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Comments on the metastable states of bosonized QED around a large-Z nucleus

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ABSTRACT

The bosonization technique has been recently applied to the study of supercritical QED around a large-Z nucleus. New charge-neutral metastable states emerge from the spectrum of the theory and their existence represents a possible explanation of the e^+e^- peaks observed in heavy-ion collisions. Actually, we suspect that such metastable states might be a mere product of the approximations introduced in the treatement of the bosonized Hamiltonian. In this work we present both quantitative and qualitative arguments to support our conjecture.

Introduction

The observation of narrow peaks in the positron and electron spectra produced from the collision of heavy ions have created much interest [1-4]. Up to now no satisfactory explanation of such structures is available. Results resembling the experimental data have been obtained by introducing some more or less convincing "ad hoc" hypoteses [5-9]. In this scenario a series of interesting papers [10-14] have been produced by Y. Hirata and H. Minakata . They have studied the problem in the non perturbative framework of a Partial-Wave-bosonized QED. In such a scheme it is possible to go beyond the external field approximation and to take into account, at least partially, the quantum fluctuations of the electromagnetic field.The form of the bosonized Hamiltonian is quite involved and its spectrum can be found at the cost of many severe approximations. Once simplified the theory predicts the existence of new neutral metastable states which are interpreted as arising from the non perturbative aspects of QED. The energy and the width of these states suggest that they might be the cause of the narrow e^+e^- peaks observed in heavy-ion collisions. Actually, we suspect that such states would be ruled out by an improved analysis of the bosonized Hamiltonian. In other words we think that their origin rests on the various approximations introduced in references [10,11] . In order to show this we shall bosonize the QED Hamiltonian in the background or external field approximation; since this problem is exactly soluble it will be straightforward to check whether the approximations adopted in [10,11] do introduce wrong states in the spectrum of the system.

Bosonized QED in the external field approximation

QED with an external charge density $Ze\,\rho(\mathbf{r},t)$ is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^{\mu}\partial_{\mu} + e\gamma^{\mu}A_{\mu} - m_e)\psi - Ze\rho(\mathbf{r},t) \qquad (1)$$

with $\rho(\mathbf{r},t)$ normalized to unity : $\int d^3r\rho(\mathbf{r},t) = 1$. In references [10-12] a spherically symmetric source is considered, the higher partial waves of the fields are omitted and only the s-wave electromagnetic field and the (j=1/2)-wave spinor field are retained. As a result the theory is casted into the form of an effective two-dimensional fermionic theory. The bosonization technique [15-17] can then be used to obtain the corresponding two-dimensional boson theory which is described by the Hamiltonian:

$$\begin{aligned} H = \int dr \Big\{ & \sum_m \tfrac{1}{2}(\Pi_m^2 + P_m^2 + \Phi'^2_m + Q'^2_m) \\ & + \sum_{m\delta} \frac{1}{2\pi r^2}\left[1 - \cos\sqrt{\pi}\left[\Phi_m + Q_m - \delta\int_r^{\infty} ds\left[\Pi_m(s) - P_m(s)\right]\right]\right] \qquad (2) \\ & + \sum_m \frac{\pi}{4} m_e^2\left[2 - \cos(2\sqrt{\pi}\,\Phi_m) - \cos(2\sqrt{\pi}\,Q_m)\right] \\ & + \frac{e^2}{8\pi r^2}\left[\left(C(r,t) - \frac{1}{\sqrt{\pi}}\sum_m (\Phi_m + Q_m)\right)^2 - C(r,t)^2\right] . \end{aligned}$$

The fields Φ_m and Q_m are Boson fields living in a (t,r) universe with $r \geq 0$. Π_m and P_m denote their canonical momenta. The index m (= ± 1) represents the z-component of the angular momentum and δ (= ± 1) corresponds to the chirality. C(r,t) is defined as :

$$C(r,t) = 4\pi Z \int_0^r r'^2 \rho(r',t) dr' \tag{3}$$

and $f=\sqrt{e}$ with e=2.718.. .

