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## ONE LOOP CORRECTIONS TO THE LIGHTEST HIGGS MASS IN THE MINIMAL $\eta$ MODEL

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#### Abstract

We have evaluated the one loop correction to the bound on the lightest Higgs mass valid in the minimal, $E_{6}$ based, supersymmetric $\eta$ model. Under the assumption that the theory remains perturbative up to the $10^{16} \mathrm{GeV}$ scale, we derive a conservative bound that decreases with the top mass for $M_{t} \leq 2 M_{W}$ and varies from $\sim 160 \mathrm{GeV}$ to $\sim 145 \mathrm{GeV}$ when $90 \leq M_{t} \leq 200 \mathrm{GeV}$.


One of the most exciting (and corageous) predictions of a large class of supersymmetric models is the existence of (at least) one light Higgs scalar [1], that would be certainly discovered at the next colliders ( alternatively, the non observation of such particle would be rather difficult to explain for the previous models).

An extremely important theoretical task becomes therefore the accurate determination of a rigorous upper bound for this light Higgs mass. For renormalizable models, this means that a calculation that includes radiative corrections e.g. at the one loop level would be relevant and welcome.

A well known and particularly illustrative example of the previous statement is provided by the so called Minimal low energy Supergravity Models [2]. Here, as a consequence of Supersymmetry, one has the famous tree level bound:

$$
\begin{equation*}
M_{H_{1}} \leq M_{z} \tag{1}
\end{equation*}
$$

where by $M_{H_{1}}$ we denote, from now on, the mass of the ligthest CP even Higgs scalar.This bound, as it has recently been stressed in several publications [3], is common to a class of "Minimal" SUSY models, that includes SUGRA models where e.g. all the SUSY breaking scalar masses are supposed to be equal at the GUT scale [4], but also models where such a constraint is not imposed and where soft breaking terms can be varied independently.

When radiative corrections to the bound of eq.(1) are computed [5], two important effects are generated. The first one is a substantial increase of the upper bound, essentially due to the top-stop contribution to the Coleman Weinberg [6] effective potential, that contains a quartic top mass dependence. The most dramatic consequence of this is that, for $M_{t} \simeq 150 \mathrm{GeV}, M_{\tilde{t}} \simeq 1 \mathrm{TeV}$, the new bound becomes now $\simeq 120 \mathrm{GeV}$, that is out of the reach of LEP2 with integrated luminosity of $500 \mathrm{pb}^{-1}$ and $\sqrt{s}=190 \mathrm{GeV}$ [7]. The second remarkable effect is the fact that both the numerical value of the improved bound and other important phenomenological features (like the possibility that the CP odd "pseudoscalar " becomes lighter than $H_{1}$ ) become now different within the previous large class of "Minimal" models [8], that would be of paramount importance in case of future Higgs(es) discovery.

A few years ago it was shown [9] that another class of models exists for which, at least over an interesting region of the parameter space, a bound for the lighthest scalar Higgs mass can be simply and elegantly computed. Such are those models where an extended
gauge symmetry generated by the exceptional group $E_{6}$, embedded into a supersymmetric minimal spectrum of 27 -plets with fermions of the conventional (quarks and leptons) type, is broken down by Hosatani type mechanisms [10] to either a rank 6 or a rank 5 low energy residual gauge group (for an exhaustive discussion of the various theorical details we defer to the existing literature [11]).

In both cases, Haber and Sher [9] derived bounds at tree level for the lightest scalar that are only slightly higher than the corresponding bound for Minimal SUSY models eq.(1). In particular, for the simplest rank 5 case with a minimal content of particles (two Higgs doublets and one Higgs singlet in the third generation acquiring vevs $v_{1}, v_{2}, v_{3}$ ) that is usually called (minimal) $\eta$ model, the result

$$
\begin{equation*}
M_{H_{1}} \leq 108 \mathrm{GeV} \tag{2}
\end{equation*}
$$

was found (similar values obtaining for the rank 6 case).
To see how one can possibly obtain a bound like that of eq.(2) and also for better understanding the philosophy of our paper, a quick review of its derivation and of the role of the parameters of the scalar sector of the models is requested. The latter ones are the coupling constant $\lambda$, that multiplies the trilinear term of the original supersymmetric superpotential ( $E_{6}$ invariance forbidding any other possible e.g. bilinear supersymmetric term), and the parameters of the SUSY breaking sector, i.e. the scalar mass terms $\mu_{1}, \mu_{2}, \mu_{3}$ and another soft breaking mass term $\lambda A$.

