(r)\right] = 0 \qquad (4.7)$$

For our purposes, we can rewrite eq. (4.5') as

$$
\begin{cases}
R_{\mu\nu} - \dfrac{1}{2}\, g_{\mu\nu} R_{\varrho}^{\varrho} = -\dfrac{8\pi}{c^4}\,(S_{\mu\nu} + t_{\mu\nu}) \, , \\[2ex]
t_{\mu\nu} \equiv -\dfrac{c^4 H}{8\pi}\,(g_{\mu\nu} + \eta_{\mu\nu} - \dfrac{1}{2}\, g_{\mu\nu} g^{\alpha\beta}\eta_{\alpha\beta}) \, ,
\end{cases}
\qquad (4.5")
$$

with $t_{\mu\nu} = 0$ for $r \gg 1$ fm. (It is immediate to recognise here that $t_{\mu\nu}$ is also a tensor (not a pseudo-tensor), provided that one remembers that $\eta_{\mu\nu}$ is nothing but an approximation for $f_{\mu\nu}$). Still at the static limit, and at the first order of an iterative procedure[21], the first one of eq. (4.7) then becomes[24, 21]

$$\exp\left[-b(r)\right] = 1 - \frac{2\ell}{r} + \left(\frac{\bar\mu \bar k^2}{r} + \frac{\bar k^2}{r^2}\right)\cdot \exp\left[-2\bar\mu r\right] = 0 \, , \qquad (4.8)$$

even if one should remember that the procedure explained in Ref. (21) applied to ordinary mass hadrons rather that to heavy hadronic fireballs. In eq. (4.8) it is $\bar\mu \equiv m_S c/\hbar$, so that for ordinary hadrons at least $\bar\mu \simeq m_\pi c/\hbar$, and $\bar k^2$, ℓ are constants as well. The second one of eq. (4.8), under the same approximations, implies the preliminary solution of the equation[24]

$$a'(r) = \frac{1}{r}\,(\exp\left[b(r)\right] - 1) \, .$$

We shall practically confine ourselves to solve eq. (4.8), which yields the radii in correspondence to which the coefficient of dr^2 diverges. It is very important to recognize that eq. (4.8) yields <u>two values</u> of r for which the coefficient of dr^2 in eq. (4.6) becomes infinite; and precisely one value not far from 2ℓ and another value much smaller: $r_1 \lesssim \lesssim 2\ell$, $r_2 \gtrsim 0$. As to the integration constant ℓ, it is expected:

<u>First choice</u> : $\quad \bar k = \ell \, , \quad$ and $\quad \ell \equiv \dfrac{Nm}{c^2} \equiv \dfrac{Gm}{c^2 \varrho} \equiv \dfrac{Ng^2}{c^2 m} \, .$

At least for ordinary hadrons, ℓ is therefore expected to be of the order of 1 fm, as it can be immediately verified (for instance, in the pion case $Ng^2/\hbar c \simeq 3$ and in the

nucleon case $Ng^2/\hbar c \simeq 15$). More precisely - taking into account that test particles with negligible strong-charge do not exist in hadronic physics - in Refs. (24, 21) it was shown that :

Second choice: $\quad \ell \equiv \dfrac{Nm^3}{(c^2m'^2)}$, $\qquad$ and $\qquad \bar{k} \equiv Nm^2/(c^2m')$,

with $N = G/\varrho$, quantity m' being the mass of the hadronic test-particles. (We can e. g. choose $m' = m_q$ = average quark mass, the "test-quark" being a priori considered as initially outside the possible horizon).

Eq. (4.8) has been solved by computer. For instance, in the case of ordinary hadrons, the two SBH radii r_S have been found approximately to be - with the first choice - such that $r_1/r_2 \simeq 3$; $r_1 \simeq 1$ fm. With the second choice, on the contrary, one finds for ordinary mesons (pions) that

$$r_1/r_2 \approx 10 , \qquad\qquad (4.9a)$$

and for ordinary baryons (nucleons):

$$r_1/r_2 \approx 30\text{-}40 . \qquad\qquad (4.9b)$$

Since the experimental data are still poor, we do not attempt here - as already mentioned - to find out also the values of r for which the coefficient of dt^2 in eq. (4.6) vanishes. However, in Refs. (21) it was verified that in correspondence with the larger radius r_1 for ordinary hadrons it is :

$$\exp\left[a(r)\right] = \exp\left\{\left[b(r)\right]\right\}^{-1} \approx 1 - \frac{2Nm^3}{rc^2m'^2} ,$$

so that the values of r satisfying the first one of eq. (4.7) did approximately verify in those cases also the second one of eq. (4.7).

