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MEASUREMENT OF THE TOP MASS NEAR THE THRESHOLD AT AN e^+e^- COLLIDER

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Measurement of the top mass near the threshold at an $e+e-$ collider

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Abstract

In this note a method for measuring the top mass at a linear e^+e^- collider near the $t\bar{t}$ production threshold is presented using properties of the $t\bar{t}$ final states. A resolution on the top mass of $400 MeV/c^2$ with an integrated luminosity of $1 fb^{-1}$ is obtained.

Contents

1 Introduction

One goal of the research at the hadron colliders LHC and SSC is to discover the top quark and to measure its mass within several GeV/c^2. But for precision measurements an electron–positron collider is a good tool. Therefore, there is interest in building an electron–positron linear collider with a center of mass energy of 500 GeV and integrated luminosity per year of $1fb^{-1}$. In this note, a method for measuring the top mass is described. One looks at the leptonic decay modes of the on shell W boson, coming from the top decay. Systematic errors from radiation and resolution of the detector are discussed.

One way to measure the top mass at such a linear collider is presented in [1]. One does an energy scan around the threshold. The toponium resonances are at an energy which is twice the top mass minus the binding energy. Part of the binding energy, which depends on α_s and Higgs contributions, can be measured using the peak cross section [2][1]. Another way to measure the top mass is to analyse the kinematics of the top decay products. The top mass which one obtains is independent of the binding energy of the toponium if no $t\bar{t}$ bound states are produced. This is the case if the beam energy is beyond the resonances, several GeV above the threshold.

If the top mass is larger than the W mass the top will dominantly decay into b and W. Running several GeV above the threshold the invariant mass of the b–quark and the W is equal to the top mass. The W energy E_W is given by (neglecting for simplicity the b mass)

$$E_W^* = \frac{m_t^2 + m_W^2}{2m_t} \qquad \beta_W^* = \frac{m_t^2 - m_W^2}{m_t^2 + m_W^2}$$

$$E_W = \gamma_t E_W^* (1 + \beta_W^* \beta_t \cos\theta^*) \quad ,$$

where $*$ stands for the rest frame of the top. θ^* is the angle between the direction of flight of the W and the boost of the top in the top rest frame. The lower edge of the W spectrum is a decreasing function of the top mass if β_t is greater than β_W^*, and a increasing one if β_t is almost zero. The upper edge is a decreasing function of the top mass for any velocity. If the distribution in $\cos\theta^*$ is flat, the mean value of the spectrum is given by $\gamma_t E_W^*$. Of course this distribution depends on the polarisation of the top. Similar formulae hold for the b energy. By measuring the b or W energy or the invariant mass of the b–W system the top mass can be estimated. This procedure can be applied above the threshold so that the measurement of the top mass is independent of the binding energy of the toponium.

At high energy, the event may be divided into two hemispheres and a jet algorithm applied. Assigning the b and the W to the jets, the top mass can be measured by looking at the invariant mass of the b–W system. Near the threshold, the use of a jet algorithm seems to be difficult because of the low velocity of t and $\bar{t}$. One way out of this problem is to use the semileptonic decays of the W boson. This is described below.

If one W decays semileptonically and the other one hadronically, then the energy of one W is given by the sum of the lepton and the neutrino energy. If there is no energy loss in the beam pipe etc., and if there are no additional neutrinos from b decay, the missing four–momentum of the event is the neutrino four–momentum. Adding the lepton momentum to it we obtain the four–momentum of the W boson and therefore the top mass. The justification of our assumption, no energy loss and no additional neutrinos, can be tested, since the missing mass has to be zero and the W mass is known. If we select events having a lepton with certain properties and low missing mass, the W energy is well measured.

Let us now study this idea in more detail by using the Monte Carlo method.

For this study the generator PYTHIA was used [3]. If it is not explicitly quoted, the top mass was chosen to be 140GeV and the beam energy 141GeV. A sample of 10000 $t\bar{t}$ events, including initial state radiation was generated. All particles within a cone of $2 \times 15°$ around the beam are ignored to simulate limited detector acceptance.

The cross sections at a beam energy of 141GeV is 1.3pb for top masses between $138GeV/c^2$ and $140GeV/c^2$ [2]. The $q\bar{q}$, WW and ZZ crossections are 8.2pb, 10.5fb and 1.0fb [4]. These events give the main contribution to the background.

