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RADIATIVELY CORRECTED BOUND ON THE LIGHT HIGGS MASS IN A MINIMAL NON MINIMAL SUPERSYMMETRIC STANDARD MODEL

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Abstract

The neutral Higgs sector of the Minimal non Minimal Supersymmetric Standard Model is considered. By effective potential and R.G.E. supported method; an upper bound of the lightest Higgs is analysed. From the request of perturbativity of the coupling in the superpotential, adding the leading stop top contributions, the absolute bound of ~ 130 GeV for $90\,GeV\ < m_t < \ 180\,GeV$ and $M_{\tilde{t}} \simeq 1000$ GeV is derived. The interesting dependence on m_t for $\tan\beta \rightarrow 1$ is discussed .

The Minimal Supersymmetric Standard Model [1] (M.S.S.M.) is a simple and attractive phenomenological alternative to the Minimal Standard Model (M.S.M.), where the disturbing problem of the divergent radiative corrections to the Higgs mass is solved in a elegant way, with a minimal content of particles. It has the nice feature that, although three neutral (and two charged) Higgs bosons appear in the spectrum, an upper bound exists for the mass M_h of one (the "light") scalar. At tree level, this reads

$$M_h \leq M_Z \tag{1}$$

and one expects that radiative corrections will not modify the bound drastically. In fact, the existing calculations at one loop [2] show that in the MSSM there will always be a relatively light scalar of a mass of order $O(v)$, where $v = \sqrt{v_1^2 + v_2^2} \simeq 174$ GeV ($v_{1,2}$ are the vevs of the two doblets H_1, H_2 in the model).

The existence of an upper bound of $O(v)$ on the lightest Higgs mass is a general property of SUSY models [3]. To fix more quantitatively the precise value depends strongly on the model and the introduction of the radiative corrections is very important for this purpose. In particular it is very interesting to study the modifications induced by minimal extensions of the MSSM. With the introduction of one additional Higgs singlet field N it is possible to form a superpotential

$$W = \lambda H_1 H_2 N - \frac{k}{3} N^3 \tag{2}$$

which can generate in a "natural" way, via the vev $x =< N >$, a term $\sim \mu H_1 H_2$ whith $\mu = \lambda x \simeq O(M_W)$, difficult to explain in the MSSM [4]. The price that one has to pay is the appearance of the extra parameters x and k (necessary to avoid the appearance of one unwanted Goldstone boson) and of two extra (a scalar and a pseudoscalar) Higgs bosons. Quite interestingly, in the neutral scalar sector a bound for the lightest Higgs mass can still be obtained. The derivation of the bound at tree level is relatively simple, and it has been provided by a number of authors [5]. It reads:

$$M_h^2 \leq (\frac{g_z^2}{2} \cos^2 2\beta + \lambda^2 \sin^2 2\beta) v^2 \tag{3}$$

where $M_z^2 = g_z^2/2\ v^2$ and $\tan\beta = v_2/v_1$, so that, in order to give a quantitative number, some information on λ is now necessary. In general, one derives bounds on λ from renormalization group equation (RGE) (and request of a perturbative treatment) arguments. Once an upper bound on λ is assumed, limits on M_h are consequently derived, that are intuitively still of $O(v)$ i.e. equal to M_z or a few more GeV.

One can also evaluate radiative corrections to the bound in the model. Naively, one would expect the appearance of a "leading" correction $\sim m_t^4/M_z^2 \log M_{\tilde{t}}^2/m_t^2$ ($m_t, M_{\tilde{t}}$ are the mass of the top and of the stop), like in the MSSM case, leading to a rising of the bound when m_t gets larger. On the contrary as shown in a recent paper by J.R.Espinosa and M.Quiros [6], the plot of the upper bound of M_h versus m_t is strongly dependent on the value of $\tan\beta$ and in particular for $\tan\beta \to 1$ it shows a pronounced decrease with m_t, contrary to the intuitive expectation. This seems very interesting from the phenomenological point of view. Therefore, I have studied in this paper this model with the method of the effective potential [7] to compare the results with those of ref. [6] based on a pure RGE approach.