After imposing the three conditions of minimum and replacing the scalar mass terms by the related vevs one is left with four free parameters, since it is still possible to relate $v_{1}^{2}+v_{2}^{2}$ to $M_{z}^{2 *}$ :

$$
\begin{equation*}
M_{z}^{2}=\frac{1}{2}\left(g_{L}^{2}+g_{y}^{2}\right)\left(v_{1}^{2}+v_{2}^{2}\right)=\frac{1}{2} g_{z}^{2} v^{2} \tag{3}
\end{equation*}
$$

A possible choice of the four parameters is given, for instance, by the set :

$$
\frac{v_{2}}{v_{1}}=\tan \beta, \quad v_{3}, \lambda, \quad \lambda A .
$$

Alternatively, one can use the Z' mass:

$$
\begin{equation*}
M_{z^{\prime}}^{2}=\frac{1}{18} g_{\eta}^{2}\left(v_{1}^{2}+16 v_{2}^{2}+25 v_{3}^{2}\right) \tag{4}
\end{equation*}
$$

[^0]( $g_{\eta}$ is the extra $U(1)$ coupling, and in practice $g_{\eta}=g_{y}$ ) and the mass of the single ( CP odd) pseudoscalar of the model:
\[

$$
\begin{equation*}
M_{p s}^{2}=\lambda A\left\{\frac{v_{1} v_{2}}{v_{3}}+\frac{v_{1} v_{3}}{v_{2}}+\frac{v_{2} v_{3}}{v_{1}}\right\} \tag{5}
\end{equation*}
$$

\]

to replace $v_{3}$ and $\lambda A$, which is often done when numerical analyses are presented. The neutral CP even sector of the model contains three physical states $H_{1}, H_{2}, H_{3}$. To obtain the expression of the physical masses one has to diagonalize a $3 \times 3$ mass matrix $M^{2}$ whose six independent elements $\left[m^{2}\right]_{i j}=a_{i j}=a_{j i}$ can be cast in the form:

$$
\begin{equation*}
a_{i j}=\alpha_{i j} v_{3}+\beta_{i j} \text { for }(i, j) \neq(3,3) \text { and } a_{33}=\gamma_{33} v_{3}^{2}+\frac{\lambda A v_{1} v_{2}}{v_{3}} \tag{6}
\end{equation*}
$$

where $(\alpha, \beta)_{i j}$ and $\gamma_{33}$ do not depend on $v_{3}$, and their explicit expression is given for instance in ref.[9]. As it was pointed out by Drees [12], the determination of a bound for the lightest Higgs mass is strongly affected by the value of the ratio $\lambda A / v_{3}$. The values $\lambda A \ll v_{3}$ correspond to a certain region in the parameters space, that we shall refer to for simplicity as the "Haber-Sher's region", where the bound of eq.(2) can be derived without enforcing extra assumptions on the non purely gauge couplings of the model. Conversely in the "Drees region", where $\lambda A$ is not $\ll v_{3}$, the derivation of a bound needs extra assumptions on the $\lambda$ parameter. Invoking reasonable renormalization group equations (RGE) arguments (and assuming $M_{t} \simeq 40 \mathrm{GeV}$ ), Drees was able to fix a somehow higher value for the bound in this region, qualitatively equal to $M_{H_{1}} \leq 170 \mathrm{GeV}$, and, strictly speaking, this should be considered as the true bound of the model (at least, for the assumed $M_{t}$ value).

