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NON LOGARITHMIC TERMS IN THE STRONG FIELD DEPENDENCE OF THE PHOTON PROPAGATOR

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ABSTRACT

Motivated by some speculations on a possible new phase of QED we study the modifications of the electromagnetic coupling in the presence of a strong external magnetic field. The anisotropy of the space induced by the applied field is explicitly taken into account. The result is described in terms of an isotropic logarithmic factor, as generally expected, plus a directional term showing an unusual linear dependence on the external field. This last term is the most important in the limit of strong static fields and it always tends to reduce the effective coupling. The present calculation provides more general grounds to an analogous statement already derived in a previous analysis.

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1. Introduction

It is well known that the coupling constants of gauge theories can be regarded as running parameters which depend on the energy scale of the processes one is dealing with. In a similar fashion, the study of particle propagators in the presence of external fields naturally leads to the introduction of effective coupling constants $g_{\text{eff}}(F_{\mu\nu})$ as functions of the background field configuration [1]. Recently, the observation of narrow e^+e^- peaks in heavy-ion collisions at GSI [2,3] has attracted much interest on the strong field dependence of the effective fine structure constant α_{eff} [4]. In principle, an appropriate choice for the external field configuration and strength could drive α_{eff} from the perturbative regime $\alpha_{\text{eff}} \ll 1$ to the strong coupling scenario $\alpha_{\text{eff}} \sim 1$, were QED is supposed to have a new confining phase [5-8]. As pointed out by some authors [9,10] this new phase would actually explain the main features of the anomalous electron-positron peaks observed at GSI.

The running of α_{eff} in a constant background field has been studied in [4] using the Schwinger's "proper time" formalism [11] and it was soon found that the structure constant exhibits a negligible logarithmic increase, so tiny that it cannot corroborate the new phase hypothesis. A similar conclusion was also drawn in [1]. However we think that the problem still deserves some clarification. In particular, more care must be devoted to the role played by the presence of preferred directions, which are necessarily introduced by the background fields. As we shall see, anisotropy has some important consequences which cannot be described by means of a single effective coupling constant. In fact, we find that the strong field dependence of the photon propagator is dominated by the presence of non logarithmic terms which heavily depend on the direction of the exchanged momentum. Since in a strong electric field it is possible to have vacuum instabilities which would give rise to a complex effective coupling, we shall take the background field such that it is purely magnetic in a suitable frame. The effect of this field configuration is in any case to reduce the

electromagnetic couplings, so that the main conclusion of Ref. [4] about the new phase of QED is unaffected.

2. Effective couplings in the long wavelength limit

In this section we provide a first evidence suggesting that the dominant effect of a strong magnetic field is a sizeable reduction of the electromagnetic couplings. The argument is a simplified version of the considerations presented in [12]. More general and rigorous results will follow in the next section. For the moment our computation is based on the so called Euler-Heisenberg Lagrangian which is known to describe the non linear effects of QED in the long wavelength limit [13]. This effective Lagrangian can be given the following integral representation [14]

$$\mathcal{L}' = \frac{m^4}{8\pi^2} \int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3} \left[-(\eta a \cotg(\eta a) \eta b \coth(\eta b) + 1 - \frac{\eta^2}{3} (a^2 - b^2)) \right] \quad (1)$$

with:

$$a = \frac{e}{m^2} \left[(\mathcal{F}^2 + \mathcal{G}^2)^{1/2} + \mathcal{F} \right]^{1/2}, \quad (2)$$

$$b = \frac{e}{m^2} \left[(\mathcal{F}^2 + \mathcal{G}^2)^{1/2} - \mathcal{F} \right]^{1/2}, \quad (3)$$

where \mathcal{F} and \mathcal{G} are the fundamental invariants of the electromagnetic field, $\mathcal{F} = (\mathbf{E}^2 - \mathbf{B}^2)/2$, $\mathcal{G} = \mathbf{E} \cdot \mathbf{B}$. When the vectors \mathbf{E} and \mathbf{B} are mutually parallel the invariants a and b have a simple physical meaning, namely:

$$a = \frac{eE}{m^2}, \quad b = \frac{eB}{m^2}. \quad (4)$$

In terms of \mathcal{L}' one can write the electric polarization \mathbf{P} of the vacuum as [14]