Starting from the Higgs superpotential (2), the tree level scalar potential is:

$$V(H_1, H_2, N) = V_F + V_D + V_{SB} \tag{4}$$

$$V_F = \lambda^2[|N|^2(|H_1|^2 + |H_2|^2) + |H_1 H_2|^2] + k^2|N|^4 - (\lambda k H_1 H_2 N^{*2} + h.c.) \tag{5}$$

$$V_D = \frac{g_z^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^+ H_2|^2 \tag{6}$$

and we have assumed the usual expression of the soft supersymmetry breaking term:

$$V_{SB} = m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_N^2|N|^2 - (A_\lambda \lambda N H_1 H_2 + h.c.) - (\frac{k}{3}A_k N^3 + h.c.) \tag{7}$$

where, in absence of explicit CP violation, λ and k are real. Minimizing the potential introduces the real vevs v_1, v_2 and x. The extra vev x is not strongly constrained from experiment (differently from the vev associated to the breaking of one extra U(1) group [8]); the most interesting bound comes from the lightest chargino mass:

$$M_{chargino} \simeq \lambda x > 45 GeV \tag{8}$$

that implies values of $x \sim O(v)$. However, when drawing the plot of M_h versus x one finds an increase with x wich is saturated for $x \gg v$, see fig.[1], so that it is convenient, to perform a calculation of the maximum, to work in the limit $x \to \infty$.

The tree level matrix elements m_{ij}^2 of the neutral scalar fields (ReH_1, ReH_2, ReN) after imposing the minimum conditions are:

$$m_{11}^2 = \hat{m}_{11}^2 + \bar{m}_{11}^2 = \lambda x \tan\beta(kx + A_\lambda) + \frac{g_z^2 v_1^2}{2} \tag{9}$$

$$m_{22}^2 = \hat{m}_{22}^2 + \bar{m}_{22}^2 = \lambda x \cot\beta(kx + A_\lambda) + \frac{g_z^2 v_2^2}{2} \tag{10}$$

$$m_{12}^2 = \hat{m}_{12}^2 + \bar{m}_{12}^2 = -\lambda x(kx + A_\lambda) + 2v_1v_2(\lambda^2 - \frac{g_z^2}{4}) \tag{11}$$

$$m_{13}^2 = \hat{m}_{13}^2 + \bar{m}_{13}^2 = 2\lambda^2 x v_1 - 2\lambda k v_2 x - \lambda A_\lambda v_2 \tag{12}$$

$$m_{23}^2 = \hat{m}_{23}^2 + \bar{m}_{23}^2 = 2\lambda^2 x v_2 - 2\lambda k v_1 x - \lambda A_\lambda v_1 \tag{13}$$

$$m_{33}^2 = \hat{m}_{33}^2 + \bar{m}_{33}^2 = 4k^2x^2 - kA_k x + \lambda A_\lambda \frac{v_1 v_2}{x} \tag{14}$$

where the terms $\hat{m}^2$ are of order $O(x)$ and $\bar{m}^2$ of order $O(v)$. In the limit $x \gg v$ and allowing the soft term A to be at most of order $O(x)$ the lightest Higgs mass results up to terms $O(v/x)$:

$$M_h^2(x \to \infty) = \frac{1}{v^2}[\,\bar{m}_{11}^2 v_1^2 + \bar{m}_{22}^2 v_2^2 + 2\bar{m}_{12}^2 v_1 v_2\,] - \frac{[v_2 \hat{m}_{23}^2 + v_1 \hat{m}_{13}^2]^2}{v^2 \hat{m}_{33}^2} = \tag{15}$$

$$M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{4}{4-a}\frac{\lambda^4}{k^2}[1 - \frac{k}{\lambda}\sin 2\beta - \frac{b}{2}\sin 2\beta]^2$$

where $A_k = kax$ and $A_\lambda = \lambda bx$ (to have a positive spectrum $b \leq 2$, $a \leq 3$, see ref.[9]). This expression has a maximum for:

$$\hat{z} \equiv \frac{k}{\lambda} = \frac{1}{\sin 2\beta} - \frac{b}{2} \tag{16}$$

This means that at tree level the bound on M_h becomes:

$$M_h^2 \leq M_z^2 \;\; for \;\; \lambda^2 < \frac{g_z^2}{2} \;\; and \;\; \tan\beta \gg 1 \tag{17}$$

$$M_h^2 \leq \lambda^2 v^2 \;\; for \;\; \lambda^2 > \frac{g_z^2}{2} \;\; and \;\; \tan\beta \to 1 \tag{18}$$

One sees here a major difference with the MSSM and a possible way out in the case of a not uncovered very light Higgs. The bound (17) is the same result as that of the MSSM whereas eq. (18) requires a careful evaluation of the coupling λ (it's interesting to note that in the case in which the extra Higgs is associated to an extra gauge group U(1) the dependence on λ disappears automatically in the maximization of M_h, see ref.[9]).

