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Abstract

In recent years quantum electrodynamics (QED) vacuum polarization corrections (at low momentum transfer) has been tested in several physical situations using muonic atoms: this has permitted a direct check for different values of $Z\alpha$ [1– 6].

The vacuum polarization test obtained by measuring the 2S–2P energy difference, by means of laser-induced transition in muonic helium, has reached the 0.17% level. Such a test is limited by the uncertainty of the helium electric form factor; in order to overcome this limit, a scheme to measure the 3D–3P energy difference in muonic helium has been suggested and a test measurement performed. In this article we wish to make some comments on this experimental method and discuss its future possibilities.

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Some years ago, at the CERN Syncro-cyclotron, a CERN–Pisa Collaboration [1] devised a scheme to perform a laser-induced transition experiment on a muonic-helium ion $(\mu^- \, {}^4He)_{2S}$. At that time this rather complex set-up, connected with the operation of the Synchro-cyclotron itself, allowed the energy level differences D_i^2 = 2S–2P, to be measured with an accuracy of 1.5×10^{-4}. In *Table-1* the experimental results and the expected theoretical values given by the quantum electrodynamics (QED) calculations [2] are reported.

It is important to note from the table that about 20% of D_i^2 is, in this case, due to corrections induced by the nuclear electric form factor $\langle r^2 \rangle^{1/2}$. Taking for this form factor the experimental value [3]

$$\langle r_e^2 \rangle^{1/2} = (1.676 \pm 0.008\ ) \text{ fm},$$

(as seen by an electron probe), and assuming muon–electron universality, one sees that the QED correction is tested to the level of 0.17%; the result of this experiment is among the best direct tests so far performed, see *Table 2*.

Later a CERN–Columbia Collaboration [7] looked into the possibility of performing an experiment of the same type (i.e. laser-induced transition in muonic ions), but on transitions between levels not belonging to the S states in order to avoid the influence of the electromagnetic form–factor uncertainties. A set-up to measure the 3D–3P energy-level differences, via laser stimulation, in muonic helium was realized. In this paper as well as giving a brief account of the apparatus built at BNL and its performances [8,9] some comments are also reported on the possibilities of this experimental method.

In *Fig. 1* the lower energy levels of the free muonic-helium ion, together with some of their characteristics, are shown. The principle of the experiment consists in observing a variation in the K_β line intensity caused by the laser-stimulated 3D–3P transition when negative muons are stopped in a multipass optical cavity filled with helium and where high-density electromagnetic radiation is stored. It is crucial to realize that the lifetimes of the muonic levels involved are all around 10^{-12} s.

The main physical background, from which the small increase due to the laser-stimulated emission has to be extracted, is the natural emission of K_β X-rays during the cascade of the negative muons.

In *Table -3* are presented the results [7,9] of a calculation for the various contributions to the 3D–3P energies obtained under the assumption that the $(\mu^- \, {}^4He)^+$ ion is free. The wavelength of the radiation to match the resonance conditions is obtained with CO_2 lasers.

From *Fig.1* one sees that the width Γ of the line is

$$\Gamma = 5 \times 10^{-4}\ \text{eV} = 0.0473\ \mu\text{m}.$$

If N_D is the number of negative muons passing the D levels of the muonic ion (about 60%), and E/V is the energy density of the radiation at the side of the stopping muon, then the fraction of transitions is given by [8,9]:

$$\varepsilon(\omega) = \frac{N\text{stim}}{N_D} = \frac{\Gamma/2}{\hbar^2 (\omega-\omega_0)^2 + \Gamma^2/4} \frac{1}{\gamma_{3D}} |F_{3D-3P}|^2 E/V, \quad (1)$$

where $\Gamma = (\gamma_{3D} + \gamma_{3P})$ is the width of the transiton, γ_{3D} and γ_{3P} are the radiative decay rate of the D and P levels of interest, respectively (see *Fig.1* ; $\gamma = \hbar/\tau$), and $|F_{3D-3P}|^2 = 2.43\ (e\ a_\mu)^2$ is the square of the electric-dipole transition-matrix element. Taking $V = 0.4$ (cm^3) and E = 4 (Joules) in Eq. (1), at the resonance frequence one gets $\varepsilon = 0.6\%$.

Figure 2 is a sketch of the target set-up; the effective target is represented by a multipass optical cavity [10], where the CO_2 laser burst was stored (for about 100 ns). The cavity is assembled in 3 atm of helium and, in the small cavity region, the burst of negative muons is stopped during the presence of the radiation.

The X-ray detecting system was composed of three Si(Li) detectors each having a sensitive area of about 1 cm^2. An isotopic $^{13}C^{18}O_2$ laser delivered the 4 J of radiation stored in the optical cavity. A particular frequency was selected by means of a grid to locate the centre of the resonance (98595 Å for $3D_{5/2}$–$3P_{3/2}$) to better than 10 Å.

At the Single Burst Extraction (SBE) Beam of the BNL AGS (28 GeV/*c*), an isolated burst of negative muons (about 50 ns wide) was brought to stop in the helium gas target. The SBE operation was synchronized with the laser firing so that muons and laser radiation would be present at the same time in the cavity target (see *Fig.3*). The relevant figures for the muon beam (10^{12} protons on target, 1 burst each 1.4s) are given below:

Momentum (MeV/*c*)	μ^- stops in cavity per burst	e/μ	Si(Li) total counts per burst per counter
25	300	8	0.65