As anticipated in the introduction, we now consider the external or background field approximation . We also assume a time indipendent external source. The Lagrangian density of QED is now given by :

$$\mathcal{L}_{ext} = \overline{\psi}(i\gamma^\mu \partial_\mu - m_e)\psi + eA_0 \overline{\psi}\,\gamma^0 \psi \tag{4}$$

where A_0 is the external potential. A straightforward application of the bosonization technique gives :

$$H_{ext} = \int dr \sum_m \frac{1}{2}(\Pi_m^2 + P_m^2 + \Phi'^2_m + Q'^2_m)$$

$$+\sum_{m\delta} \frac{1}{2\pi r^2}\left[1-\cos\sqrt{\pi}\left[\Phi_m + Q_m - \delta\int_r^\infty ds[\Pi_m(s) - P_m(s)]\right]\right] \tag{5}$$

$$+\sum_m \frac{\pi}{4} m_e^2 \left[2-\cos(2\sqrt{\pi}\,\Phi_m) - \cos(2\sqrt{\pi}\,Q_m)\right] + \frac{e^2}{4\pi\sqrt{\pi}r^2} C(r) \sum_m (\Phi_m + Q_m)$$

We see that the relevant effect of the external field approximation is to remove from H the term:

$$H_{fluct} = \frac{e^2}{8\pi r^2}\left(\frac{1}{\sqrt{\pi}}\sum_m (\Phi_m+Q_m)\right)^2 \quad (6)$$

which then describes the fluctuations of the electromagnetic field. Actually, this term corresponds to the quantum fluctuations of the longitudinal degrees of freedom since only the s-wave electromagnetic field has been kept.

To explore the spectrum of H_{ext} we now follow closely the methods suggested in [10-12]. We first look for the configuration (Φ_{cl},Q_{cl}) which minimizes H_{ext} by solving the classical equation of motion. We take the symmetric ansatz $\Phi_{cl}=Q_{cl}$ and we work in the approximation of vanishing canonical momenta $\Pi_{cl}=P_{cl}=0$, see references [10-12] for details. As expected we find two local minima corresponding to the neutral and charged vacuum respectively. The energies of these vacua are plotted in fig. (1) as a function of the central charge Z. In our external field approximation the transition from the neutral-undercritical vacuum to the charged-supercritical one takes place at $Z_{cr}\sim 170$. This value agrees with that of [10-12] and with the results obtained by more conventional tools [18-19].

In order to study the dynamics of the system (3) we expand the Bose fields around their background configuration:

$$\Phi_m=(\Phi_m)_{cl}+\phi_m\,,\ Q_m=(Q_m)_{cl}+q_m,\ \Pi_m=\pi_m,\ P_m=p_m\,,$$

where the small letters represent the quantum fluctuations. Correspondingly, the Hamiltonian H_{ext} is expanded around its minimum up to the quadratic terms of the small fluctuations.

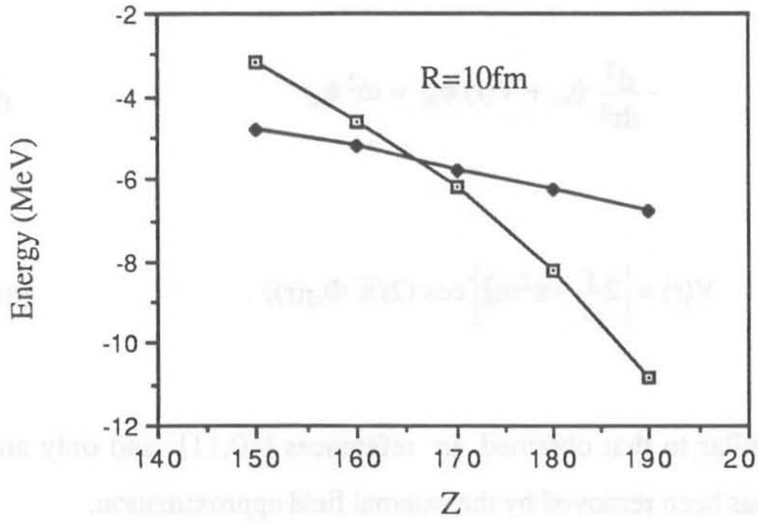

Fig. 1. The energy of the normal (solid square) and the supercritical (open square) vacuum are plotted as functions of the nuclear charge Z. The external source is a uniformly charged sphere of radius R=10fm