In the case of our mesonic fireballs, namely of the Centauro and Chiron-Geminion decay modes of $M^{(1)}$ and $M^{(2)}$, since the Centauro decay-products seem to come from evaporating objects with radius $r_1 \approx 0.1$ fm, our eq. (4.8) yields approximately with the first choice

$$r_1/r_2 \approx 5 , \qquad\qquad (4.9c)$$

and with the second choice

$$r_1/r_2 \approx 15 . \qquad\qquad (4.9d)$$

(but a higher ratio, in the latter case, if the mesonic fireballs are considered as exot ic hadrons). For computing (4.9c, d) we set $\bar{\mu} \cong 10 \, m_\pi c/\hbar$.

In conclusion, the results obtained for the radii of our SBHs suggest that hadrons could be identified with the SBH - solutions of our field equations.

As a consequence, mesons and baryons should possibly exist in two states, the second one corresponding to a smaller radius (in particular $r_S^{(2)} \lesssim 0.1$ fm for "collapsed" ordinary mesons, and $r_S^{(2)} \approx 0.01$-0.1 fm for "collapsed" ordinary baryons).

Before passing to our conclusions, let us observe that the relevant constants appearing in eq. (4.8) can in general be written, in the $N = 1$ units, as[24, 21]

$$\bar{\mu} \equiv m_S c/\hbar ; \qquad \bar{k} \equiv g^2/(c^2 m'); \qquad \ell \equiv g^2 m/(c^2 m'^2) , \tag{4.10}$$

or even

$$\mu \equiv m_S c/h; \qquad \bar{k} \simeq \ell \simeq N g^2/(c^2 m) \equiv g^2/(c^2 m) , \tag{4.10'}$$

with g (the strong-charge of the generic source-hadron considered) to be determined independently of our rough evaluations of ϱ^{-1} or N. Especially for generic, high mass hadrons precise calculations are possible, provided that one has direct information on the hadron-strong-charge distribution or at least on its g. In such a context, it is worth while to clarify the following: if we borrow from experience the information that quarks appear to be the true carriers of the strong-charge, then g ought to be defined as $g \equiv$ $\equiv n g_0 (n = 2, 3)$, the quantity g_0 being the average magnitude[21] of the quark rest strong- -charges g_i, with $g_i = s_i g_0$; $\Sigma s_i = \Sigma g_i = 0$; $g_0 = |g_i|$. Attention should be paid also to the fact that g_0 is the rest-value; inside a hadron the strong-charge of quarks will a priori depend on their speed in a relativistic way[21].

4. 3. - Consequences

The consequences of the previous subsections seem to be:

i) The fireballs $M^{(1)}$ and $M^{(2)}$ correspond to mesonic SBHs, and can exist with radii r_1 and r_2 such that

$$r_2 \simeq \frac{1}{10} r_1 . \tag{4.11}$$

ii) By analogy with the black-body behaviour, such SBHs are expected to "evaporate" emitting only hadrons (one can make recourse to arguments similar to the one in Ref. (18)).

iii) Following the spirit of Refs.(20) - e. g. by assuming a statistical evaporation - one then derives that our SBHs are expected to emit a thermal spectrum of hadrons with the temperature[21] (k = Boltzmann constant):

$$T = \frac{\hbar c}{4\pi k r_S} \quad , \tag{4.12}$$

which is of the order[21] of $T \simeq 2 \times 10^{11}$ K, i.e. of $kT \simeq 20$ MeV, for $r \simeq 1$ fm. In correspondence with the pion, eq. (4.12) yields the "maximal temperature" of <u>ordinary</u> physics. These (unstable) SBHs are expected to evaporate with a life-time of the order of[21]

$$\Delta t \simeq (10^{18}/10^{41}) \text{ s} \simeq 10^{-23} \text{ s} .$$

We assume eq. (4.12) to hold also for the "collapsed" states (corresponding to the smaller radii $r_2 \equiv r_S^{(2)}$); so that "the same" hadronic SBH can exist with two different temperatures (still maintaining the same mass, a priori). For both our $M^{(1)}$ and $M^{(2)}$ mesonic SBHs, we get from eqs. (4.11), (4.12) that their own possible states correspond to temperatures such that