$$\mathbf{P} = \frac{\partial \mathcal{L}'}{\partial \mathbf{E}} \quad (5)$$

moreover, if we expand \mathcal{L}' in powers of \mathbf{E} and we keep only the lowest order terms, it follows that \mathbf{P} is a linear function of \mathbf{E} , $P_i = \chi_{ij}(\mathbf{B})E_j$, so that one can define the dielectric permeability tensor $\epsilon_{ij} = \delta_{ij} + \chi_{ij}(\mathbf{B})$. Starting from eq. (1), a simple algebra gives

$$\epsilon_{ij} = \delta_{ij} + 2\left(\frac{e}{m^2}\right)^2 \left[-\frac{\partial \mathcal{L}}{\partial b^2} \delta_{ij} + \left(\frac{\partial \mathcal{L}}{\partial a^2} + \frac{\partial \mathcal{L}}{\partial b^2}\right) n_i n_j \right] \quad (6)$$

where $\mathbf{n} = \mathbf{B}/B$ and all the derivatives are taken at $\mathbf{E}=0$ (and consequently $b=eB/m^2$):

$$\frac{\partial \mathcal{L}}{\partial a^2} = \frac{m^4}{8\pi^2} \int_0^\infty d\eta \frac{e^{-\eta}}{3\eta} [\eta b \coth(\eta b) - 1], \quad (7,a)$$

$$\frac{\partial \mathcal{L}}{\partial b^2} = \frac{m^4}{8\pi^2} \int_0^\infty d\eta \frac{e^{-\eta}}{\eta^2} \left[-\frac{1}{2b} \coth(\eta b) + \frac{\eta}{2 \sinh^2(\eta b)} + \frac{\eta}{3} \right]. \quad (7,b)$$

The problem is now reduced to find the asymptotic behaviour, for $b \rightarrow \infty$, of $\partial \mathcal{L} / \partial a^2$ and $\partial \mathcal{L} / \partial b^2$. From eqs. (7,a,b) it is not difficult to recognize that

$$\frac{\partial \mathcal{L}}{\partial b^2} \sim \frac{m^4}{8\pi^2} \cdot \frac{1}{6} \ln b^2 \equiv \frac{m^4}{8\pi^2} \cdot \frac{1}{6} \ln \left(\frac{\alpha B^2}{m^4} \right), \quad (8,a)$$

$$\frac{\partial \mathcal{L}}{\partial a^2} \sim \frac{m^4}{8\pi^2} \cdot \left(\frac{b}{3} - \frac{1}{6} \ln b^2 \right) \equiv \frac{m^4}{8\pi^2} \cdot \left[\frac{eB}{3m^2} - \frac{1}{6} \ln \left(\frac{\alpha B^2}{m^4} \right) \right]. \quad (8,b)$$

Inserting eqs. (8,a,b) into eq. (6) we obtain:

$$\epsilon_{ij} = \left[1 - \frac{\alpha}{6\pi} \ln \left(\frac{\alpha B^2}{m^4} \right) \right] \delta_{ij} + \frac{\alpha}{3\pi} \frac{eB}{m^2} n_i n_j \quad (9)$$

Obviously, there exists a closed relationship between the \mathcal{D}_{00} component of the photon propagator and the dielectric tensor ϵ_{ij} ; for $k=(0,\mathbf{k})$ (static limit) and $k^2 \rightarrow 0$ (large distance limit), the Fourier transform of \mathcal{D}_{00} reads:

$$\mathcal{D}_{00}(k) = - \frac{i}{\mathbf{k} \cdot \boldsymbol{\epsilon} \cdot \mathbf{k}} \quad (10)$$

The matrix ϵ_{ij} has the eigenvalues

$$\lambda_{\perp} = 1 - \frac{\alpha}{6\pi} \ln(\alpha B^2/m^4), \quad \lambda_{\parallel} = 1 - \frac{\alpha}{6\pi} \ln(\alpha B^2/m^4) + \frac{\alpha}{3\pi} \frac{eB}{m^2} \quad (11)$$

with eigenvectors perpendicular and parallel to \mathbf{B} respectively. Such eigenvalues, in turn, correspond to the effective coupling constants $\alpha_{\perp} = \alpha/\lambda_{\perp}$ and $\alpha_{\parallel} = \alpha/\lambda_{\parallel}$. The last one describes a reduction of the effective coupling and this effect turns out to be the dominant one in a strong magnetic field.