It is important to note that since the muon beam was instantaneous compared to the integration time of the detectors, only one count could be accepted during a single burst. The optimum integration time was a compromise between the need for good energy resolution and the minimization of the pile-up. In *Fig. 4* is shown the K_β X-ray yield [8] obtained in the experiment (laser off). It is easy to see that a useful quantity is the ratio K_β/K_α: from the experiment, the following value was deduced for this ratio:

$$Y_{\mathrm{exp}} = \frac{\left(\frac{K_\beta}{K_\alpha}\right)_0 - \left(\frac{K_\beta}{K_\alpha}\right)_f}{\left(\frac{K_\beta}{K_\alpha}\right)_f} = 0.012 \pm 0.014$$

where 0 means laser on, f laser off. The expected value for Y, assuming the use of the correct wavelength, is

$$Y_{\mathrm{th}} = 0.017 \ .$$

It appears evident that in order to perform a significant measurement, i.e. comparable or competitive with the 2S–2P difference measurement [1], much higher statistics would have been required. The experiment faced pratical limits owing to the size of the sensitive area of the Si(Li) detectors. A large array of X-ray detectors having high segmentation, good energy resolution, and fast response would allow the experiment to be performed.

Meanwhile, the processes occurring when a negative muon is stopped in a helium gas target at a pressure of few atmospheres were studied (see Refs. [11–13]). These studies have shown that most probably at these densities, very soon (i.e. within the cascade time), the single charged system $(\mu^- \, {}^4\mathrm{He})^+ = \mathrm{M}_n^+$ remains bound in a helium–ionic molecular system: i.e. the M_n^+ formed in the gas (which appears as a heavy hydrogen ion), does not remain free and forms bound muon–helium molecular ions $[(\mathrm{M}_n^+ \, \mathrm{He})^+\mathrm{He}]_i^+$. As a consequence, electron screening effects on the muonic energy levels should be expected.

All possible bound muon-helium molecular ions (ground and excited ones) from Ref. [13] are shown in *Fig. 5*. The possible prompt formation of various bound muonic molecular ions, unfortunately, changes the prospects of using the 3D–3P energy difference measurements, suggested in Ref. [7], as a high-precision QED vacuum-polarization test.

We wish to discuss and point out the substantial correction $\Delta_{\mathrm{corr},3}$ due to the electrons screening in the molecules $[(M_n^+\mathrm{He})^+\mathrm{He}]_i^+$, to the 3D–3P energy level difference. The correction will, in general, depend on the molecular–ion formed : however the correction can be written

$$\Delta_{\mathrm{corr},3} = -\,|\rho|\,\frac{e^2}{a_0}\left(\frac{a_{\mu\mathrm{He}}}{a_0}\right)^2 \frac{2^3}{3^7 \times 5} \int e^{-2/3 r_\mu}\, r_\mu^4 \, d^3 \vec{r}_\mu = -\,4 \times 10^{-3} |\rho| \ (\mathrm{eV})$$

where

$|\rho|$ is the electron screening density at the muon's site in e/a_0^3 units,
a_0 is the Bohr radius of the hydrogen atom,
$a_{\mu\mathrm{He}}$ is the Bohr radius of the muonic helium;
e is the electron's charge.

An estimation of $\Delta_{corr,3}$ taking for the muonic molecular ion the ground system $X'\Sigma^+$ (see *Fig. 5*) gives as a correction to the values of *Table 3*

$$\Delta_{corr,3} = -\,10^{-3}\ \mathrm{eV} = 0.0860\ \mu\mathrm{m} \simeq 2\Gamma .$$

Interpolating the data given in Ref.[14] for the $[H^+\,He]_i^+$ molecular excited ions, values for $\Delta_{corr,3}$ below Γ were obtained for the excited muonic systems, depending on the level considered.

The correction to the 2S–2P energy difference, $\Delta_{corr,2}$, due to the screening of the molecular electrons can be written

$$\Delta_{corr,2} = -\,|\rho|\,\frac{e^2}{a_0}\left(\frac{a_{\mu He}}{a_0}\right)^2 \frac{1}{3\times 2^2}\int e^{-r_\mu}\, r_\mu^2\, d^3\vec{r}_\mu$$

$$= -\,10^{-3}\,|\rho|\ (\mathrm{eV}),$$

and on the assumption that the molecular ion is the $X'\Sigma^+$ ground system one gets the following corrections to the values in *Table 1*:

$$\Delta_{corr,2} = -\,2.4\times 10^{-4} = 1.3\ \text{Å},$$

i.e. very small compared with the uncertainty caused by the form factor and a fraction 1/7 of the width (~ 8 Å).

Conclusions

Before carrying on with the experiment discussed in Ref. [7,8,9], as a QED vacuum polarization test, further work is required. At first glance it appears that the formation at early times of muon-molecular ions makes this experimental method unlikely to give results competitive with the 2S–2P experiment.

However looking at the experimental results of the Columbia test is has to be stressed that:

i) the clean observation of the K_β line in an extremely intense pulsed low energy muon beam (25 MeV/*c*) has been a success,

ii) with the multipass cavity-target technique it has been possible to make a (fast) muon-beam laser-pulse coincidence.

These facts suggest that it is worth while employing this technique to study the neutral μ^-p system formed by stopping negative muons in hydrogen gas: in particular to look at the 3D–3P energy difference. In *Fig. 6* the energy spectrum of the μ^-p *K* transitions is presented as measured by a Xe gas scintillation proportional detector [15,16]. For orientation we have calculated the energy differences of the 3D–3P levels (see *Fig.7*) and found the values shown in *Table 4*: the corresponding line width Γ is

$$\Gamma = \frac{\hbar}{\tau_P} + \frac{\hbar}{\tau_D} = 2 \times 10^{-5}\,\text{eV}\,,$$

i.e. we have transitions at frequencies around 1000 GHz with a width of about 6 GHz.

We note that for the 3D-3P levels the transition probabilities in the μ^-p case are, for the same energy density, about 100 times higher than for the $(\mu^-\cdot{}^4He)^+$.

