ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Trieste

INFN/AE-91/04

16 aprile 1991

G. Calucci and D. Treleani

MINIJETS: TRANSVERSE ENERGY FLOW IN VERY-HIGH-ENERGY NUCLEAR COLLISION

Servizio Documentazione
dei Laboratori Nazionali di Frascati

MINIJETS: TRANSVERSE ENERGY FLOW IN VERY-HIGH-ENERGY NUCLEAR COLLISIONS

G. Calucci and D. Treleani

Dipartimento di Fisica Teorica dell'Università and INFN Sezione di Trieste

Trieste, I 34014 Italy

ABSTRACT

We estimate the transverse energy produced by semi-hard partonic interactions in nuclear collisions at c.m. energies of the order of one TeV per nucleon. Keeping properly into account multiple partonic interactions the transverse energy spectrum has no longer any power divergence for small values of the cut off p_t^{min} which separates the semi-hard region from the soft one.

I. Introduction

A parton that has exchanged a transverse momentum of a few GeV, in hadronic collisions with C.M. energies in the TeV range, will produce a small jet of particles in the final state (mini-jet). The events with at least one mini-jet in the final state define the semi-hard cross section. In the kinematical range for mini-jet production the interaction is in a perturbative regime, but the momentum transfer is negligible with respect to the total energy available. It is then possible that the lower threshold p_t^{min}, which defines the scattered parton as a mini-jet, might be considered as an infrared cut off, at least with respect to some physical observables. In fact, since the regime is such that a large fraction of the interaction is likely to fall in the domain allowed to the perturbation theory, one may forsee the possibility to handle major contributions to global characteristics of an inelastic event without dealing with the problem of confinement, which is related to a momentum scale much smaller than the lower threshold for mini-jet production. More precisely one may expect to construct physical observables, inside the framework of perturbation theory, whose dependence on the choice of the threshold for mini-jet production becomes less and less important when increasing the C.M. energy.

While large p_t phenomena in high energy hadronic collisions are successfully described by the QCD parton model, it was realized soon that the QCD parton model, in the form that is used to describe large p_t phenomena, is not adequate for mini-jets because it is based on a single scattering expression. In fact the expression for the inclusive cross section to produce mini-jets provided by the QCD parton model gives the average number of partonic collisions multiplied by the semi-hard cross section[1] and the actual value can easily exceed by far the total cross section in hadronic collisions with very large C.M. energies[2]. When looking to high energy nuclear collisions the effect is largely emphasized by the dependence on the atomic mass numbers: if A and B are the atomic mass numbers of the colliding nuclei, the QCD parton model expression for the integrated mini-jet cross section goes as $A \times B$, while the total cross section goes rather as $(A^{1/3} + B^{1/3})^2$. As a consequence, in heavy ion collisions with very high energies in the C.M. system, quite a large number of partons are interacting with a momentum transfer larger than the threshold p_t^{min} defining the mini-jet region[3].

In order to describe more detailed aspects of the interaction than just the average number of partonic collisions multiplied by the semi-hard cross section, one has to face the complicated task of dealing with multiple partonic collisions. There are two different kinds of multiple partonic collisions. One kind is the disconnected collision (from the point of view of the hard interaction), where each parton pair interacts independently. These are collisions localized at different points in the transverse plane, since the semi-hard interaction is localized in a region of order $(p_t^{min})^{-1}$ in size and the interacting pairs are rather at a distance of the order of the radius of the interacting objects. These collisions, at a given order in the number of interactions, maximize the incoming flux and therefore are the most likely ones in a regime where the incoming parton flux is large and the elementary interaction probability is small. The second kind of multiple parton interaction is represented by rescatterings, where each parton interacts more than once with the target. When the parton population grows the combinatorics of the possible interactions grows much faster and the regime where each parton undergoes multiple interactions becomes important. The general case will be a combination of the two possibilities.

In order to deal explicitly with multiple partonic collisions one does not need only to handle the variety of all possible semi hard partonic interactions, one needs also to provide the non-perturbative input, namely the multi-parton distributions, where one is facing the difficulty that multi-parton distributions are quantities independent[4] of the single parton distributions, which represent the usual non perturbative input in the QCD-parton-model.

The simplest possibility one may consider is the following:

a- Assume independence among different partonic collisions, so that the over all interaction probability can be expressed in a factorized form in terms of the probability of interaction of a parton pair. (In most of the cases studied in the literature one moreover makes the simplification of taking into account disconnected partonic collisions only).

b- Take a Poissonian form for the parton distribution, so that all multiple parton distributions can be expressed in terms of the single parton distribution and a dimensional parameter (multiple parton distributions are dimensional quantities[5]).

With this input a number of results have been obtained both in the context of high energy hadronic and nuclear collisions: A series of papers[6] have included the semi-hard component in an eikonal picture of high energy hadronic interactions adding to the eikonal phase a contribution proportional to the QCD-parton-model expression of the large p_t parton production cross section. As it has been discussed in Ref.7 this procedure is equivalent to the assumptions a (with the inclusion of disconnected partonic collisions only) and b; a and b represent the assumptions for the discussions on the semi-hard cross section in Ref.8 and 9. The case of the semi-hard cross section in high energy nuclear collisions has been studied in the same framework in Ref.3 and 10 where the transverse energy flow from the semi-hard component of the interaction is estimated.

In all these references the semi hard interaction is represented taking into account only disconnected parton collisions. In Ref.7, however, we have made the observation that, for nuclear collisions with energies of the order of 1 TeV per nucleon in the C.M. system, the amount of semi-hard partonic rescattering in sizeable. The average value of the fraction of energy going into semi-hard interactions and its dispersion were then estimated keeping rescatterings explicitly into account. It was also found that as the semi-hard cross section, once evaluated including multiple partonic collisions, has only a smoother dependence on the cut-off p_t^{min}[8,9] in a similar way the average energy fraction and its dispersion are also weakly dependent on p_t^{min} after including parton rescattering in the interaction[7].

The purpose of the present paper is to gain a better insight on a further physical quantity as far as its relation with the choice of the lower threshold for mini-jet production is concerned. In fact we will focus on the semi-hard contribution to the transverse energy spectrum. Although a more general approach is possible[11], in the present paper we will remain in the framework of the simplest assumptions discussed above, keeping however parton rescattering explicitly into account. Presently available estimates of the semi-hard contribution to the transverse energy flow in high energy nuclear collisions do not include parton rescattering in the interaction[3,10]. The resulting transverse energy spectrum, as a consequence, depends as an inverse power on the cut off p_t^{min}. The unpleasant feature is that a small variation on the choice of p_t^{min} produces dramatic changes in the whole semi-hard transverse energy spectrum even at very large energies in the C.M. system. Such a dramatic dependence is not a physical effect, it is rather the consequence of a too drastic simplification in the interaction. Keeping explicitly into account parton rescattering, we will derive a semi-hard transverse energy spectrum with no inverse power singularity as function of the cut off. In order to give a quantitative feeling on the regime of interest for the present analysis we will present some numerical estimates in the final part of the paper. A few remarks on parton correlations will be presented in appendix C.