The origin of the difference between the bounds in the two regions can be easily understood if one uses a simplified procedure based on the assumption $v_{3} \gg v_{1}, v_{2}$ (no statements about $\left.\lambda A / v_{3}\right)$. The latter choice is phenomenologically motivated by the most recent bounds on the mass of the extra Z of the model, that can be derived either via direct CDF limits [13] or via indirect analyses of LEP1 data [14], [15], both leading to the result:

$$
M_{Z^{\prime}} \geq 300 \mathrm{GeV} \simeq 0.4 v_{3}
$$

from wich $v_{3} \gg v_{1}, v_{2}$ already emerges (note that future negative searches of the extra Z at CDF and LEP2 would soon improve the previous bound by a factor 2 [16]). In this configuration, one can show that one "light" Higgs exists i.e. one whose mass, which does
not become of $\mathrm{O}\left(v_{3}\right)$, is given by the following expression:

$$
\begin{equation*}
M_{H_{1}}^{2}=\left[\beta_{11} \cos ^{2} \beta+\beta_{22} \sin ^{2} \beta+\beta_{12} \sin 2 \beta-\frac{\left[m_{23}^{2} \sin \beta+m_{13}^{2} \cos \beta\right]^{2}}{\gamma_{33} v_{3}^{2}}\right] \tag{7}
\end{equation*}
$$

In the Haber-Sher's region,$v_{3} \gg \lambda A$, the negative contribution coming from the $(1,3)$ and $(2,3)$ non diagonal matrix elements of eq.(7) does never vanish. This brings a (negative) term of $O\left(\lambda^{4}\right)$ that, when combined with the positive $O\left(\lambda^{2}\right)$ contribution of the remaining matrix elements, produces a maximum becoming, in the limit $\tan \beta \rightarrow \infty$, the bound of eq. (2).

In the Drees region for $\lambda A \simeq O\left(v_{3}\right)$ it is conversely possible for the previous negative contribution to vanish. This leaves a new bound that contains a positive quadratical $\mathrm{O}\left(\lambda^{2}\right)$ term and has also a $\tan \beta$ dependence.

For $\lambda^{2}<g_{Z}^{2} / 2+g_{\eta}^{2} / 3$ the maximum is the same as in the Haber-Sher's case and corresponds to $\tan \beta=\infty$.

But for $\lambda^{2}>g_{Z}^{2} / 2+g_{\eta}^{2} / 3$, the maximum is obtained for:

$$
\begin{equation*}
\tan ^{2} \beta=\frac{\lambda^{2}-\frac{g_{Z}^{2}}{2}+\frac{g_{\eta}^{2}}{12}}{\lambda^{2}-\frac{g_{Z}^{2}}{2}-\frac{g_{\eta}^{2}}{3}} \tag{8}
\end{equation*}
$$

and a numerical evaluation of this expression requires a RGE approach to fix a maximum value of $\lambda$. This led to a bound on $M_{H_{1}}$ of qualitatively 170 GeV at the time of the original Drees derivation in which, we insist, the top mass was assumed to be of approximately 40 GeV and the hypothesis $\lambda^{2} / 4 \pi \leq 1$ at the scale $10^{16} \mathrm{GeV}$ was used.

As one can see, the role of the ratio $\lambda A / v_{3}$ is thus rather crucial in this game, since the difference between the two bounds is (from an experimental point of view) indeed dramatic. The aim of this paper is that of reconsidering the whole problem of the determination of a bound for $M_{H_{1}}$, in the most general configuration in the parameter space for the $\eta$ model, at the next one loop order of perturbation theory, particularly taking into account the fact that the top is now known to be heavier than $\sim 90 \mathrm{GeV}$ from the last CDF limits [17]. For sake of comparison with the previous tree level estimates, we shall still divide the parameter space into two regions, corresponding to whether the condition $\lambda A / v_{3} \ll 1$ is satisfied or
not, although in fact the "true" bound should always be derived in the Drees' region. ${ }^{\dagger}$
In pratice, the expected modification of the bound at one loop in the Haber-Sher's region is rather obvious if one believes that, in the large $v_{3}$ limit, the exotic sector should simply decouple from the conventional one. In this case, the radiative corrections to the bound should simply come from the top-stop sector. The actual proof of this statement requires a number of technicalities, that have been given in a previous note [18]. The result is that, as expected, the one loop bound in the Haber-Sher's region becomes:

$$
\begin{equation*}
M_{H_{1}}^{2} \leq M_{z}^{2}\left[1+\frac{16 g_{\eta}^{2}}{9 g_{z}^{2}}\right]+\frac{3 \alpha}{2 \pi c_{w}^{2} s_{w}^{2}} \frac{M_{t}^{4}}{M_{z}^{2}} \ln \frac{M_{t}^{2}}{M_{t}^{2}} \tag{9}
\end{equation*}
$$

showing that the full one loop correction is just the large $\tan \beta$ limit of the corresponding correction in the case of the minimal SUSY models ${ }^{\ddagger}$.