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Z. Phys. A283, 139 (1978).

Table 1

Contributions to the n = 2 energy splittings in the $(\mu^{-}\ {}^{4}He)^{+}_{2S}$ system

Contribution	Transition energies (meV) $2P_{3/2}-2S_{1/2}$	$2P_{1/2}-2S_{1/2}$
Dirac contribution with Coulomb potential and point like charges	145.70	0
Nuclear polarizability	3.1 ± 0.6	3.1 ± 0.6
Finite size*	-289.5 ± 2.8	-289.5 ± 2.8
Electronic vacuum polarization		
Uehling term: first iteration	1664.44	1664.17
higher iteration	1.70	1.70
Kallen–Sabry term ($\alpha^2 Z\alpha$)	11.55	11.55
$\alpha(Z\alpha)^n$, $n > 3$	-0.02	-0.02
$\alpha^2(Z\alpha)^2$	0.02	0.02
Muon vacuum polarization	0.33	0.33
μ–e vacuum polarization	0.02	0.02
Hadron vacuum polarization	0.15	0.15
Vertex corrections and (g–2)		
$\alpha(Z\alpha)$	-10.52	-10.85
$\alpha(Z\alpha)^n$, $n > 1$	-0.16	-0.16
$\alpha^2 Z\alpha$	-0.03	-0.03
Recoil terms		
Breit	0.28	0.28
Two photons	-0.44	-0.44
Weak contribution	0.00002	0.00002
Sum theory	1526.6 ± 2.8	1380.3 ± 2.8
Experiment	1527.5 ± 0.3	1381.3 ± 0.5

* Value obtained from the results of Ref. [3] $\langle r_e^2 \rangle^{1/2}$ = (1.676±0.008) fm.

Table 2

Summary of the most accurate tests of QED vacuum polarization correction.

EXPERIMENT	TOTAL EFFECT	VACUUM POLARIZATION	TOTAL UNCERTAINTY	MAJOR SOURCE OF UNCERTAINTY	RELATIVE UNCERTAINTY IN VACUUM POLARIZATION	REF.
Lamb Shift H	1×10^3 MHz	26 MHz	$^{0.02}_{0.05}$ MHz	Calculation	$\sim 1.7 \times 10^{-3}$	[17]
$(g\text{-}2/2)_e$	1.1×10^{-3} eV	1×10^{-7} eV	2×10^{-10} eV	Value of α	$\sim 1.7 \times 10^{-3}$	[18]
$(g\text{-}2/2)_\mu$	1.1×10^{-3} eV	6×10^{-6} eV	2×10^{-8} eV	Strong Int. Cont. + Expt. error	$\sim 1.7 \times 10^{-3}$	[19]
Muonic Atoms High Z	4×10^5 eV	2×10^3 eV	8 eV	Expt. error + Electron screening	4×10^{-3}	[20]
Muonic Atoms, He Laser Induced	1.5 eV	1.7 eV	4×10^{-3} eV	$\langle r^2 \rangle^{1/2}$	1.7×10^{-3}	[1]
Muonic Atoms Low $Z \leq 13$	^{24}Mg 5.6×10^4 eV ^{28}Si 7.7×10^4 eV	1.8×10^2 eV 2.8×10^2 eV	3 ppm; 0,16 eV 3 ppm; 0.25 eV	Electron screening + Calculation	0.95×10^{-3}	[6]

Table 3

Contributions to the 3D–3P energy level difference for the free $(\mu^- \, ^4He)^+$ system.

Transition	Vacuum polarization α (Uehling–Serber) (meV)	Vacuum polarization α^2 (Kallen–Sabry) (meV)	Fine structure (Dirac) (meV)	Hyperfine structure (meV)	Total (meV)	λ (μm)
$3D_{3/2}-3P_{1/2}$	110.458	0.905	43.164	0	154.528	8.0235
$3D_{5/2}-3P_{3/2}$	110.458	0.905	14.389	0	125.751	9.8595
$3D_{3/2}-3P_{3/2}$	110.458	0.905	0	0	111.363	11.1334

$\Gamma = 5 \times 10^{-4}$ eV $\simeq$ 0.047 μm

Table 4

Various contributions to the 3D–3P energy level differences for the $(\mu^- p)$ neutral system. Width $\Gamma \simeq 2 \times 10^{-5}$ eV.

Transition	Dirac (meV)	Hyperfine (meV)	Vehling (meV)	Kallen (meV)	Total (meV)	λ (mm)
$3D^3_{3/2}-3P^5_{3/2}$	0	-0.6757171	4.649347	0.0442757	4.017960	0.308582
$3D^5_{3/2}-3P^5_{3/2}$	0	-0.1351433	"	"	4.558480	0.271988
$3D^3_{3/2}-3P^3_{3/2}$	0	+0.2252390	"	"	4.918862	0.252061
$3D^5_{5/2}-3P^5_{3/2}$	0.831152	-0.5405736	"	"	4.984201	0.248756
$3D^7_{5/2}-3P^5_{3/2}$	"	-0.1930620	"	"	5.331713	0.232543
$3D^5_{3/2}-3P^3_{3/2}$	0	+0.7650126	"	"	5.459436	0.227103
$3D^5_{5/2}-3P^3_{3/2}$	0.831152	+0.3603822	"	"	5.885157	0.210674
$3D^3_{3/2}-3P^3_{1/2}$	2.493455	-0.9009561	"	"	6.286122	0.197363
$3D^5_{3/2}-3P^3_{1/2}$	"	-0.360382	"	"	6.826695	0.181618
$3D^3_{3/2}-3P^1_{1/2}$	"	+1.351434	"	"	8.538511	0.145270

Notation, nL_J^{2F+1} ; $\vec{J} = \vec{\ell} + \vec{s}$; $\vec{F} = \vec{J} + \vec{I}$; I = nuclear spin, s = lepton spin.