II. Average transverse energy and dispersion

Following Ref.7 we consider the case of a Poissonian partonic distribution; namely we assume that the probability density for having n partons with fractional momenta $x_1, \ldots, x_n$ and with transverse coordinates $\mathbf{b}_1, \ldots, \mathbf{b}_n$ in a nucleus of atomic mass A, is given by:

$$\frac{1}{n!}\Gamma_A^{f_1}(x_1, \mathbf{b}_1)\ldots\Gamma_A^{f_n}(x_n, \mathbf{b}_n) \times exp\Big[-\int\sum_f \Gamma_A^f(x, \mathbf{b})dx d^2b\Big] \qquad (1)$$

where $\Gamma_A^f(x, \mathbf{b})$ is the average number of partons with longitudinal momentum fraction x (scaled with respect to the nucleon momentum), $\mathbf{b}$ is the parton transverse coordinate and the index f counts the various species of partons. The normalization of $\Gamma_A^f(x, \mathbf{b})$ is A times that of the nucleon parton distributions and the integral in Eq.(1) is regularized with a cut off related to p_t^{min}. Multiparton distributions are dimensional quantities, in our scheme dimensionality is provided by the explicit dependence of Γ on the transverse partonic coordinate $\mathbf{b}$ and therefore it is related to the geometrical size of the nucleus.

We express the semi-hard cross section as:

$$
\begin{aligned}
\sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1\dots f_n} & \frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\dots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)e^{-\int\sum_f \Gamma_A^f(x,\mathbf{b})dxd^2b} \\
\times \sum_{l=1}^{\infty} \sum_{f_1'\dots f_l'} & \frac{1}{l!}\Gamma_B^{f_1'}(x_1',\mathbf{b}_1'-\beta)\dots\Gamma_B^{f_l'}(x_l',\mathbf{b}_l'-\beta) \\
\times e^{-\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}')dx'd^2b'} & \times \left[1-\prod_{i=1}^{n}\prod_{j=1}^{l}\prod_{f_i,f_j'}(1-\hat{\sigma}_{ij}^{f_if_j'})\right] \\
\times dx_1d^2b_1\dots dx_nd^2b_ndx_1'd^2b_1'\dots dx_l'd^2b_l' &
\end{aligned}
\tag{2}
$$

where $\hat{\sigma}_{ij}^{f_if_j'} \equiv \hat{\sigma}_{ij}^{f_if_j'}(x_ix_j,\mathbf{b}_i-\mathbf{b}_j')$ is the probability for the parton f_i from nucleus A to have a semi-hard interaction with parton f_j' from nucleus B ($\hat{\sigma}$ will depend on x_ix_j, on the difference of the transverse relative distance $\mathbf{b}_i-\mathbf{b}_j'$ and on the indices f_i, f_j').

The square bracket in Eq.(2) represents the probability of having at least one semi-hard partonic interaction between nucleus A and nucleus B, and the cross section is constructed summing over all possible partonic configurations of the two nuclei and integrating on the nuclear impact parameter β. Eq.(2) represents the incoherent sum of all possible interactions between partons of nucleus A and partons of nucleus B including all possible semi-hard rescatterings in nuclear matter.

Disconnected collisions, being localized at different points in the transverse plane, add incoherently when one evaluates the semi-hard cross section. Incoherence is less obvious in the case of rescatterings. The rescattering contribution to the integrated cross section is obtained summing over all possible discontinuities of the rescattering diagram. It has been shown that, in QCD, the leading contribution of each cut in the rescattering diagram is the same[12], apart from a weight factor that is the one given by the AGK rules[13]. The sum of all contributions amounts, as a consequence, to the introduction of an absorptive correction which is given by the iteration of the single scattering term. The interaction, in the form given by the square bracket in Eq.(2), represents therefore the semi-hard cross section in a consistent way.

From the expression for the semi-hard cross section one can obtain the average number of wounded partons (actually partons that have suffered at least one semi-hard interaction). In fact, following Ref.7, one can write the average number of wounded partons of nucleus B as:

$$
\begin{aligned}
\langle l\rangle\sigma_H^{AB} = &\int d^2\beta\sum_{n=1}^{\infty}\sum_{f_1\dots f_n}\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathrm{b}_1)\dots\Gamma_A^{f_n}(x_n,\mathrm{b}_n)\\
&\times\sum_{l=1}^{\infty}\sum_{f'_1\dots f'_l}\frac{1}{l!}\sum_{s=1}^{l}\sum_{f'_s}\Gamma_B^{f'_1}(x'_1,\mathrm{b}'_1-\beta)\dots\\
&\dots\Gamma_B^{f'_s}(x'_s,\mathrm{b}'_s-\beta)\dots\Gamma_B^{f'_l}(x'_l,\mathrm{b}'_l-\beta)\\
&\times e^{-\int\sum_f\Gamma_A^f(x,\mathrm{b})dxd^2b}\times e^{-\int\sum_{f'}\Gamma_B^{f'}(x',\mathrm{b}')dx'd^2b'}\\
&\times\left[1-\prod_{m=1}^{n}\prod_{f_m}(1-\hat{\sigma}_{ms}^{f_m f'_s})\right]\\
&\times dx_1d^2b_1\dots dx_nd^2b_ndx'_1d^2b'_1\dots dx'_sd^2b'_s\dots dx'_ld^2b'_l.
\end{aligned}
\tag{3}
$$

This expression shows explicitly the content in terms of multiple collisions on the wounded parton: the factor $\left[1-\prod_{m=1}^{n}\prod_{f_m}\left(1-\hat{\sigma}_{ms}^{f_m f'_s}\right)\right]$ represents all possible interactions of one parton from B (labelled with s) and $\langle l\rangle$ is obtained summing over all configurations where the parton s interacts with A at least once. In analogy with Eq.(3) one can evaluate the average number of wounded partons with transverse momentum $\mathbf{K}$ in the final state multiplied by the semi-hard cross section, $\mathcal{D}(\mathbf{K})$, expanding the product and requiring the total momentum transferred to the wounded parton to have a fixed value. After having dropped the flavour indices to simplify the notation, we obtain:

$$
\begin{aligned}
\mathcal{D}(\mathbf{K}) = &\int d^2\beta \sum_{n=1}^{\infty} \frac{1}{n!}\Gamma_A(x_1,\mathbf{b}_1)\ldots\Gamma_A(x_n,\mathbf{b}_n)e^{-\int \Gamma_A(x,\mathbf{b})dxd^2b} \\
&\times \sum_{l=1}^{\infty}\frac{1}{l!}\sum_{s=1}^{l}\Gamma_B(x_1',\mathbf{b}_1'-\beta)\ldots\Gamma_B(x_s',\mathbf{b}_s'-\beta)\ldots\Gamma_B(x_l',\mathbf{b}_l'-\beta) \\
&\times e^{-\int\Gamma_B(x',\mathbf{b}')dx'd^2b'}\sum_{\nu_s=1}^{n}\binom{n}{\nu_s}\frac{d^2\hat{\sigma}_{s,1}}{d^2\mathbf{k}_1}\ldots\frac{d^2\hat{\sigma}_{s,\nu_s}}{d^2\mathbf{k}_{\nu_s}}\left(1-\hat{\sigma}_{s,\nu_s+1}\right)\ldots \\
&\ldots\left(1-\hat{\sigma}_{s,n}\right)\times\delta^2(\mathbf{K}-\mathbf{k}_1-\ldots-\mathbf{k}_{\nu_s})d^2k_1\ldots d^2k_{\nu_s} \\
&\times dx_1d^2b_1\ldots dx_nd^2b_ndx_1'd^2b_1'\ldots dx_s'd^2b_s'\ldots dx_l'd^2b_l'
\end{aligned}
\tag{4}
$$

The factors $d^2\hat{\sigma}/d^2\mathbf{k}$ are understood as probabilities for successive rescatterings of the wounded parton; we will however neglect in the following the kinematical correlations induced by the successive collisions, the validity of the assumption will be discussed in appendix A.