To evaluate the modification to the bound in the Drees' region, one has to start from the expression of the modified eq.(7) at one loop. This requires the explicit expression of the relevant matrix elements at that order. To perform the calculation, we have followed the effective potential approach [20] and we have first evaluated the contribution to the $M^{2}$ matrix that is obtained by considering all the fermion and sfermion content of the model (including the eleven exotic states). We have used the expressions of the masses of ref [21]; whenever this was possible, we have systematically neglected the various $D$-terms and/or terms of $\mathrm{O}\left(v / v_{3}\right)$. Within these approximations, the starting expressions of the various masses become :

For the top-stop system:

$$
\begin{align*}
M_{t}^{2} & =h_{t}^{2} v_{2}^{2}  \tag{10}\\
M_{\tilde{t}_{R}}^{2} & =m_{\tilde{t}_{R}}^{2}+h_{t}^{2} v_{2}^{2}+\frac{5}{18} g_{\eta}^{2} v_{3}^{2}+h_{t} A v_{2}+h_{t} v_{1} v_{3}  \tag{11}\\
M_{\tilde{t}_{L}}^{2} & =m_{\tilde{t}_{L}}^{2}+h_{t}^{2} v_{2}^{2}+\frac{5}{18} g_{\eta}^{2} v_{3}^{2}-h_{t} A v_{2}-h_{t} v_{1} v_{3} \tag{12}
\end{align*}
$$

[^1]with $h_{t}$ the top Yukawa coupling and $M_{\tilde{t}}$ the corresponding soft mass Susy breaking (we shall assume, as it is usually done, that $\left.M_{\tilde{t}}=\mathrm{O}(\mathrm{TeV})\right)$. Analogous expressions can be given for the $b, \tilde{b}$ system, whose contribution turns out to be negligible at realistic $\tan \beta$ values and will therefore be omitted in this first evaluation .

The quark exotic sector contributes with:

$$
\begin{align*}
M_{h}^{2} & =h_{h}^{2} v_{3}^{2}  \tag{13}\\
M_{h_{R}}^{2} & =m_{\tilde{h}}^{2}+h_{h}^{2} v_{3}^{2}-\frac{5}{36} k g_{\eta}^{2} v_{3}^{2}  \tag{14}\\
M_{\tilde{h}_{L}}^{2} & =m_{\tilde{h}}^{2}+h_{h}^{2} v_{3}^{2}-\frac{5}{9} k g_{\eta}^{2} v_{3}^{2} \tag{15}
\end{align*}
$$

where $h_{h}$ is the exotic Yukawa coupling which in principle can be large and $m_{\tilde{h}}$ the soft mass; in eq.(14),(15) k is a finite number that does not contribute appreciably to the result in any case.

The contribution of the other scalar mass sparticles for which the mixing is negligible i.e. $\tilde{f}_{i}=\left(\tilde{v}, \tilde{v^{c}}, \tilde{e}, \tilde{e}^{c}\right)$ is :

$$
\begin{equation*}
M_{f_{i}}^{2}=m_{f_{i}}^{2}+m_{f_{i}}^{2}+\frac{5}{6} g_{\eta}^{2} Y_{1}^{i} v_{3}^{2} \tag{16}
\end{equation*}
$$

with $Y_{1}^{i}$ the extra $\mathrm{U}(1)$ hypercharges [22].
Starting from eq. (10)-(16), inserting them in the effective potential written as usually in the form [6]

$$
\begin{equation*}
\Delta V\left(Q^{2}\right)=\frac{1}{64 \pi^{2}} S \operatorname{tr} M^{4} \ln \left[\frac{M^{2}}{Q^{2}}-\frac{3}{2}\right] \tag{17}
\end{equation*}
$$

and reevaluating the minimum conditions, one arrives at the one loop expressions of the $M^{2}$ matrix elements.

