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AN OVERVIEW OF THE NONLINEAR QED EFFECTS IN THE CONTEXT OF MEASUREMENTS OF VACUUM BIREFRINGENCE IN THE PVLAS EXPERIMENT - EARLY ESTIMATES

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An overview of the nonlinear QED effects in the context of measurements of vacuum birefringence in the PVLAS experiment - early estimates

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1. Introduction.

The PVLAS experiment, in preparation at the INFN Laboratori Nazionali di Legnaro [1] is designed to measure for the first time the effects of the interaction of photons with external magnetic fields. QED predicts that external electromagnetic fields produce virtual electron-positron pairs; what is experimentally observed as photon-photon interactions is in fact the EM interaction of the latter with the virtual pairs. In the PVLAS experiment the photons of a linearly polarized laser beam cross a region in which is present a constant and fairly homogenious external magnetic field $B_0 \approx 10^5$G); the dominating effect - most naturally interpreted in terms of optical quantities - is expected to be the *birefringence of the vacuum* induced by the external field. This effect, analogous to the Delbrück scattering of photons in the Coulomb field of atomic nuclei [2], is very hard to detect because the magnetic fields B_0 that can actually be created in a laboratory are by far below the critical value $B_{cr} = c^3 m^2/eh \approx 4.10^{13}$ G.

In the language of optics, the effects of photon-photon interactions when a linearly polarized monochromatic beam propagates through a constant magnetic field could give rise to:

a) non-zero elipticity of the laser beam polarization;

b) rotation of the polarization plane;

c) diffusion and dispersion of the beam, etc.

Our attention in the present note will be focused - in the context of the PVLAS experiment - on the elipticity acquired by an initially linearly polarized beam along its path through the birefringent vacuum. The experimental setup allows - in principle - for measurments with a high relative accuracy of 10^{-2} and better; the interpretation of the results requires a careful analysis of all the physical phenomena that contribute at this level. Our consideration will be very often based on and make use of results in the years 1960-1970 and even earlier with - in the meantime - the specific features of the PVLAS experiment being kept in mind. Of primary importance will be the facts that the optical path of the photons lies within a Fabry-Perot resonator of very high finesse (corresponding to a magnification factor for the observable effects of upto 30000) and that the magnetic field (except for the boundary regions) is fairly homogenious. A detailed description of the PVLAS experiment can be find in [1].

2. A survey of the theorist's approaches

2a. Perturbative expansion in a series of Feynman graphs.

One possible way of describing the phenomena occuring when monochromatic light propagates in vacuum in presence of external fields, is to consider the scattering amplitude of photons by classical electromagnetic fields and to evaluate the probability for the transition from an initial state with certain polarization λ into a final state with polarization λ'. In terms of Feynman graphs the leading order effect comes from the graphs a) and b) on Fig.1. (the wave lines with a cross sign at the end standing for classical photons):