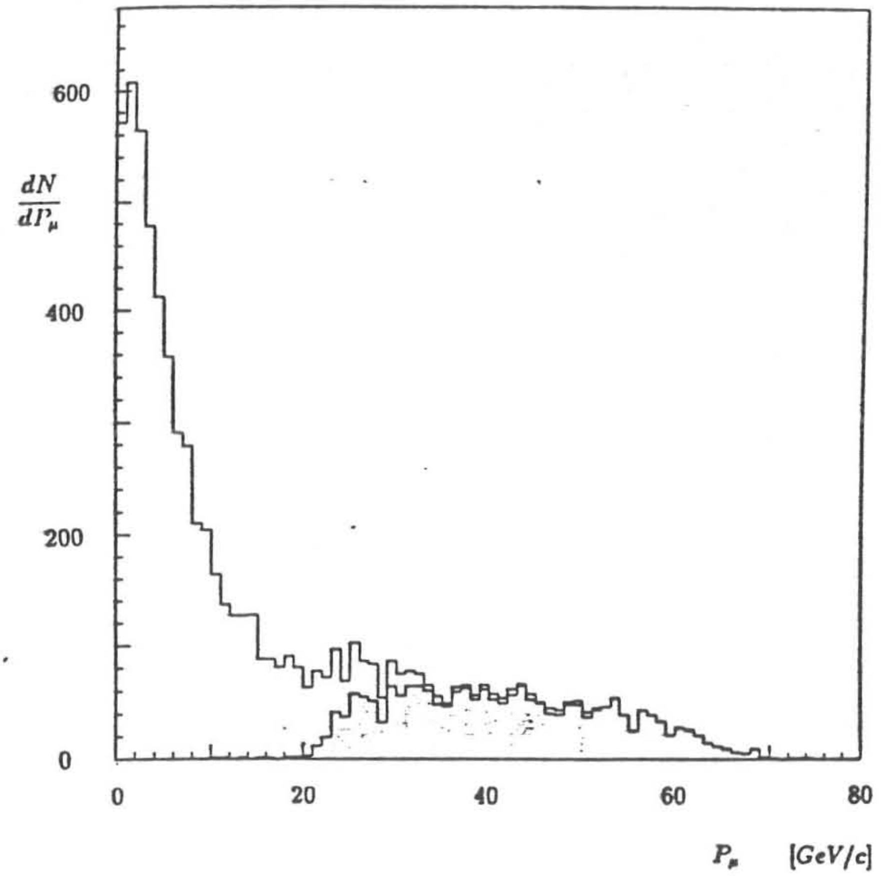

Figure 1: The muon momentum spectrum. The contribution of muons from semileptonic W decay is shadowed.

2 Selection of semileptonic decay modes

In this section the selection of the top decay mode

$$ee \rightarrow t\bar{t} \qquad t \rightarrow bW^+ \qquad \bar{t} \rightarrow \bar{b}W^-$$

$$W^+ \rightarrow \mu^+\nu_\mu \qquad W^- \rightarrow had.$$

or the charge–conjugated state is studied.

As can be seen on figure 1, muons in $t\bar{t}$ events coming from the leptonic decay of the W boson show a momentum which is above 22GeV/c, whereas the muons from other sources are peaked at 2GeV/c. So we ask for a muon with a momentum above 22GeV/c. The identification of the muon should not be a problem, because as it will be shown later, they are well isolated. The efficiency of this cut, normalised to the $t\bar{t}$ events is 20.3% and 91.0% normalised to the desired decay mode. The $q\bar{q}$ background is reduced by a factor of fourteen.

