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THE "HIDE-AND-SEEK" OF THE Z'

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The "hide-and-seek" of the Z′

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Abstract

We discuss about the possibilities of decoupling the Standard Model dynamic from the effects due to extra gauge symmetries. We generalize the Georgi-Weinberg hypotesis allowing a standard Z coupling in $q^2 = 0$ interactions and show that, analogous conditions on the higgses may cause, at $q^2 = M_Z^2$, the decoupling between the virtual currents. E_6 derived canonical models can fit the requested Higgses decoupling condition and reproduce, both at $q^2 = 0$ and at $q^2 = M_Z^2$, standard couplings for the Z vector boson and the photon; this Higgs representation can justify an eventually large difference of mass between the Z and the Z'.

1 Introduction

Last LEP 1 run as well as q^2=0 APV experiments, have given no evidence up to now about the existence of the extra intermediate vector boson foreseen in most of the Grand Unified theories; in particular in almost all of the $U(1)'$ expanded models, the amount of $Z - Z'$ mixing as it comes from experiments, results to be less than 1% and next experiments will probably thin this already small interval around the zero[1, 2].

At this rate the hope to find a new intermediate neutral vector boson goes away from present accelerators forward the next generation of more energetic ones; but, in the meantime, one can try to guess some special mechanism which could hide the Z' manifestation within the context of extended gauge theories.

Already in 1978 Georgi and Weinberg[3] tried to explain the absence of extra neutral gauge currents in q^2=0 neutrino interactions, by a very peculiar vacuum breaking Higgs structure, and in Sec. 2 we will analyze some models which have found able to satisfy to those implications even without strictly submitting to such restrictive conditions on the Higgses. However, as LEP 1 physics describes the e^+e^- interactions at $q^2 = M_Z^2$, it would be quite interesting to generalize the validity of the theorem even to the case of charged particles interacting at energies higher than zero.

With this aim in mind, in Sec. 3 we firstly seek for the eigenfunctions of the squared mass matrix and analyze, in particular, the rank three extended models. The experimental request of a vanishing $Z_0 - Z_0'$ mixing angle θ_M in L-R models, imposes an infinite limit for the Z' mass. In canonical models the phenomenological condition $\theta_M = 0$, occurs with the block diagonalization of the mass matrix at the χ neutral generator which causes in turn, the automatic decoupling of the extra gauge dynamic from that of the Standard Model. The only not trivial way to simulate a $Z - Z'$ decoupling, is to expand to rank 4 models. In this case, in fact, one can require a "$\theta_M = 0$" behaviour of the $\vec{Z}$ eigenvector by imposing some simply relations among the various elements of the mass matrix.

In Sec. 4 we show as, in analogy with Ref. [3], the whole neutral propagator can be expressed by means of the squared mass matrix and its eigenvectors in such a way that, when the exchange of each neutral vector boson becomes dominant, the non-resonant neutral Hamiltonian connects some "fictitious" currents to the associated reduced mass-matrix. The dynamical currents have a feature analogous to that described by Georgi and Weinberg, so that in the q^2=0 sector, there is complete equivalence between the two descriptions and the S.M. limit can be easily checked. In this way we get an extension of the $q^2 = 0$ dynamic described by Georgi and Weinberg, also at the scale of the mass squared of each neutral intermediate vector boson.

As it will observed in section 5, the rank 4 decoupled models get a standard Z_0 coupling. Moreover, if the reduced mass matrix is block diagonal, at the scale of the Z resonance each fictitious virtual current will interact independently from the other; as for canonical models, one of the virtual fictitious currents is the electromagnetic current this causes the dynamical decoupling between the ordinary and the extended physics. For an "Inert" Z', we can justify the cancellation of the extra virtual current so that the dynamic will exactly reproduce the Standard Model predictions.

2 The Georgi-Weinberg theorem

Trying to explain the absence of non standard neutral current effects in neutrino scattering, Georgi and Weinberg argued that, if the gauge symmetry is constituted of a direct product of the kind $G_1 \otimes G_2 \otimes U(1)$, broken by the *VEV*s of two sets of scalar higgses ϕ_1 and ϕ_2 transforming respectively non (and) trivially under G_1 and G_2 , then in an elastic process involving electrically neutral and G_2 neutral fermions, at zero momentum transfer, the neutral current effects due to the G_2 symmetry would completely disappear.

The proof of the theorem (we refer here to Ref. [3] for notation) is based on the fact that, as a consequence of the assumed higgses' proprieties, the inverse of the $U(1)$ reduced squared mass matrix $[\mu^{-2}]_{ij}$ can be decomposed into block-diagonal components of separate G_1 and G_2 origin. Combining these results with the main observation that $[\mu^{-2}]_{ij}$ is strictly connected with the propagator of the effective four fermion neutral current at low momentum transfer

$$\Delta_{\alpha\beta} = \sum_{i\neq 0} m_i^{-2} u_{i\alpha} u_{i\beta} \tag{1}$$

by the relation

$$\Delta_{\alpha\beta} = \sum_{i,j\neq 0} \zeta_{\alpha i}\zeta_{\beta j}[\mu^{-2}]_{ij} \tag{2}$$

it results that the neutral current hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_{\alpha\beta}(\bar\psi\gamma^\mu t_\beta\psi)(\bar\psi\gamma_\mu t_\alpha\psi) g_\alpha g_\beta \Delta_{\alpha\beta} \tag{3}$$

transforms into

$$\mathcal{H} = \frac{1}{2}\sum_{ij\neq 0}(\bar\psi\gamma^\mu n_i\psi)(\bar\psi\gamma_\mu n_j\psi)[\mu^{-2}]_{ij} \tag{4}$$

and can be then divided into two pieces related to G_1 and G_2 respectively; if the fermions involved are both electrically neutral and neutral under G_2, the second term will not contribute to the interaction.