In order to evaluate $\mathcal{D}(\mathbf{K})$ one proceeds writing the δ function as an exponential and introducing accordingly the Fourier transforms of $d^2\hat{\sigma}/d^2\mathbf{k}$:

$$
\begin{aligned}
&\delta^2\left(\sum\mathbf{k}-\mathbf{K}\right)=\frac{1}{(2\pi)^2}\int exp\left[i\left(\sum\mathbf{k}-\mathbf{K}\right)\cdot\mathbf{u}\right]d^2u \\
&\tilde{\sigma}(\mathbf{u})=\int d^2K\frac{d^2\hat{\sigma}}{d^2\mathbf{K}}e^{i\mathbf{K}\cdot\mathbf{u}}.
\end{aligned}
\tag{5}
$$

The differential distribution of Eq.(4) can then be expressed as:

$$
\begin{aligned}
\mathcal{D}(\mathbf{K}) = &\int d^2\beta \sum_{n=1}^{\infty} \frac{1}{n!}\Gamma_A(x_1,\mathbf{b}_1)\ldots\Gamma_A(x_n,\mathbf{b}_n)e^{-\int \Gamma_A(x,\mathbf{b})dxd^2b} \\
&\times \sum_{l=1}^{\infty}\frac{1}{l!}\sum_{s=1}^{l}\Gamma_B(x_1',\mathbf{b}_1'-\beta)\ldots\Gamma_B(x_s',\mathbf{b}_s'-\beta)\ldots\Gamma_B(x_l',\mathbf{b}_l'-\beta) \\
&\times e^{-\int\Gamma_B(x',\mathbf{b}')dx'd^2b'}\frac{1}{(2\pi)^2}\int d^2ue^{-i\mathbf{K}\cdot\mathbf{u}} \\
&\times\sum_{\nu_s=1}^{n}\binom{n}{\nu_s}\tilde{\sigma}_{s,1}(\mathbf{u})\ldots\tilde{\sigma}_{s,\nu_s}(\mathbf{u})\left(1-\hat{\sigma}_{s,\nu_s+1}\right)\ldots\left(1-\hat{\sigma}_{s,n}\right) \\
&\times dx_1d^2b_1\ldots dx_nd^2b_ndx_1'd^2b_1'\ldots dx_s'd^2b_s'\ldots dx_l'd^2b_l'.
\end{aligned}
\tag{6}
$$

where the notation $\tilde{\sigma}(0) \equiv \hat{\sigma}$ is used. Keeping into account the symmetry of parton distributions one can write

$$
\begin{aligned}
\sum_{\nu_s=1}^{n} \binom{n}{\nu_s} \tilde{\sigma}_{s,1}(\mathrm{u}) \dots \tilde{\sigma}_{s,\nu_s}(\mathrm{u})(1-\hat{\sigma}_{s,\nu_s+1}) \dots (1-\hat{\sigma}_{s,n}) = \\
= (1+\tilde{\sigma}_{s,1}(\mathrm{u})-\hat{\sigma}_{s,1}) \dots (1+\tilde{\sigma}_{s,n}(\mathrm{u})-\hat{\sigma}_{s,n}) - \\
-(1-\hat{\sigma}_{s,1}) \dots (1-\hat{\sigma}_{s,n}).
\end{aligned}
\tag{7}
$$

All the terms give the same contribution when summing over s, the result is a factor l and the $1/l!$ is replaced by $1/(l-1)!$. The sum over l can be done giving the average number of partons of nucleus B as a result; the sum over n exponentiates the terms in Γ_A. Since the range in b of Γ is of the order of the nuclear dimension while the range in $\mathrm{b}-\mathrm{b}'$ of $\hat{\sigma}$ is rather of the order of $(p_t^{min})^{-1}$, we use the approximation

$$
\int \sum_{f'} \Gamma_A^{f'}(x',\mathrm{b}) \hat{\sigma}^{ff'}(xx',\mathrm{b}-\mathrm{b}') dx' d^2b \approx \int \sum_{f'} \Gamma^{f'}(x',\mathrm{b}') \hat{\sigma}^{ff'}(xx') dx'
$$

where $\hat{\sigma}^{ff'}(xx')$ represents the elementary cross section integrated on the polar angle in the partonic C.M. system with the cut off provided by p_t^{min}. The term $\tilde{\sigma}\Gamma_A$ is treated analogously. The resulting expression takes the simple form:

$$
\begin{aligned}
\mathcal{D}(\mathbf{K}) = \int d^2\beta dx' d^2b' \Gamma_B(x',\mathrm{b}'-\beta) \frac{1}{(2\pi)^2} \int d^2u e^{-i\mathbf{K}\cdot\mathbf{u}} \\
\times \Big\{ exp\Big[\int dx (\tilde{\sigma}(\mathrm{u})-\hat{\sigma}) \Gamma_A(x,\mathrm{b}')\Big] - exp\Big[-\int dx \hat{\sigma} \Gamma_A(x,\mathrm{b}')\Big]\Big\}.
\end{aligned}
\tag{8}
$$

Integrating over d^2K and d^2b' we verify that the average number of wounded partons is expressed[7] as:

$$
\langle l \rangle \sigma_H^{AB} = \int d^2\beta dx' d^2b' \sum_{f'} \Gamma_B^{f'}(x',\mathrm{b}'-\beta) \times \eta_A^{f'}(x',\mathrm{b}') \tag{9}
$$

where

$$\eta_A^f(x,\mathbf{b}') \equiv 1 - exp\Big(-\int \sum_f \Gamma_A^{f'}(x',\mathbf{b}')\hat{\sigma}^{ff'}(xx')dx'\Big). \tag{10}$$

In Eq.(8) only the first exponential describes the actual distribution in the transverse momentum $\mathbf{K}$ of the scattered parton, the second one being proportional to a δ function. In fact, the presence of a δ function for $\mathbf{K} = 0$ is an indication that the present estimate is too rough at small $\mathbf{K}$. In appendix B a lower limit of validity for the small $\mathbf{K}$ region will be obtained. Let us consider the argument of the first exponential:

$$\chi(\mathbf{u}) \equiv \int dx\Big(e^{i\mathbf{k}\cdot\mathbf{u}} - 1\Big)\frac{d^2\hat{\sigma}}{d^2\mathbf{k}}\Gamma_A(x,\mathbf{b})d^2k$$

Since $d^2\hat{\sigma}/d^2\mathbf{k}$ depends only on the modulus* of $\mathbf{k}$ the angular integration can be done giving as a result a Bessel function. In perturbative regime $d^2\hat{\sigma}/d^2\mathbf{k}$ is roughly $1/k^4$ at small k, one can then notice that the expression above has only a logarithmic divergence in the lower limit of integration. More explicitly

$$\chi = \int dx\Gamma_A(x,b)\Big[J_0\Big(u\frac{\sqrt{xx's}}{2}sin\theta^*\Big) - 1\Big]\frac{d\hat{\sigma}}{dcos\theta^*}dcos\theta^* \tag{11}$$

and

$$\frac{d\hat{\sigma}}{dcos\theta^*} = \frac{1}{xx's}g(cos\theta^*)$$

where s is the nucleon-nucleon C.M. system energy squared, θ^* is the polar scattering angle in the partonic C.M. system and $g(cos\theta^*)$ is a pure angular function which depends on the spin of the partons.