$$T_2 \simeq 10 \, T_1 \; ; \tag{4.13}$$

these two states can correspond to the Centauro and Chiron decay-modes. The temperatures evaluated in our Section 3 tell us finally that

$$r_1 \equiv r_S^{(1)} \approx 0.1 \text{ fm} ; \qquad r_2 \equiv r_S^{(2)} \approx 0.01 \text{ fm} , \tag{4.14}$$

in agreement with the $\langle p_t \rangle$ distributions (cf. eq. (4.1) and Table I). Concluding, for the $M^{(1,2)}$ cluster we can say that the Centauro and Chiron decay-products come from the evaporation of the following two SBH states:

$$\begin{cases} M^{(1,2)} \text{ Centauros} \quad : \quad r_1 \approx 0.1 \text{ fm} ; \quad T_1^{(2)} \approx 0.2 \text{ GeV} ; \\ \\ \left. \begin{array}{l} M^{(2)} \quad \text{Chiron} \\ M^{(1)} \quad \text{Geminion} \end{array} \right\} \; : \; r_2 \approx 0.01 \text{ fm}; \quad T_2^{(2)} \approx 10 \, T_1^{(2)} \approx 2 \text{ GeV} , \end{cases}$$

respectively. The fact that these temperatures are significantly different from the experimental ones (1.5 GeV and 10 GeV respectively) might have been expected from the fact that eq. (4.12) refers to <u>Schwarzchild</u>-type black-holes, whilst our SBH hadrons are to be associated with more sophisticated SBH solutions - e. g. of the Kerr-Newman type - corresponding to

$$T = \frac{\hbar c}{4\pi k r_S} F(e, J, \ldots) , \qquad (4.12')$$

where F is a suitable function of all the hadrons "charges" or quantum numbers (see the following).

iv) Even more, since the products at least of the $M^{(2)}$ Chiron decay does not seem to be ordinary baryons (they appear to have a range about one third of the normal hadron range[17], and their mass is of the order of 10 GeV), they might themselves be regarded as "collapsed" baryons. In other words, they can be considered not ordinary baryons, but baryons corresponding to the second (smaller) possible radius; see eq. (4.9"). (Unless one did not like to try to associate the Chiron-decay "baryons" to quark themselves). But we leave this problem completely open, also due to the scarcity of the experimental data.

v) Let us recall that our SBHs can be characterised also by subnuclear charges or "quantum numbers" (besides mass, electric charge, angular momentum, and maybe a pseudoscalar charge), since the short range fields become long range at their scale, - just as it happens for the strong-mass itself.

vi) Our SBHs do not necessarily have to evaporate; they can also be stable against the strong interaction decays (just as nucleons, and probably the "baryons" mentioned under point (iv)), according to the value of the strong-gravity field at their surface. Their temperature, in other words, instead of being expressed by the rough relation (4.12), can be given by equations of the type[25]

$$T = \frac{\hbar c}{4\pi k r_S} \frac{2(M^2 - J^2 - E^2)^{1/2}}{\left[M - \frac{1}{2}\frac{E^2}{M} + (M^2 - J^2 - E^2)^{1/2} \right]} ; \qquad (N \equiv \varrho^{-1} G) , \qquad (4.12')$$

which holds e.g. for Kerr-Newmann SBHs. In eq. (4.12') it is

$$M \equiv Nm^2/c ; \qquad E \equiv \bar{K}e^2 M/c ; \qquad \bar{K} \equiv (4\pi\varepsilon_0)^{-1} ,$$

and J is the angular momentum of the considered hadron. The temperature given by eq. (4.12') vanishes, e.g., for hadrons lying on Regge trajectories[21], but not for our $M^{(1)}$, $M^{(2)}$ fireballs.

vii) Let us repeat that the SBH solutions of Einstein equations in their strong version (i.e. obtained merely by changing G into N) cannot represent hadrons, since their strong-gravity field would not fade away over the distance of a few fm; in particular -

at the static limit and in the "intermediate region" - it would behave as $-Ng/r$ without any exponential damping. In our approach, on the contrary, we meet such a Yukawa damping, and our strong-gravity metric is expected to vanish over the correct distance from the source-hadron, leaving at its place the pure gravitational metric (that is to say, our total metric smoothly "fades" into the pure gravitational metric).