It is useful to introduce the fields $\psi_m = \phi_m + q_m$ and $\chi_m = \phi_m - q_m$. As one can easily verify the ψ_m and χ_m fluctuations decouple under the symmetrical ansatz $\Phi_{cl} = Q_{cl}$. Moreover only ψ_m is coupled to the charge, then we focus our attention on this mode, freezing out the χ_m degrees of freedom. The effective Hamiltonian for the ψ_m fluctuations reads

$$H_\psi = \frac{1}{2} \sum_m \left(\frac{\partial \psi_m}{\partial t}\right)^2 + \left(\frac{\partial \psi_m}{\partial r}\right)^2 + \left[2\frac{f}{r^2} + \pi^2 m_e^2\right] \cos(2\sqrt{\pi}\ \Phi_{cl}(r))\ \psi_m^2 \qquad (7)$$

and the fields equations are :

$$\frac{\partial^2 \psi_m}{\partial t^2} - \frac{\partial^2 \psi_m}{\partial r^2} + \left[2\frac{f}{r^2} + \pi^2 m_e^2\right] \cos(2\sqrt{\pi}\ \Phi_{cl}(r))\ \psi_m \ . \qquad (8)$$

Setting $\psi_m = e^{\pm i\omega t}\,\phi_m(r)$ we obtain a Schrödinger-type equation for $\phi_m(r)$:

$$-\frac{d^2}{dr^2}\phi_m + V(r)\,\phi_m = \omega^2\,\phi_m \tag{9}$$

where V(r) is given by:

$$V(r) = \left[2\frac{f}{r^2} + \pi^2 m_e^2\right]\cos\left(2\sqrt{\pi}\,\Phi_{cl}(r)\right). \tag{10}$$

This potential is very similar to that obtained in references [10,11] and only an additive term $\Delta V \sim \frac{e^2}{\pi r^2}$ has been removed by the external field approximation.

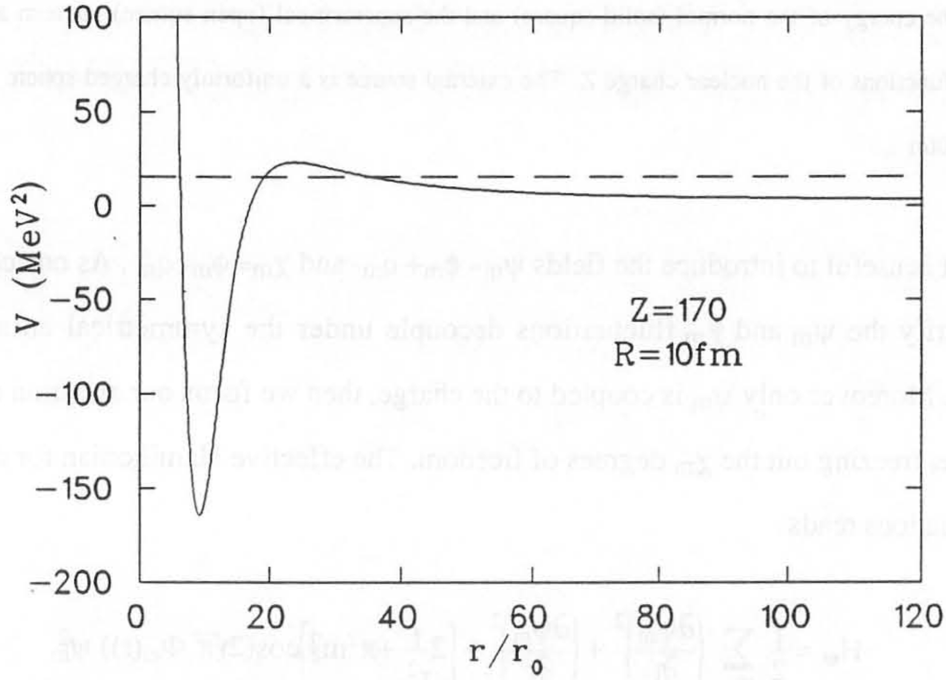

Fig. 2 The potential (10) felt by the small fluctuations is plotted for a uniformly charged sphere of radius R=10fm and Z=170. The radial coordinate is measured in r_0 units, r_0 being the classical electron radius. The dashed line correspond to the energy squared of the state trapped in the potential well.