---

* We do not consider polarization effects: everything is summed, or averaged, over spin variables

The effect of the Rutherford like power singularity of the elementary cross section at $\theta^* = 0$ is to induce a (much smoother) logarithmic dependence of χ as a function of $(p_t^{min})^2/s$. The leading contribution to the integration in $cos\theta^*$ is obtained from the most singular part of $d\hat{\sigma}/dcos\theta^*$ and the leading behaviour of $J_0 - 1$ for small values of the argument. The result of the integration is a constant multiplying the logarithm of the ratio of the integration limits for $cos\theta^*$ (actually 0 and $\left[1 - 4p_c^2/xx's\right]$, with $p_c = p_t^{min}$). As a consequence one may express χ, in the large energy-small cut off regime, as

$$\chi = -u^2 H_A \equiv -u^2 \times \left[a \times \int_{\frac{4p_c^2}{x's}}^{1} dx \Gamma_A(x, b) \times ln\Big(\frac{xx's}{4p_c^2}\Big) \right] \tag{12}$$

and a is a constant, flavour dependent, proportional to α_s^2. In the case of gluon gluon interactions $a = 9\pi\alpha_s^2/8$.

The dominant contribution to $\mathcal{D}(\mathbf{K})$ is finally expressed as

$$\begin{aligned} \mathcal{D}(\mathbf{K}) &= \int d^2\beta dx' d^2b \Gamma_B(x', \mathrm{b} - \beta) \frac{1}{(2\pi)^2} \int e^{-i\mathbf{K}\cdot\mathbf{u}} e^{-H_A u^2} d^2u \\ &= \int d^2\beta dx' d^2b \Gamma_B(x', \mathrm{b} - \beta) \frac{1}{4\pi H_A} exp\Big(-\frac{K^2}{4H_A}\Big) \end{aligned} \tag{13}$$

The average transverse momentum acquired by a wounded parton and its dispersion at fixed x' and b are

$$\begin{aligned} \langle K \rangle &= \sqrt{\pi H_A} \\ \langle K^2 \rangle - \langle K \rangle^2 &= (4 - \pi) H_A \end{aligned} \tag{14}$$

One can estimate the transverse energy flow keeping into account that the average transverse energy is given by the average number of wounded partons multiplied by the average transverse momentum acquired by each one of them and the dispersion is obtained combining the dispersion in K and the dispersion in the number of wounded partons. More precisely

$$
\begin{aligned}
&\langle E_t^2(\beta)\rangle - \langle E_t(\beta)\rangle^2 = \\
&= \int dx' d^2b \Gamma(x', \mathrm{b}-\beta) \times \left(\langle K^2\rangle - \langle K\rangle^2\right) + \\
&+ \int dx' d^2b \Gamma(x', \mathrm{b}-\beta) \times \langle K\rangle^2
\end{aligned} \tag{15}
$$

where the first term is the average number of partons multiplied by the dispersion in the transverse momentum acquired by each wounded parton (Eq.14) and the second is the dispersion in the number of partons (actually Γ, given the Poissonian distribution) multiplied by the transverse momentum squared acquired by a wounded parton. The average transverse energy, at a given value of the nuclear impact parameter β, and its dispersion are then:

$$
\begin{aligned}
\langle E_t(\beta)\rangle &= \int dx' d^2b \Gamma_B(x', \mathrm{b}-\beta) \times \sqrt{\pi H_A} + (A \leftrightarrow B) \\
D_t(\beta) &= \int dx' d^2b \Gamma_B(x', \mathrm{b}-\beta) \times 4H_A + (A \leftrightarrow B).
\end{aligned} \tag{16}
$$

The transverse energy distribution is finally expressed (by means of the Central Limit Theorem) as

$$
\frac{d\sigma_H}{dE_t} = \int d^2\beta \left(1 - e^{-\langle n(\beta)\rangle}\right) \frac{1}{\sqrt{2\pi D_t(\beta)}} exp\left[-\frac{\left(E_t - \langle E_t(\beta)\rangle\right)^2}{2D_t(\beta)}\right] \tag{17}
$$

where $1 - exp(-\langle n(\beta)\rangle)$ represents the semi hard cross section at fixed impact parameter β.

We emphasize the different dependence on the cut off p_t^{min} of the transverse energy spectrum, as expressed by Eq.(17), with respect to previous estimates. In fact the averages $\langle E_t(\beta)\rangle$ and $D_t(\beta)$, as given by Eq.(16), are functions of the $ln(s/(p_t^{min})^2)$ while in previous estimates the dependence was as an inverse power of p_t^{min}.

III. Conclusions

We have estimated the contribution from semi-hard parton collisions to the transverse energy distribution in heavy ion collisions with C.M. energies at the scale of $1TeV$ per nucleon. The main point in our analysis has been to improve previous estimates that are affected by a strong dependence on the cut off that defines the semi-hard regime. In fact, in previous estimates, the average transverse energy was characterized by an inverse power dependence as a function of the cut off. The origin was the dimensional behaviour characteristic of the quantities computed using the single scattering expression of the QCD parton model. Even if the semi-hard component of the interaction was constructed adding incoherently many elementary partonic interactions, the average semi hard transverse energy was still obtained by the single scattering expression of the QCD parton model, since rescatterings were not taken into account[3,10]. While the single scattering expression is in fact proportional to the average number of partonic collisions (rescatterings included)[7], the measurable physical quantities are rather related to the wounded partons, namely the partons that have suffered at least one semi-hard interaction. With an analogous observation, measurable physical quantities in heavy-ion collisions where obtained, in Ref.14, from averages involving the wounded nucleons. In a regime where the average number of partonic rescatterings is sizeable, the average transverse energy is not any more linked to the average number of partonic collisions in a simple way. The procedure we have followed, to estimate the average transverse energy, has been to evaluate the average transverse momentum acquired by each wounded parton, after several rescatterings with the target and to multiply it by the average number of partons. The main result is that the average transverse momentum acquired by each wounded parton, when rescatterings have been included, has a much smoother dependence on the cut off p_t^{min} than previous estimates that did not include any rescattering. It is in fact expressed by Eq.(14) and (12) and it is a function of the $\left[ln\left(s/(p_t^{min})^2\right)\right]^{1/2}$. The transverse energy spectrum that follows becomes less and less sensible to the choice of the cut off at larger and larger C.M. energies as a consequence. It may be noted that, in the present case, the cut off dependence is softened by the unitarization of the semi-hard parton-nucleus interaction that induces, in the wounded parton, a random walk in transverse momentum though successive rescatterings.

