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How large can be direct CP violation in $K \to 3\pi$ from Chiral Perturbation Theory?

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Abstract

Using a general parametrization, we discuss the size of direct CP violation in charged $K \to 3\pi$ decays, as it should be expected from chiral perturbation theory including chiral corrections to order p^4. These terms are required in order to remove the $\Delta I = 3/2$ suppression. We argue that the magnitude of these effects for the slope asymmetry, although enhanced by the $\Delta I = 1/2$ rule, should be at most of the order of 10^{-5}.

In the Standard Model of weak interactions direct CP violation is predicted to be different from zero. As an alternative to $K \to 2\pi$ decays, where so far the experimental results are contradictory,[1,2] it is interesting to study the charge asymmetries in the decays $K^{\pm} \to 3\pi$,[3,4] which are nonvanishing only if there is direct CP violation. These studies seem very suitable at a Φ-factory, where with $10^{10} \div 10^{11}$ Φ's one could expect a statistical error on the asymmetry of about $10^{-3} \div 10^{-4}$.[5]

In $K \to 2\pi$ there are only two independent isospin amplitudes, namely $A_{I=0}$ and $A_{I=2}$. Therefore, direct CP violation is suppressed in this mode by a factor 1/22 since it can occur only through the interference between these two amplitudes. In $K^{\pm} \to 3\pi$ there are three independent amplitudes, i.e. $A_{I=1S}$, $A_{I=1M}$ and $A_{I=2}$. Thus one might hope to overcome the 1/22 suppression by the interference between the two $\Delta I = 1/2$ amplitudes. In what follows we discuss this aspect of direct CP violation in the general framework of Chiral Perturbation Theory (CHPT). We find results different from those obtained by previous authors,[6] and we discuss the possible origin of this difference.

Making reference to the conventional parametrization of the Dalitz plot distribution[7] of $K \to 3\pi$

$$|A(K \to 3\pi)|^2 \propto 1 + gY + jX, \tag{1}$$

one can define two CP-odd observables, i.e. the partial rate asymmetry:

$$\Delta\Gamma = \frac{\Gamma(K^+ \to 3\pi) - \Gamma(K^- \to 3\pi)}{\Gamma(K^+ \to 3\pi) + \Gamma(K^- \to 3\pi)}, \tag{2}$$

and the slope asymmetry:

$$\Delta g = \frac{g(K^+ \to 3\pi) - g(K^- \to 3\pi)}{g(K^+ \to 3\pi) + g(K^- \to 3\pi)}. \tag{3}$$

In Eq.(1) we have defined the Dalitz variables $Y = \frac{s_3 - s_0}{m_\pi^2}$ and $X = \frac{s_1 - s_2}{m_\pi^2}$, where $s_i = (p - p_i)^2$ with p and p_i the four-momenta of the kaon and of the pion i ($i = 3$ indicates che "odd charge" pion), and $s_0 = \frac{1}{3}(s_1 + s_2 + s_3)$.

We will concentrate on the decay channel $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$, which can be directly related to $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ by using the isospin decomposition,[8] and in particular on the slope asymmetry Δg where the CP-odd effect should be larger.

The isospin decomposition of $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$ decays up to linear terms, in the notation of Refs.[7,9,10], is written as follows:

$$\begin{aligned} A(K^+ \to \pi^+\pi^+\pi^-) &= (2\alpha_1 - \alpha_3)e^{i\delta_1} + (\beta_1 - \frac{1}{2}\beta_3)e^{i\delta_M}Y + \sqrt{3}\gamma_3 e^{i\delta_2}Y, \\ A(K^- \to \pi^-\pi^-\pi^+) &= (2\alpha_1^* - \alpha_3^*)e^{i\delta_1} + (\beta_1^* - \frac{1}{2}\beta_3^*)e^{i\delta_M}Y + \sqrt{3}\gamma_3^* e^{i\delta_2}Y, \end{aligned} \tag{4}$$

where α_i, β_i and γ_i correspond to the three different final states: $I = 1$ symmetric, $I = 1$ with mixed symmetry and $I = 2$. The subscript $i = 1,3$ refers to the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions respectively. We have included the phases due to final state strong interactions, which are necessary in order to induce nonvanishing CP violating asymmetries. These phases are expected to be very small, due to the smallness of the available phase space: indeed they have been estimated both in the non relativistic limit[11] and in CHPT.[4] The two calculations coincide at the center of the Dalitz plot, and the result is:

$$\delta_1 - \delta_M = \frac{\delta_1 - \delta_2}{2} \simeq .07 \tag{5}$$

From Eq.(4) we obtain:

$$\Delta g = \frac{Im[(2\alpha_1 - \alpha_3)^*(\beta_1 - \frac{1}{2}\beta_3)]\sin(\delta_1 - \delta_M) + Im[(2\alpha_1 - \alpha_3)^*\sqrt{3}\gamma_3 \sin(\delta_1 - \delta_2)]}{Re[(2\alpha_1 - \alpha_3)^*(\beta_1 - \frac{1}{2}\beta_3)]\cos(\delta_1 - \delta_M) + Re[(2\alpha_1 - \alpha_3)^*\sqrt{3}\gamma_3 \cos(\delta_1 - \delta_2)]} \tag{6}$$