This is typically the case for the L-R symmetric breaking pattern $S(B)$ described in Ref. [4],

$$S(B): \qquad SO(10) \rightarrow SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

where the two sets ϕ_1 and ϕ_2 of the breaking higgses with *VEVs* $v_{1,2}$ are chosen to transform according the $\underline{16}$ representation of $SO(10)$:

$$\begin{array}{lcl} \phi_1 & \equiv & (\; I_{L1} = \frac{1}{2} \;\; ; \;\; I_{R1} = 0 \;\; ; \;\; B - L = 1 \;) \\ \phi_2 & \equiv & (\; I_{L2} = 0 \;\; ; \;\; I_{R2} = \frac{1}{2} \;\; ; \;\; B - L = 1 \;) \end{array}$$

one can see that the left or right isospin, alternatively vanish thus reproducing the requested hypothesis on the higgses. In literature however, one can find other models which still satisfy the Georgi-Weiberg implications even without requiring the strong over described higgses proprieties.

In canonical models, for example, the spontaneous symmetry breaking $S(A)$,

$$S(A): \qquad SO(10) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y_W} \otimes U(1)_\chi$$

may occur by means of a couple of higgses transforming as :

$$\begin{array}{lcl} \phi_1 & \equiv & (\; I_{L1} = \frac{1}{2} \;\; ; \;\; Y_{W1} = -\frac{1}{2}\sqrt{\frac{3}{5}} \;\; ; \;\; \chi_1 = \frac{3}{2\sqrt{10}} \;) \\ \phi_2 & \equiv & (\; I_{L2} = 0 \;\; ; \;\; Y_{W2} = 0 \;\; ; \;\; \chi_2 = -\frac{5}{2\sqrt{10}} \;) \end{array}$$

which do not satisfy the above condition. Anyway in this case a very lucky situation occurs, because the ϕ_2 field does not broke the S.M., which makes it possible to reproduce the implications of the theorem as well.

In fact the vanishing of the isospin-left in ϕ_2, allows us to divide the inverse reduced squared mass-matrix into two components :

$$\Sigma_{ij}^{-2} = \frac{1}{g_1^2 I_{L1}^2} \begin{pmatrix} \frac{1}{v_1^2} & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{v_2^2 \chi_2^2} \begin{pmatrix} \frac{1}{g_1^2}\frac{\chi_1^2}{I_{L1}^2} & -\frac{1}{g_1 g_2}\frac{\chi_1}{I_{L_1}} \\ -\frac{1}{g_1 g_2}\frac{\chi_1}{I_{L_1}} & \frac{1}{g_2^2} \end{pmatrix} \tag{5}$$

of which the first term reproduces the S.M. neutral current, while the second one turns out to be a perfect squared quantity when contracted between the current doublet (n_1, n_2) in this way we get the interaction hamiltonian as :

$$\mathcal{H} = \frac{1}{2} \left[n_1 \left(\frac{1}{g_1 v_1 t_1} \right)^2 n_1^+ + \frac{1}{v_2^2 \chi_2^2} \left(n_1 \frac{\chi_1}{g_1 t_1} - n_2 \frac{1}{g_2} \right)^2 \right] \tag{6}$$

with $n_1 = g_L \bar{\psi}\gamma^\mu (I_L - \sin^2\theta\, Q)\psi$ the standard neutral current and $n_2 = g_\chi \bar{\psi}\gamma_\mu \chi_f \psi$ the extra neutral one. Considering that the neutrino transforms like ϕ_1 , we will get for the interactions involving this neutral particle an automatic cancellation of the second term.

Unfortunately this is not always the case when we extend our analysis to a generic pattern of S.S.B. Already in the well known E_6 models derived from string theory, ordinary (and exotic) particles can be represented within the $\underline{27}$ of E_6 [5]; as a consequence, also the number of necessary higgses becomes larger and they will generally not satisfy neither the G-W hypothesis, nor the peculiar cancellation mechanism above described.

In the following we will find a more general condition on the Higgses which still may cause the decoupling of the virtual currents in $q^2 = 0$ interactions, moreower we will proove an interesting generalization of the theorem at the scale of the Z exchange.

3 The squared mass matrix

Let us now consider a Gauge theory for an effective symmetry group given by the direct product $G = G_1 \otimes G_2 \otimes U(1)$ with G_1 and G_2 being arbitrary groups. Be this theory broken by the *VEV*s of a set of higgs scalars ϕ_i each of them having charge $y_{\alpha i}$ with respect to the αth neutral generator T_α but still leaving the electric charge unbroken

$$Q = \sum_\alpha c_\alpha T_\alpha \tag{7}$$

the higgses' charges $y_{\alpha i}$, are then asked to satisfy to the relation

$$\sum_\beta c_\beta y_{\beta i} = 0 \tag{8}$$

The main problem arising in spontaneously broken gauge theories, is that gauge and mass eigenstates generally do not coincide. The most general squared mass matrix results to be

$$\mu^2_{\alpha\beta} = g_\alpha g_\beta \sum_i v_i^2 y_{\alpha i} y_{\beta i} \tag{9}$$

where the i index runs over the all the Higges involved. In order to get the mass eigenstates, we diagonalize it by means of an orthogonal transformation U. Calling $u_{\alpha t}$ the generic element of the U matrix, we can write:

$$\sum_\beta \mu^2_{\alpha\beta} u_{\beta t} = M_t^2 u_{\alpha t} \tag{10}$$

For the photon eigenvector $M_\gamma^2 = 0$, and we can easily take:

$$u_{\alpha 0} = N \frac{c_\alpha}{g_\alpha} \tag{11}$$

which automatically annihilates, by virtue of eq. (8), the mass matrix eigenvalues equation. The normalization constant N, results to be

$$N = e = \left[\sum_{\alpha}\left(\frac{c_\alpha}{g_\alpha}\right)^2\right]^{-\frac{1}{2}} \tag{12}$$

and has the meaning of the elementary electric charge e.