We point out that there is a kinematical condition which is always present in our treatment, i.e. the requirement that the transverse momenta have to be small with respect to the longitudinal ones (see also appendix A). This gives a limit of applicability to the results: they cannot refer to the very central rapidity part of the partonic spectrum, because, in that region, the longitudinal momenta of both the interacting partons are of the order of the transverse momenta. Moreover, in the central rapidity region, the radiation process, that we have neglected on the grounds that it represents a higher order correction to the elementary partonic interaction, plays its major role. The inclusion in our scheme of the radiation processes is a non trivial step and it will not be addressed here. As far as the kinematical limitations we remark that the central rapidity region, where our treatment is not an adequate one, does not grow with the C.M. energy of the reaction. As a consequence, when integrating over all the kinematical range, as we have done in obtaining the transverse energy spectrum (Eq.17), our error becomes relatively less important when increasing the C.M. energy.

On the purpose to give a quantitative indication on the regime of interest for the present analysis, we have done some numerical estimates of the transverse energy spectrum as given by Eq.(17). The averages $\langle E_t(\beta)\rangle$ and $D_t(\beta)$ are evaluated using Eq.(16). We made an approximate estimate of $\langle n(\beta)\rangle$ keeping into account disconnected partonic collisions only. More explicitly:

$$\langle n(\beta)\rangle = \int \sum_{ff'} \Gamma_A^f(x,\mathrm{b})\Gamma_B^{f'}(x',\mathrm{b}-\beta)\hat{\sigma}^{ff'}(xx')dxdx'd^2b \tag{18}$$

For the average number of partons $\Gamma^f(x,\mathrm{b})$ we have used the factorized expression:

$$\Gamma^f(x,\mathrm{b}) = AG^f(x)\frac{3}{2\pi R^3}\sqrt{R^2-b^2}\,\theta(R^2-b^2)$$

where $G^f(x)$ is the average number of partons with flavour f and fractional momentum x in a nucleon, A is the atomic mass number and the dependence on b corresponds to a uniform spherical distribution. R is the nuclear radius and we take $R = r_0A^{1/3}$ with $r_0 = 1.12fm$. Effectively we have considered the case of two flavors only, gluons and quarks, the elementary cross sections being equal apart from a relative scale factor. In performing the calculations we have used the parton distributions from ref.15. As a scale factor in the parton distributions and in α_s we have used $Q^2 = (p_t^{min})^2$. In the elementary partonic interaction a $k = 2$ factor has been assumed.

The results are presented in two figures: In the first one the differential cross section as a function of the transverse energy produced is shown in the case of Pb against Pb with a C.M. energy of $100GeV$ per nucleon and for two different cut off values $p_t^{min} = 4GeV$ (dashed curve) and $p_t^{min} = 2GeV$ (continuous curve). In the second figure the same cross sections is plotted after increasing the C.M. energy to $1TeV$ per nucleon. While at $100GeV$ the dependence on the cut off is dramatic, when the TeV scale is reached most of the spectrum is rather insensitive to the choice of the cut off.

Our conclusion is that at C.M. energies of the order of $1TeV$ per nucleon a value of p_t^{min} of $2-4GeV$ can be considered as an infrared cut off, with the meaning that most of the contribution to the transverse energy spectrum from semi-hard partonic collisions shows little sensitivity to this choice.

Appendix A

When we wish to take into account the effect of multiple scattering among partons we encounter the following problem: even if we start by taking all partons aligned, i.e. we neglect their intrinsic transverse momenta,we are not allowed to neglect the transverse momentum they acquire in the successive hard scatterings. In fact the interpretation we suggest is that the final transverse momentum is precisely built up in many hard-scattering processes, but, if this happens for the parton which in the description acts as projectile, the same happens for the partons playing the role of targets. The barycentric frame of the hadronic system is not the barycentic frame of the colliding partons, because they may have different fractional momenta x, x'. In fact we may even expect that the main contribution to the rate of the whole process will come from the configurations where either x or x' is small, because there the parton density is larger. While in the simplest situations the two frames are related by a purely longitudinal Lorentz transformation, so that the transverse momenta remain unchanged, this is not the present case. Fortunately in all the processes the momentum transfer, although large with respect to the intrinsic transverse momentum, remains, in the mean, small in comparison to the longitudinal momenta of the two partons when we go in a frame where the energies of the two scattering partons are equal; it is therefore meaningful to distinguish between "forward" and "backward" particles. It is convenient to follow one "forward" parton in its subsequent scatterings by "backward" partons.

After some collisions the parton carries some transverse momentum,so its four momentum is $p_\mu = (E; P_a, -E)$ $(a = 1, 2)$ up to terms of the order P^2/E. It scatters against a backward,probably not very energetic parton,which already suffered some scattering and has,therefore, a fourmomentum $q_\mu = (q_0; Q_a, q_3)$, possibly with $q_0, q_3 \simeq Q_a$. By means of a longitudinal boost we can reach the frame F where the two fourvectors become:

$$p'_\mu = (E'; P_a, -E')$$
$$q'_\mu = (E'; Q_a, E')$$

with $(2E')^2 = (2E + q_0 - q_3)(q_0 + q_3)$, and we consider only the processes with $E' >> Q_a, P_a$. By means of a Lorentz transformation one can eliminate,to first order,the transverse components of both p and q. We write

$$p''_\mu = p'_\mu + \epsilon_{\mu\nu} p'^\nu$$
$$q''_\mu = q'_\mu + \epsilon_{\mu\nu} q'^\nu$$

with

$$\epsilon_{a0} = -\epsilon_{0a} = (Q_a + P_a)/2E'$$
$$\epsilon_{a3} = -\epsilon_{3a} = (Q_a - P_a)/2E'$$
$$\epsilon_{ab} = 0$$
$$\epsilon_{03} = 0.$$

We get $P''_a = 0$ and $Q''_a = 0$ and we are,up to second order terms,in the barycentric frame B .Here a further scattering takes place and the two momenta become,up to second-order corrections:

$$k_\mu = (E'; K_a, -E')$$
$$l_\mu = (E'; L_a, E')$$

When we work out their components in the frame F,the result is:

$$k'_\mu = (E'; K_a + P_a, -E')$$
$$l'_\mu = (E'; L_a + Q_a, E')$$

From this we learn that, for a forward parton, the transverse momenta of the backward partons have no effect,within the kinematical limitations already discussed,whereas the momenta acquired in the subsequent collisions add up,as intuitively expected. Clearly the same result holds symmetrically for the backward partons. This explains why in the multiparton scattering we can ignore the previous history of the target partons, provided in every process the transverse energy be small with respect to the longitudinal energy.