In fig. (2) we plot the potential V(r) for a uniformly charged sphere of radius R=10fm and Z=170. Again we found a good agreement with the results of [10,11]. In particular , even in the external field approximation, the potential develops the pocket structure responsible for the trapping of the boson excitation. If we employ a WKB approximation to solve eq. (9) we explicitly verify that the first excitation is trapped in the potential well . Both the energy and the width of this state agree with the values found in [10,11].

Drawbacks of bosonized QED

We are now in a position to draw some conclusions from the foregoing results. The crucial point is that the metastable states of references [10,11] are still present in our external field approximation. Within such an approximation, QED is nothing but a text-book subject and it is well known that no e^+e^- resonance can appear. Actually , as shown in references [18,19] , only positron resonances are present in the spectrum of supercritical QED. Thus we are forced to conclude that the metastable states found in [10,11] are simply a mere product of the several approximations introduced there.

It is now useful to identify the approximation which brings the wrong states into the spectrum of QED. As far as the vacuum state is concerned, our results seem to be resonable. As shown in fig. (1) the transition from the neutral vacuum to the charged one is clearly reproduced. Moreover the value of the critical charge Z_{cr} lies in the expected range. It is then natural to search the bug in the "small fluctuation" approximation, that is in the expansion of the bosonized Hamiltonian up to the quadratic terms of the boson excitations. We now give a simple argument supporting this hypotesis. Let us consider , from a classical point of view, the field equations (8) and let $\xi(r,t)$ be the classical fluctuation defined as :

$$\xi(r,t) = e^{-i\omega t}\xi_\omega(r) + e^{i\omega t}\xi_\omega^*(r) \,, \qquad (11)$$

where ω is the energy of the trapped state and ξ_ω satisfies equation (9). From the effective Hamiltonian (7) we get the classical energy E_{cl} of the fluctuation:

$$E_{cl} = 2\omega^2 \int_0^\infty dr \left|\xi_\omega(r)\right|^2 . \qquad (12)$$

Since ξ_ω (r) is confined in a region of lenght L ~ 50fm. (the width of the potential well) , we can write:

$$E_{cl} \cong 2\,\omega^2 \,\langle\, |\xi_\omega|^2 \rangle\, L \qquad (13)$$

where $\langle\, |\, \xi_\omega \,|^2 \rangle$ is the average of $|\, \xi_\omega \,|^2$ in the potential well. Consequently:

$$\langle \xi_\omega \rangle \equiv \sqrt{\langle |\xi_\omega|^2 \rangle} \;\cong \sqrt{\frac{E_{cl}}{2\omega^2 L}} \qquad (14)$$

and this relation gives us a rough estimate of the fluctuation amplitude as a function of its energy E_{cl} . In the exact Hamiltonian the boson fields appear as argument of a cosine function. The condition $\langle\, \xi_\omega \,\rangle \ll 1$ should then be fullfilled in order to rely on the small fluctuation approximation. Using eq. (14) we obtain :

$$E_{cl} \ll 2\, m \left(\frac{\omega}{m_e}\right)^2 \left(\frac{L}{r_0}\right) \alpha \qquad (15)$$

where r_0 is the classical electron radius. Inserting the numerical values of L~50fm and ω~$2m_e$, the last inequality boils down to $E_{cl} \ll 2m_e$. Since $2m_e$ is a lower bound for the physical boson excitations , it is very hard to have $\langle\, \xi_\omega \,\rangle \ll 1$ at the quantum level.

Conclusions

We have applied the methods developed in references [10,11] to bosonize the lowest partial wave of QED within the external field approximation . As a result we have verified that the approximation scheme adopted in [10,11] introduce non existing states in the spectrum of QED, namely the metastable states assumed to be the origin of the narrow e^+e^- peaks observed in heavy-ion collisions. We have also given a hint to identify the approximation responsible for the described drawback: it is the "small fluctuation" one, that is the expansion of the bosonized Hamiltonian up to the quadratic terms of the Boson excitations.

As a by-product of our analysis we have found a rather simple expression for the quantum fluctuations of the electromagnetic field in terms of the Boson variables. Our results suggest that such fluctuations cannot influence dramatically the spectrum of QED around a highly charged source. When confirmed by an improved analysis of the overall bosonization strategy, this conjecture will turn in a strong argument against the hypotesis of the e^+e^- peaks as a pure QED effect . For this reason we think that the framework developed in references [10,11] can still provide us useful informations about non perturbative Quantum Electrodynamics.

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