In the case of non-vanishing eigenvalues, things appear less simple. Looking for a solution of eq. (10), we also want to take into account of the physical request about the Higgses' neutrality expressed in eq. (8). With this aim in mind we sum over the α index in the eigenvectors equation (10).

$$\sum_{\beta}(\sum_{\alpha} g_\alpha g_\beta \sum_{i} v_i^2 y_{\alpha i} y_{\beta i} - M_t^2) u_{\beta t} = 0 \tag{10'}$$

In the same way we can generalize eq. (8) by multiplying it with the term $v_i^2 g_\alpha y_{\alpha i}$ and then summing over the α and i indices

$$\sum_{\beta}\frac{c_\beta}{g_\beta}\sum_{\alpha}\mu_{\alpha\beta}^2 = 0 \tag{8'}$$

we try to derive the generic t eigenvector by requesting equality for each member of the sum, and get, for such models in which the coefficients $c_i \neq 0$ and the squared mass matrix would be not block diagonal:

$$u_{\beta t} = N_t \frac{c_\beta}{g_\beta}\frac{\sum_\alpha \mu_{\alpha\beta}^2}{\sum_\alpha \mu_{\alpha\beta}^2 - M_t^2} \tag{13}$$

where the normalization coefficient N_t, which allows $\sum_\beta (u_{\beta t})^2 = 1$, is

$$N_t = \left[\sum_{\beta}\left(\frac{c_\beta}{g_\beta}\right)^2\left(\frac{\sum_h \mu_{h\beta}^2}{\sum_h \mu_{h\beta}^2 - M_t^2}\right)^2\right]^{-\frac{1}{2}} \tag{14}$$

consistently with the definition of the electric charge given above (12).

We now observe that the $u_{\beta t}$ obtained in this way, are, by construction, a solution of the eigenvalues equation (10) and therefore, because of uniqueness, they represent, in the case of no degeneration of the bosons' masses, the searched solution to the problem. The orthogonality conditions can be used to check the eigenfunctions of the model under consideration; they suggest some useful relations among the masses of the neutral vector bosons as ($t \neq t'$):

$$\sum_{\alpha} u_{\alpha t} u_{\alpha t'} = 0 \Rightarrow \sum_{\alpha}\left(\frac{c_\alpha}{g_\alpha}\right)^2 \frac{(\sum_h \mu_{h\alpha}^2)^2}{(\sum_h \mu_{h\alpha}^2 - M_t^2)(\sum_h \mu_{h\alpha}^2 - M_{t'}^2)} = 0 \tag{15}$$

Relation (15) comes out to be very useful when applied to the photon, in fact, in this case it simplifies to:

$$\sum_\alpha \left(\frac{c_\alpha}{g_\alpha}\right)^2 \frac{\sum_h {\mu_{h\alpha}}^2}{\sum_h {\mu_{h\alpha}}^2 - M_t^2} = 0 \tag{16}$$

from which one can get informations about the masses of the others neutral vector bosons starting from the mass matrix and the definition of the electric charge.

In practical cases, one restrict his research to just one extra non-standard Z' coming from residual Gauge groups like $SU(2)_L \otimes U(1)_Y \otimes U(1)_\chi$ or $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ which are supposed to appear in the low energy behaviour of more general Grand Unified models like E_6 [6].

In rank three models, the mass matrix can be written as

$$\mu^2 = \begin{pmatrix} g_0^2 \sum_i v_i^2 y_{0i}^2 & g_0 g_1 \sum_i v_i^2 y_{0i} y_{1i} & g_0 g_2 \sum_i v_i^2 y_{0i} y_{2i} \\ g_0 g_1 \sum_i v_i^2 y_{0i} y_{1i} & g_1^2 \sum_i v_i^2 y_{1i}^2 & g_1 g_2 \sum_i v_i^2 y_{1i} y_{2i} \\ g_0 g_2 \sum_i v_i^2 y_{0i} y_{2i} & g_2 g_1 \sum_i v_i^2 y_{2i} y_{1i} & g_2^2 \sum_i v_i^2 y_{2i}^2 \end{pmatrix} \tag{17}$$

where the index 0 conventionally refers to the $U(1)$ symmetry in the Gauge group as for example the weak hypercharge gauge symmetry in canonical theories.
The gauge fields will be mixed into the physical ones

$$\begin{pmatrix} Y_0 \\ Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\psi & -(\cos\psi\sin\theta\cos\phi + \sin\phi\sin\psi) & -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi \\ \sin\theta & \cos\theta\cos\phi & -\cos\theta\sin\phi \\ \sin\psi\cos\theta & \sin\phi\cos\psi - \cos\phi\sin\theta\sin\psi & (\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) \end{pmatrix} \begin{pmatrix} \gamma \\ Z \\ Z' \end{pmatrix} \tag{18}$$

for canonical and L-R models we can respectively take $(Y_0\,; Y_1\,; Y_2) \sim (Y\,; T_{3L}\,; \chi) \sim (B-L\,; T_{3L}\,; T_{3R})$. Comparing this equation with the general eigenvector equation (13), we get for the "Weinberg" angle,

