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ARE CLASSICAL TACHYONS SLOWER-THAN-LIGHT QUANTUM PARTICLES? (*)

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ABSTRACT

After having studied the shape that a tachyon T (e.g., intrinsecally spherical) would take up, we show in an explicit example that the characteristics of classical tachyons are similar to those of the ordinary (slower-than-light) quantum particles. In particular, a realistic tachyon is associated with a "phase-speed" V ($V^2 > c^2$), but with a "group-speed" $v = c^2/V$ ($v^2 < c^2$).

1. - FOREWORD

Research about faster-than-light objects(1) has far origins, which go back at least to Lucretius(2). To confine ourselves to post-relativistic times, the first attemp to extend Special Relativity (SR) to the case of Superluminal objects is probably due to Somigliana(3).

Some authors limited themselves to consider objects both subluminal (Bradyons) and Superluminal (Tachyons), always referred - however - to subluminal observers (s).

Other authors tried on the contrary to extend SR by introducing both subluminal observers s and Superluminal observers (S), and then by generalizing the Relativity Principle.

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This second approach meets of course the greatest obstacles. Such an "Extended Relativity" (ER), in fact, can be straightforwardly built up only in a "Minkowskian" space-time M(n, n). For instance, very simple - and instructive - it happens to be the model-theory in two dimensions, i.e. in the M(1, 1) space-time. Due to the known difficulties met in the ordinary Minkowski space M(1, 3), we had first to sketch the classical theory of ER in an auxiliary space M(3, 3), and later on to go back to four dimensions[4].

However, in order to go on with our analysis we have only to assume that "Superluminal Lorentz transformations" (SLT) exist[1,4], which transform time-like into space-like quantities, and vice-versa. Actually, the SLTs are to be characterized by that property, and must differ from the ordinary Lorentz transformations (LT) only in it. As a consequence, the quadratic forms must be scalar under the ordinary LTs, and pseudo-scalars under the SLTs. No other hypotheses and no further deviations from standard SR will be required.

Let us add that the reasons which justify studying the classical theory of tachyons can be divided into a few cathegories, two of which will be mentioned here: (a) the larger scheme that one is building, in order to extend relativistic theories to space-like objects, allows to understand better many aspects of the ordinary relativistic physics[1,5], even if tachyons would not exist in our cosmos as "asymptotically free" objects; (b) we may hope to be eventually able to reproduce at a classical level the quantum-like behaviour[5], provided that classical physics is extended to include Superluminal particles (and suitable "extended models"[7] are associated with elementary particles).

In this Letter we put forth a concrete example corroborating the point (b) above, which can be stimulating in the present period when the interpretative foundations of quantum mechanics are under scrutiny and relevant results are going to be produced in experiments so as Rapisarda's and so as Aspect's[8].

2. - TACHYON SHAPE

Let us consider[9] an ordinary bradyon B which for simplicity is intrinsecally spherical, so that - when at rest - its "world-tube" in Minkowski space is represented by $0 \leqslant x^2 + y^2 + z^2 \leqslant r^2$. When B moves with subluminal speed v along the x-axis (see Fig. 1), the world-tube equation becomes ($\beta \equiv v/c$):

$$0 \leqslant \frac{(x - ct\beta)^2}{1 - \beta^2} + y^2 + z^2 \leqslant r^2 \qquad (\beta^2 < 1) \qquad (1)$$

and, in Lorentz-invariant form[9],

$$0 \leqslant \frac{(x_\mu u^\mu)^2}{u_\mu u^\mu} - x_\mu x^\mu \leqslant r^2 \, , \qquad (\beta^2 < 1) \qquad (2)$$

where $x_\mu = (ct, x, y, z)$ and u_μ is the 4-velocity.

(Fig. 3b); when it is Superluminal, this surface becomes a two-sheeted hyperboloid (Fig. 3d)[10].