viii) We have always associated the $M^{(2)}$ Chiron decay with the $M^{(1)}$ Geminion decay, since in both cases $\langle p_T \rangle_B \simeq 15$ GeV/c, and since the Geminion-decay can be regarded as a Chiron decay with $\langle n \rangle_B = 2$ simply for energy-conservation reasons.

ix) Till now we have not attempted to explain the pion decay modes of the $M^{(1)}$ and $M^{(2)}$ objects. We may suggest pion production to occur when a hot-spot is formed in the high energy collision of two nucleons but the energy density is not enough to collapse the colliding matter into its possible strong-Schwarzchild radius. This explanation seems reasonable in view of the fact that the emission temperature is small with respect to the "baryonic" decay modes.

x) The above analysis seems to suggest that real "phase transitions" may take place in the high energy collisions. These "phase transitions" can be associated with the collapsing of the fundamental matter beyond the strong-Schwarzchild radii, which yield, through a Hawking-like relation, the temperature of the transitions. At these very high temperatures the emission of heavy objects is enhanced, as it is observed experimentally.

Let us close this section 4 by saying the following: The gravitational black-holes do not seem to have been observed yet with certainity; it would be nice if Chirons and Centauros could be associated with the actual observation of strong black-holes.

5. - CONCLUSIONS.

The analysis developed in the previous sections seems to indicate clearly the relevance of fireballs of discrete masses in the phenomenon of multiple particle production in very high energy collisions.

Indeed, we showed that the existence of these objects and their decay modes are compatible with, and a natural explanation for, the data obtained in pion production and baryon production by BJECC. The nature of such objects and of the direct baryon production is a real challenge to current physical theories; and it is indeed interesting that our description of Chirons and Centauros as evaporating "strong black-holes"

yields at least the right order of magnitude for the temperature of these events and for the "fireball" radii, and also gives a hope to understand their decay modes.

The overall picture seems promising, but we need more theoretical and experimental studies to understand better these new phenomena. In particular, and prelimi narly,it is important that Chirons and Centauros be found also at the SPS-collider experiments.

Note added :

In a recent UA1 series of experiments with $p\bar{p}$ Collider, an increase with energy of the mean transverse momentum $\langle p_t \rangle$ of charged particles has been found, as well as that $\langle p_t \rangle$ is correlated with the charged particle multiplicity n_c so that values of $\langle p_t \rangle$ greater than 1 GeV/c are not unusual, as was observed by the BJCC experiments (See G. Arnison et al. , CERN/EP/82-120, 121 and 125 (1982); M. Barner et al. , Phys. Letters 115B, 59 (1982)).

REFERENCES

(1) - C. M. G. Lattes, Y. Fujimoto and S. Hasegawa, Phys. Rep. 65, 151 (1980), and references therein.

(2) - UA2 Collaboration (M. Banner et al.), Phys. Letters 115B, 59 (1982).

(3) - Y. Sato, H. Sugimoto and T. Saito, J. Phys. Soc. Japan 41, 1821 (1976).

(4) - W. Thomé et al. , Nuclear Phys. B129, 365 (1977).

(5) - UA5 Collaboration (K. Alpgard et al.), Phys. Letters 107B, 315 (1982).

(6) - AU1 Collaboration (A. Arnison et al.), Phys. Letters 107B, 320 (1982).

(7) - UA1 Collaboration, Proceedings of XXI Intern. Conf. on High-Energy Physics, Paris 26-31 July, 1982.

(8) - UA5 Collaboration (K. Alpgard et al.), Phys. Letters 115B, 71 (1982).

(9) - W. A. Rodrigues Jr. and A. Turtelli Jr. , Nuovo Cimento 23A, 227 (1974).

(10) - W. A. Rodrigues Jr. , A. Turtelli Jr. and M. Lakys, An. Acad. Brasil. Cienc. 46, 197 (1974).

(11) - C. A. P. Ceneviva and W. A. Rodrigues Jr. , Acta Phys. Pol. B8, 475 (1977).

(12) - J. Dias De Deus and W. A. Rodrigues Jr. , Nuovo Cimento 55A, 34 (1980).

(13) - T. Tati, Preprint RRK-79-7 (Research Instituto of Theoretical Physics, Hiroshima University, 1979).

(14) - J. D. Bjorken and L. D. McLerran, Phys. Rev. D20, 2353 (1979).

(15) - D. G. Sutherland, Proceedings Conf. on Cosmic Rays and Particle Physics (Bartol, D. E. , 1978), Amer. Inst. Phys. Conf. Proc. No. 49 (A. I. P. , New York, 1979), p. 503.