Consider now the introduction of the leading radiative corrections, coming from the top stop sector. For the upper bound only the corrections to the elements (11), (12), (22) are required becouse the maximum at tree level contains only the first term on the right side of eq.(15). The field dependent masses which one must introduce in the effective potential are:

$$M^2_{\tilde{t}_{R,L}} = m^2 + (-)\Delta^2 \tag{19}$$

$$m_t^2 = h_t^2 v_2^2 \tag{20}$$

$$m^2 \equiv m^2_{soft} + m_t^2 + \frac{g_z^2}{8}(v_1^2 - v_2^2) \; ; \; \Delta^2 \equiv h_t(\lambda x v_1 - A_t v_2) \tag{21}$$

with h_t the top Yukawa coupling and m^2_{soft}, A_t the soft mass SUSY breaking terms. The radiative corrections to each element are:

$$\delta m^2_{11} = \frac{3}{32\pi^2}[(\frac{g_z^4}{16}v_1^2 + \lambda h_t^2 A_t x \tan\beta)Z + g_z^2 \lambda h_t v_1 x \frac{\Delta^2}{m^2}] \tag{22}$$

$$\delta m^2_{12} = \frac{3}{32\pi^2}[((2h_t^2 - \frac{g_z^2}{4})\frac{g_z^2}{4}v_1 v_2 - \lambda h_t^2 A_t x)Z + 2\frac{\Delta^2}{m^2}(\lambda h_t v_2 x(2h_t^2 - \frac{g_z^2}{4}) - \frac{g_z^2}{4}h_t A_t v_1)] \tag{23}$$

$$\delta m^2_{22} = \frac{3}{32\pi^2}[(\frac{g_z^4}{16}v_2^2 - g_z^2 h_t^2 v_2^2 + \lambda h_t^2 A_t x \cot\beta)Z + 4h_t^4 v_2^2(\log\frac{M^2_{\tilde{t}_R}}{m_t^2} + \log\frac{M^2_{\tilde{t}_L}}{m_t^2}) - 4\frac{\Delta^2}{m^2}h_t A_t v_2(2h_t^2 - \frac{g_z^2}{4})] \tag{24}$$

here I have used the simplification $\log(M_{\tilde{t}_R}/M_{\tilde{t}_L}) \simeq 2\Delta^2/m^2$ and the position $Z = \log(M^2_{\tilde{t}_R}/M_z^2) + \log(M^2_{\tilde{t}_L}/M_z^2)$.

The radiatively corrected mass is then

$$M_h^2 = \frac{1}{v^2}[\bar{m}^2_{11}v_1^2 + \bar{m}^2_{22}v_2^2 + 2\bar{m}^2_{12}v_1 v_2 + \delta m^2_{11}v_1^2 + \delta m_{22}v_2^2 + 2v_1 v_2 \delta m^2_{12}] \tag{25}$$

In agreement with the screening equation of ref. [3],valid in the large x limit, the radiative corrections of order $O(\alpha x)$ cancel and the corrected upper bound becomes :

$$M_h^2 = M_z^2\cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + \frac{3}{32\pi^2}v^2[(\frac{g_z^4}{16}\cos^2 2\beta + g_z^2 h_t^2 \sin^2\beta\cos 2\beta)Z + \frac{\Delta^4}{v^2 m^2}(8h_t^2\sin^2\beta + g_z^2\cos 2\beta) + 4\frac{m_t^4}{v^4}(\log\frac{M^2_{\tilde{t}_R}}{m_t^2} + \log\frac{M^2_{\tilde{t}_L}}{m_t^2})] \tag{26}$$

A precise evaluation of the maximum of M_h goes through an analysis of the possible value of λ. It becomes necessary, for such purpose, to perform a careful analysis of the RGE [5] and of the parameter space involved in the numerical evaluation of the mass of the light Higgs. The R.G.E. promote the configurations in which the free parameters of the superpotential (2), k and λ, are in the ratio

$$\frac{\lambda^2}{k^2} = 2 \tag{27}$$

as shown by the paper of P.Binetruy and C.A.Savoy [5]; at the same time a contourplot of M_h (radiatively corrected) on the (k, λ) plane shows that a long this direction the maximum is reached most quickly, see fig.[3]. So imposing this constraint, the RGE reduce to:

$$\frac{dh_t}{dt} \simeq \frac{h_t}{8\pi}(3h_t^2 - \frac{8}{3}g_s^2) \tag{28}$$

$$\frac{d\lambda}{dt} \simeq \frac{\lambda}{8\pi}(\frac{5}{2}\lambda^2 + \frac{3}{2}h_t^2 - \frac{3}{2}g^2 - \frac{1}{2}g'^2) \tag{29}$$

$(t = \log \frac{\mu}{M_z})$, with g_s, g and g' the couplings of the gauge groups SU(3), SU(2), U(1).