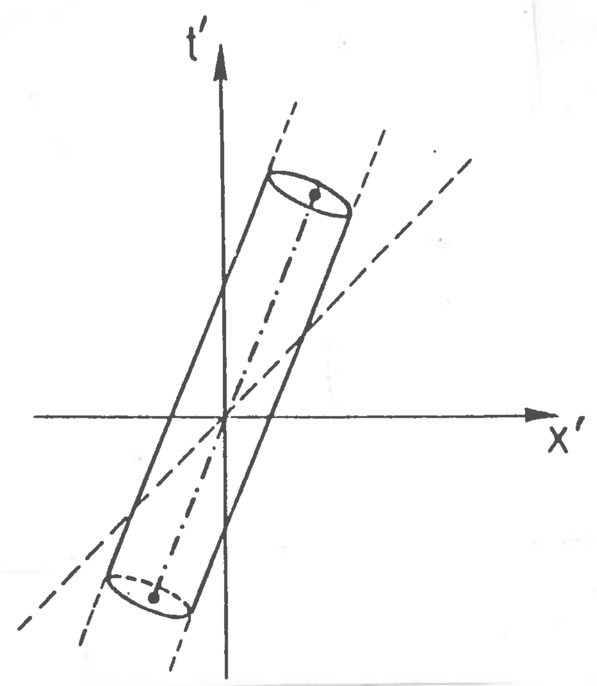

FIG. 1 - The "world-tube" of an ordinary (bradyonic) particle B, assumed to be spherical in its rest-frame. For simplicity, it is supposed particle B to move along the x-axis and the world-line of the center C of B to pass through the space-time origin, so that $C \equiv 0$ for $t = 0$.

Let us now take into consideration also the space-like values u'_μ of the 4-velocity (even if in ER the light-speed c in vacuum goes on being the invariant speed, and cannot be crossed[1,4], neither from the left, nor from the right). Let us consider, however, only subluminal observers s: As a consequence, we shall in the following regard the SLTs, as well as the ordinary LTs, only from the active point of view.

By an "active" SLT, let us transform the initial bradyon B into a final tachyon T, endowed with the Superluminal speed V' along x; eq. (2) transforms then into ($\beta' \equiv V'/c$):

$$0 \leqslant x'_\mu x'^\mu - \frac{(x'_\mu u'^\mu)^2}{u'_\mu u'^\mu} \leqslant r^2 , \qquad (\beta'^2 > 1) \tag{3}$$

where the 4-velocity u'_μ for tachyons is to be defined[4] $u'_\mu \equiv dx'_\mu/(c d\tau_o)$, (if we want it to behave as a 4-vector also under SLTs). Eq. (3) refers to the "world-shape" of tachyon T, in the sense that the events x'_μ satisfying relations (3) form the 4-dimensional extension of T. In our frame, eq. (3) explicitly writes:

$$0 \geqslant - \frac{(ct'\beta' - x')^2}{\beta'^2 - 1} + y'^2 + z'^2 \geqslant - r^2 , \qquad (\beta'^2 > 1) \tag{4}$$

where the l. h. s. equality yields an unlimited double-cone $\mathscr{C}$ and the r. h. s. equality yields a two-sheeted rotation hyperboloid $\mathscr{H}$, asymptotical to the cone $\mathscr{C}$. Eq. (4), as time elapses, yields the relativistic shape of a tachyon T travelling along x with speed V': see Fig. 2.

Let us examine the external surface of the object (initially B, finally T). When it is at rest, the surface is spherical; when it is subluminal, such a surface becomes an ellipsoid (Fig. 3b); when it is Superluminal, this surface becomes a two-sheeted hyperboloid (Fig. 3d)[10].