Figure captions

Fig. 1: Scheme of the first energy levels of the $(\mu^{-}\,{}^{4}He)^{+}$ muonic ion.

Fig. 2: Lay out of the target and optical cavity-system.

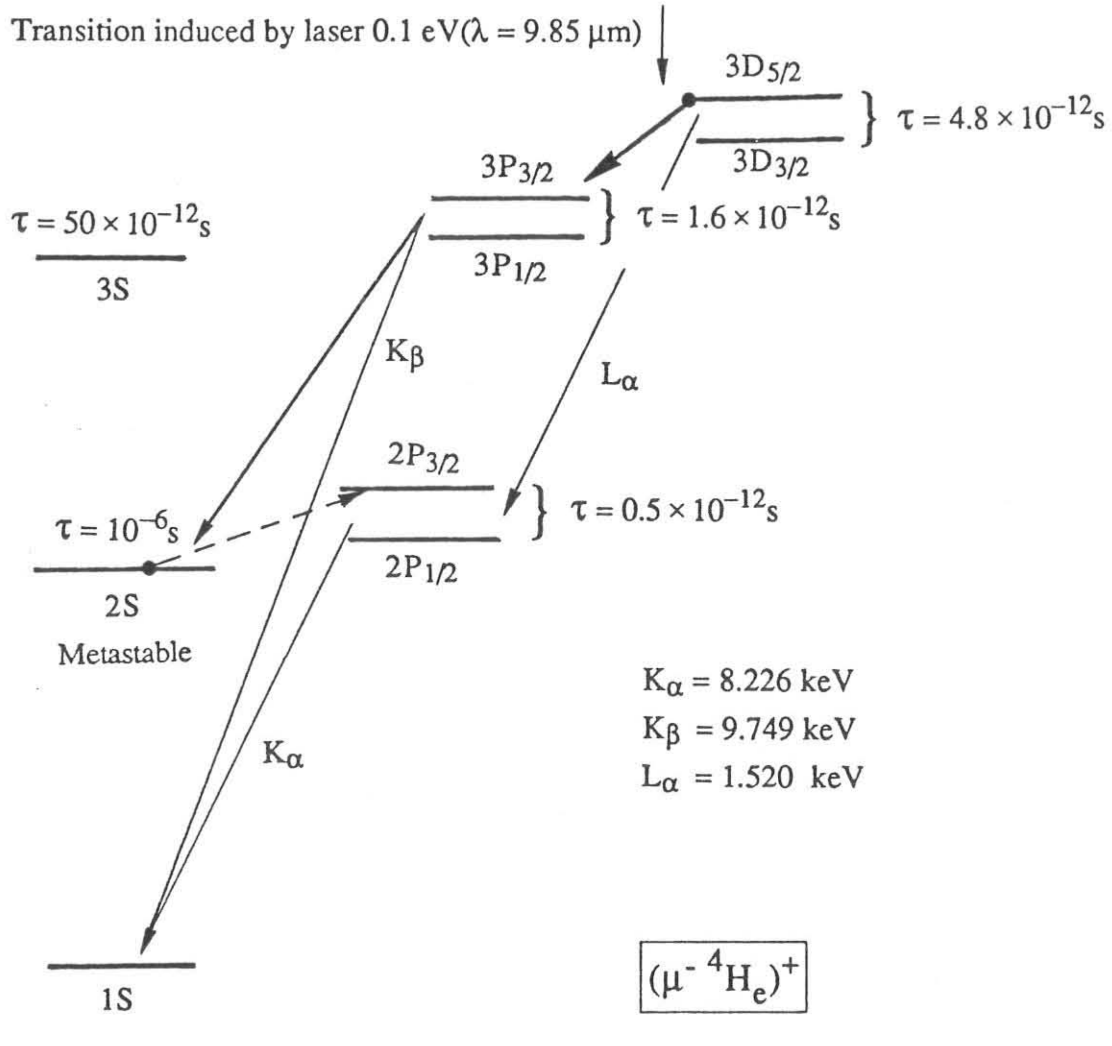
Fig. 3: Timing sequence of the experiment.

Fig. 4: Energy spectrum recorded in a Si(Li) detector of the prompt K X-rays from the negative muons stopped in the He target at 3 atm. The smooth curve is a fit to the data, using Gaussian shapes with fixed centres and widths for the K_{α}, K_{β}, K_{γ}, including also a combination of higher-K X-rays and a flat background, [9].

Fig. 5: Possible bound muonic-helium molecular ions $[(\mu^{-}\,{}^{4}He)^{+}He]_{i}^{+}$.

Fig. 6: K transition of the $\mu^{-}p$ system as measured by a Xe gas scintillation detector; the FWHM is about 20% at 2 keV.

Fig. 7: Scheme of the first energy levels of the $(\mu^{-}p)$ muonic atoms.