$$\sin\theta = u_{10} = e\frac{c_1}{g_1} \tag{19}$$

that is, for $c_L = 1$, $\sin\theta_W = \frac{e}{g_L}$ consistently with the definition of the Standard Model; this is the quantity experimentally measured in the ratio $\sin^2\theta_W = 1 - M_W^2/M_Z^2 \simeq 0.2339 \pm 0.0022$[7]. For the other two Euler angles, we have the relations:

$$\tan\psi = \frac{u_{20}}{u_{00}} = \frac{c_2}{c_0}\frac{g_0}{g_2} \tag{19'}$$

which vanishes in canonical and in $g_2 \to \infty$ models, and

$$\tan\phi = -\frac{u_{12}}{u_{11}} \sim \frac{N_2}{N_1}\frac{M_Z^2 - \sum_h \mu_{h1}^2}{\sum_h \mu_{h1}^2 - M_{Z'}^2} \tag{19''}$$

The ϕ angle gives the mixing between the S.M. Z_0 and the extra gauge boson Z'_0, in fact $\tan\phi$ results to be proportional to the non-diagonal element of the Z_0, Z'_0 mass matrix:

$$M^2 = \begin{pmatrix} M_Z^2 \cos^2\phi + M_{Z'}^2 \sin^2\phi & \Delta M^2 \sin\phi\cos\phi \\ \Delta M^2 \sin\phi\cos\phi & M_{Z'}^2 \cos^2\phi - M_Z^2 \sin^2\phi \end{pmatrix} \tag{20}$$

with $\Delta M^2 = M_{Z'}^2 - M_Z^2$. Defining M_0^2 the first term of the M^2 matrix, we find the well known relation

$$\tan^2\phi = \frac{M_Z^2 - M_0^2}{M_0^2 - M_{Z'}^2} \tag{20'}$$

When $\phi = 0$, the M^2 matrix diagonalizes and the physical boson masses coincide with the mathematical ones.

In our analysis we take into account of all the neutral generators entering into the squared mass matrix before diagonalizing it and therefore it is quite different from the one done in Ref. [8], where the authors only consider the U(1) reduced mass matrix in order to get the physical mass eigenstates of the two neutral vector bosons. The M^2 mass matrix of the Z_0 and Z'_0 in our description comes out by the double rotation of the μ^2 matrix with the two Euler angles θ and ψ. For this reason, our elements of the M^2 matrix are, in general, more complex, and in particular the mixing angle does not result any more proportional to μ_{12}^2. However for canonical (ψ=0) models (to which we will dedicate a great part of our paper) the two descriptions are coincident.

The present experimental limits on the Z-Z' mixing angle θ_M [1, 2] suggest values very close to the zero, for most all the canonical and L-R models, and even compatible with it at 1% confidence level. It would be thus interesting to look for assumed conditions on the Higgses which could induce an exact decoupling between the Z and the Z'. We will call "Spontaneous Decoupled" [9] all those models whose $Z - Z'$ mixing angle is a-priori posed equal to zero, already into the bare Lagrangian.

Seeking for spontaneous decoupled rank three models, we try to compare the mass eigenvectors $\vec{u}_Z$ and $\vec{u}_{Z'}$ from eq. (13) with the respective ones of eq. (18) at $\phi = 0$. Note that, in this way, it is not possible to get any Higgs' decoupling condition without incurring into indetermination. In fact the same request for a vanishing $u_{1Z'}$ term, which is proportional to $\sin\phi$

$$u_{1Z'} \simeq \frac{c_1}{g_1} \frac{\sum_h \mu_{h1}^2}{\sum_h \mu_{h1}^2 - M_{Z'}^2} = 0 \quad , \tag{21}$$

together with the assumed behaviour of the $\vec{Z}$ eigenvector

$$\vec{Z}_{\phi=0} = (-\cos\psi\sin\theta\, ; \, \cos\theta\, ; \, -\sin\psi\sin\theta) \quad , \tag{22}$$

requires, in a real model with a finite value of $M_{Z'}^2$, either the degeneration of the masses, or the block-diagonalization of the mass-matrix.

In the limit of large Z' mass, which can be reached by shifting the g_2 coupling constant to infinity, both the $u_{2\gamma}$ and the u_{2Z} need to vanish because of the exploding third column of the mass matrix. On the other side, in canonical models, the spontaneous decoupling condition $\theta_M = 0$ always requires the off-diagonal M^2 matrix element, proportional to the μ_{12}^2 term of the mass matrix, to vanish:

$$g_L\, g_\chi \sum_i v_i^2 T_{Li} \chi_i = g_Y\, g_\chi \sum_i v_i^2 y_i \chi_i = 0 \quad . \tag{23}$$

This automatically causes, in this case, the complete block-diagonalization of the mass matrix at the $M_{Z'}$ eigenvalue. The $\vec{u}_{Z'}$ eigenvector is therefore coincident with the extra gauge current: $\vec{u}_{Z'} \simeq (0\,;0\,;1)$ and we fall into the previous case. This is the only spontaneous decoupled rank three model allowing a Z' of finite mass; in fact the block-diagonalization of the mass matrix makes the extra gauge dynamic independent from the standard one.