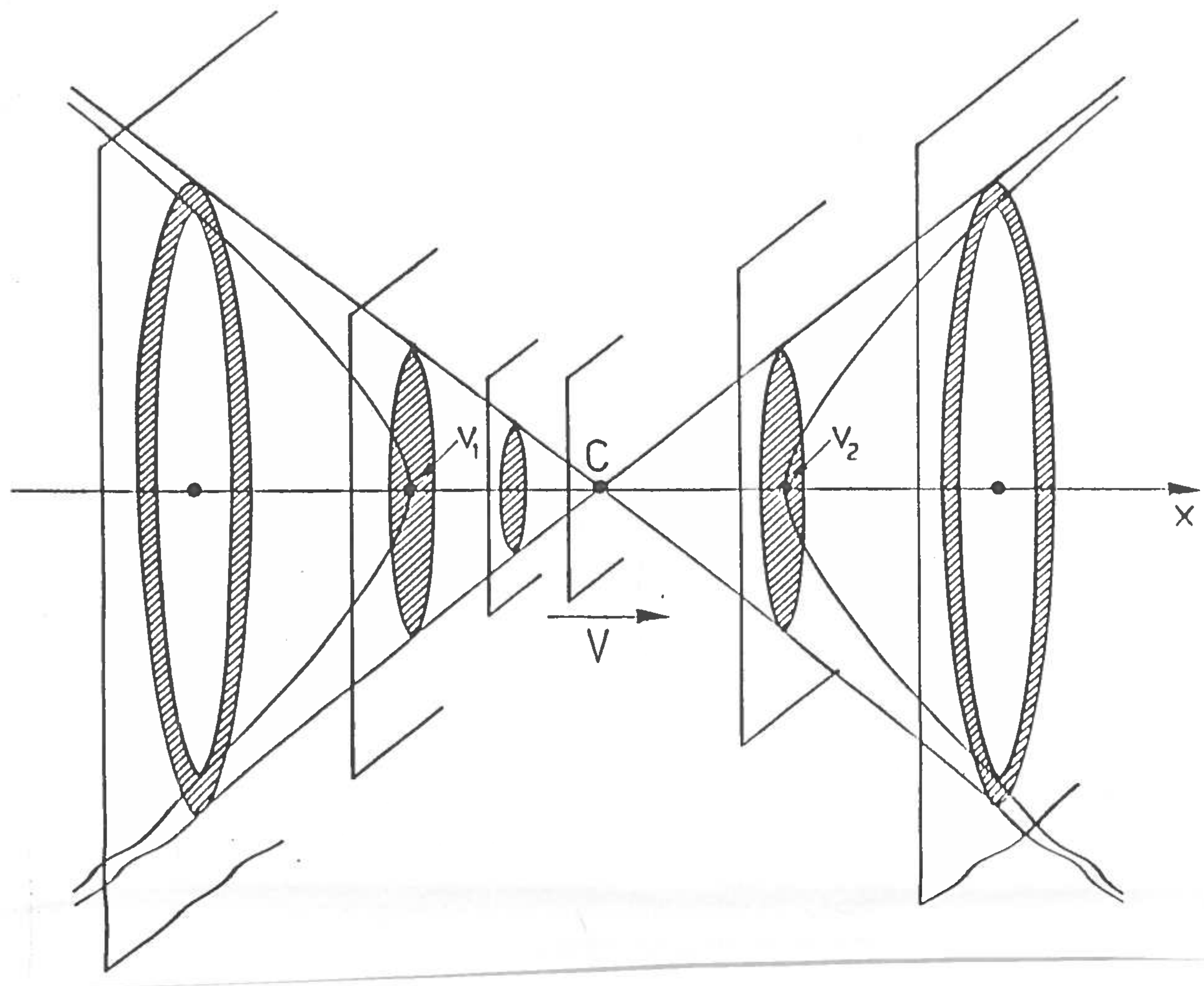

FIG. 2 - Shape of a particle, intrinsecally spherical, when seen from a Superluminal frame. See the text. The intersections are in particular shown of the tachyon--shape with (two-dimensional, spatial) planes orthogonal to the tachyon motion-line.