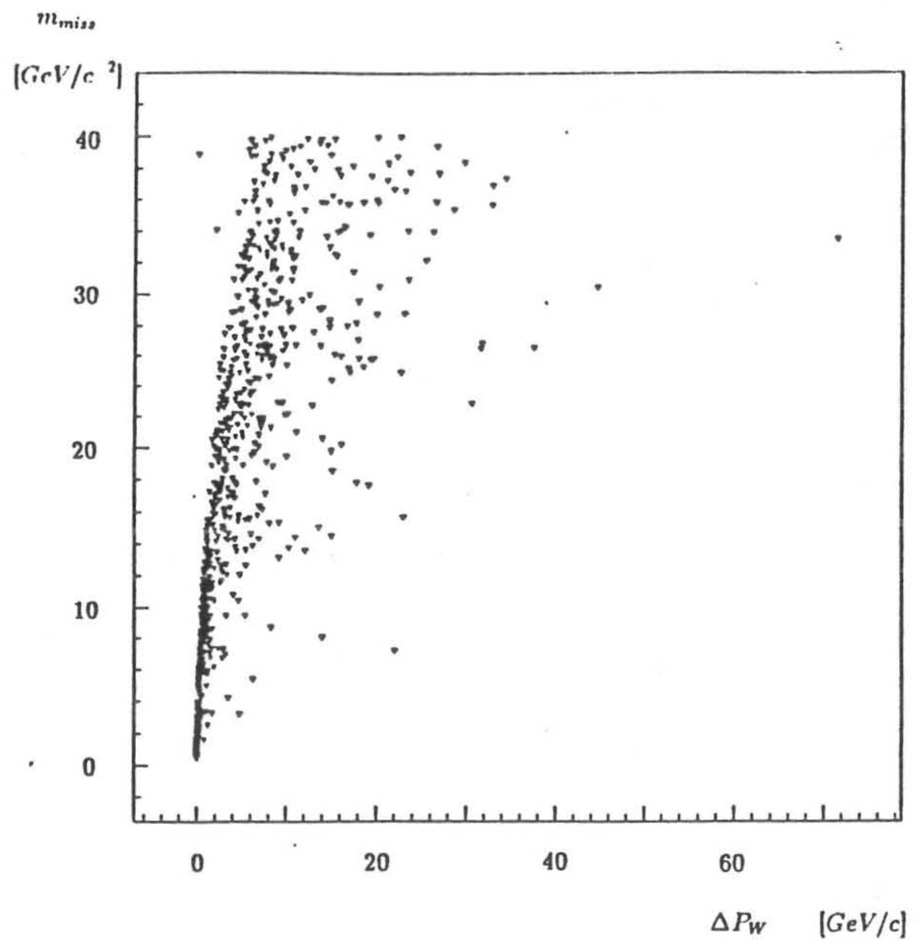

Figure 2: Scatter plot of missing mass m_{miss} versus resolution ΔP_W of the W momentum. The resolution is given by the difference between the reconstructed and the generated W momentum.

If there are two leptons (electron or muon) with momenta above 22GeV/c then the event is rejected to avoid ambiguities in assigning the right lepton to the W boson. The τ lepton is ignored for that cut, because it will be rejected by the cut in the missing mass of the event as described below, since it has an additional neutrino. The efficiency of that cut, normalised to the semileptonic decay mode, is 96.6%. This cut also reduces the $q\bar{q}$ background by a factor of three.

Figure 2 shows the missing mass of the event as a function of the reconstructed W momentum. minus the generated W momentum. The W momentum was reconstructed by taking the sum of the missing momentum and the muon momentum and excluding the particles in the beam pipe as described above. As one can see, the reconstructed W momentum tends to be larger then the generated one, because one loses a fraction of the total momentum due to the beam pipe. If one wants to analyze the full W momentum spectrum, a cut in the missing mass between 5 and $20GeV/c^2$ is needed to guarantee a small error on the W momentum (see next section). For the moment we apply a cut at $20GeV/c^2$ (see section 5). The efficiency of this cut is 28.9% normalised to the $t\bar{t}$ events which are left after the cut on the muon momentum. The efficiency normalised to the semileptonic decay mode is 32.7%.

To guarantee good muon identification and to further reduce muons coming from b decays, we ask for an isolated muon. A muon is to be considered as isolated if there is no charged particle within 10°.

After these cuts we are left with 3.8% of the $t\bar{t}$ events. All of them have a semileptonic decay mode so there is no background from the $t\bar{t}$ events. The efficiency for that decay mode is 25.0%, which is mainly given by the cut in the missing mass. Two out of 10000 $q\bar{q}$ events and 6.5% of the WW events are left.

To get a reduction of the $q\bar{q}$ background, the reconstructed W mass is restricted to be between 70 and $90GeV/c^2$.

A reduction of the WW background can be obtained by doing a cut in the total invariant mass excluding the highest energetic lepton. For WW events, where one W decays leptonically and the other one hadronically, this mass is equal to the W mass, whereas for $t\bar{t}$ events it is equal to the invariant mass of one b and one W. So we reject the event if this mass is lower than $170GeV/c^2$ (see figure 3). The WW background is reduced by a factor of 280. No $t\bar{t}$ event was rejected.

It is worth pointing out that the W momentum is of the order of 110GeV/c for WW events with a small tail down to 50GeV/c, so most of the events are outside the kinematical range of the $t\bar{t}$ events. On the other hand, the events which are left after the cut in the hadronic mass of the event have to belong to that tail.

With this procedure one obtains an efficiency of $(3.4 \pm 0.5)\%$ normalised to $t\bar{t}$ events and $(22.7 \pm 1.2)\%$ normalised to the semileptonic decay mode. The error contains only the statistical error on the Monte Carlo sample. Less than $2 \cdot 10^{-5}$ of the $q\bar{q}$ events are left and 10 of 50000 WW events survive. Taking the cross sections into account, the signal to $q\bar{q}$ background ratio is greater than 272. For WW events that ratio is 21 ± 7. The error is statistical only. The ZZ background can be neglected.