The constants α, β and γ must be estimated in a theoretical model. Since $K \to 3\pi$ is a low energy process involving would-be Goldstone bosons of the strong interaction symmetry $SU(3)_L \times SU(3)_R$, the natural framework to estimate the relevant hadronic matrix elements $< 3\pi|H_W|K >$, where H_W is the effective electroweak nonleptonic Hamiltonian, is represented by CHPT.[12] In this approach transition amplitudes are expanded in powers of pseudoscalar meson masses and momenta, by means of phenomenological chiral Lagrangians[13,10]. Such Lagrangians are particularly convenient computational tools to evaluate hadronic matrix elements in agreement with the low energy theorems of current algebra and PCAC, and allow the extrapolation of these theorems to higher orders in momenta consistently with the chiral symmetry of strong interactions.

At the leading order p^2, neglecting electromagnetic corrections, there are only two $\Delta S = 1$ meson operators: one octect for $\Delta I = 1/2$ transitions and a 27-plet for $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions. These operators relate the $K \to 3\pi$ amplitudes to the $K \to 2\pi$ amplitudes both for the CP conserving and the CP violating parts. The fact that there are only two operators implies that there is only one relative CP violating (electroweak) phase, so that we cannot have, at this order p^2, any interference between the two unsuppressed $\Delta I = 1/2$ amplitudes. Consistently with the usual parametrization of the CKM matrix, we attribute this phase to the octect operator, and accordingly we have:

$$\frac{ImA_0}{ReA_0} = \frac{Im\alpha_1}{Re\alpha_1} = \frac{Im\beta_1}{Re\beta_1}. \tag{7}$$

$\dfrac{ImA_0}{ReA_0}$ is related to the CP violating parameter ϵ' of $K \to 2\pi$:

$$\epsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} Im\left(\frac{A_2}{A_0}\right) \simeq -\frac{e^{i\pi/4}}{\sqrt{2}}\omega\frac{ImA_0}{ReA_0}(1-\Omega_t), \tag{8}$$

where $\omega = \dfrac{ReA_2}{ReA_0} \simeq \dfrac{1}{22}$, and Ω_t takes into account isospin breaking and electromagnetic effects, which we will neglect.* Taking $Re(\epsilon'/\epsilon) \simeq 2.3 \times 10^{-3}$ from Ref.[1], and using it as an upper bound, we obtain

$$\left|\frac{ImA_0}{ReA_0}\right| \lesssim 1.6 \times 10^{-4}. \tag{9}$$

Then, using lowest order results of CHPT for $K^\pm \to 3\pi$ and eq.(5), expression (6) for Δg reduces to

$$|\Delta g| \simeq (\delta_1 - \delta_M)\frac{\omega}{\sqrt{2}}\left|\frac{ImA_0}{ReA_0}\right|\left(\frac{63}{4} + \frac{9}{4}\frac{m_\pi^2}{m_K^2 - m_\pi^2}\right) \lesssim 0.7 \times 10^{-5}. \tag{10}$$

We emphasize that the large coefficient $\dfrac{63}{4}$ is just due to a large $\Delta I = 3/2$ Clebsch-Gordan coefficient.

Since in the CP conserving amplitudes there is a 20% − 30% discrepancy between lowest order theoretical predictions of CHPT and experimental results,[7] it is important to consider higher order corrections both in the real and in the imaginary parts of the amplitudes. By power counting we see that the numerator of eq.(6) is at least of order p^6, resulting from an order p^2 for each amplitudes and an extra p^2 for the final state interaction phases δ. The next order, which will include loops and counterterms, is of order p^8. Since higher order operators can have different electroweak phases one might hope to beat the 1/22 suppression factor by chiral corrections. For this reason

* These corrections may be relevant only in the interference between $\Delta I = 3/2$ and $\Delta I = 1/2$ amplitudes.[14] Since we are interested only in the order of magnitude of Δg we can disregard them.

we will concentrate our analysis only on the $\Delta I = 1/2$ amplitudes (so that in what follows we drop the subscript 1 in α and β).