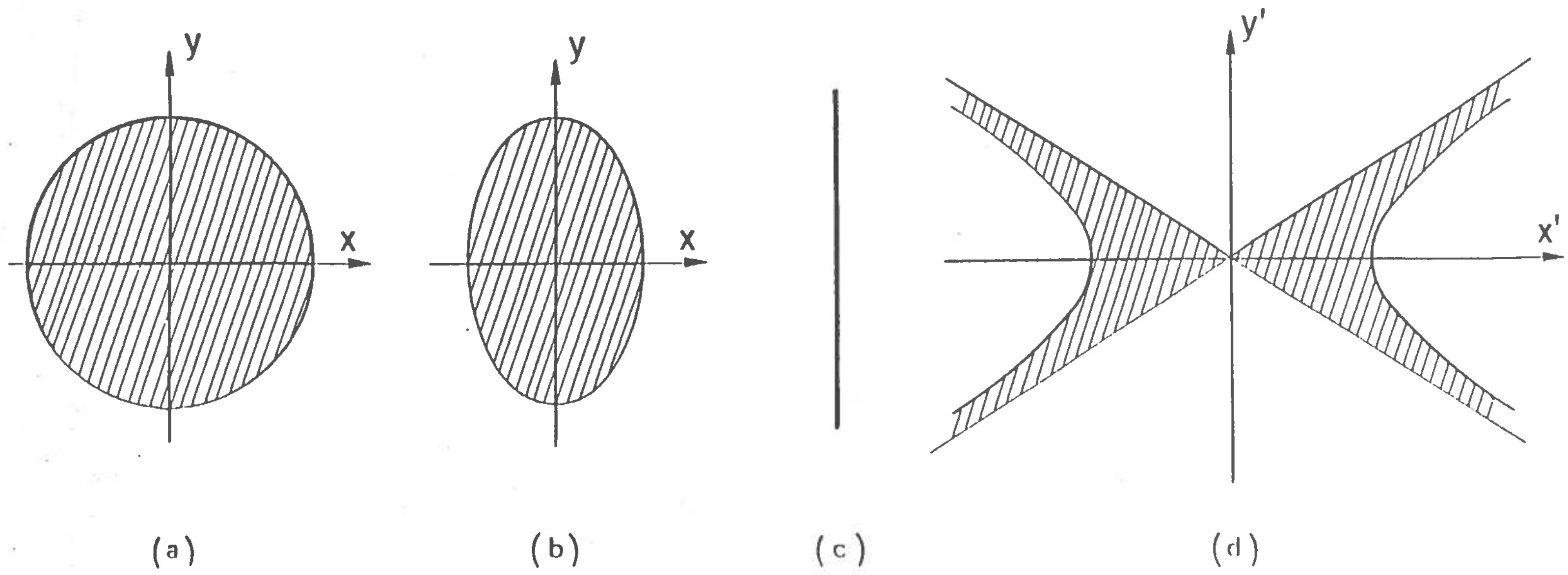

FIG. 3 - Let a particle be spherical in its own rest-frame (Fig. a). Under a subluminal x-boost it appears as ellipsoidal (Fig. b). Under a Superluminal x-boost it will appear so as in Fig. d. Fig. c refers to the limiting case when the boost-speed u $\to$ c, either from the left or from the right. (For simplicity, a space-axis is skipped).

3. - DO TACHYONS BEHAVE AS SUBLUMINAL (QUANTUM) OBJECTS ?

Thus - by applying an "active" SLT to a bradyon B having an infinite life-time - we have got a tachyon T infinitely extended in space; in fact, T occupies the whole space-region confined between $\mathscr{C}$ and $\mathscr{H}$. Actually, tachyons were expected to be unlocalizable in the ordinary 3-space since, when considering the Poincaré Group representations for the space-like case, the Localization Group ("litthe group") happens to be SO(2, 1) and not SO(3).

However, it is easy to find out that - if the initial bradyon B possesses a finite life-time, e. g. in its own rest-frame it is created at time $\bar{t}_1$ and absorbed at time $\bar{t}_2$ - the corresponding tachyon T then possesses a finite space-extension.

Under the present hypotheses, in fact, one has to associate suitable limiting space-like hypersurfaces with eqs. (1)-(2). In the case when B is at rest, such hypersurfaces can simply be the space hyperplanes $t = \bar{t}_1$ and $t = \bar{t}_2$ (Fig. 4). The generic Lorentz-invariant equation for a hyperplane is

$$x_\mu u^\mu = K . \qquad (K = \text{constant}) \qquad (5)$$

By remembering once more that the 4-vector products (scalar under LTs) are pseudo-scalars under SLTs, we get that eq. (5) under the action of an "active" SLT still keeps its form : $x'_\mu u'^\mu = K'$. The relevant fact is that one goes from a time-like u_μ to a space-like u'_μ ; so that the hyperplanes $x'_\mu u'^\mu = K'$ are now to be referred to one temporal and two spatial basis-vectors (cf. Fig. 5). Such hyperplanes represent ordinary space planes (orthogonal to the x-axis,