Transition induced by laser 0.1 eV(λ = 9.85 μm)
3D5/2
τ = 4.8 × 10^-12 s
3D3/2
3P3/2
τ = 1.6 × 10^-12 s
3P1/2
τ = 50 × 10^-12 s
3S
Kβ
Lα
2P3/2
τ = 0.5 × 10^-12 s
τ = 10^-6 s
2P1/2
2S
Metastable
Kα = 8.226 keV
Kβ = 9.749 keV
Lα = 1.520 keV
Kα
1S
(μ- 4He)+

Fig. 1

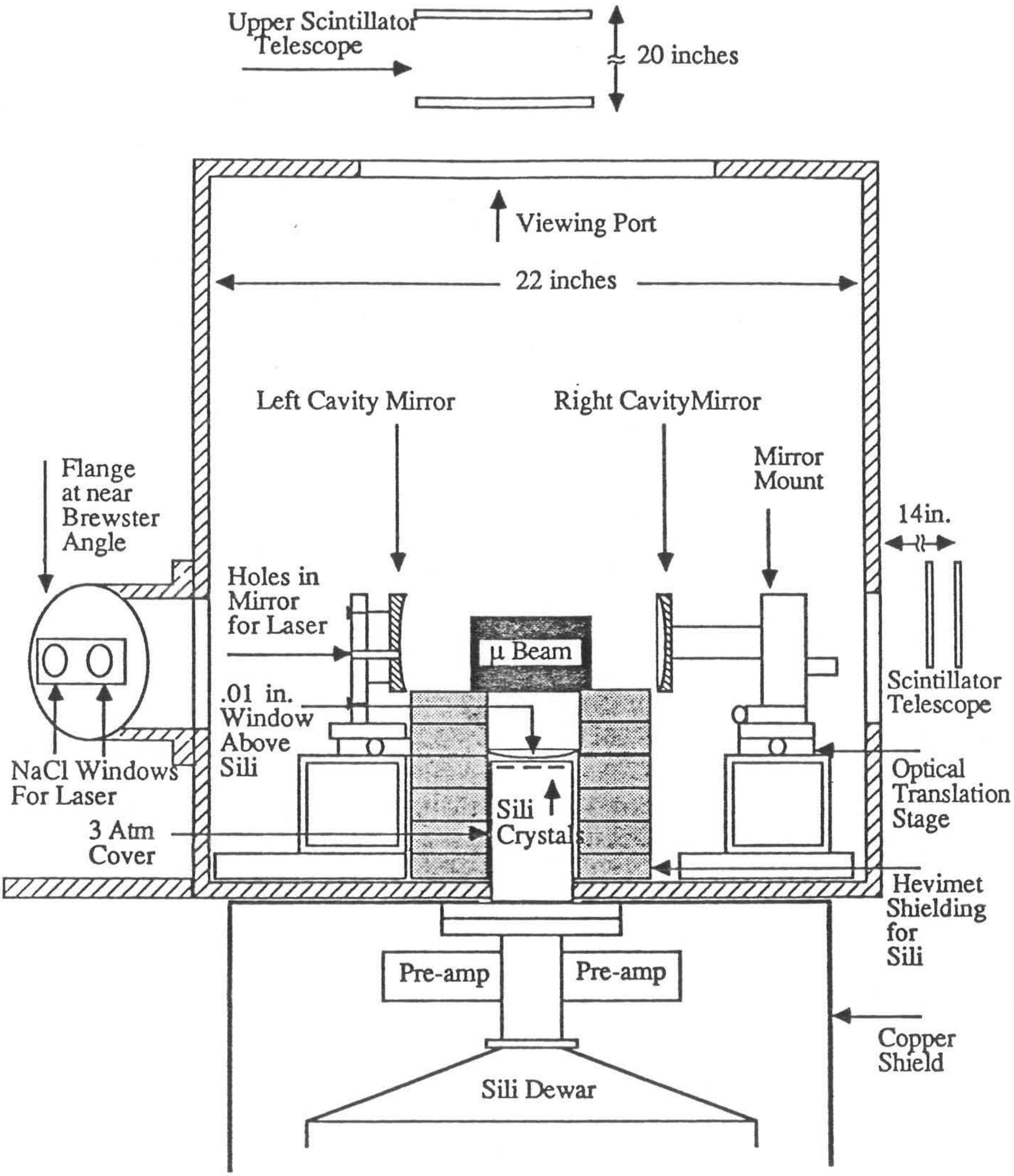

Upper Scintillator Telescope
20 inches
Viewing Port
22 inches
Left Cavity Mirror
Right CavityMirror
Mirror Mount
14in.
Flange at near Brewster Angle
Holes in Mirror for Laser
μ Beam
.01 in. Window Above Sili
Scintillator Telescope
NaCl Windows For Laser
Sili Crystals
Optical Translation Stage
3 Atm Cover
Hevimet Shielding for Sili
Pre-amp
Pre-amp
Copper Shield
Sili Dewar