3. Beyond the long wavelength limit

The most striking feature of eq. (11) is the presence of the term $\frac{\alpha}{3\pi} \frac{eB}{m^2}$ which tends to reduce the electromagnetic couplings in a strong magnetic field. The aim of this section is to confirm the presence of such an effect via a computation of wider validity, which goes beyond the long wavelength limit discussed above. More precisely, we are going to compute the one loop vacuum polarization tensor $\omega_{\mu\nu}(k, \mathbf{B})$ to all orders in the external field \mathbf{B} . This is achieved replacing the free electron propagator by the Green function $S_F(x, y; \mathbf{B})$, the "exact" electron propagator in a homogeneous magnetic field. A rather compact form for $S_F(x, y; \mathbf{B})$ is known from the old paper by Schwinger [11] on the so called proper time formalism. For \mathbf{B} along the z direction one has [15]:

$$S_F(x, y; \mathbf{B}) = \Phi(x, y) G(x-y) \quad (12, a)$$

with

$$\begin{aligned} G(x) &= \frac{1}{(4\pi)^2} \int_0^\infty \frac{ds}{s} e^{-ism^2} \frac{z}{\sin z} e^{i\sigma_3 z} \\ &\times \exp \left[-\frac{i}{4s} \left(x_0^2 + z \cot z x_1^2 \right) \right] \\ &\times \left[m + \frac{1}{2s} \left(\gamma x_0 + \frac{z}{\sin z} e^{-i\sigma_3 z} \gamma x_1 \right) \right], \end{aligned} \quad (12, b)$$

$$\Phi(x, y) = \exp \left[-ie \int_x^y A(\zeta) d\zeta \right], \quad (12, c)$$

$$b_{\parallel}=(b^0,b^3), b_{\perp}=(b^1,b^2), z=esB. \quad (12,d)$$

In the fermion loop required for computing $\omega_{\mu\nu}$ the phase factors $\Phi(x,y)$ appear in the combination $\Phi(x,y)\Phi(y,x)=1$ and one is left with the product of two translationally invariant functions G , whose Fourier transforms \mathcal{G} depend on one momentum only. In the Euclidean space the function $\mathcal{G}(p)$ can be given the following integral representation [15]:

$$\begin{aligned} \mathcal{G}(p) = & \int_0^{\infty} ds \exp \left[-s \left(m^2 + p_{\parallel}^2 + \frac{\tanh z}{z} p_{\perp}^2 \right) \right] \\ & \times \left[(1 + \sigma_3 \tanh z) (m - \gamma_{p_{\parallel}}) - \frac{1}{\cosh^2 z} \gamma_{p_{\perp}} \right] \end{aligned} \quad (13)$$

which should be inserted in

$$\omega_{\mu\nu}(k) = e^2 \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left[\gamma_{\mu} \mathcal{G}(p) \gamma_{\nu} \mathcal{G}(p-k) \right] \quad (14)$$

in order to get the strong field dependence of the photon propagator. However we find it useful to proceed in a simplified way first. To start with we approximate the propagator $\mathcal{G}(p)$ with a more compact form $\mathcal{G}_a(p)$ which holds in the asymptotic region we are interested in, namely $\frac{eB}{m^2} \gg 1$. This approximated Green function $\mathcal{G}_a(p)$, once inserted in eq. (14), gives rise to a simple expression which can be analytically integrated to obtain the asymptotic behaviour of the vacuum polarization tensor as $\frac{eB}{m^2} \rightarrow \infty$.

Let us now focus on the first step, that is, the strong field limit of eq. (13). Since the integrand is limited from the above as $esB \rightarrow \infty$, the leading contribution to $\mathcal{G}(p)$ comes from the region $esB \gg 1$, where we can safely set $\tanh(esB) \sim 1$ and were we neglect the term $1/\cosh^2(esB)$ as well. As a result, the propagation function $\mathcal{G}(p)$ is cast into the form

$$\mathcal{G}_a(p) = \exp[-p_{\perp}^2/eB] (1+\sigma_3) \frac{m-\gamma p_{\parallel}}{m^2+p_{\parallel}^2} \quad (15)$$