The results of our procedure are shown in the next formulae. One sees that the formal $v_{3}$ dependence of the tree level expressions is retained, and one can still write :

$$
\begin{gather*}
a_{i j}^{(1)}=\alpha_{i j}^{(1)} v_{3}+\beta_{i j}^{(1)} \quad(i, j) \neq(3,3)  \tag{18}\\
a_{33}^{(1)}=\gamma_{33}^{(1)} v_{3}^{2}+\delta_{33}^{(1)} \tag{19}
\end{gather*}
$$

where $(\alpha, \beta, \gamma)_{i j}$ do not depend on $v_{3}$ (and $\delta_{33} \sim 1 / v_{3}$ ). In particular, we find (the upper 1-index denotes the complete quantity at one loop; the same quantity without upper index
is meant to be the tree level expression, with renormalized couplings):

$$
\begin{align*}
& \frac{\alpha_{11}^{(1)}}{\alpha_{11}}=\frac{\alpha_{12}^{(1)}}{\alpha_{12}}=\frac{\alpha_{22}^{(1)}}{\alpha_{22}}=1-\frac{3}{16 \pi^{2}} h_{t}^{2} \ln \frac{M_{t}^{2}}{Q^{2}}  \tag{20}\\
& \alpha_{13}^{(1)}=\alpha_{13}+\frac{3}{8 \pi^{2}} h_{t}^{2} \lambda^{2} v_{1} \ln \frac{M_{t}^{2}}{Q^{2}}  \tag{21}\\
& \alpha_{23}^{(1)}=\alpha_{23}+\frac{5}{24 \pi^{2}} h_{t}^{2} g_{\eta}^{2} v_{2} \ln \frac{M_{t}^{2}}{Q^{2}}  \tag{22}\\
& \gamma_{33}^{(1)}=\gamma_{33}+\frac{3}{4 \pi^{2}} h_{h}^{4} \ln \frac{M_{\hat{h}}^{2}}{M_{h}^{2}}+(D-\text { terms }>0)  \tag{23}\\
& \frac{\beta_{11}^{(1)}}{\beta_{11}}=\frac{\beta_{12}^{(1)}}{\beta_{12}}=\frac{\delta_{33}^{(1)}}{\delta_{33}}=1  \tag{24}\\
& \beta_{22}^{(1)}=\beta_{22}+\frac{3}{4 \pi^{2}} v_{2}^{2} h_{t}^{4} \ln \frac{M_{t}^{2}}{M_{t}^{2}}  \tag{25}\\
& \beta_{13}^{(1)}=\beta_{13}+\frac{3}{16 \pi^{2}} h_{t}^{2} \lambda A v_{2} \ln \frac{M_{t}^{2}}{Q^{2}}  \tag{26}\\
& \beta_{23}^{(1)}=\beta_{23}+\frac{3}{16 \pi^{2}} h_{t}^{2} \lambda A v_{1} \ln \frac{M_{t}^{2}}{Q^{2}} \tag{27}
\end{align*}
$$

and in all the logarithms $M_{\tilde{t}_{L}}^{2}=M_{\tilde{t}_{R}}^{2} \equiv M_{\tilde{t}}^{2}$ is used. In eq.(23) we have called " $D$-terms" the sum of all the contributions of this kind coming from the corresponding terms in the sfermion masses. They will have little role in the numerical results, and we do not write down their explicit form.

From the previous expression of the relevant matrix elements it would be possible to compute numerically the value of the light Higgs mass for any given choice of the parameters. For what concerns the determination of a bound, though, the approach is relatively simpler since one immediately realizes that the contribution to eq.(7) coming from the ( 1,3 ) and $(2,3)$ non diagonal elements is still definitely negative and as such it can be neglected for this specific purpose. Since the exotic contribution is essentially contained in these terms, this means that for the specific determination of the bound it will still be possible to ignore it. This remarkable simplification allows us to concentrate our analysis on the reduced matrix $M_{i, j}^{2}, \mathrm{i}, \mathrm{j}=1,2$, and to look for its maximum in the considered region.