In this sense, rank three models cannot allow a vanishing ϕ angle without requesting at the same time the complete decoupling between two physics, the standard and the extra one, thus preventing the last one to take part into the low energy dynamic (that is the Z dynamic) even at the virtual level; its manifestation is therefore submitted to the only possibility of direct production of extra gauge bosons. However in rank 4 models, to which typically belong all the E_6 models, it is possible to get the hoped $Z_{\phi=0}$ behaviour without necessary imposing a vanishing value to $u_{1Z'}$. This is because it's possible to think about successive rotations of the fourth gauge group which could just influence the $u_{1Z'}$ term making it no more proportional to $\sin\phi$. Note that, in this expanded case, the ϕ angle , which defines the $L - \chi$ rotation, will not coincide any more with the $Z_0 - Z'_0$ mixing angle θ_M, on the contrary, the most general relation between the two quantities will be generally far from a simple proportionality. In fact, because of the rank-4 rotation on the $\vec{u}_{Z'}$ eigenvector, relation (19″) defining the ϕ angle in rank three models, looses its validity. However, we observe that, if the $\vec{Z}$ mass eigenvector assumed the typical rank three $\phi = 0$ behaviour, by looking at the Z resonant physics, there would be little room to realize the presence of other symmetries.

The eigenvector which simulates a rank three vanishing $Z - Z'$ mixing angle, can be expressed as :

$$\vec{Z}_{\phi=0} = \frac{e}{\sqrt{\left(\frac{c_0}{g_0}\right)^2 + \left(\frac{c_2}{g_2}\right)^2}} \left(-\frac{c_0}{g_0}\frac{c_1}{g_1}\,;\ \left(\frac{c_0}{g_0}\right)^2 + \left(\frac{c_2}{g_2}\right)^2\,;\ -\frac{c_2}{g_2}\frac{c_1}{g_1}\,;\ 0 \right) \tag{24}$$

which, by means of eq. (13), requires

$$\sum_h \mu_{h0}^2 = \sum_h \mu_{h2}^2 = \frac{1}{2}\, M_Z^2 \tag{25}$$

and

$$\sum_h \mu_{h1}^2 = \frac{\left(\frac{c_0}{g_0}\right)^2 + \left(\frac{c_2}{g_2}\right)^2}{\left(\frac{c_0}{g_0}\right)^2 + \left(\frac{c_2}{g_2}\right)^2 - \left(\frac{c_1}{g_1}\right)^2} M_Z^2 \tag{26}$$

In the following we will see that the class of models defined by the last two relations, which simulates $\theta_M = 0$, gives exactly the standard coupling for the Z current.

After having deduced the neutral vector bosons' eigenfunctions, we show now, how it's possible to generalyze the Georgi-Weinberg theorem to the scale of the Z exchange, that is LEP 1 physics.

4 The neutral current interaction

Let's now consider the neutral current interaction Hamiltonian

$$\mathcal{H}_{NC} = \frac{1}{2}\sum_{\alpha\beta}(\bar\psi\gamma^\mu t_\alpha\psi)(\bar\psi\gamma_\mu t_\beta\psi)\, g_\alpha g_\beta\, \Delta_{\alpha\beta} \tag{27}$$

where as usually, g_α is the coupling constant of the canonically normalized neutral gauge field A_α coupled to the T_α current whose representation on the fermions we call t_α and

$$\Delta_{\alpha\beta} = A_\alpha{}^\mu\, A_{\beta_\mu} \tag{28}$$

is the "propagator" of the neutral gauge bosons. The U matrix allows us to transform the gauge eigenstates A_α into the physical ones Z_i, so that, by means of completeness we can specify in eq. (28) $A_\alpha = \sum_i u_{\alpha i} Z_i$

$$\Delta_{\alpha\beta} = \sum_{ij} u_{\alpha i} u_{\beta j}\; Z_i{}^\mu Z_{j_\mu} \quad . \tag{29}$$

At the tree level the propagator of the "physical" vector bosons, can be written

$$\langle Z_i^\mu, Z_{j\mu}\rangle = \frac{\delta_{ij}}{M_i^2 - q^2 + i\epsilon} \tag{30}$$

where M_i is the physical mass of the neutral boson Z_i. Substituting this term into eq. (29) and summing over the j index, we get

$$\Delta_{\alpha\beta} = \sum_i \frac{1}{M_i^2 - q^2 + i\epsilon}\, u_{\alpha i} u_{\beta i} \quad . \tag{31}$$

When the q^2 is off of a resonance scale, one can neglect the imaginary ϵ term into the denominator, but considering the case of $q^2 = M_i^2$ for a fixed value of i, one need this imaginary part to define eq. (31). The main observation is that the propagator $\Delta_{\alpha\beta}$ as obtained in eq. (31), is simply related, to the squared mass matrix μ^2, in the following way

$$\Delta_{\alpha\beta} = (\mu^2 - q^2 I)^{-1}{}_{\alpha\beta} \tag{32}$$

in fact if $u_{\alpha i}$ is eigenvector of the mass matrix with eigenvalue M_i^2, also it will be eigenvector of the matrix $(\mu^2 - q^2 I)$ with eigenvalue $M_i^2 - q^2$; the inverse of this eigenvector equation brings, after a little algebra to the expression

$$(\mu^2 - q^2 I)^{-1}_{\alpha\beta} = \sum_t \frac{1}{M_t^2 - q^2} u_{\alpha t} u_{\beta t} \tag{33}$$

which proves the above relation (32).