We write any of the $K \to 3\pi$ amplitudes α or β in the following form:

$$A = A_{tree}^{(2)+(4)} + A_{loop}^{(4)+(6)}, \tag{11}$$

where superscripts (2), (4) and (6) denote the order in the chiral expansion and of course the *tree* amplitudes include also the order p^4 counterterms. To separate out the effect of final state strong interactions, we further decompose A_{loop} into absorptive plus dispersive parts:

$$A_{loop}^{(4)+(6)} = iA_{abs}^{(4)+(6)} + A_{disp}^{(4)+(6)}, \tag{12}$$

so that

$$\begin{aligned} \alpha_{abs}^{(4),(6)} &= \delta_1 \alpha_{tree}^{(2),(4)}, \\ \beta_{abs}^{(4),(6)} &= \delta_M \beta_{tree}^{(2),(4)}. \end{aligned} \tag{13}$$

As a consequence of the fact that there is only one $\Delta I = 1/2$ operator at order p^2, we have the following identities:

$$\frac{Im\alpha_{tree}^{(2)}}{Re\alpha_{tree}^{(2)}} = \frac{Im\alpha_{disp}^{(4)}}{Re\alpha_{disp}^{(4)}} = \frac{Im\alpha_{abs}^{(4)}}{Re\alpha_{abs}^{(4)}} = \frac{Im\beta_{tree}^{(2)}}{Re\beta_{tree}^{(2)}} = \frac{Im\beta_{disp}^{(4)}}{Re\beta_{disp}^{(4)}} = \frac{Im\beta_{abs}^{(4)}}{Re\beta_{abs}^{(4)}}. \tag{14}$$

Using these relations one can easily verify that at order p^8, extracting the final state interaction $O(p^2)$, only the interference between $A_{tree}^{(2)}$ and $A_{tree}^{(4)}$ remains in the numerator of Δg in Eq.(6). We observe that $A_{loop}^{(4)}$ does not contribute, because it has the same electroweak phase as $A_{tree}^{(2)}$, and $A_{disp}^{(6)}$ contributes only at $O(p^{10})$.

Finally, we obtain:

$$\Delta g = (\delta_1 - \delta_M)\frac{ImA_0}{ReA_0}\left[\frac{Re\beta^{(4)}}{Re\beta^{(2)}} - \frac{Re\alpha^{(4)}}{Re\alpha^{(2)}} + \frac{Im\alpha^{(4)}}{Im\alpha^{(2)}} - \frac{Im\beta^{(4)}}{Im\beta^{(2)}}\right], \tag{15}$$

where for simplicity we have dropped the subscript "*tree*" in α and β.

We know from experiments that the chiral expansion works pretty well for the CP conserving amplitudes, implying that $\left|\frac{Re\beta^{(4)}}{Re\beta^{(2)}}\right|, \left|\frac{Re\alpha^{(4)}}{Re\alpha^{(2)}}\right| \leq 1$.

Now, if we make the plausible assumption that the chiral expansion also works for the CP violating amplitudes, we see that

$$|\Delta g| \leq 4.5 \times 10^{-5}. \tag{16}$$

This assumption, though not verified yet by experiments, is justified theoretically if the electroweak phase arises mainly from the gluon penguin operator,[15] which separately satisfies current algebra relations together with the current-current quark operators.[9]

Our estimate for Δg is consistent with the conclusions of Refs.[16,17], but not with Ref.[6] which gives a result about 30 times larger. The reason of this discrepancy could be that in Ref.[6] the weak mesonic Lagrangian is derived by applying a hadronization procedure to the quark effective nonleptonic Lagrangian. It appears from Eqs.(13) and (10) of Ref.[6] that the resulting matrix element of the gluon penguin operator $< \pi\pi|O_5|K >$ does not vanish in the flavour-$SU(3)$ limit, in contradiction with the Cabibbo Gell-Mann theorem,[18] and this incorrect chiral behaviour is also present in the matrix element $< \pi\pi\pi|O_5|K >$. In this way the current-current quark operators O_1, O_2, O_3 have different chiral behaviour from O_5, so that there are two different meson operators at order p^2 which could lead to a large interference between two unsuppressed $\Delta I = 1/2$ amplitudes for $K^{\pm} \to 3\pi$. Indeed, in the subsequent Ref.[19] by the same authors, devoted to $K^0 \to 3\pi$ decays, the correct behaviour of O_5 is implemented, while the amplitudes of O_1, O_2 and O_3 remain the same. One can see that in this case the contribution from O_5 becomes different by a factor about 1/20, and that a result for $|\Delta g|$ quite compatible with our Eq.(16) is obtained.

In conclusion, although probably beyond the reach of present experimental capabilities, we believe it still interesting to improve the existing, rather poor limits on Δg (and $\Delta\Gamma$) at a Φ-factory and/or at intense kaon beams, in order to test the prediction in Eq.(16). Any experimental result for the slope asymmetry larger than 10^{-5} (in order of magnitude) is unexpected, and would represent a great surprise for Chiral Perturbation Theory.

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