Appendix B

The final expression for the inclusive spectrum seems to be well defined also for very small values of the transverse momentum. However we must remember that an infrared cutoff is necessary to define the theory, our aim was not to ignore it but to find the conditions in which the dependence on this arbitrary parameter is less important. The expression of the inclusive one-particle spectrum contains a delta singularity with negative coefficient which requires an interpretation.This may be obtained giving a spread to the distribution,the easiest form is a Gaussian shape:

$$\mathcal{F}_K(p) = \frac{1}{\pi K^2} \int F(q) e^{-(p-q)^2/K^2} dp$$
$$\lim_{K \to 0} \mathcal{F}_K(p) = F(p)$$

So the singularity is now transformed into a regular expression,which is still negative. The request that the spectrum remains positive puts conditions on the minimum allowed spread. To evaluate it we take the expression for the spread out spectrum around $p = 0$,where the situation is the worst;by approximating $\tilde{\sigma}(u)$ with $\hat{\sigma} - Cu^2$ we get for K the condition

$$K^2 > 4Ce^{-\int\Gamma\hat{\sigma}}\left(1-e^{-\int\Gamma\hat{\sigma}}\right)^{-1}$$

In the small cutoff limit both C and the exponent diverge (actually the exponent faster than C) and so the limitation tends to the trivial form $K > 0$. A more dangerous configuration may arise when one looks at the border of the hadronic system,where Γ is very small,we must however remember that in these configurations all the resummation procedure becomes questionable, because the mean number of collisions may become less than one. The relevance of these configurations is expected not to be very large,at least for large nuclei because the area relevant for them grows as $A^{1/3}$ while the total area grows as $A^{2/3}$.

Appendix C

In this appendix we present a few remarks on parton correlations. We start from the average number of parton pairs with transverse momenta $\mathbf{K}$ and $\mathbf{K}'$ in the final state $\mathcal{D}(\mathbf{K},\mathbf{K}')$. The expression is the straightforward generalization of Eq.(4):

$$\begin{aligned}\mathcal{D}(\mathbf{K},\mathbf{K}') = &\int d^2\beta \sum_{n=1}^{\infty}\frac{1}{n!}\Gamma_A(x_1,\mathbf{b}_1)\ldots\Gamma_A(x_n,\mathbf{b}_n)e^{-\int\Gamma_A(x,\mathbf{b})dxd^2b}\\ &\times\sum_{l=1}^{\infty}\frac{1}{l!}\sum_{\substack{r,t=1\\ t\neq r}}^{l}\Gamma_B(x_1',\mathbf{b}_1'-\beta)\ldots\Gamma_B(x_l',\mathbf{b}_l'-\beta)e^{-\int\Gamma_B(x',\mathbf{b}')dx'd^2b'}\\ &\times\sum_{vw}^{n}\binom{n}{v_r}\binom{n}{w_t}\frac{d^2\hat{\sigma}_{r,1}}{d^2\mathbf{k}_1}\ldots\frac{d^2\hat{\sigma}_{r,v_r}}{d^2\mathbf{k}_{v_r}}(1-\hat{\sigma}_{r,v_r+1})\ldots(1-\hat{\sigma}_{r,n})\\ &\times\delta^2\left(\mathbf{K}-\sum\mathbf{k}_i\right)\frac{d^2\hat{\sigma}_{t,1}}{d^2\mathbf{k}_1'}\ldots\frac{d^2\hat{\sigma}_{t,w_t}}{d^2\mathbf{k}_{w_t}'}(1-\hat{\sigma}_{t,w_t+1})\ldots(1-\hat{\sigma}_{t,n})\\ &\times\delta^2\left(\mathbf{K}'-\sum\mathbf{k}_j'\right)dx_1d^2b_1\ldots dx_nd^2b_ndx_1'd^2b_1'\ldots dx_l'd^2b_l'\\ &\times\prod d^2k_id^2k_j'\end{aligned} \qquad (C.1)$$

This expression can be elaborated along the same ways already used in order to discuss in detail the dispersion in the number of wounded partons in Ref.7 (appendix B). Beyond the combinatorial complications,what is explicitly said is that there are some forward partons each of which is scattered by more than one backward partons on the same line of flight.In this situation the zero range approximation of the hard interaction in no longer allowed. The two-body spectrum is in fact decomposed into a factorized part plus a non factorizable term:

$$\begin{aligned}\mathcal{D}(\mathbf{K},\mathbf{K}') = \int d^2\beta dx' dx'' d^2b' d^2b'' \Gamma_B(x',\mathbf{b}'-\beta)\Gamma_B(x'',\mathbf{b}''-\beta) \\ \times \frac{1}{(2\pi)^4}\int d^2u d^2u' e^{-i\mathbf{K}\cdot\mathbf{u}-i\mathbf{K}'\cdot\mathbf{u}'}\Big[F(\mathbf{u},\mathbf{u}')+N(\mathbf{u},\mathbf{u}')\Big]\end{aligned} \qquad (C.2)$$

The term F is dealt with as in the previous calculations and is brought to the form:

$$\begin{aligned}F(\mathbf{u},\mathbf{u}') =& \Big\{exp\Big[\int dx\big(\tilde{\sigma}(\mathbf{u})-\hat{\sigma}\big)\Gamma_A(x,\mathbf{b}')\Big] - exp\Big[-\int dx\hat{\sigma}\Gamma_A(x,\mathbf{b}')\Big]\Big\} \\ &\times\Big\{exp\Big[\int dx\big(\tilde{\sigma}(\mathbf{u}')-\hat{\sigma}\big)\Gamma_A(x,\mathbf{b}'')\Big] - exp\Big[-\int dx\hat{\sigma}\Gamma_A(x,\mathbf{b}'')\Big]\Big\}.\end{aligned} \qquad (C.3)$$

The term N requires a better treatment, because, in the zero range limit for the interaction, we would get terms like $exp\big[\delta(b-b')\big]$. In practice, since the interaction range τ, which depends on p_t^{min}, is anyhow small with respect to the typical dimensions of Γ, we can write:

$$\hat{\sigma}(b-b',x,x') \approx \theta\Big(2\pi(b-b')^2-\tau\Big)$$

and using the relation:

$$exp\Big[A\theta\Big(2\pi(b-b')^2-\tau\Big)\Big]-1 = \theta\Big(2\pi(b-b')^2-\tau\Big)\Big[e^A-1\Big]$$

the non factorized term can be put into the form:

$$
\begin{aligned}
N(\mathbf{u},\mathbf{u}') =&\theta\Big(2\pi(b''-b')^2-\tau\Big)\sum_{cd=1}^{2}(-)^{c+d}\\
&\times exp\Big[\int dx s_c(\mathbf{u})\Gamma_A(x,\mathbf{b}')+\int dx s_d(\mathbf{u}')\Gamma_A(x,\mathbf{b}'')\Big]\\
&\times\Big\{exp\Big[\int dx s_c(\mathbf{u})s_d(\mathbf{u}')\Gamma_A(x,\mathbf{b}')/\tau\Big]-1\Big\}
\end{aligned}
\tag{C.4}
$$

with

$$
\begin{aligned}
s_1(\mathbf{u}) &= \tilde{\sigma}(\mathbf{u})-\hat{\sigma}\\
s_2(\mathbf{u}) &= -\hat{\sigma}
\end{aligned}
$$

The term N may be considered perturbative,because it is directly proportional to the elementary hard cross section,in fact, within the approximation, the term θ is the same as a term $\tau\times\delta$. This correlation term is produced by the dynamics even if one starts from a totally uncorrelated parton distribution,as in the present case.