To derive a maximum for the "reduced" component of eq.(7) we have proceeded as follows. First, we have computed the modified relevant quantity at one loop, i.e. including the (largely dominant) top-stop contribution, that coincides with the one appearing in
eq.(9). The modified one loop bound becomes therefore the following:
$M_{H_{1}}^{2} \leq \frac{M_{z}^{2}}{\left(1+\tan ^{2} \beta\right)^{2}}\left[\left(\left(\tan ^{2} \beta-1\right)^{2}+\frac{g_{\eta}^{2}}{9 g_{Z}^{2}}\left(4 \tan ^{2} \beta+1\right)^{2}\right)+\frac{8 \lambda^{2}}{g_{Z}^{2}} \tan ^{2} \beta\right]+\frac{3 \alpha}{2 \pi c_{w}^{2} s_{w}^{2}} \frac{M_{t}^{4}}{M_{z}^{2}} \ln \frac{M_{t}^{2}}{M_{t}^{2}}$

The maximization of eq.(28) follows from the same assumptions that were used at tree level, with one crucial difference coming from the extra positive term that always increases with $M_{t}$ as $\sim M_{t}^{4}$. On the contrary, the contribution to the maximum coming from the first two terms on the r.h.s. of eq.(28) decreases with $M_{t}$ for not too large $M_{t}$ values. This is due to the fact that, in order to maximize $\lambda$, we shall follow the convention of imposing "perturbative saturation" i.e. $\lambda^{2} / 4 \pi \leq 1$ at the $\Lambda=10^{16} \mathrm{GeV}$ scale. From the relevant RGE for $\lambda$ and $h_{t}$ :

$$
\begin{gather*}
\frac{d \lambda}{d t} \simeq \frac{\lambda}{8 \pi^{2}}\left(-\frac{3}{2} g_{L}^{2}-\frac{1}{2} g_{Y}^{2}-\frac{7}{6} g_{\eta}^{2}+\frac{3}{2} h_{t}^{2}+2 \lambda^{2}\right)  \tag{29}\\
\frac{d h_{t}}{d t} \simeq \frac{h_{t}}{8 \pi^{2}}\left(-\frac{8}{3} g_{\text {strong }}^{2}-\frac{3}{2} g_{L}^{2}-\frac{13}{18} g_{Y}^{2}-\frac{2}{3} g_{\eta}^{2}+3 h_{t}^{2}+\frac{1}{2} \lambda^{2}\right) \tag{30}
\end{gather*}
$$

where $t=\log Q / \Lambda$, it follows then immediately that the value of $\lambda(Q)$ decreases with $h_{t}$ in a way which is shown in Fig.[1] ${ }^{\S}$. Since $\tan \beta$ at the maximum is related to $\lambda$ from eq.(8), and $h_{t}$ is related to $m_{t}$ and $\tan \beta$ by the equation:

$$
\begin{equation*}
M_{t}^{2}=h_{t}^{2} v^{2} \frac{\tan ^{2} \beta}{1+\tan ^{2} \beta} \tag{31}
\end{equation*}
$$

this means that at the maximum all the various parameters i.e., $\lambda, \tan \beta$ and $h_{t}$ will be expressed in terms of $M_{t}$. In particular, the $\lambda$ dependence on $M_{t}$, that will crucial for our conclusions, is shown in Fig.[2].

One can now proceed in the following way. Of the three terms that appear in eq.(28), the third one is clearly increasing with $M_{t}$. The second $\mathrm{O}\left(\lambda^{2}\right)$ terms decreases with $M_{t}$; this term would reproduce essentially the Drees' bound at tree level, and is numerically dominant for relatively small $M_{t}$ values. The first term increases with $M_{t}$, but remains relatively depressed compared to the second one for small $M_{t}$ values. This would exactly reproduce the Haber-Sher's bound for large $M_{t}$ values (corresponding to $\tan \beta \rightarrow \infty$ ) when it becomes conversely dominant over the second one.