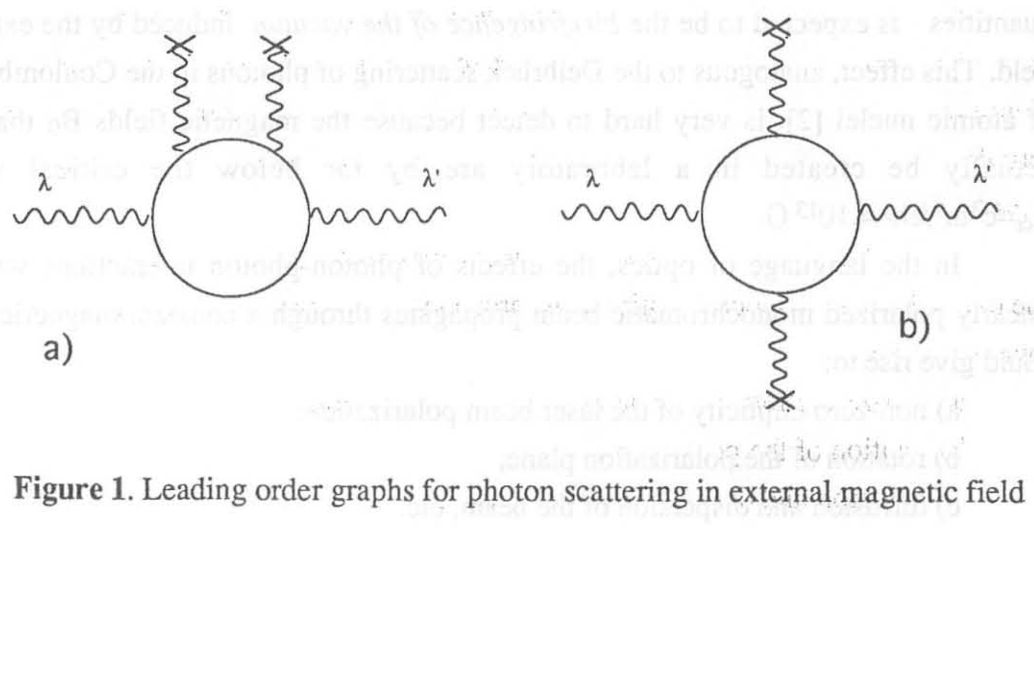


Figure 1. Leading order graphs for photon scattering in external magnetic field

Radiative corrections to the leading term - graphs a), b) and c) in Fig.2 - and higher order terms with more classical field insertions - graph d) - are all suppressed by powers of α (a factor α for any internal photon line or pair of classical insertions) and factors of (B_0/B_{cr}) for any extra external magnetic field line.

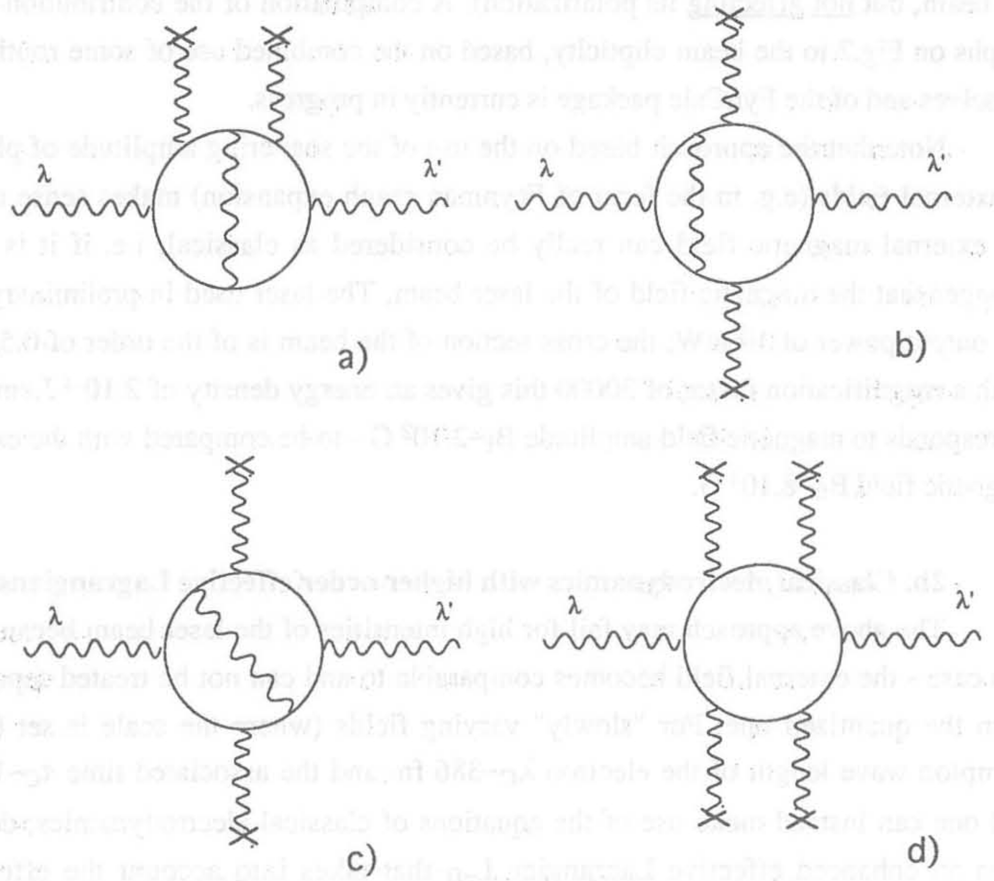


Figure 2. Graphs for radiative and multiphoton corrections for photon scattering in external magnetic field

Most of these graphs have been evaluated as early as in the seventies: the exact expression for the graphs in Fig.1 has been calculated by Costantini et al. [2], ring diagrams like d) on Fig.2 have been considered by Adler [3], etc. (we are far from trying to give a full bibliography on the subject); nowadays there also exists a variety of programs for the evaluation of rather complex graphs by computer (e.g. the *Mathematica* FeynCalc package by R.Mertig [4]). The evaluation of the elipticity of the polarization of a low intensity laser beam within the context of PVLAS is - in some sense - simpler than calculating the general amplitude for the transition $\lambda \rightarrow \lambda'$ because we are only interested in nearly forward scattering (acquiring momentum from the external field would bring the photon either out of resonance or out of the optical axis of the Fabry-Perot cavity so that it will certainly be lost, thus reducing the intensity of the beam, but not affecting its polarization). A computation of the contribution of the graphs on Fig.2 to the beam elipticity, based on the combined use of some routines of ourselves and of the FynCalc package is currently in progress.