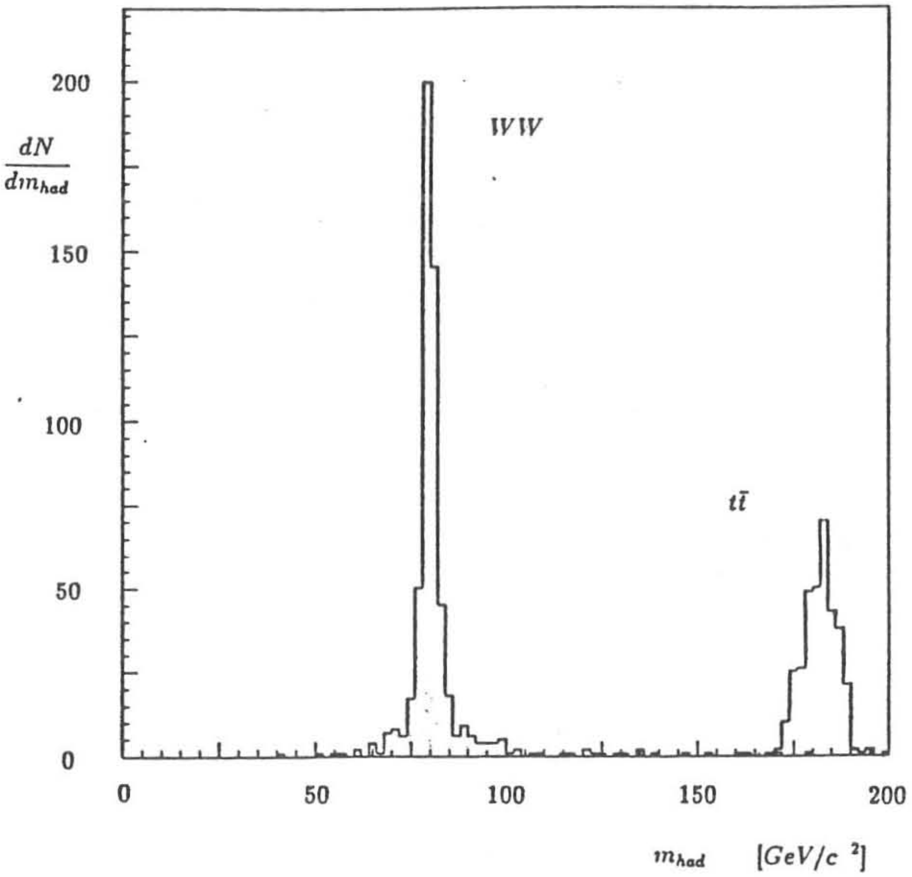

Figure 3: Distribution of the total hadronic mass. The WW background is shadowed.

3 Measurement of the top mass

Figure 4 shows the lower and upper bound of the W momentum at a beam energy of 141GeV as a function of the top mass using the formulae above. The W mass is varied within its partial width. Using the momentum spectrum instead of the energy spectrum of the W, a lower dependence of the reconstructed top mass from the W mass is obtained. To express the top mass as a function of that momentum an inversion of a cubic equation is needed. This requires care because, due to finite momentum resolution, the equation must also be converted outside the kinematical limits. To avoid this, an approximation is used,

$$\frac{P_W}{GeV/c} \approx 48 + a \cdot \sqrt{141 - \frac{m_t}{GeV/c^2}} \quad ,$$

with a=-12.0 (+10.4) for the lower (upper) bound of the spectrum. As can be seen, the mean value of the spectrum is roughly 48GeV/c and independent from the top mass, so one is only interested in the width of the spectrum.

Figure 5 shows the W momentum for different top masses. The dependence of the width of the W momentum spectrum from the top mass was studied by generating 10,000 events with top masses beetween $140GeV/c^2$ and $139GeV/c^2$ in steps of $250MeV/c^2$ and one sample at $138GeV/c^2$. It can be seen that there is a small tail at the lower and upper edge of the spectrum. The endpoint can be obtained within $200MeV/c^2$ by fitting a straight line at the lower and upper bounds of the spectrum. To check, a physicist, not knowing what I was doing, was asked to read the endpoint by eye. We also got the top mass within $200MeV/c^2$. The edges of the spectrum can be also obtained in the following way. First 1% of the events at the lower and upper edge are rejected. Then the event with the lowest (highest) W momentum is taken as the boundary of the spectrum. The top mass could be well measured. The variation of the number of events which are rejected at the edges from 1% to 2% does not change the result. This tells us that the tail can be easily handled.

If the spectrum is flat, the r.m.s. is equal to the width divided by the square root of 12. Using the r.m.s., the top mass could be obtained