Fig.[5] gives the result of the numerical analysis in which I have imposed the constraint of perturbativity on λ ($\frac{\lambda^2(\Lambda)}{4\pi} \leq 1$ with $\Lambda \sim 3\,10^{16}$ GeV) and shown how $\lambda[M_z]$ strongly depends on the values of $h_t[M_z]$ setting a severe bound on it.

In this way the dependence of M_h on m_t is double; an explicit one, coming from the effective potential, and an implicit one in the running of the elements at tree level. Whereas the first one tends to increase the upper bound for growing m_t, the second one, on the contrary, tends to decrease it. The relative weight of this two contributions depends on the values of $\tan\beta$ and the first is larger for $\tan\beta \gg 1$, (the parameters space of eq. (17)), the second for $\tan\beta \to 1$. In such a way there are two different pattern of M_h on m_t, one increasing for $\tan\beta \gg 1$, very similar to the values of the MSSM bound, another one strongly decreasing for $\tan\beta \to 1$. At intermediate regions this two effects compensate, from which the relative flatness of the intermediate curves of fig.[5] emerges. Making the envelope of all the curves, M_h results always smaller than 130 GeV for $90GeV < m_t < 180GeV$, a result that shows a difference of $\sim$ 8 percent from that obtained with the method of ref. [2]confirming the relative stability ot the outcome with respect to various reasonable theoretical imputs.

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FIGURE CAPTIONS

Fig.[1] Plot of M_h versus x from a numerical diagonalization of the 3x3 neutral mass matrix, radiatively corrected, for different values of λ [k=0.5; $A_k = A_\lambda = 100$ GeV, see ref.[5]; m=1000 GeV; Δ=400 GeV; m_t=150 GeV; $\tan\beta = 20$].

Fig.[2] The same as before but with k=0.6; $\tan\beta$=1

Fig.[3] Contour Plot of M_h in the plane (k,λ); x = 1000 GeV; $A_k = A_\lambda$ =100 GeV; m=1000 GeV; Δ=400 GeV; m_t=150 GeV; $\tan\beta$= 1. The dotted straight line represents the fixed ratio $\lambda^2/k^2 = 2$.

Fig.[4] The same as before but with $\tan\beta$=20

Fig.[4] Plot of $\lambda^2[M_z]$ versus $h_t[M_z]$ from the request of perturbativity at the scale $\Lambda \sim 3\,10^{16}$ GeV and with the constraint λ^2/k^2=2.

Fig.[6] Plot of the upper bound (eq.(26)) versus m_t with λ running (fig. [4]) for different values of $\tan\beta$.