Note that the approach based on the use of the scattering amplitude of photons in external fields (e.g. in the form of Feynman graph expansion) makes sense only if the external magnetic field can really be considered as classical, i.e. if it is much stronger that the magnetic field of the laser beam. The laser used in preliminary tests has output power of 10 mW; the cross section of the beam is of the order of 0.5 mm^2. With a magnification factor of 30000 this gives an energy density of 2.10^{-4} $J.cm^{-3}$ that corresponds to magnetic field amplitude $B_l \sim 2.10^2$ G - to be compared with the external magnetic field $B_0 \sim 8.10^4$ G.

2b. Classical electrodynamics with higher order effective Lagrangians.

The above approach may fail for high intensities of the laser beam because - in this case - the external field becomes comparable to and can not be treated separately from the quantized one. For "slowly" varying fields (where the scale is set by the Compton wave length of the electron $\lambda_C \sim 386$ fm and the associated time $\tau_C \sim 10^{-21}$ s [5]) one can instead make use of the equations of classical electrodynamics, derived from an enhanced effective Lagrangian L_{eff} that takes into account the effects of interaction of photons with virtual electron-positron pairs; unlike Maxwell's, these equations are no longer linear. You can think of L_{eff} as of a sum of the quadratic term L_0 and a series of correction terms $L_{n,k}$ of order $O(\alpha^{n+1} (B_0/B_{cr})^k)$ each:

$$L_{eff} = L_0 + L_{1,2} + L_{1,4} + \ldots + L_{2,2} + L_{2,4} + \ldots,$$
$$L_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{8\pi}(\vec{E}^2 - \vec{B}^2) \quad (1)$$

The main effects are described by the the correction term $L_{1,2}$

$$L_{1,2} = au + bv; \; a = A, \; b = 7A, \; A = \frac{\alpha^2}{90\pi} \frac{\lambda_e^3}{m_e c^2};$$
$$u = (\vec{E}^2 - \vec{B}^2)^2, \; v = (\vec{E}.\vec{B})^2. \tag{2}$$

As a matter of fact $L_{1,2}$ is the first term in the weak field expansion of the well known Euler-Heisenberg Lagrangian L_{HE} [3]:

$$L_{HE} = \frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s} e^{-ism^2} \left(e^2 uv \frac{ch(eus)\cos(evs)}{sh(eus)\sin(evs)} - \frac{1}{s^2}\right) = L_{1,2} + L_{1,4} + \ldots \tag{3}$$

Effective Lagrangians can be derived in various ways [6]. One possibility is to ask the effective Lagrangian (of a given class of processes) to have matrix elements that coincide with the S-matrix elements of these processes, calculated in the framework of perturbation theory (see, for instance, [7]); in this context $L_{eff} = L_0 + L_{1,2}$ of Eq.2 is the effective Lagrangian of photon-photon scattering in the one loop approximation of graph a) on Fig.3.

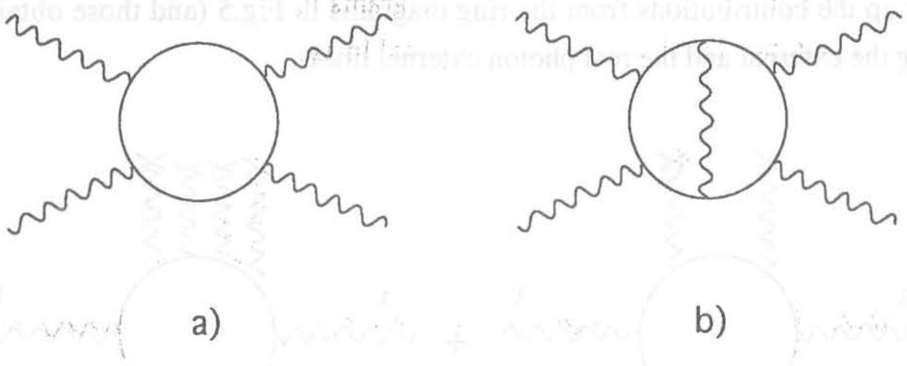


Figure 3. Leading order graphs for the scatering of real photons.