Fig. 2

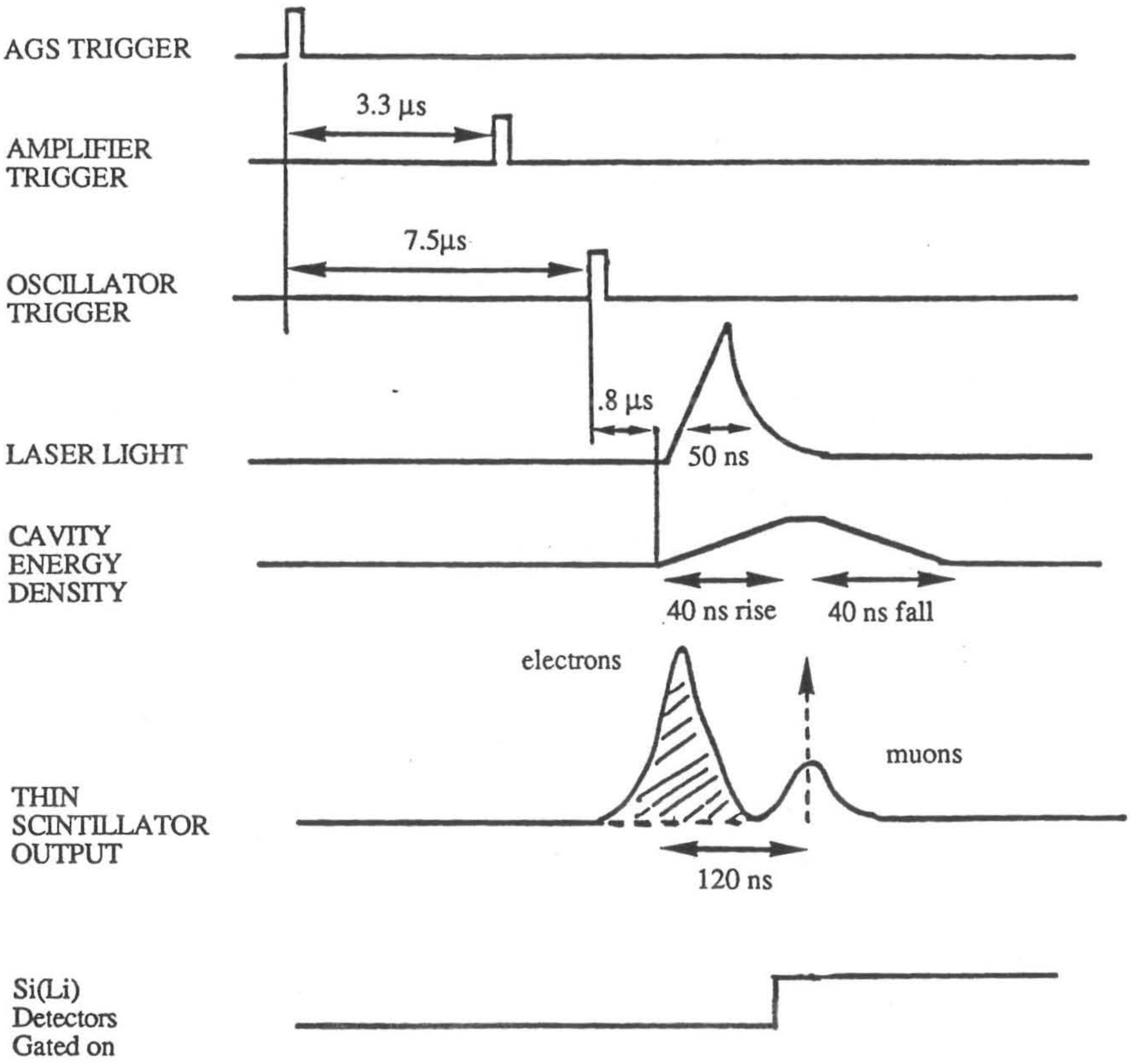

AGS TRIGGER
3.3 µs
AMPLIFIER TRIGGER
7.5µs
OSCILLATOR TRIGGER
.8 µs
LASER LIGHT
50 ns
CAVITY ENERGY DENSITY
40 ns rise
40 ns fall
electrons
muons
THIN SCINTILLATOR OUTPUT
120 ns
Si(Li) Detectors Gated on

Fig. 3

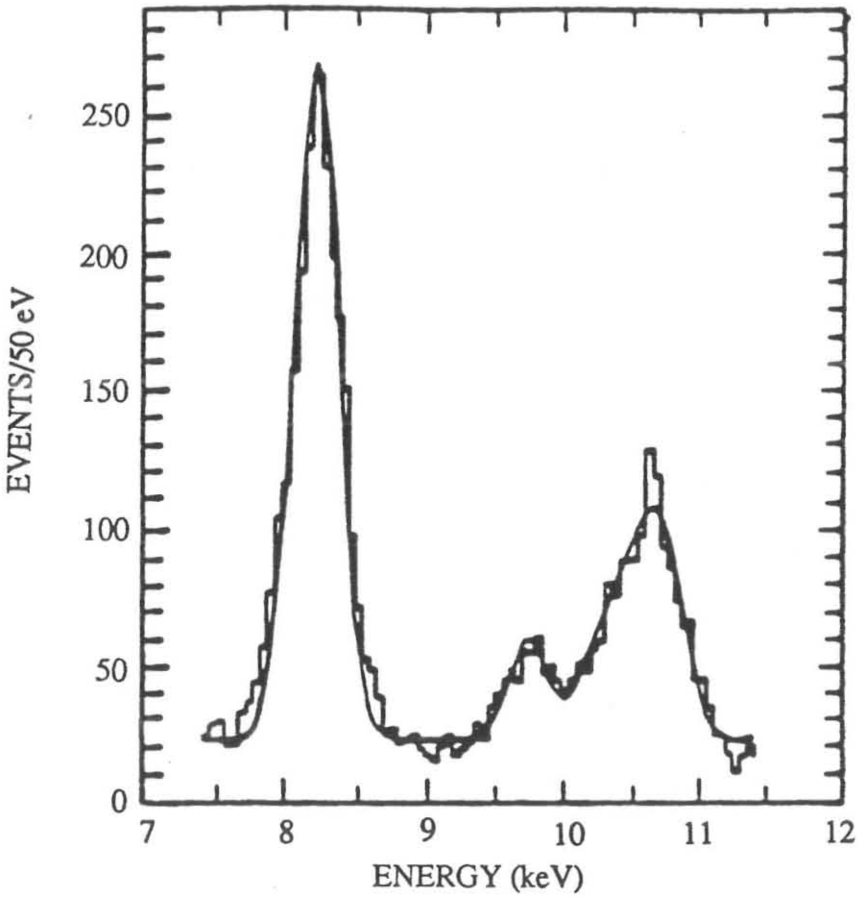


Fig. 4

J. Chem. Phys. 44, 3834 (1966)
J. Chem. Phys. 61, 5186 (1974)
J. Chem. Phys. 64, 395 (1976)
J. Chem. Phys. 69, 101 (1978)