(16) - M. Hama, M. Nagasaki and H. Suzuki, Progr. Theor. Phys. 57, 160 (1977).

(17) - Brazil-Japan Collaboration on Chacaltaya Emulsion Chamber Experiments (J. A. Chinellato et al.), in 'Proton-Antiproton Collider Physics', 1981, Madison, Wisconsin (American Inst. of Physics, New York, 1982).

(18) - S. W. Hawking, Nature 248, 30 (1974); Comm Math. Phys. 43, 199 (1975); Sci. Amer. 236, 34 (1977).

(19) - See, e. g. , R. Hagerdon, in 'Cargése Lectures in Physics' (Gordon and Breach, 1973), Vol. 6, p. 643 ; See also E. Fermi, Progr. Theor. Phys. 5, 570 (1950); Phys. Rev. 63, 137 (1943); 66, 149 (1944).

(20) - Cf. W. Bardeen, B. Carter and S. W. Hawking, Comm. Math. Phys. 31, 161 (1973); J. B. Hartle and S. W. Hawking, Phys. Rev. D13, 2188 (1976); E. Recami and P. Castorina, Lett. Nuovo Cimento 15, 347 (1976); C. Sivaram and K. P. Sinha, Phys. Rev. D16, 1975 (1977); Phys. Rep. 51, 111 (1979); P. Castorina and E. Recami, Lett. Nuovo Cimento 22, 195 (1978).

(21) - See E. Recami, in 'Quarks and the Nucleus', Vol. 8 of 'Progress in Particles and Nuclear Physics', ed.by D. H. Wilkinson (Pergamon, Oxford, 1982), p. 401; Found. of Phys. 13, 341 (1983); in 'Old and New Questions in Physics, Cosmology,... : Essays in honour of W. Yourgrau', ed. by A. Van Merwe (Plenum, New York, 1982, in press), p. 377; E. Recami, in 'Relativity, Quanta and Cosmology in the development of the Scientific thought of A. Einstein', ed. by

F. De Finis and M. Pantaleo (Johnson Rep. Co. , New York, 1979), Vol. 2,Chap
ter 16, p. 581; P. Caldirola and E. Recami, Lett. Nuovo Cimento 24, 565(1979);
P. Caldirola, M. Pavšič and E. Recami, Nuovo Cimento B48, 205 (1978); Phys.
Letters A66, 9 (1978); E. Recami and P. Castorina, Lett. Nuovo Cimento 15,
347 (1976). See also R. Miguani, Lett. Nuovo Cimento 16, 6 (1976); V. De Sab
bata and P. Rizzati, Lett. Nuovo Cimento 20, 525 (1977); V. De Sabbata and
M. Gasperini, Lett. Nuovo Cimento 25, 489 (1979); E. Recami and K. T. Shan,
Lett. Nuovo Cimento 24, 115 (1979); P. F. Gonzalez Dias, Lett. Nuovo Cimen
to 31, 39 (1982).

(22) - See   e. g. A. Salam, in 'Proceedings 19th Intern. Conf. on High-Energy Phy-
sics' (Tokyo, 1978), p. 937; A. Salam and J. Strathede, Phys. Rev. D18, 4596
(1978). See also C. Sivaram and K. P. Sinha, Phys. Rep. 51, 111 (1979); D. D.
Ivanenko, in 'Relativity, Quanta and Cosmology', ed. by F. De Finis and M.
Pantaleo (Johnson Rep. Co. , New York, 1979), Vol. 1, Chapter 9, p. 295;
P. Roman and J. Haavisto, Int. J. Theor. Phys. 16, 915 (1978); W. Drechler
and M. E. Mayer, Fibre Bundle Techniques in Gauge Theories (Springer, Ber
lin, 1977), 2nd part, p. 146; J. E. Charon, Théorie de la Relativité Complexe
(A. Michel, Paris, 1977); Compt. Rend. A. S. P., A266, 750 (1968); M. A.
Markov, Zh. Exp. Theor. Fiz. 51, 878 (1966).

(23) - G. N. Fowler, E. M. Friedlander and R. M. Weiner, Phys. Letters B116, 203
(1982).

(24) - See D. K. Roos, Nuovo Cimento A8, 603 (1972); E. Recami and P. Castorina,
Lett. Nuovo Cimento 15, 347 (1976).

(25) - See for e. g. D. N. Page, Phys. Rev. D13, 198 (1976).