This method has the advantage that only the zero momentum transfer values of the graphs are needed, because the general form of L_{eff} is strictly determined from relativistic symmetry upto only 2 arbitrary constants, that can be extracted from the values of graph a) for forward scattering for two different initial polarizations of the incident photons [7]. Of course, in the case of weak quantum fields (compared to the external classical ones) the results coincide with the contribution of the graphs on Fig.1.

Alternative approaches developed in [6]-[9] start from the vacuum-to-vacuum transition amplitude in one- or more- loop approximation calculated with the exact electron propagators (see Fig.4):

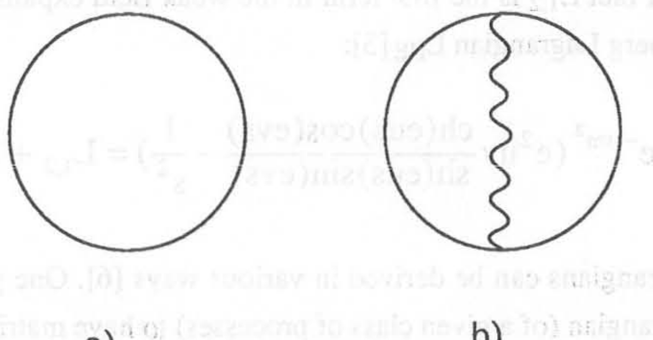


Figure 4. Vacuum conservation graphs in one- and two-loop approximation.

The one-loop approximation now gives the Heisenberg-Euler effective Lagrangian L_{HE}, of Eq.3. In the weak field limit the use of L_{HE} is equivalent to summing up the contributions from the ring diagrams in Fig.5 (and those obtained by permuting the external and the real photon external lines).

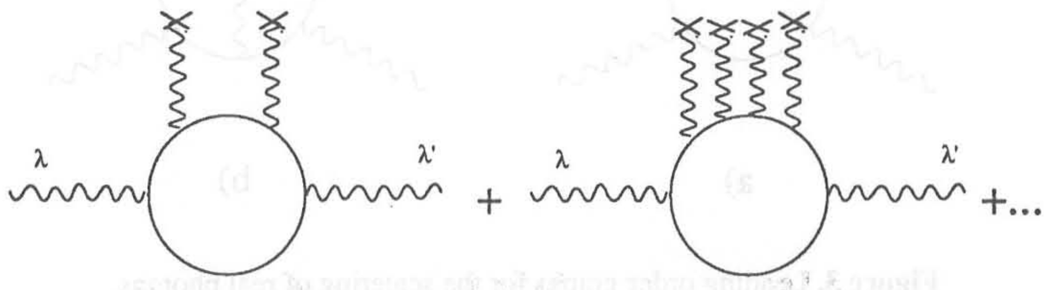


Figure 5. One loop ring diagrams for photon scattering by external fields.

The two-loop approximation, i.e. the contribution of graph b) on Fig.3 , first calculated in [10,11], gives rise to a correction term L_R to be added to the effective Lagrangian $L_{eff} = L_0 + L_{HE}$. (The explicit expression of L_R is too lengthy to be listed here). Its inclusion into the effective Lagrangian $L_{eff} = L_0 + L_{HE} + L_R$ is equivalent - in the weak field limit - to summing up the contribution of the graphs, obtained by adding one internal photon line (in all possible ways) to all the graphs in Fig.5, following the examples of graphs a), b) and c) on Fig.2.

In practical calculations in the weak field limit, the integral expressions of L_{HE} and L_R have to be expanded in series in powers of **E** and **B**; for the typical values $(B_0/B_{cr}) \sim 10^{-8}$ it is enough to keep only the lowest order terms in these expansions. As already mentioned,

$$L_{HE} = L_{1,2}(1 + O((\tfrac{B_0}{B_{cr}})^2)) \tag{4}$$

with $L_{1,2}$ being defined in Eq.2, while for L_R we got (after some algebra)

$$L_R = L_{2,2}(1 + O((\tfrac{B_0}{B_{cr}})^2))$$

$$L_{2,2} = \frac{\alpha^3}{\pi m^4}(\tfrac{16}{81}u + \tfrac{263}{162}v) \tag{5}$$