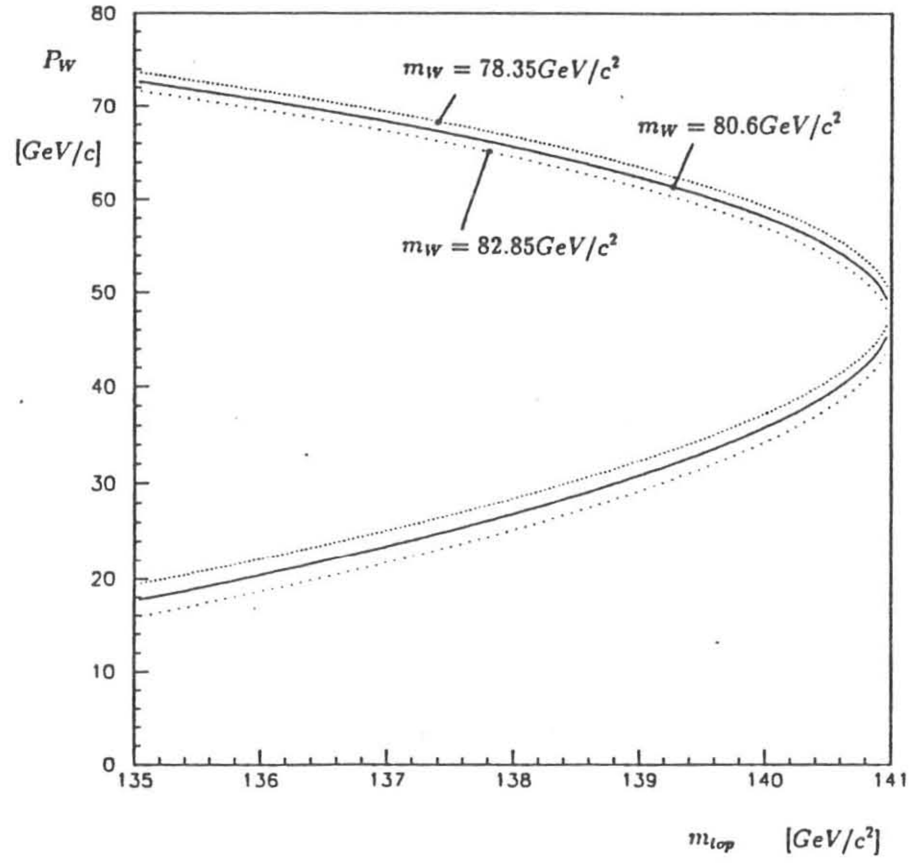

Figure 4: The kinematical boundary of the W momentum spectrum as a function of the top mass for different W masses.

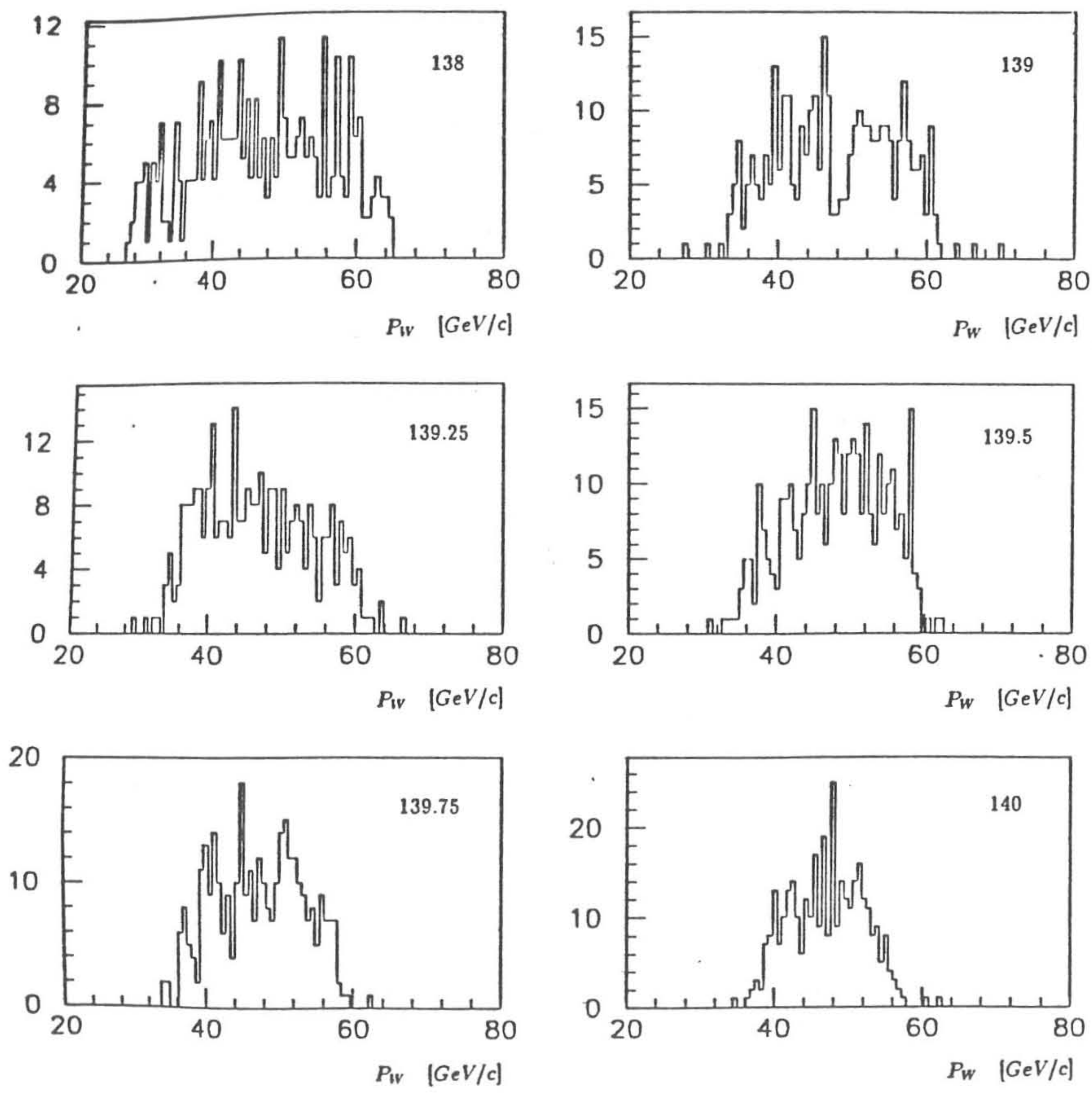

Figure 5: The W momentum spectrum for top masses $138 GeV/c^2$, $139 GeV/c^2$, $139.25 GeV/c^2$, $139.5 GeV/c^2$, $139.75 GeV/c^2$ and $140 GeV/c^2$.

within $200 MeV/c^2$, but with a shift of $500 MeV/c^2$, since the spectrum is not completly flat.