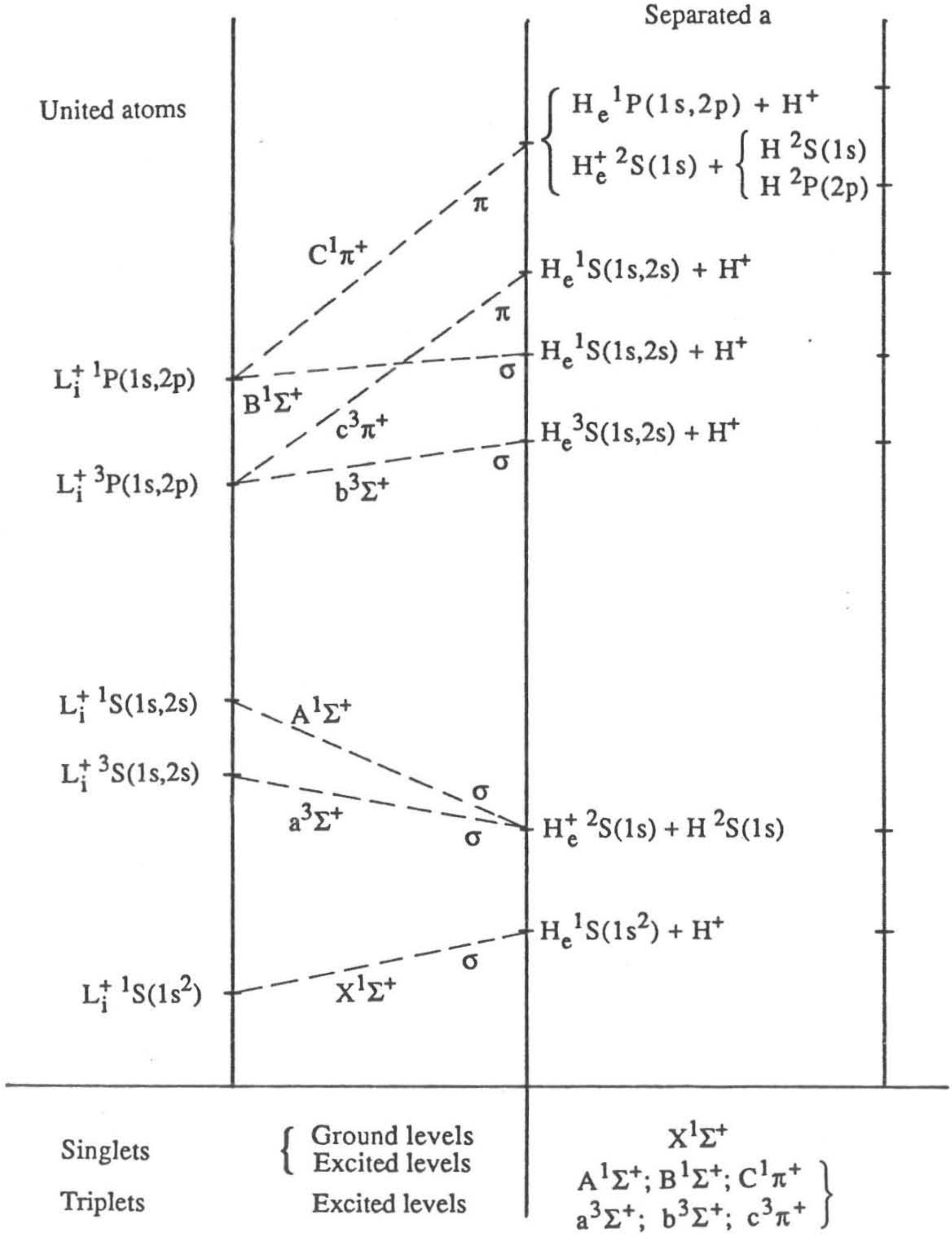


Fig. 5

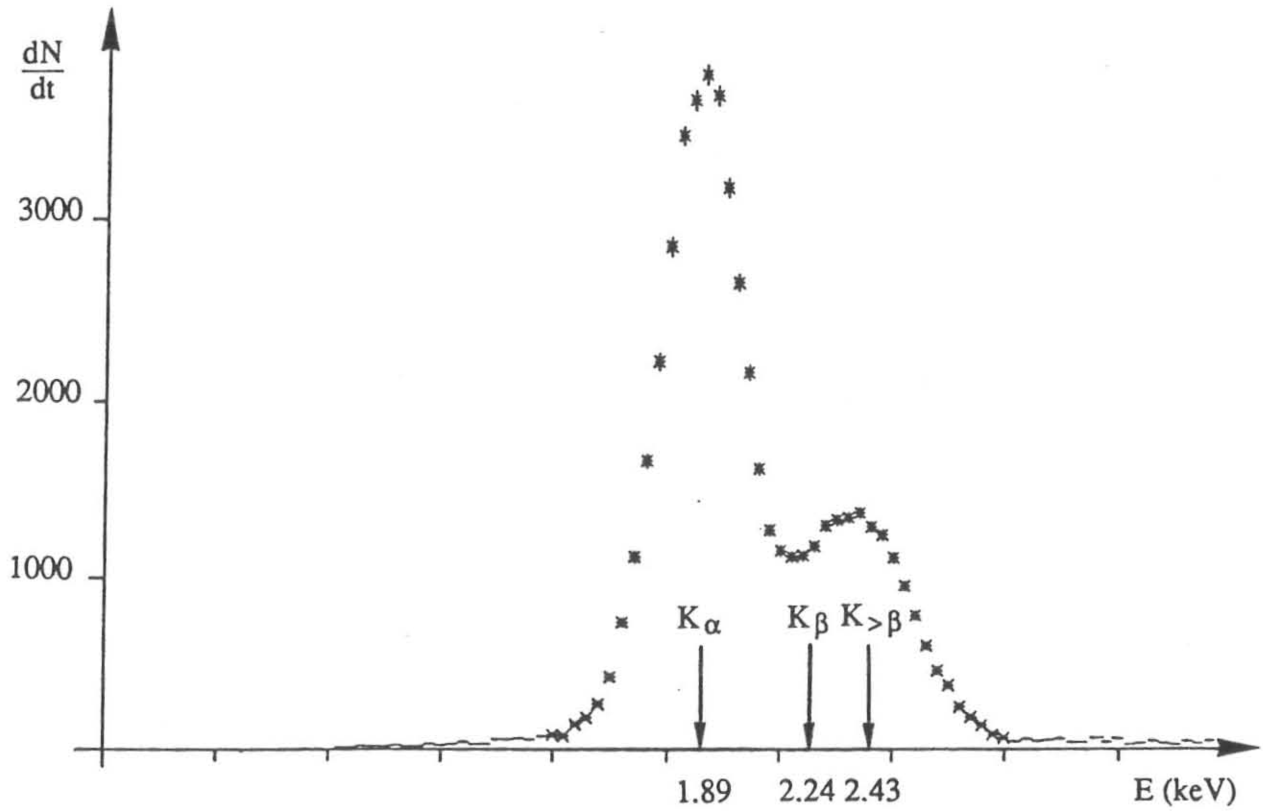
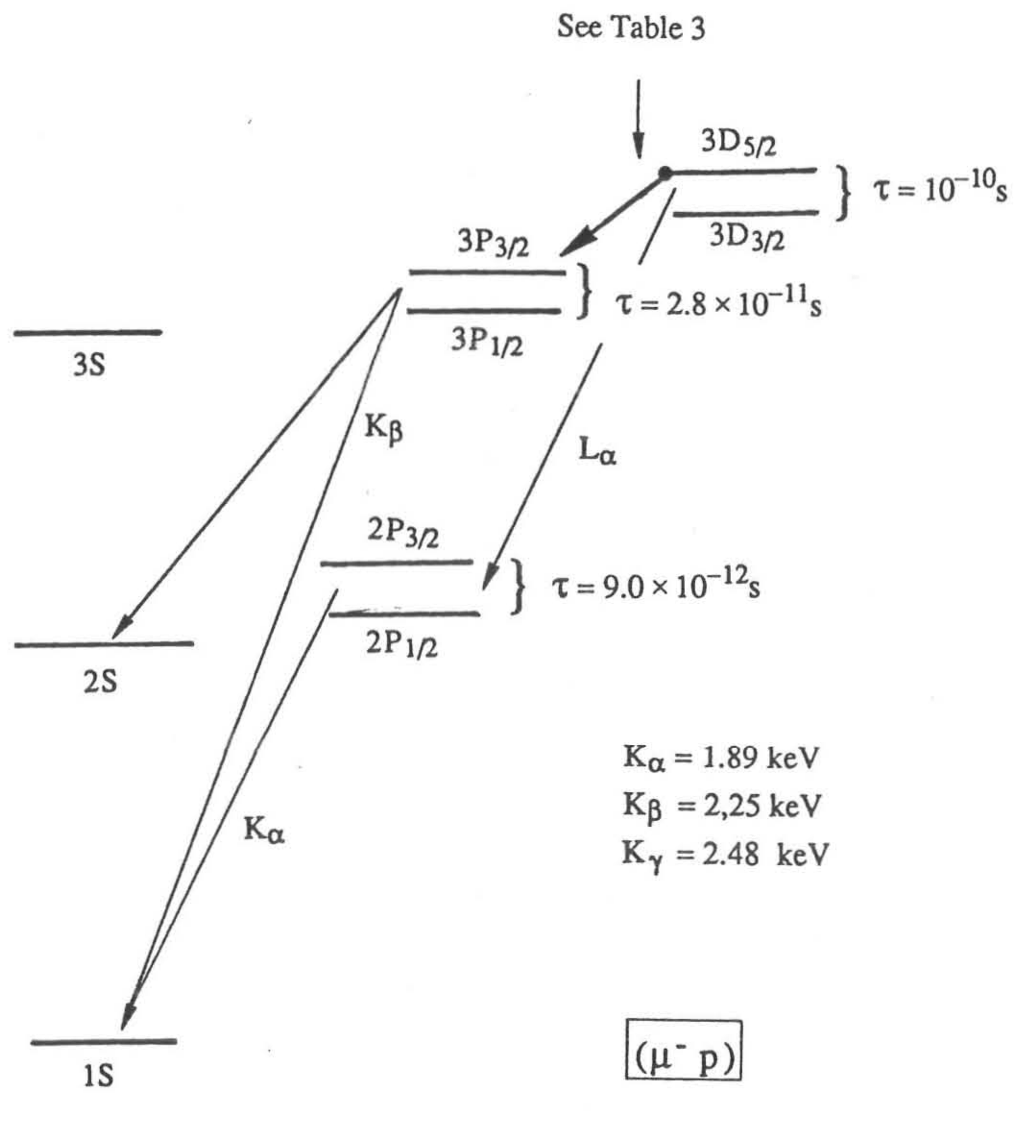


Fig. 6



See Table 3
$3D_{5/2}$
$3D_{3/2}$
$\tau = 10^{-10}$s
$3P_{3/2}$
$3P_{1/2}$
$\tau = 2.8 \times 10^{-11}$s
3S
K_β
L_α
$2P_{3/2}$
$2P_{1/2}$
$\tau = 9.0 \times 10^{-12}$s
2S
$K_\alpha = 1.89$ keV
$K_\beta = 2,25$ keV
$K_\gamma = 2.48$ keV
K_α
1S
$(\mu^- p)$

Fig. 7