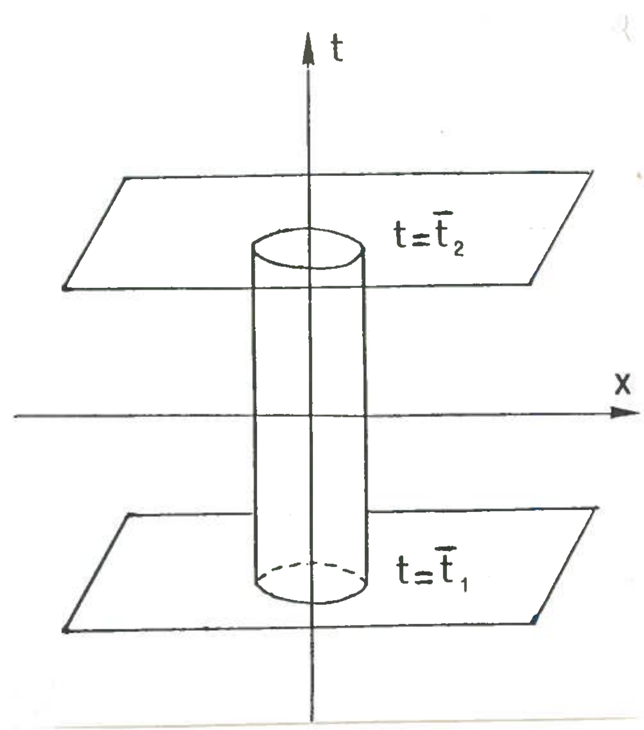

FIG. 4 - When the initial bradyon B has a finite life-time, so as depicted in this figure, then the corresponding tachyon T (obtained by applying a SLT) gets a finite space-extension. See the text.

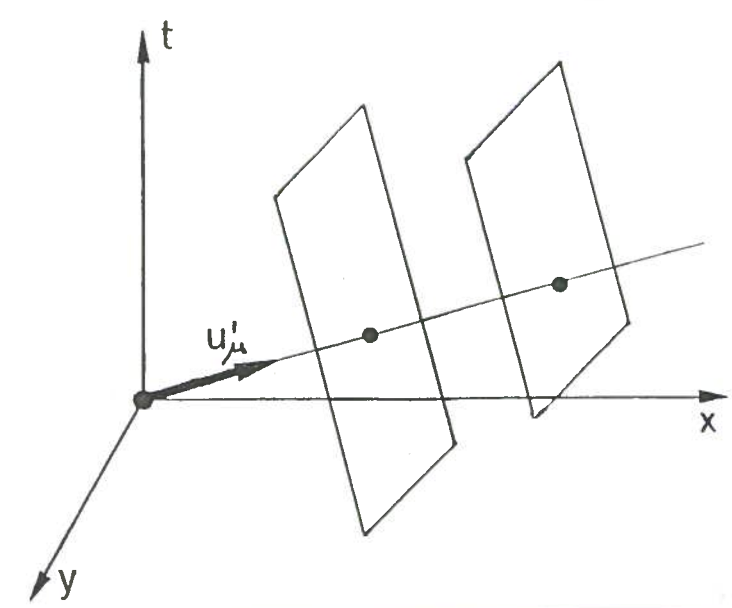

FIG. 5 - Under the hypotheses of Fig. 4, the tachyon T is constituted not by the whole structure in Fig. 2, but only by its portion confined inside two planes $x = x'_1$ and $x = x'_2$. Such a "window" is mobile; this figure shows that the "window" will travel with the subluminal speed $v' \equiv c^2/V'$, dual to the tachyonic speed V'.

in our case) which move parallely to themselves with the subluminal speed $v' = c^2/V'$; as it follows from their being orthogonal to u'_μ.

In conclusion, in the $\beta'^2 > 1$ case, one has to associate with eqs. (3)-(4) the additional constraints (in natural units : c = 1) :

$$-\bar{t}_2 \sqrt{V'^2 - 1} + x'V' \leqslant t' \leqslant -\bar{t}_1 \sqrt{V'^2 - 1} + x'V' ;$$

this means that the shape of a realistic tachyon (obtained from a finite life-time bradyon) is got by imposing on the structure $\mathscr{C} + \mathscr{H}$ in Fig. 2 the following contraints

$$\bar{t}_1 \frac{\sqrt{V'^2 - 1}}{V'} + \frac{t'}{V'} \leqslant x' \leqslant \bar{t}_2 \frac{\sqrt{V'^2 - 1}}{V'} + \frac{t'}{V'} . \qquad (v'^2 > 1) \qquad (6)$$

It follows that the realistic tachyon is constituted not by the whole structure in Fig. 2, but only by its portion confined inside a suitable, mobile "window" (i. e. , bound by the two planes $x = x'_1$ and $x = x'_2$).