We finally conclude that the statistical error on the top mass is $200 MeV/c^2$ for 10000 $t\bar{t}$ events, which corresponds to an integrated luminosity of $7.7 fb^{-1}$. This error is in agreement with a calculation by hand of the error on the r.m.s. of a flat distribution. Since we asked for well isolated muons, electrons might also be used with the same efficiency to reduce the statistical error.

There are two points worth noting: first, for this analysis the b mass was not ignored because it shifts the top mass by $100 MeV/c^2$. The error coming from the width of the W boson is negligible. Second, the method discribed here can also be used to analyse $t\bar{t}$ bound states. One has in semileptonic decays an easy method to measure the W momentum. If in addition a vertex detector is used for reconstructing b mesons one has a complete information about the kinematic of the $t\bar{t}$ decay products.

4 Error on Energy flow

The method described above relies on a good total energy–flow measurement. The contributions to the error on the total four momentum are the momentum and energy resolution of the tracking device and of the calorimeter, double counting and energy loss due to cracks and overlap between charged and neutral clusters in the calorimeter. This affects the efficiency and the signal to background ratio of the cut in missing mass at $20 GeV/c^2$. Furthermore it determines the error on the W energy. It is worth noting, that an energy loss due to cracks increases the reconstructed W energy as well as the missing mass, which means that such an event will be rejected. The central question is how the efficiency changes due to errors in the energy flow and how well a good resolution in the W energy is guaranteed by doing the missing mass cut at $20 GeV/c^2$.

To measure the total four momentum of the event, the tracking device is used for charged particles. Doing so, we run into problems if there is neutral energy deposited near or inside the shower of the charged particle, because the neutral particle might not be found. This was tested by rejecting all neutrals which are within a cone of $\pm 1^\circ(2^\circ)$ around a charged particle. The efficiency decreases by a factor 0.60 (0.25). If one rejects only neutrals within $\pm 1^\circ$ and having an energy which is less then 30% of the charged particle, then the efficiency decreases only by a factor of 0.94. The change of the efficiency due to an overlapping cluster is small, because only the loss of high energy neutrals contributes to the missing mass, but high energy neutrals are well measured.

After the cut in the muon momentum, the polar angle distribution of the missing momentum is flat. This indicates that there is no need for a cut in that angle to reduce the contribution of the beam pipe.

The error on the W energy due to the momentum resolution of the tracking device and due to the energy resolution of the calorimeters was calculated by adding all errors in quadrature. We obtain a resolution of 2GeV/c on the W momentum, assuming

$$\frac{\delta p}{p^2} = 10^{-3} \qquad \frac{\delta E}{E} = 8\%/\sqrt{E} + 2\% \qquad \frac{\delta E}{E} = 60\%/\sqrt{E} + 2\%$$

for the tracking device and the electromagnetic and hadron calorimeter respectively in units of GeV. This corresponds to an uncertainty of around $500MeV/c^2$ on the top mass for an event which is just at the lower or upper bound of the W momentum spectrum This error is statistical. For a luminosity of $1fb^{-1}$ one obtains an error on the top mass of $300MeV/c^2$.

The method described here relies strongly on the availability of the energy flow which can be well obtained by constructing a homogeneous detector with good spatial resolution of the calorimeter. A more detailed simulation of the energy flow is needed to study various detector effects. This will be done in the following section.

5 Detector effects

A detector simulation program containing smearing of the momentum and energy measurement, spatial resolution of the calorimeters and identification of leptons was used. Two sets of parameters were used: a standard and an optimistic one. For the standard (optimistic) tracking device the following was assumed: a geometrical acceptance $\cos\theta$ of 0.9 (0.95), a momentum resolution of $\delta p/p^2 = 10^{-3}GeV/c^{-1}$ ($\delta p/p^2 = 2 \cdot 10^{-4}GeV/c^{-1}$), a track efficiency of 99.5% (99.9%) and an angular resolution of $2 \times 0.5mrad^2$ ($1 \times 0.2mrad^2$). The acceptance of the calorimeter is 0.985 (0.995). The energy resolution is given by

$$\delta E/E = \sqrt{a^2 \cdot (E/GeV)^{-1} + b^2} \quad ,$$

with a=8%(1%), b=2%(1%) for the electromagnetic calorimeter and a=60%(30%), b=2%(2%) for the hadron calorimeter. The granularity is $2^\circ \times 2^\circ (1^\circ \times 1^\circ)$ and $4^\circ \times 4^\circ (2^\circ \times 2^\circ)$ for the electromagnetic and hadron calorimeter. The spatial resolution of the electromagnetic calorimeter was assumed to be $2 \times 2mrad^2$ ($1 \times 1mrad^2$). The efficiency for muon and electron identification is 95% (99%). The fraction of hadrons identified as muons or electrons is 1% (0.5%) for each lepton species.