More in general intrinsic correlations may be found,which exist already at the level of parton distributions,in this case,however,the typical size of the correlation should be provided by the hadron scale because we expect that one to be the dimension characterizing the non perturbative dynamics. As a consequence, for any acceptable value of the cutoff, we expect the dynamical correlations,of the type discussed above, to be overwhelmed by the intrinsic correlations,if they are there. A general treatment of the intrinsic correlations among parton could be performed,e.g.,by means of the functional formalism for the parton distributions discussed elsewhere[11].

The two body correlations of the partons could,clearly,influence the observed quantities, but the process of hadronization can heavily blur the signal. It must be noted, however, that the correlations influence also less detailed observables, like the transverse energy. If we start from the set of exclusive cross sections S defined in such a way that:

$$\sum_n \frac{1}{n!} \int S_n(p_1 \dots p_n) \prod dp = \sigma_H$$

and:

$$\sum_n \frac{n}{n!} \int S_n(p_1, p_2 \dots p_n) \prod_{i \geq 2} dp_i = \mathcal{F}(p_1);$$
$$\sum_n \frac{n(n-1)}{n!} \int S_n(p_1, p_2, p_3 \dots p_n) \prod_{i \geq 3} dp_i = \mathcal{J}(p_1, p_2)$$

are the one-body and the two-body inclusive spectra,we get for the mean transverse energy:

$$\langle E \rangle \sigma_H = \int \mathcal{F}(p) p dp = \langle p \rangle \langle n \rangle \sigma_H$$

and for the mean squared energy:

$$\begin{aligned} \langle E^2 \rangle \sigma_H &= \int \mathcal{F}(p) p^2 dp + \int \mathcal{J}(p_1, p_2) p_1 p_2 dp_1 dp_2 \\ &= \Big[\langle p^2 \rangle \langle n \rangle + \langle p_1 p_2 \rangle \langle n(n-1) \rangle \Big] \sigma_H \end{aligned}$$

If the two body spectrum is not factorized, then we must write:

$$\langle p_1 p_2 \rangle = \langle p \rangle^2 + C_2$$

and finally the dispersion of the energy contains a term directly related to the correlations:

$$\langle E^2 \rangle - \langle E \rangle^2 = \Big(\langle p^2 \rangle - \langle p \rangle^2 \Big) \langle n \rangle + \langle p \rangle^2 \left(\langle n^2 \rangle - \langle n \rangle - \langle n \rangle^2 \right) + C_2 \langle n(n-1) \rangle. \qquad (C.5)$$

References

1. M. Jacob and P.V. Landshoff, *Mod. Phys. Lett.* **A1** (1986) 657.
2. G. Pancheri and Y. Srivastava *Phys. Lett.* **B182** (1986)199; S. Lomatch, F.I. Olness and J.C. Collins *Nucl. Phys.* **B317**(1989)617.
3. K.J. Eskola, K. Kajantie, and J. Lindfors, *Nucl. Phys.* **B323**(1989) 37.
4. H.D. Politzer, *Nucl. Phys.* **B172** (1980) 349; R.K. Ellis, R. Petronzio and W. Furmanski *Nucl. Phys.* **B207** (1982) 1.
5. N. Paver and D. Treleani, *Nuovo Cimento* **A70** (1982)215; *Z. Phys.* **C28** (1985)187; B. Humpert, *Phys. Lett.* **B131** (1983)461; B. Humpert and R. Odorico, *Phys. Lett.* **B154** (1985)211; T. Sjostrand and M. Van Zijl, *Phys. Rev.* **D36** (1987)2019.
6. L. Durand and H. Pi, *Phys. Rev. Lett.* **58**(1987)303; A. Capella, J. Tran Thanh Van and J. Kwiecinski, *Phys. Rev. Lett.* **58**(1987)2015; B. Margolis, P. Valin, M.M. Block, F. Halzen and R.S. Fletcher, *Phys. Lett.* **B213** (1988) 221; W.R. Chen and R.C. Hwa, *Phys. Rev.* **D39** (1989)179; T.K. Gaisser and T. Stanev, *Phys. Lett.* **B219** (1989)375.
7. G. Calucci and D. Treleani, *Phys. Rev.* **D41** (1990) 3367.
8. Ll. Ametller and D. Treleani, *Int. J. Mod. Phys.* **A3** (1988) 521.
9. E. Gotsman and D. Lissauer, *Phys. Lett.* **B245** (1990) 258.
10. K. Kajantie, P.V. Landshoff and J. Lindfors, *Phys. Rev. Lett.* **59**(1987) 2527.
11. G. Calucci and D. Treleani, *Nucl. Phys.* B (Proc. Suppl.) 18C (1990) February 1991, p.187.

12) L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Reports **100** (1983) 1.

13. V.A. Abramovskii, V.N. Gribov and O.V. Kancheli, Yad. Fiz. **18** (1973) 595, (Sov. J. Nucl. Phys. **18** (1974) 308); J.L. Cardy and G.A. Winbow, Phys.

Lett. **52B**(1974)95; C.E. De Tar, S.D. Ellis and P.V. Landshoff, Nucl. Phys. **B87** (1975) 176.

14. A. Bialas, M. Bleszynski and W. Czyz, *Nucl. Phys.* **B111** (1976)461.

15) P.N. Harriman, A.D. Martin, R.G. Roberts, W.J. Stirling, preprint DTP-90-04.

Figure Captions

Fig 1. Contribution to the transverse energy spectrum from semi-hard partonic collisions in Pb against Pb with a C.M. energy of 100 GeV per nucleon. The continuous curve refers to a cut off $p_t^{min} = 2GeV$ while the dashed curve to a cut off $p_t^{min} = 4GeV$.

Fig 2. Same as in Fig. 1 after increasing the C.M. energy to the value of 1 TeV per nucleon.