The next step is the insertion of eq. (15) into eq. (14). For the sake of simplicity we compute only the ω_{44} component of the polarization tensor (ω_{00} in the Minkowski space) since it is strictly connected with the effective charges discussed in the previous section. Moreover we focus our attention on the case $\mathbf{k} \parallel \mathbf{B}$ where the maximum deviation from the logarithmic increase of the couplings is expected. We get:

$$\begin{aligned} \omega_{44}(k, \mathbf{B}) = & \int \frac{d^4 p}{(2\pi)^4} \exp[-p_{\perp}^2/eB] \int_0^{\infty} ds_1 \int_0^{\infty} ds_2 \exp[-m^2 s - s_1 p_{\parallel}^2 - s_2 (p-k)_{\parallel}^2] \\ & \times \text{tr} \{ \gamma_0 (1+\sigma_3) (m-\gamma p_{\parallel}) \gamma_0 (1+\sigma_3) [m-\gamma(p-k)_{\parallel}] \} \end{aligned} \quad (16)$$

with $s=s_1+s_2$ and $k_1=k_2=0$. A simple calculation gives:

$$\omega_{44}(k, \mathbf{B}) = \frac{e^2}{4\pi^2} eB \int_0^{\infty} ds_1 \int_0^{\infty} ds_2 \left[\frac{s_1 s_2}{s^3} (k_4^2 - k_3^2) + \frac{m^2}{s} \right] \exp \left[-m^2 s - \frac{s_1 s_2}{s} k^2 \right] \quad (17, a)$$

where the factor proportional to B comes from the Gaussian integration over p_{\perp} . The techniques required for the integration of (17) can be found in [16] and it is possible to express ω_{44} in terms of a one parameter integral representation:

$$\omega_{44}(k, \mathbf{B}) = \frac{e^2}{4\pi^2} eB \int_0^1 dx \frac{x(1-x)(k_4^2 - k^2) + m^2}{m^2 + x(1-x)k^2}. \quad (17,b)$$

According to eq. (17,b) the ω_{44} component of the vacuum polarization tensor does not vanish as $k^2 \rightarrow 0$. In the following we shall recognize the origin of this drawback. For the moment we blindly regularize eq. (17,b) by subtracting $\omega_{44}(0, \mathbf{B})$:

$$\omega_{44}^R(k, \mathbf{B}) = \omega_{44}(k, \mathbf{B}) - \omega_{44}(0, \mathbf{B}). \quad (18)$$

With this prescription one obtains:

$$\begin{aligned} \omega_{44}^R(k, \mathbf{B}) &= \frac{e^2}{4\pi^2} eB \int_0^1 dx \frac{x(1-x)(k_4^2 - k^2)}{m^2 + x(1-x)k^2} = \\ &= \frac{e^2}{4\pi^2} eB (k_4^2 - k^2) \left[\frac{m^2}{k^2} + 2 \left(\frac{m^2}{k^2} \right)^2 \frac{1}{\sqrt{1+4m^2/k^2}} \ln \frac{\sqrt{1+4m^2/k^2} - 1}{\sqrt{1+4m^2/k^2} + 1} \right]. \end{aligned} \quad (19)$$

In the limit of small k , eq. (19) gives:

$$\omega_{44}^R(k, \mathbf{B}) \cong \frac{\alpha}{3\pi} \frac{eB}{m^2} (k_4^2 - k^2) \quad (20)$$

in perfect agreement with the result derived in the previous section from the Euler-Heisenberg effective Lagrangian.

As promised above, we now compute $\omega_{\mu\nu}$ using the full integral representation (13). Once again we start from the ω_{44} component with $\mathbf{k} \parallel \mathbf{B}$, so that here $k^2 = k_4^2 + k_3^2$. A lengthy but simple algebra gives:

$$\omega_{44}(\mathbf{k}, \mathbf{B}) = \frac{e^2}{4\pi^2} eB \int_0^\infty ds_1 \int_0^\infty ds_2 \exp\left[-m^2 s - \frac{s_1 s_2}{s} k^2\right] \times \left[\frac{m^2}{s \tanh(esB)} + \frac{s_1 s_2}{s^3 \tanh(esB)} (k_4^2 - k_3^2) + \frac{eB}{s \sinh^2(esB)} \right] \quad (21)$$

where the last contribution comes from the term $\gamma p_\perp / \cosh^2(z)$ which was neglected in deriving the asymptotic propagation function $G_A(p)$ (see eq. (13) and (15)). In the strong field limit $eB/m^2 \gg 1$, one can verify that the only effect of the new term is to make $\omega_{44}(0, \mathbf{B})=0$ as required. This completely justify the naive prescription (18) together with eqs. (19) and (20). Moreover, it is worthy to cast $\omega_{44}(\mathbf{k}, \mathbf{B})$ in a form which explicitly exhibits charge conservation. This constraint is satisfied when ω_{44} is proportional to the combination $k_4 k_4 - \delta_{44} k^2 = k_4^2 - k^2$. To show this let us arrange the factor $(k_4^2 - k_3^2)$ as $k_4^2 - k_3^2 = 2(k_4^2 - k^2) + k^2$. With this splitting the unwanted contribution to the vacuum polarization tensor takes on the form:

$$\bar{\omega}_{44}(\mathbf{k}, \mathbf{B}) = \frac{e^2}{4\pi^2} eB \int_0^\infty ds_1 \int_0^\infty ds_2 \exp\left[-m^2 s - \frac{s_1 s_2}{s} k^2\right] \times \left[\frac{m^2}{s \tanh(esB)} + \frac{s_1 s_2}{s^3 \tanh(esB)} k^2 + \frac{eB}{s \sinh^2(esB)} \right]. \quad (22)$$

The integrand shows a unique singularity for $s \rightarrow 0$ of the form ds/s^2 which is eliminated by a suitable Pauli-Villars regularization. Then, by means of techniques analogous to those presented in Ref. [16], the regularized integral is found to be vanishing and we are left with the simple expression:

$$\omega_{44}(k, \mathbf{B}) = \frac{e^2}{4\pi^2} eB (k_4^2 - k^2) \int_0^\infty ds_1 \int_0^\infty ds_2 \exp\left[-m^2 s - \frac{s_1 s_2}{s} k^2\right] \frac{2s_1 s_2}{s^3 \tanh(esB)} \quad (23)$$

which holds for any value of \mathbf{B} . In particular, the strong field limit $\tanh(esB)=1$ gives both eqs. (19) and (20).

So far we have examined the ω_{44} component of the vacuum polarization tensor only. However, the remaining components can be worked out in a similar manner. Since no new idea is involved in their computation, we simply state the final result for the full tensor $\omega_{\mu\nu}(k, \mathbf{B})$ without any restriction on the direction of \mathbf{k} . By exploiting charge conservation as for eq. (23), $\omega_{\mu\nu}(k, \mathbf{B})$ turns out to be:

$$\begin{aligned} \omega_{\mu\nu}(k, \mathbf{B}) = & \frac{e^2}{4\pi^2} eB \int_0^\infty ds_1 \int_0^\infty ds_2 \exp\left[-sm^2 - \frac{s_1 s_2}{s} k_\parallel^2 - \frac{\sinh(es_1 B) \sinh(es_2 B)}{\sinh(esB)} k_\perp^2 / eB\right] \\ & \times \left[2 \frac{s_1 s_2}{s^3 \tanh(esB)} (k_{\parallel\mu} k_{\parallel\nu} - \delta_{\mu\nu}^{\parallel} k_\parallel^2) + 2 \frac{\sinh(es_1 B) \sinh(es_2 B)}{s \sinh^3(esB)} (k_{\perp\mu} k_{\perp\nu} - \delta_{\mu\nu}^{\perp} k_\perp^2) \right. \\ & \left. + \frac{s_1 \sinh(2es_2 B) + s_2 \sinh(2es_1 B)}{2 s^2 \sinh^2(esB)} (k_{\parallel\mu} k_{\perp\nu} + k_{\perp\mu} k_{\parallel\nu} - \delta_{\mu\nu}^{\parallel} k_\perp^2 - \delta_{\mu\nu}^{\perp} k_\parallel^2) \right] \quad (24) \end{aligned}$$

where

$$\delta_{\mu\nu}^{\parallel} = 1 \text{ if } \mu=\nu=0,3 \text{ and } \delta_{\mu\nu}^{\parallel} = 0 \text{ otherwise,} \quad (25,a)$$

$$\delta_{\mu\nu}^{\perp} = 1 \text{ if } \mu=\nu=1,2 \text{ and } \delta_{\mu\nu}^{\perp}=0 \text{ otherwise.} \quad (25,b)$$

A careful inspection of eq. (24) shows that only the longitudinal components of the vacuum polarization tensor exhibit the unexpected linear dependence on the external field strength as $eB/m^2 \gg 1$. Finally, we remark that the standard perturbative series is obtained by expanding the hyperbolic functions in powers of eB .