Energy deposits in the calorimeters are merged together if the entry point of the particles into the calorimeters is within the granularity. If one of the particles is a charged one then the cluster is called a charged cluster. Otherwise it is called a neutral cluster. Two charged clusters are not merged together, so a charged cluster contains only one charged particle and maybe some neutrals.

The total momentum of the event was calculated by summing up the momentum measured by the tracking device and the energy of neutral clusters in the calorimeters. For a charged electromagnetic cluster where the barycenter is more than 0.2° away from the entry point of the track into the calorimeter and the energy of the cluster is two sigma higher than the momentum, the deposited energy was used insted of the momentum.

The dependence of the efficiency on the assumed detector is mainly given by the efficiency of the cut in the missing mass of the event. A cut at $20GeV/c^2$ was chosen, which is roughly the expected resolution. The error on the missing mass is given by the r.m.s. of the variable reconstructed missing mass minus the total invariant mass of all neutrinos generated. For the standard detector one obtains $18.7GeV/c^2$. Using the standard design but with the optimistic geometrical acceptance, granularity and spatial resolution of the calorimeters, the r.m.s. is $14.2GeV/c^2$. For the standard (optimistic) detector design the efficiency of the missing mass cut for semileptonic decays is 22%

(32%). This should be compared with 32.7% obtained in the previous chapter. Using the parameter of the standard design but with the geometrical acceptance of the optimistic detector once obtains an efficiency of 25%. Taking also the optimistic granularity and spatial resolution of the calorimeter the efficiency is 31%. Using the electron and muon decay mode, the final efficiency for selecting $t\bar{t}$ events is for the four designs 4.2%, 7.0%, 5.0% and 6.5%.

The resolution on the W boson momentum is for the four different designs (given in the order as above) 4.5GeV/c, 2.9GeV/c, 4.3GeV/c and 4.0GeV/c, which corresponds for an event which is just at the lower or uper bound of the spectrum to an error on the top mass of $400MeV/c^2$, $260MeV/c^2$, $380MeV/c^2$ and $360MeV/c^2$. The overall statistical error on the top mass, including the statistical error coming from the detector was calculated by dividing the selected events in subsamples of 100 events. For the standard design with the optimistic geometrical acceptance, granularity and spatial resolution, the r.m.s. of the top masses obtained from the subsamples was $220MeV/c^2$ and each subsample corresponds to a luminosity of $1.2fb^{-1}$.

6 Beam radiation and bremsstrahlung

The formula which is used to calculate the top mass requires knowledge of the velocity of the top. Initial state radiation and beam radiation due to the collimators affect the precision on the top mass, as well the absolute scale of the beam. An uncertainty in the beam energy scale of 200MeV (500MeV) is equivalent to an error on the width of the W momentum spectrum of around 2.3GeV/c (7GeV/c), which gives a shift in the top mass of $200MeV/c^2$ ($500MeV/c^2$).

Photons which are produced by initial or beam radiation are lost in the beam pipe, so they give a contribution to the missing mass by

$$m_{mis}^2 \rightarrow m_{mis}^2 + 2E_\gamma E_{mis}(1-\cos\theta) \qquad ,$$

where E_γ is the photon energy, E_{mis} the missing energy if there were no radiation and θ is the angle between them. The distribution of E_{mis} is similar to the energy spectrum of the muon (see figure 1), so one may set $E_{mis} = 35GeV$ and $\cos\theta = 0$ for a short calculation. With these numbers the photon energy has to exceed 5.7GeV to obtain a contribution to the missing mass more than $20 GeV/c^2$. This tells that the missing mass variable cannot be used to avoid the errors on the W energy due to radiation.

Initial state radiation was studied by switching it on and off in the Monte Carlo generator. There was no change within the statistical error obtained. The generator which was used does not contain the $t\bar{t}$ resonances. Since these resonances have a radiative tail, they should be implemented.

The influence of beam radiation was done by using a program written by T. Barklow and installed at CERN by W Kozanecki [5]. The Desy–Darmstadt and the Palmer F design was used with the default value quoted by W Kozanecki [6]. Figure 6 shows the distribution of the electrons after beam radiation. Most of the electrons are peaked within 50MeV of the nominal beam energy. Below that peak there is an almost flat tail. Roughly 25% of the events, normalised to the peak, with a photon energy above 50MeV are within 1GeV.