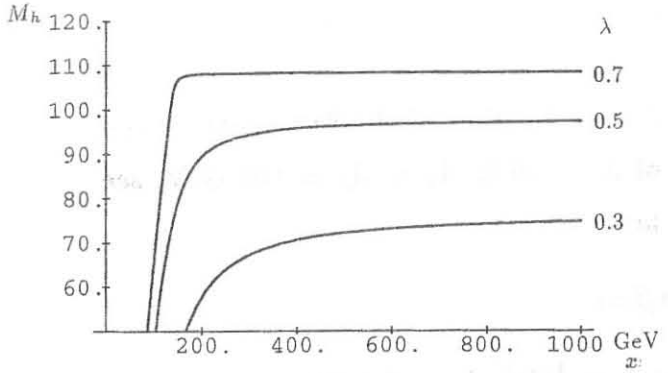

Fig.1

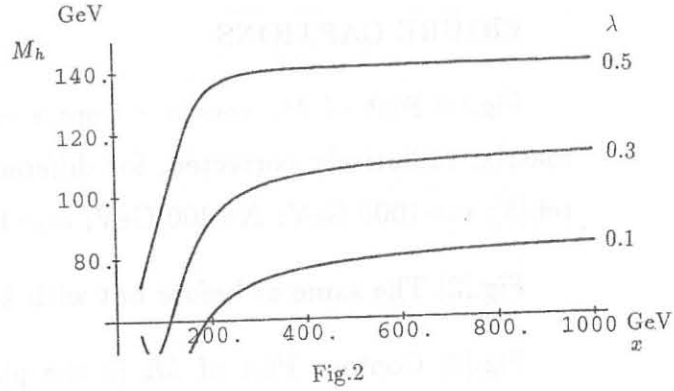

Fig.2

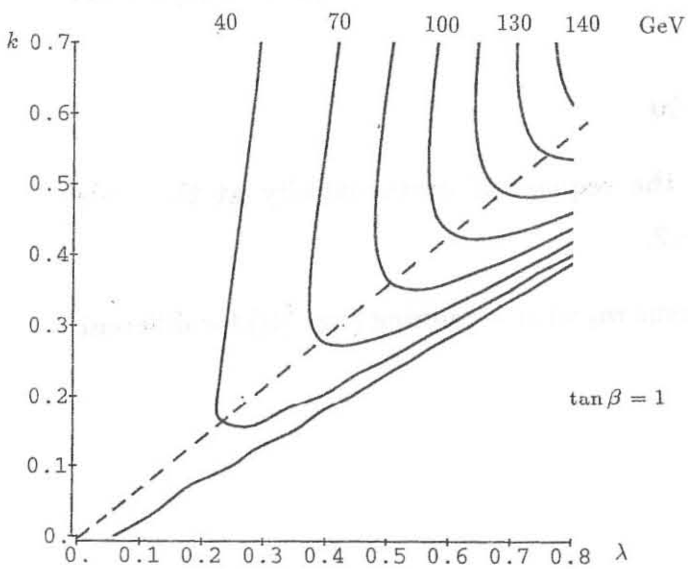

Fig.3

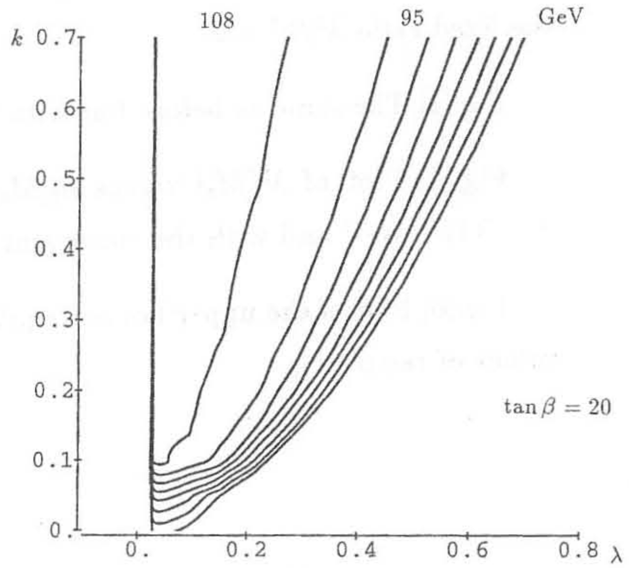

Fig.4

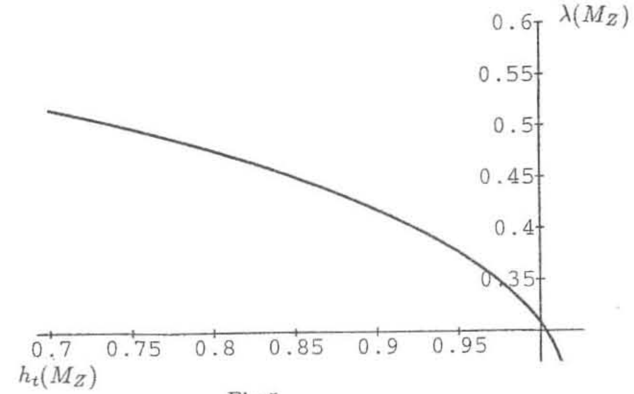

Fig.5

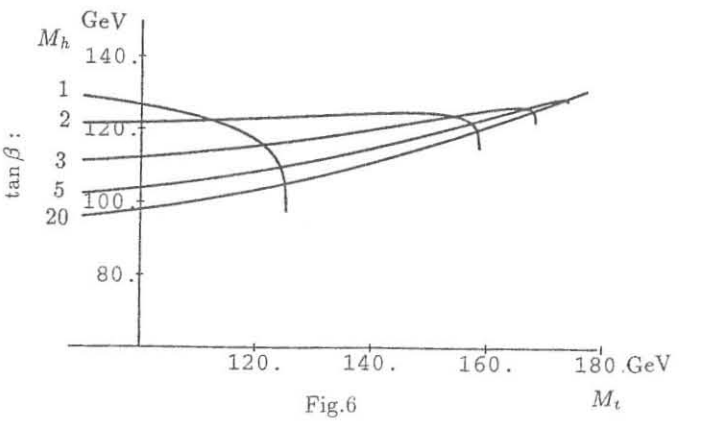

Fig.6