This window, as we saw, travels with the speed v' dual to the tachyon speed V':

$$v' = 1/V' ; \qquad (V'^2 > 1; \; v'^2 < 1) \qquad (7)$$

and, if V' is constant, its width is constant too ($\bar{\Delta t} \equiv \bar{t}_2 - \bar{t}_1$):

$$\Delta x' = \bar{\Delta t} \sqrt{1 - v'^2} . \qquad (v' = 1/V') \qquad (8)$$

Chosen a fixed position $x = \bar{x}'$, such a "window" will take to cross the plane $x = \bar{x}'$ a time-duration independent of $\bar{x}'$ (if V' is still constant):

$$\Delta t' = \bar{\Delta t} \sqrt{V'^2 - 1} = \bar{\Delta t} \sqrt{1 - v'^2}/v' . \qquad (V'^2 > 1; \; v'^2 < 1) \qquad (9)$$

We have thus found that a realistic tachyon, even if associated to a structure $\mathscr{C} + \mathscr{H}$ travelling with the Superluminal speed V, would appear actually to travel with the subluminal, dual speed $v \equiv 1/V$. The magnitude of its "group-velocity" (i. e. , the speed of its "wave-front"), in fact, is given by eq. (7). Within the "window" confining the real portion of the tachyon (which probably carries the tachyon energy and momentum), there will be visible - however - a structure evolving at Superluminal speed, associable to a tachyonic "phase-velocity".

4. - COMMENTS

Such characteristics call to our memory those of an ordinary quantum particle, associated with its own "de Broglie wave" (in that case too, e. g. , phase-velocity and group-velocity are connected through eq. (7)).

To investigate this connection, let us define the new "wave-length" λ:

$$\lambda \equiv \frac{\hbar}{E/c} = \lambda_C \sqrt{1-\beta^2}\,, \qquad (\beta^2<1) \qquad (10a)$$

and recall the definitions of the Compton wave-length λ_C and the de Broglie wave-length λ_{dB} $(\beta^2<1)$:

$$\lambda_C \equiv \frac{\hbar}{m_o c}\;; \qquad \lambda_{dB} \equiv \frac{\hbar}{|\vec{p}|} = \lambda_C \sqrt{1-\beta^2}/\beta = \lambda/\beta\,, \qquad (10b)$$

where $1/\lambda^2 = 1/\lambda^2_{dB} + 1/\lambda^2_C$. Eqs. (10) suggest the following kinematical interpretation of the previous wave-lengths: let λ_C represent the intrinsic size of the considered (subluminal, quantum) particle; then $\lambda = \lambda_C \sqrt{1-\beta^2}$ is the particle size along its motion-line in the frame where it travels with speed $v \equiv \beta c$; and $\lambda_{dB}/c = \lambda/v$ will then be the time spent in the same frame by the particle to cross a plane orthogonal to its motion-line.

Let us now examine our own eqs. (8), (9). In eq. (8) it is natural to identify:

$$\Delta x' \equiv \lambda' = \lambda'_C \sqrt{1-\beta'^2}\,, \qquad (v' \equiv \frac{1}{V'};\; \beta'^2 \equiv (\frac{v'}{c})^2 < 1) \qquad (11a)$$

wherefrom it follows

$$\lambda'_C = c\bar{\Delta t}\,. \qquad (11b)$$

Then, from eq. (9) one gets $(v' = 1/V')$:

$$\lambda'_{dB} = \lambda'_C \frac{\sqrt{1-\beta'^2}}{\beta'} = \frac{\lambda'}{\beta'}\,. \qquad (\beta'^2<1) \qquad (11c)$$

By comparing eqs. (11) with eqs. (10), one recognizes that the characteristics of a classical tachyon T actually fit the "de Broglie relations", eqs. (10), provided that one attributes to the tachyon (or, rather, to its portion confined within the mobile, subluminal "window") a proper mass m_o such that

$$\lambda'_C/c = \frac{\hbar}{m_o c^2} = \bar{\Delta t} \Longrightarrow E\cdot\Delta x' = \hbar \qquad (E \equiv \frac{m_o c^2}{\sqrt{1-\beta'^2}})$$

where $\bar{\Delta t}$ is the intrinsic life-time of the initial bradyon B.