The latter expression coincides with the weak field limit results of [10] and [11]. Combining Eqs. 5 and 6, we get for the effective Lagrangian that will be used in further calculations the polynomial form

$$L_{eff} = L_0 + a'u + b'v \tag{6}$$

where the relativistic invariants u and v were defined in Eq.2, and

$$a' = A(1 + \tfrac{40}{9}\tfrac{\alpha}{\pi}) = a(1 + \tfrac{40}{9}\tfrac{\alpha}{\pi}),$$

$$b' = A(7 + \tfrac{1315}{36}\tfrac{\alpha}{\pi}) = b(1 + \tfrac{1315}{252}\tfrac{\alpha}{\pi}) \tag{7}$$

Note that the tensor structure of L_{eff} remains the same when summing up terms $L_{1,n}$, n=1,2,... The coefficients a' and b' can be expanded in series in powers of α, the details of the convergence of these series being discussed in [10].

3. Vacuum birefringence in strong external magnetic field upto terms of order $O(\alpha^3)$.

The equations of motion that follow from the Lagrangian L_{eff} of Eqs.6,7 involve nonlinear terms. Because of the small coefficients a' and b' (both of order $O(\alpha^2)$), in the weak field limit ($B_1 << B_0$) and in the particular case of monochromatic plane waves, these terms lead to effects that can be interpreted in terms of the optical characteristics of vacuum (considered as optical media). By combining the canonical definitions of the electrical displacement **D** and of the magnetic vector **H**

$$\vec{D} = 4\pi \frac{\partial L_{eff}}{\partial \vec{E}}; \quad \vec{H} = -4\pi \frac{\partial L_{eff}}{\partial \vec{B}}$$

with the material equations

$$\vec{D} = \varepsilon \vec{E}; \quad \vec{H} = \mu^{-1} \vec{B}$$

you get (see [1])

$$\vec{D} = \varepsilon \vec{E} = \vec{E} + 16\pi a' (\vec{E}^2 - \vec{B}^2)\vec{E} + 8\pi b' (\vec{E}.\vec{B})\vec{B}$$

$$\vec{H} = \mu^{-1}\vec{B} = \vec{B} + 16\pi a' (\vec{E}^2 - \vec{B}^2)\vec{B} - 8\pi b' (\vec{E}.\vec{B})\vec{E} \qquad (8)$$

In the presence of constant homogenious external magnetic field B_0 the equations of motion have approximate plane wave solutions in the form $\vec{E} = \vec{E}_1$, $\vec{B} = \vec{B}_1 + \vec{B}_0$ that - in the weak field limit $|\vec{B}_1| << |\vec{B}_0|$, $|\vec{E}_1| << |\vec{B}_0|$ - lead to:

$$\varepsilon_\perp = 1 - 16\pi a' B_0^2, \quad \mu_\perp = 1 + 48\pi a' B_0^2, \quad n_\perp = \sqrt{\varepsilon_\perp \mu_\perp} = 1 + 16\pi a' B_0^2$$

for $\vec{E}_1.\vec{B}_0 = 0$ and to

$$\varepsilon_\parallel = 1 - 16\pi a' \vec{B}_0^2 + 8\pi b' \vec{B}_0^2, \quad \mu_\parallel = 1 + 16\pi a' \vec{B}_0^2, \quad n_\parallel = \sqrt{\varepsilon_\parallel \mu_\parallel} = 1 + 4\pi b' \vec{B}_0^2$$

for $\vec{B}_1.\vec{B}_0 = 0$.

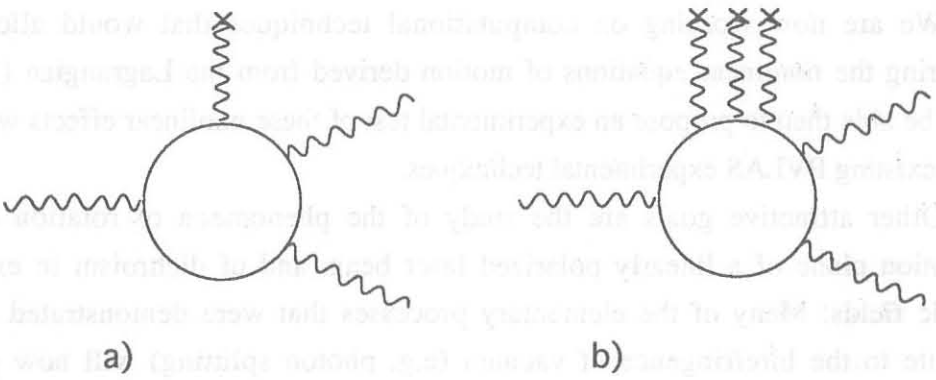
These expressions reproduce the results of [1] with the only difference that the constants a and b of Eq.2 have been substituted by a' and b' from Eq.7. With the contribution from the two-loop correction taken into account, the theoretical prediction for the vacuum birefringence to be measured in the PVLAS experiment becomes:

$$\Delta n = n_\parallel - n_\perp = 4\pi(b' - 4a')\vec{B}_0^2 \approx 12\pi(1 + \tfrac{25}{4}\tfrac{\alpha}{\pi})\vec{B}_0^2 \qquad ..(9)$$

The two-loop correction is of order $2\alpha \sim 1.5\%$ and will be measured when the relative accuracy of 10^{-2} will be achieved; contributions from three and higher loop graphs will be of relative order $O(\alpha^2)$ and cannot be measured for the moment being.

We also want to enumerate those effects which - according to our preliminary estimates - will not contribute to vacuum birefringence at this level of accuracy, though it was not obvious from the outset.

First has to be mentioned the elementary process of photon splitting in external magnetic fields, studied in details in [3]. The lowest order graphs for the amplitude of photon splitting are listed in Figure 6.



Fugure 6. Lowest order one-loop ring graphs for photon splitting.

Compared to the graphs for photon scattering in an external magnetic field of Fig.1, the above amplitudes have one suppressing factor (B_0/B_{cr}) less; the detailed analysis of [3] shows, however, that the amplitude a) vanishes for constant and homogenious magnetic field. The amplitude b) does not vanish, but - in the lowest order approximation - affects only the intensity of the beam and not its phase, thus not contributing to the birefringence value of Eq.9.

The effects due to the inhomogenity of the external magnetic field, and in particular - in the boundary regions, where the gradient of B_0 may reach 10^5 G.cm^{-1} - were also estimated and were found to be of order $O((B_1/B_0)^2)$, so that - for the energy densities currently achieved - they can be neglected.

4. Concluding remarks.

All our treatment of the vacuum birefringence induced by an external magnetic field has been essentially based on the assumption that the energy density in the Fabry-Perot cavity are such that the amplitude of the staying waves B_l is much smaller than the external field amplitude B_0: $\rho=|B_l|/B_0\sim 1/400<<1$. Things will significantly change, however, if ρ is augmented either by augmenting the finesse of the Fabry-Perot cavity, or by increasing the power of the laser, or by reducing the cross section of the beam. Values of $\rho\sim 1/10$ would already make the leading nonlinear effects (quadratic in $|B_l|/B_0$) comparable to the two-loop corrections to the vacuum birefringence and therefore - observable within the same experimental accuracy.

We are now working on computational techniques that would allow for considering the nonlinear equations of motion derived from the Lagrangian (7); we hope to be able then to propose an experimental test of these nonlinear effects with the already existing PVLAS experimental techniques.

Other attractive goals are the study of the phenomena of rotation of the polarization plane of a linearly polarized laser beam and of dichroism in external magnetic fields. Many of the elementary processes that were demonstrated not to contribute to the birefringence of vacuum (e.g. photon splitting) will now play a dominating role. The experimental set-up of the PVLAS experiment allows for measuring these effects with the same relative accuracy of $\sim 10^{-2}$ and this is a nice challenge for theorists.

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References.

1. D.Bakalov et al. Nucl.Phys.B (Proc.Suppl.) 35 (1994) 180.
2. V.Costantini, B. De Tollis, G.Pistoni. Nuovo Cim. 2A,3 (1971) 733.
3. S.Adler. Ann.Phys. 67 (1971) 599.
4. R.Mertig. Comp.Phys.Comm. 64 (1991) 345.
5. I. Bialynicki-Birula. Physica Scripta T21 (1988) 22.
6. W.Dittrich, M.Reuter. "Effective Lagrangians in QED". Lecture Notes in Physics vol. 220, Springer Verlag. 1971.
7. A.Akhiezer, V.Berestetsky. "Quantum Electrodynamics". Moscow, 1969.
8. W.Heisenberg, H.Euler. Z.Phys. 98 (1936) 714.
9. J.Schwinger. Phys.Rev. 82 (1951) 664.
10. V.I.Ritus. Sov.Phys.-JETP 42,5 (1976) 774
11. W.Dittrich. J.Phys.A10,5 (1977) 833.