Inserting eq. (31) into the Hamiltonian eq. (27), we can derive the coupling constant associated with the fermion-antifermion vertex relative to the exchange of a vector boson of mass M_t, as a function of the mass eigenvectors:

$$\sum_\alpha g_\alpha t_\alpha u_{\alpha t} \quad . \tag{34}$$

When the interaction scale q^2 coincides with the mass squared of a neutral vector boson Z_t, its exchange, expressed in the propagator $\langle Z_t, Z_t \rangle$, will dominate the whole neutral current Hamiltonian. By virtue of eq. (31) one can then distinguish two terms in eq. (27) coming respectively from the contributions of the dominant Z_t exchange and of the other vector bosons.

$$\begin{aligned} \mathcal{L}_{NC} &= \frac{1}{2}\sum_{\alpha\beta} (\bar\psi\gamma^\mu t_\alpha \psi)(\bar\psi\gamma_\mu t_\beta\psi)\, g_\alpha g_\beta\, \Delta^R_{\alpha\beta} \\ &+ \frac{1}{2}\sum_{\alpha\beta} (\bar\psi\gamma^\mu t_\alpha \psi)(\bar\psi\gamma_\mu t_\beta\psi)\, g_\alpha g_\beta\, u_{\alpha t} u_{\beta t}\, \langle Z_t, Z_t \rangle \end{aligned} \tag{35}$$

with $\Delta^R_{\alpha\beta} = \sum_{i\neq t} \frac{1}{M_i^2 - q^2} u_{\alpha i} u_{\beta i}$.

The second term of eq. (35) has its pole at the physical mass of the resonant vector boson. It can be easily shown that at $q^2 = 0$ ($Z_t = \gamma$) the couplings of the zero mass boson with the fermionic current are, as expected, exactly electromagnetic. In the case in which the scale of the interaction corresponds to a non-vanishing eigenvalue M_t^2, one has to check the couplings with the proper eigenfunction $\vec{u}_t$; in the S.M. limit, as like as in $\phi = 0$ canonical models, we find the expected standard fermion-Z_0 coupling.

Considering now the first term of eq. (35), which is relative to the exchange of the off scale neutral vector bosons, it is possible to generalize the relation already got in Ref. [3] eq. (15) to the case $q^2 = M_t^2$. In fact, observing that $\Delta^R_{\alpha\beta} = [\mu^2 - q^2 I]^{-1}_R$ is the inverse of the $i, j \neq t$ submatrix $[\mu^2 - q^2 I]_{ij}$, and defining the quantity

$$R^t{}_{\alpha\beta} = \sum_{ij\neq t} \zeta_{\alpha i}\zeta_{\beta i} [\mu^2 - q^2 I]^{-1}_{R\ ij} \tag{36}$$

where $\zeta_{\alpha i} = \delta_{\alpha i} - u_{\alpha t} u_{it}$, we have the relation $\Delta^R_{\alpha\beta} = R^t{}_{\alpha\beta}$.

To demonstrate it one can use the same arguments and note that $R^t{}_{\alpha\beta}$ defined in eq. (36), satisfies, at the resonance scale, the two properties

$$[(\mu^2 - q^2 I)R^t]_{\alpha\beta} = \sum_{i \neq t} u_{\alpha i} u_{\beta i} = \zeta_{\alpha\beta} \qquad (a)$$

and

$$\sum_\alpha u_{\alpha t} R^t{}_{\alpha\beta} = \sum_\alpha R^t{}_{\alpha\beta} u_{\beta t} = 0 \qquad (b)$$

which uniquely define also the $\Delta^R_{\alpha\beta}$ inverse matrix.

By means of eq. (36) one can rewrite the first part ($\neq Z_t$) of the neutral current Hamiltonian as

$$\mathcal{H}_{NC \neq Z_t} = \frac{1}{2} \sum_{\alpha\beta} (\bar\psi \gamma^\mu t_\alpha \psi)(\bar\psi \gamma_\mu t_\beta \psi) \sum_{ij \neq t} \zeta_{\alpha i} \zeta_{\beta j} \, [\mu^2 - q^2 I]^{-1}_{R\ ij} \qquad (37)$$

which gives

$$\mathcal{H}_{NC \neq Z_t} = \frac{1}{2} \sum_{ij \neq t} (\bar\psi \gamma^\mu n_i \psi)(\bar\psi \gamma_\mu n_j \psi) \, [\mu^2 - q^2 I]^{-1}_{R\ ij} \qquad (38)$$

where the "fictitious" currents n_i, are given by the couplings

$$n_i = \sum_\alpha g_\alpha (\delta_{\alpha i} - u_{\alpha t} u_{it}) \, t_\alpha \qquad (39)$$

Specifying them by means of the eigenfunctions $\vec{u_t}$, one can see that, when the t^{th} vector boson is the photon (that is $q^2 = 0$), the second term in eq. (39) comes out to be proportional to the electric charge and the coupling n_i reproduces the general definition already given in Ref. [3]. Unfortunately, when the $q^2 = M_t^2$ of the interaction is different from zero, the couplings do not generally reproduce any more the expected standard-generalized behaviour of the kind $n_i \sim T_i - \sin^2\theta_i \, Q$, because the not-vanishing Z_t mass prevents the second term from being proportional to the electric charge generator. Note that there is no reason a-priori for which the "fictitious" fermion couplings n_i which come from the mass matrix and the fermions gauge currents, should be equal to the physical ones even at a well defined interaction scale, and therefore, the above generalization of the standard couplings at $q^2 = 0$ should be considered as a dynamical effect due to the fact that the photon has exactly a vanishing mass. However, checking with this method the Standard Model limit, both at $q^2 = 0$ and at $q^2 = M_Z^2$, one can easily verify the equivalence of the non-resonant currents n_L and n_Y, to the standard physical couplings of J_Z and J_{em} respectively.