Before closing, let us recall a few further examples of a possible rôle of tachyons in (relativistic) quantum theory, examples that can be found scattered in the existing literature: (i) classical tachyons are kinematically allowed to be the (realistic) carriers of mutual interaction between elementary particles (e.g., in the c.m.s., infinite-speed tachyons can mediate the elastic scattering interactions), and would give rise to quantum-like interactions[6,1]; (ii) the hadron virtual clouds can be described in terms of classical techyons:

e. g. , the Yukawa potential can be regarded as a "continuous" (spherical-wave) flux of out going tachyons and of incoming anti-tachyons (remember that tachyons can move in an oscillatory way, reversing their motion direction (and their particle/antiparticle character) when reaching the infinite speed, i. e. the zero total energy state)[11,1]; (iii) within the O. P. E. models, old theoretical tests seemed to confirm that the exchanged particles can be realistically regarded as travelling faster than light[1]; (iv) tachyons can allow for classical vacuum-fluctuations (e. g. the vacuum can decay in a couple of infinite-speed tachyons)[12]; (v) a Lorentz invariant bootstrap theory for elementary particles can be apparently build up - as it was shown by Corben[13] - only by adding tachyons to bradyons[11]; (vi) from ER one can derive (besides the CPT theorem) also the "crossing relations"[1]; (vii) even within the realm of non-relativistic quantum theory, recent results[8] suggest that any realistic interpretation seems to imply the existence - inside quantum systems - of tachyonic interactions.

We acknowledge very stimulating discussions with M. Pavšič and particularly with A. O. Barut, as well as the kind collaboration of L. R. Baldini.

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(4) - Cf. G. D. Maccarrone and E. Recami, preprint PP/693 (Phys. Inst., Catania Univ., 1982), submitted for publication; Lett. Nuovo Cimento 34, 251 (1982); G. D. Maccarrone, M. Pavšič and E. Recami, Nuovo Cimento B73, 91 (1983).

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(6) - Cf. e.g. G. D. Maccarrone and E. Recami, Nuovo Cimento A57, 85 (1980), and references therein, in particular Refs. (8).

(7) - See e.g. E. Recami, in "Quarks and the Nucleus", vol. 8 of 'Progress in Particles and Nuclear Physics', ed. by D. Wilkinson (Pergamon Press, 1982), p. 401, and references therein.

(8) - See e.g. A. Aspect, J. Dalibard and G. Roger, preprint (Orsay, July 1982), submitted to Phys. Rev. Letters; V. Rapisarda, Lett. Nuovo Cimento 33, 437 (1982); F. Falciglia, A. Garuccio and L. Pappalardo, Lett. Nuovo Cimento 34, 1 (1982); A. Garuccio and V. Rapisarda, Nuovo Cimento A65, 269 (1981).

(9) - Cf. A. O. Barut, G. D. Maccarrone and E. Recami, Nuovo Cimento A71, 509 (1982); E. Recami and G. D. Maccarrone, Lett. Nuovo Cimento 28, 151 (1980); P. Caldirola, G. D. Maccarrone and E. Recami, Ref. (3).

(10) - Incidentally, this fact is useful to interpret the imaginary units entering an explicit form of the SLT transverse components. In fact, if we call x_o, y_o, z_o the intersections of the ellipsoid in Fig. 3b with the positive semi-axes x, y, z, respectively, then for the corresponding quantities of the tachyon in Fig. 3d one must get - and one gets[4,1] - that $x'_o = x_o\sqrt{V'^2 - 1}$; $y'_o = iy_o$; $z'_o = iz_o$.

(11) - See e.g. E. C. G. Sudarshan, Phys. Rev. D1, 2428 (1970); P. Castorina and E. Recami, Lett. Nuovo Cimento 22, 195 (1978), and references therein.

(12) - See e.g. R. Mignani and E. Recami, Phys. Letters B65, 148 (1976), and references therein.

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