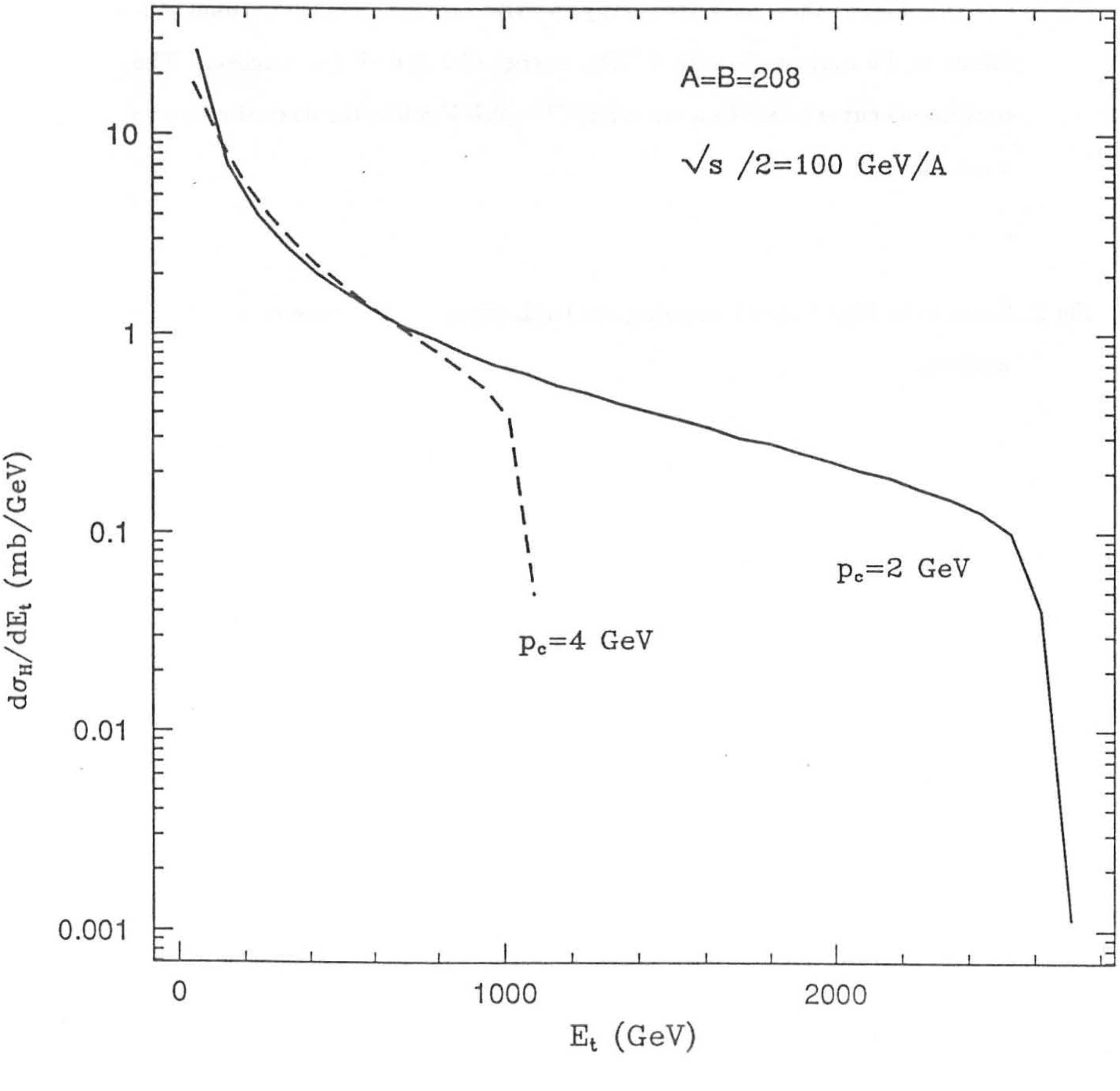
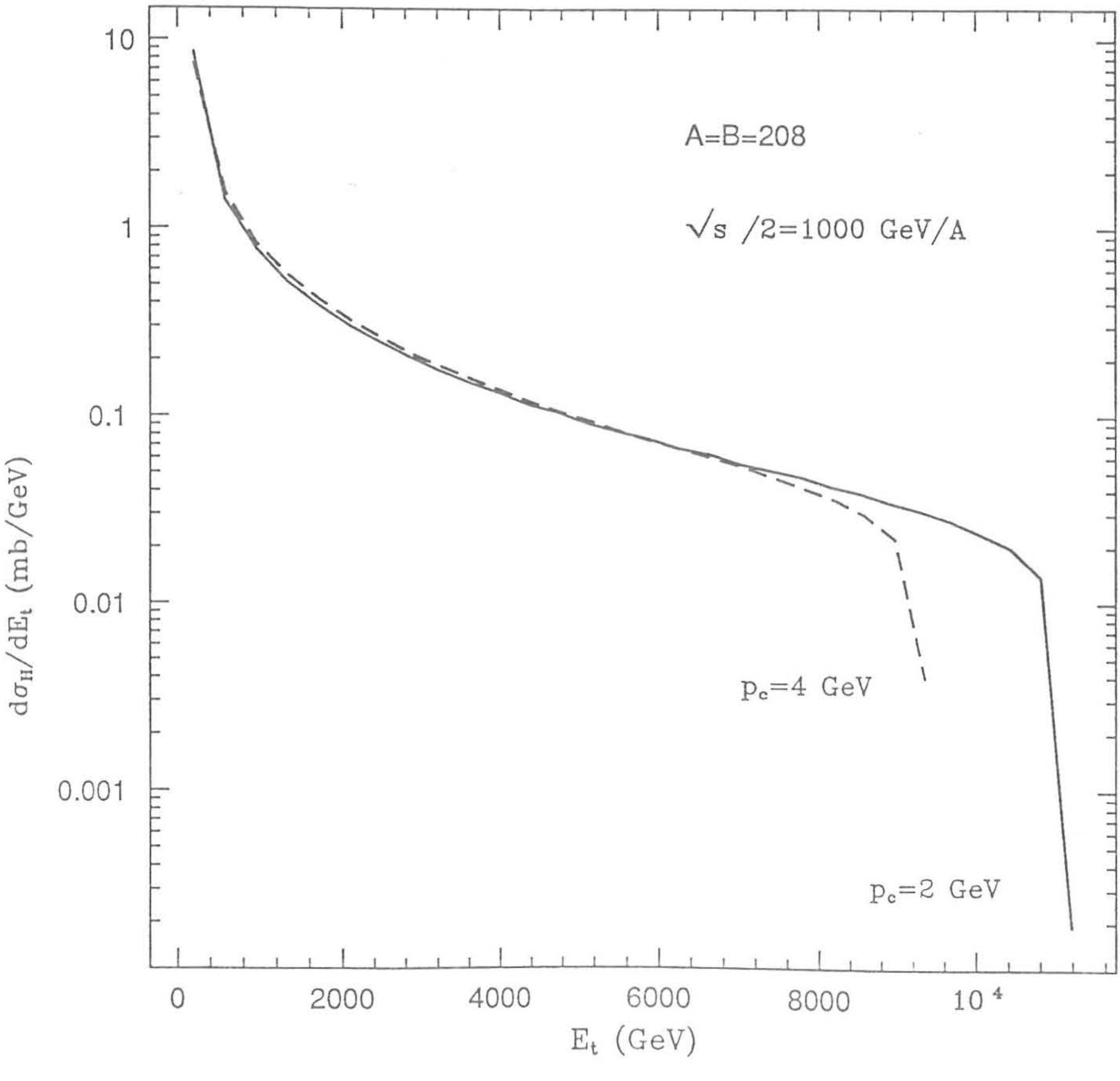

A=B=208
√s /2=100 GeV/A
p_c=2 GeV
p_c=4 GeV
$d\sigma_H/dE_t$ (mb/GeV)
E_t (GeV)
10
1
0.1
0.01
0.001
0
1000
2000

Fig. 1



A=B=208
√s /2=1000 GeV/A
p_c=4 GeV
p_c=2 GeV
10
1
0.1
0.01
0.001
dσ_II/dE_t (mb/GeV)
0
2000
4000
6000
8000
10 ⁴
E_t (GeV)

Fig. 2