In general, for those models in which all the off-diagonal terms of the Δ^R propagation matrix vanish, the non resonant Hamiltonian results in the sum of non-interacting currents

$$\mathcal{H}_{NR} = \sum_{i \neq l} n_i \left(\mu^2 - q^2 I\right)_{ii}^{-1} n_i \tag{40}$$

corresponding to a "virtual" exchange of intermediate particles whose masses and couplings do not necessary coincide with the ones of the physical vector bosons. However this dynamical description can be found very useful in such models in which, for some peculiar reason, one can reproduce the standard couplings as in the case described by Georgi and Weinberg, or such that, under very common hypotheses on the fermions' representation, some of them may vanish; in these cases, the suppression of the currents could cause the cancellation of any extra gauge effect.

In the following we will seek for such models in which this situation can occur according to given properties of the fermions involved in the interaction.

5 The hiding Z′

In the previous chapter we have seen how, in the four fermion interactions at the physical mass of each neutral intermediate gauge boson, the Hamiltonian can be divided into the resonant and not-resonant terms. The exchange of the neutral vector bosons occurs by means of the resonant coupling of eq. (34), and the virtual currents n_i of eq. (39).

When the photon exchange is dominant, the virtual couplings are shifted from the gauge ones of a term proportional to the $\gamma - Z_{0i}$ mixing angle $\sin\theta_i = e\frac{c_i}{g_i}$, consistently with the standard physical Z coupling. For the case in which the 0-reduced squared mass matrix is diagonal, that is $\sum_i v_i^2 y_{ki} y_{hi} = 0 \quad (h \neq k; h, k \neq 0)$, all them decouple together, so that, for G_2-neutral neutrino interactions, any extra gauge dynamical effect will be absent, according to how has been already observed, limitately to the case of canonical models, in a previous paper [9]. We like here interpreting this peculiar massive boson decoupling as due to an orthogonality condition in the spontaneous symmetry breaking mechanism among the neutral generators of the two main groups, which enter into the definition of the electric charge; it requires, in fact, the weight Higgs charge vectors $(v_1 y_1, v_2 y_2., ..., v_n y_n)_i$, associated with the breaking of each Y_i neutral generator, to be orthogonal each other.

In the rank three models analyzed in Sec. 2, we have already discussed about the possibilities of cancelling the extra-gauge effects in $q^2 = 0$ neutrino interactions; the first of them, requiring

$$\sum_i v_i^2 t_{Li} t_{Ri} = 0 \quad , \tag{41}$$

obviously obeys to the diagonality of 0-reduced mass matrix hypothesis. As over already observed, in canonical models (where $SU(2)_R \sim U(1)_\chi$ and $c_\chi = 0$) this is equivalent to ask

$$\sum_i v_i^2 y_i \chi_i = 0 \quad . \tag{42}$$

and coincide with the request of vanishing $Z - Z'$ mixing angle. Note that both these conditions orthogonalize the extra gauge symmetry respectively to the T_L and Y reduced Standard Model; this cause the complete decoupling of the S.M. from the $U(1)_\chi$ symmetry and allows the complete cancellation of the $U(1)_\chi$ physical effects, even in presence of Q-charged (χ-neutral) fermions, because of the annihilation of the second term (proportional to c_2) in the extra coupling.

The previous considerations refer to fermion interactions at $q^2 = 0$. Let's now try to extend these results to LEP 1 physics where pairs of e^+e^- interact by means of neutral currents at the scale of the Z exchange. In rank three models with $\mu_{02}^2 = 0$, the non-resonant neutral current Hamiltonian, because of the mass-matrix diagonalization, turns out to be the sum of two independent terms

$$\mathcal{H}_{NC\neq Z} = \frac{1}{2}(\bar{\psi}\gamma^\mu n_0 \psi)(\bar{\psi}\gamma_\mu n_0 \psi)(\mu^2 - \hat{q}^2)^{-1}_{00} + \frac{1}{2}(\bar{\psi}\gamma^\mu n_2 \psi)(\bar{\psi}\gamma_\mu n_2 \psi)(\mu^2 - \hat{q}^2)^{-1}_{22} \tag{43}$$

with $\hat{q}^2 = M_Z^2$; the virtual couplings $n_0 = g_0 T_0 - u_{0Z}\, n_1$ and $n_2 = g_2 T_2 - u_{2Z}\, n_1$ essentially derive from a modification of the T_Y and T_χ generators caused by the n_1 coupling of the resonant vector boson

$$n_1 = g_0\, u_{0Z}\, T_0 + g_1\, u_{1Z}\, T_1 + g_2\, u_{2Z}\, T_2 \quad . \tag{44}$$

These currents n_i do not generally bring to standard couplings, but as we will see in the following, the standard behaviour is reproduced in canonical models and can be reached in the L-R one by imposing the asymptotic condition $g_R \to \infty$. May be interesting to look for real models which can admit the standard Z_0 coupling for the resonant current, even without necessary imposing an infinite Z' mass. In E_6 derived models, for example, the $\vec{Z}_{\phi=0}$ eigenvector in eq. (24), gives rise, at any interaction scale, to the Z current:

$$n_Z = \frac{e}{\sqrt{\left(\frac{c_Y}{g_Y}\right)^2 + \left(\frac{c_R}{g_R}\right)^2}} \frac{c_L}{g_L} \left[-c_Y T_Y + \left(\frac{c_L}{g_L}\right)^2 \left(\left(\frac{c_Y}{g_Y}\right)^2 + \left(\frac{c_R}{g_R}\right)^2 \right) c_L T_L - c_R T_R \right] \tag{45}$$

which, by adding and subtracting $c_L T_L$, brings exactly to the standard Z_0 coupling

$$n_Z = \frac{1}{\cos\theta} \frac{g_L}{c_L} \left(c_L T_L - e^2 \frac{{c_L}^2}{{g_L}^2} Q \right) \quad ; \tag{46}$$

For the virtual currents, however, we do not generally find any coupling corresponding to the photon exchange.