The overall picture is represented in Fig.[3], where we show the mass bound as a function of the top mass in the full line, which is the representation of eq.(28) with the constraint

[^2]on $\tan \beta$ of eq.(8) ". The dotted line represents the upper bound of eq.(28) without the explicit contribution of the radiative correction $\propto M_{t}^{4}$ and shows the importance of the latter expecially for large values of $M_{t}$. Finally, the point-dotted line represents the radiative corrected upper bound in the Haber-Sher's region, eq.(9), showing that for large $M_{t}$ values the bounds, in the two regions, merge into one identical result.

One seees that for $M_{i} \leq 2 M_{W} \mathrm{GeV}$ there exists a bound for the model that would correspond to the modification of the Drees' bound and decreases with $M_{t}$, remaining always smaller than $\sim(160 \mathrm{GeV})$ for $M_{t} \geq 90 \mathrm{GeV}$. This would be in qualitative agreement with a general statement recently made by Kane et al. [24]. For larger $M_{t}$ values, the bound begins to increase with $M_{t}$. Assuming the limit on $M_{t}$ derivable from the last LEP1 data $M_{t} \leq 200 \mathrm{GeV}$, we obtain for this model the corresponding perturbatively saturated bound

$$
\begin{equation*}
M_{H_{1}} \leq 160 \mathrm{GeV} \tag{32}
\end{equation*}
$$

Thus, if the next LEP2 analyses failed to find a light scalar and set a qualitative limit $M_{H_{1}} \geq M_{Z}$, the model would predict a light scalar in a region that might be thoroughly ( and particularly well ) investigated by future higher energy $e^{+} e^{-}$linear colliders [25].

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[^3]
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## FIGURE CAPTIONS

## Fig. 1

Values of $\lambda\left(M_{Z}\right)$ for variable $h_{t}\left(M_{Z}\right)$ calculated with the RGE of eq.(29),(30) with the saturation condition $\lambda^{2}(\Lambda) / 4 \pi=1$ and $\Lambda=10^{16} \mathrm{GeV}$.

Fig. 2
Dependence of $M_{t}$ on $\lambda\left(M_{Z}\right)$ calculated after imposing the condition of eq.(9) on $\tan \beta$ and the relation between $\lambda$ and $h_{t}$ fixed by by the RGE shown in Fig.[1].

## Fig. 3

Values of the light Higgs mass bound at variable $M_{t}$ for $M_{\tilde{t}}=1 \mathrm{TeV}$. The full upper line represents the bound of eq.(28) with the constraint of eq.(8). The dotted line represents the same equation without the last contribution $\propto M_{t}^{4}$. The point-dotted line shows the bound in the Haber-Sher's region, eq.(9).


Fig.[1]


Fig.[2]


Fig.[3]


[^0]:    *We identify the physical $\mathrm{Z}, \mathrm{Z}$ ' states with the mathematical $Z_{0}, Z_{0}^{\prime}$ gauge eigenstates. Given the existing bound on $M_{Z^{\prime}}$, this makes no practical difference

[^1]:    ${ }^{\dagger}$ Assuming $v_{3} \gg v_{1}, v_{2}$ as from the previous discussion, the two different regions can be classified from the approximate expression, valid under this assumption, of the ratio

    $$
    \frac{\lambda A}{v_{3}}=\frac{25}{18} g_{\eta}^{2} \frac{\tan \beta}{1+\tan \beta^{2}} \frac{M_{p s}^{2}}{M_{z^{\prime}}^{2}}
    $$

    showing that the Drees' region does correspond to a substantially heavy ( $M_{P S} \geq M_{Z^{\prime}}$ ) pseudoscalar.
    ${ }^{\ddagger}$ The fact that the bound remains finite i.e. the radiative correction is not of $\mathrm{O}\left(\alpha v_{3}\right)$ is not occasional but is the consequence of a general "screening" property valid for a large class of SUSY models [19].

[^2]:    ${ }^{\S}$ To derive this conclusion we have neglected in the RGE the effect coming from the exotic coupling $h_{E}$, since the latter would give in any case negative contributions.

[^3]:    IIt is interesting to note that the $M_{t}$ dependence of our bound is very similar to the corresponding one in the "Minimal non Minimal" SUSY model recently examined in ref.[23].