4. Concluding summary

In this work we have discussed the the photon propagator in the presence of a strong and slowly varying magnetic field. In contrast with the common belief we have shown that the correction to the photon propagator cannot be absorbed in a logarithmically increasing effective coupling constant. Actually, the strong field dependence of the vacuum polarization tensor turns out to be dominated by non isotropic terms which gives a sizeable reduction of the electromagnetic couplings. In the first section of the paper, our statement has been supported by a simple calculation relying on the Euler-Heisenberg effective Lagrangian which describes the non linear effects of QED in the long wavelength limit, that is, for small exchanged momenta. In the strong field limit $eB/m^2 \gg 1$, the eigenvalues of dielectric permeability tensor $\epsilon_{ij}(\mathbf{B})$ have been explicitly estimated. One of them, with eigenvectors perpendicular to \mathbf{B} , corresponds to a logarithmically increasing coupling constant. The other one, with eigenvectors parallel to \mathbf{B} , shows an unusual linear dependence on the external field strength and describes the above mentioned reduction of the electromagnetic coupling. More general conclusions have been derived in the second section, where the exchanged momenta are not assumed to be small. Starting from the exact electron propagator in a homogeneous magnetic field we have obtained a quite simple

integral representation for the one loop vacuum polarization tensor to all orders in the external field \mathbf{B} . The limit of this representation, for $eB/m^2 \gg 1$, confirms the presence of the unexpected terms characterized by the linear dependence on external field strength. Moreover, as a by-product of our analysis, we have obtained a very compact approximation for the electron propagator in a high external field. Further comments on this approximation will be given in the appendix.

In the literature, the strong field corrections to the effective fine structure constant α_{eff} have been discussed in connection with the anomalous electron-positron pair production observed in heavy-ion collision at GSI (the so called e^+e^- puzzle or Darmstadt effect). In principle, the unusual field environment induced by the colliding ions could drive α_{eff} up to the strong coupling regime $\alpha_{\text{eff}} \sim 1$, where a new phase of QED is likely to occur. Such a phase would actually explain the main features of the anomalous e^+e^- peaks. Some evidence against this mechanism was provided in Ref. [4] where a negligible logarithmic increase of α_{eff} is predicted. Although our results differ to some extent from those of Ref. [4] we have to draw the same conclusions about the new QED phase, since the unexpected terms appearing in the vacuum polarization tensor does not describe any increase of the electromagnetic couplings. Obviously, the role played by time dependent background fields is still an open question [17].

Appendix

It may be useful to give an interpretation of eq. (15) in terms of intermediate electronic states. The electronic Green function is $S_F(x,y) = G(y-x)\Phi(x,y)$ so that in momentum space one gets:

$$\tilde{S}_F(p,q) = \int dk \mathcal{G}(k) \tilde{\Phi}(p-k,q-k) \quad (\text{A.1})$$

Moreover in the particular gauge $A_1=A_3=A_0=0$, $A_2=Bz$, the actual expression for the Fourier transform of the phase is:

$$\tilde{\Phi}(p,q) = \delta^2(p_{\parallel})\delta^2(q_{\parallel})\delta(p_1+q_1)\delta(p_2-q_2) \exp[2iq_2p_1/eB] (2\pi)^3/eB \quad (\text{A.2})$$

while in Minkowski space it results:

$$\mathcal{G}(k) = (1+\sigma_3) \frac{\gamma p_{\parallel} - m}{p_{\parallel}^2 - m^2} \exp[-k_{\perp}^2/eB] \equiv \mathcal{G}_{\parallel}(k_{\parallel}) \exp[-k_{\perp}^2/eB] \quad (\text{A.3})$$

The actual integration, indicated in eq. (A.1) yields

$$\begin{aligned} \tilde{\mathcal{S}}_F(p,q) &= \frac{1}{\sqrt{eB}} \delta^2(p_{\parallel}-q_{\parallel})\delta(p_2-q_2) \mathcal{G}_{\parallel}(p_{\parallel}) \\ &\times \exp[-p_1^2/2eB + ip_1p_2/eB] \exp[-q_1^2/2eB + iq_1q_2/eB] \end{aligned} \quad (\text{A.4})$$

This expression shows poles for $p_0^2=p_3^2+m^2$, whose position is independent of B . Remembering that both $(1+\sigma_3)/2$ and $(m-\gamma p_{\parallel})/2m$ are projectors and that they commute among themselves it appears evident that the residuum of $\tilde{\mathcal{S}}_F$ decomposes into the product of the wave functions corresponding to the lowest level of a spinor in a static and uniform magnetic field. So one can conclude that the limit leading to eq. (15) has the effect of pushing all the electronic Landau levels to infinity keeping only the the first one which is, in fact, independent of the magnetic field.

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