In rank 4 models, the hypothesis of a large Z'' mass, due, for example, to a large $\mu^2_{\psi\psi}$ value, let us to neglect its influence in the low energy behaviour and, in particular, the low mass-matrix eigenvalues will result unchanged. In canonical decoupled models, by eqs. (41) and (42), because of the ψ reduced mass-matrix block diagonalization, one can get rational values of the bosons' masses and compute, by means of eq. (13), the $\vec{Z}$ eigenvector $e\,(\frac{c_L}{g_L}\,;\,-\frac{c_Y}{g_Y}\,;\,0\,;0\,)$ and with it, the respective real and virtual couplings. We find, together with the Standard resonant Z current, the fictitious current n_0

$$n_0 = n_\gamma = g_Y T_Y - u_{YZ}(g_Y T_Y u_{0Z} + g_L T_L u_{LZ}) = e^2 Q \frac{c_Y}{g_Y} \tag{47}$$

which, once simplified with the mass matrix contribution $1/M_Z^2 \cos^2\theta_W$, gives the exact electromagmetic current; on the other side, the extra gauge current n_2

$$n_2 = g_\chi T_\chi \tag{48}$$

results uniquelly associated with the Z' exchange.

Trying to connect the above considerations with the fact that LEP1 experiments have given no evidence (up to now) about the existence of any extra gauge symmetry effects, we guess an ambiguous situation for which, in the fermion representation, the e_L and the e_R get opposite χ-charges: in non polarized beams, a destructive interference between opposite amplitudes could cancel any χ symmetry signal. Taking into account a Z' coming from E_6 , resulting from the mixing of the ψ and χ symmetries:

$$\tilde{Z}' = Z'(\theta) = Z_\psi \cos\vartheta + Z_\chi \sin\vartheta \ , \tag{49}$$

the above described situation, then asks $\sin\vartheta = -\sqrt{10}/4$ which corresponds to the so called "Inert" vector boson Z_I[6].

An other question arise whether such models with a vanishing $\mu^2_{Y\chi}$ make sense at all. The problem shifts now to the seek for the ideal Higgses representation which could carry out the asked annihilation of the non-diagonal χ mass matrix terms. Within the context of the 27 representation of E_6, one can find five neutral candidates for the symmetry breaking [5], which trasform with respect to the canonical breaking in $SU(2)_L \otimes U(1)_{Y_W} \otimes U(1)_{\tilde{Q}}$, according to the scheme:

$$\begin{aligned}
h_{\nu_E} &\equiv (\ I_L = +\tfrac{1}{2}\ ;\ I_{Y_W} = -1\ ;\ \tilde{Q} = +\tfrac{1}{\sqrt{6}}\cos\vartheta - \tfrac{1}{\sqrt{10}}\sin\vartheta\ ) \\
h_{\bar{\nu}_E} &\equiv (\ I_L = -\tfrac{1}{2}\ ;\ I_{Y_W} = +1\ ;\ \tilde{Q} = +\tfrac{1}{\sqrt{6}}\cos\vartheta + \tfrac{1}{\sqrt{10}}\sin\vartheta\ ) \\
h_{\nu_e} &\equiv (\ I_L = +\tfrac{1}{2}\ ;\ I_{Y_W} = -1\ ;\ \tilde{Q} = -\tfrac{1}{2\sqrt{6}}\cos\vartheta + \tfrac{3}{2\sqrt{10}}\sin\vartheta\ ) \\
h_{N} &\equiv (\ I_L = 0\ ;\ I_{Y_W} = 0\ ;\ \tilde{Q} = -\tfrac{1}{2\sqrt{6}}\cos\vartheta - \tfrac{5}{2\sqrt{10}}\sin\vartheta\ ) \\
h_{n} &\equiv (\ I_L = 0\ ;\ I_{Y_W} = 0\ ;\ \tilde{Q} = -\tfrac{2}{\sqrt{6}}\cos\vartheta\ )
\end{aligned}$$

Theoretical argumentations, let us to suppose that the first three of them, h_{ν_E}, $h_{\bar{\nu}_E}$ and h_ν, should be associated with "small" *VEVs* "m_i", while the two I_L singlets, $h_{\bar{N}}$ and h_n, contribute to the breaking with "large" *VEVs* M_i. Hypothesizing, for simplicity, equal "small" *VEVs* "m" to the first class of Higgses, the vanishing condition for

$$I_{\nu_E}\bar{Q}_{\nu_E} + I_{\bar{\nu}_E}\bar{Q}_{\bar{\nu}_E} + I_{\nu_e}\bar{Q}_{\nu_e} \ ,$$

again requires the value $\sin\vartheta = -\sqrt{10}/4$. Moreover, in this representation, the vanishing standard charges of the Higges associated with large *VEV* "M", may also justify a large difference between the bosons' masses.

In conclusion, we have showed that, an orthogonality condition on the Higgs breaking mechanism can cause the dynamical decoupling of the virtual currents in the $q^2 = M_Z^2$ fermion interactions. At this scale, we find that the Inert canonical model is able to exactly reproduce the standard couplings and to justify the absence of the extra gauge effects in non polarized electrons beams. In this sense, we hope that next experiments with polarized beams at LEP physics could finally feel the hoped signal of the extra Z_I current. If this will not be the case we cannot find, in the E_6 context, a scenario capable of vanishing both the e_L and the e_R $\bar{Q}$-charges, and therefore, a very large Z' mass is required, otherwise, may be others, unknown mechanisms hiding the Z' manifestation are playing the game.

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