The width $P_{max} - P_{min}$ of the W momentum spectrum can be well approximated by

$$\frac{P_{max} - P_{min}}{GeV/c} \approx 22 \cdot \sqrt{\frac{E_{beam}}{GeV} - 140} \quad .$$

Assuming a flat beam energy spectrum between 140GeV and 141GeV (figure 6) and approximating the $t\bar{t}$ cross section by a step function which starts at 140GeV, the expectation value of the width can be calculated by hand. It is found that this expectation value is decreased by a factor 3/4 compared to the width without beam radiation. Adding to it the events which radiate photons of less than 50MeV, one finally obtains a change of the width by a factor of 0.94, corresponding to a shift in the top mass of $150 MeV/c^2$. This is roughly the shift obtained if one uses the r.m.s. of the W momentum distribution to measure the top mass. If the difference between the lower and upper endpoint of the W momentum distribution is used, the contribution of beam radiation can be neglected, because a radiative event has a lower P_{max} and a higher P_{min}. It is like shifting the curves in figure 1 for some events to the left.

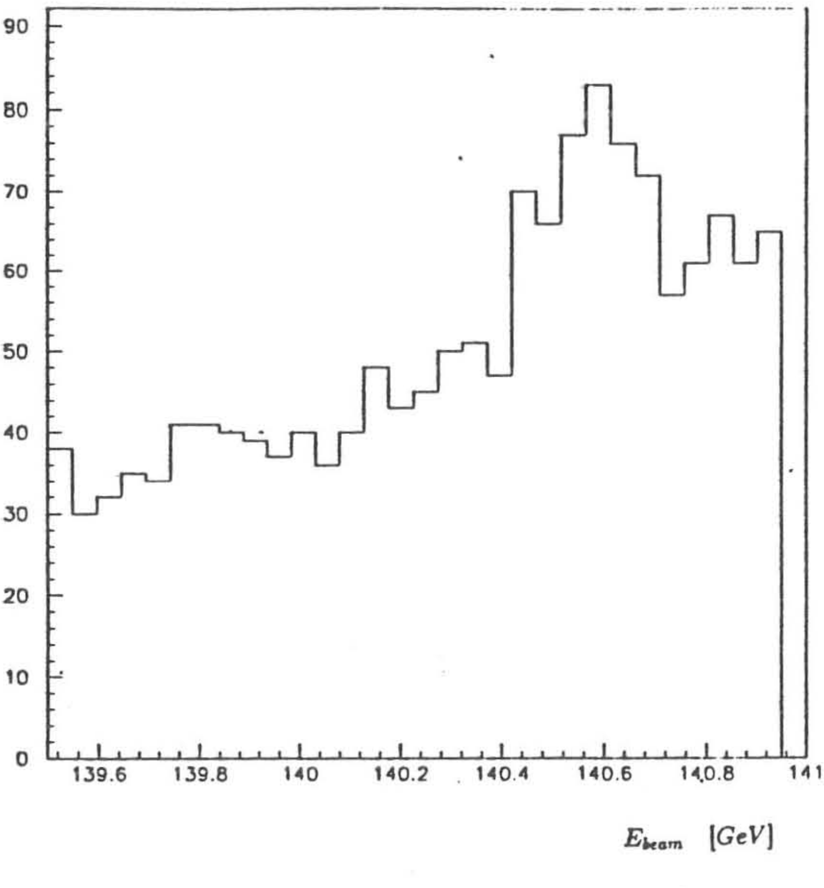

Figure 6: Distribution of the beam energy, where beam radiation is taken into account. Events radiating less than 50MeV are not drawn.

For the Tesla design the effect of beamradiation is decreased by a factor 10.

In addition to beamradiation also the smearing of the beam energy contributes to the error on the top mass. A Monte Carlo sample of $t\bar{t}$ events with a smeared beam energy according to beamradiation and energy spread was generated. The energy spread was assumed to be gaussian with $\delta E/E_{beam} = 0.17\%$ (0.3%, 1.5%) for the Palmer F (Tesla, DESY–Darmstadt) design. Using the standard detector with an optimistic geometrical acceptance, granularity and spatial resolution one obtains at a luminosity of $1.2 fb^{-1}$ a statistical error on the top mass of $280 MeV/c^2$ ($420 MeV/c^2$ $470 MeV/c^2$) due to the fluctuation of the beam energy. The mean value of the top masses was shifted by $-170 MeV/c^2$ ($-280 MeV/c^2$ $-440 MeV/c^2$), so it should be corrected by that.

7 Conclusion

Using the muon and electron decay modes of the W boson the top mass can be measured at an integrated luminosity of $1.fb^{-1}$ with a statistical error of $240 MeV/c^2$. The tau decay mode cannot be used because of the additional neutrino. The identification of the leptons is not difficult, because they are well isolated. A beam spread of $\delta E/E_{beam} = 0.17\%$ gives a contribution of $310 MeV/c^2$ at a luminisity of $1.fb^{-1}$. Contributions from bremsstrahlung and beam radiation can be neglected. Adding the errors in quadrature one obtains $390 MeV/c^2$. The systematic errors are limited by the energy flow measurement and the beam spread. In addition the error on the top mass caused by the error on the absolute scale of the beam energy is equal to the error on the scale. The method relies strongly on the energy flow, so a homogeneous detector with a good geometrical acceptance and a high granularity is preferred.

I am very grateful for all the inspiring discussions in the top group, which was founded at [7] especially for those with Peter Igo–Kemenes and Luigi Rolandi.

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