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RECENT DELPHI RESULTS ON MULTIPLICITY FLUCTUATIONS IN HADRONIC FINAL STATES FROM THE DECAY OF THE Z^0 BOSON¹

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ABSTRACT

An analysis of the fluctuations in the phase space distribution of hadrons produced in the decay of the Z^0 boson has been carried out, using the method of factorial moments. The high statistics collected by the DELPHI experiment at LEP during 1990 (around 80,000 events after the cuts) allowed studies not only for the global event sample, but also in intervals of p_t and multiplicity, and for different jet topologies.

1. Introduction

This paper presents results of the study of fluctuations in the phase space distribution of hadrons produced in e^+e^- collisions around the Z^0 energy, using the DELPHI detector at LEP. It follows, complements and extends to a larger statistics previous studies from the same collaboration [1].

In order to provide a quantitative test of anomalous multiplicity fluctuations (spikes) in variable intervals of rapidity, Bialas and Peschanski[2] proposed in 1986 to analyze the distributions of multiplicity in terms of normalized factorial moments. Given an experimental distribution of particles in the rapidity interval from $-Y/2$ to $Y/2$, the interval Y is divided into M equal subintervals, each of size $\delta y = Y/M$. By defining N to be the number of particles in the whole rapidity interval, n_m to be the number of particles in the m -th bin ($m = 1 \dots M$), the factorial moment of (integer) rank j of the distribution with respect to the partition is defined as

$$F_j(\delta y) = \frac{M^{j-1}}{\langle N \rangle^j} \left\langle \sum_{m=1}^M n_m(n_m-1)\dots(n_m-j+1) \right\rangle \quad (1)$$

where the average is taken over many events. The factorial moment of rank j for a rapidity interval δy acts as a filter for selecting events with j particles or more

¹Presented by A. De Angelis.

in at least one bin and is therefore highly sensitive to events with large density fluctuations.

Simple models representing the hadronization process as a random cascade with selfsimilar structure predict a powerlike increase of the factorial moments in the limit in which the bin size δy goes to zero, i.e.

$$F_j \propto (1/\delta y)^{j^2} \quad (2)$$

and the validity of the above relation was taken by the authors of Ref. [2] as definition of *intermittency*, a term mutated from hydrodynamics, as most of the mathematical techniques used in this field [3]. We will take in the following (2) as definition of intermittency; it should be underlined, however, that a universally accepted definition does not exist in the literature².

The first direct measurement of factorial moments in e^+e^- annihilations was published by the TASSO collaboration [4], at a centre of mass energy of around 35 GeV, and claimed an intermittency effect that could not be explained by the JETSET Parton Shower Monte Carlo [5] (JETSET PS in the following), nor by the Marchesini-Webber [6] and the Hoyer model [7]. The TASSO work was confirming the results of an indirect analysis of the HRS data [8] at $\sqrt{s} = 29$ GeV (more recently, the HRS collaboration has provided a direct analysis [9], whose conclusions are in agreement with the previous one, but no explicit comparison with Monte Carlos is done). The predictions of various models for e^+e^- interactions differ considerably at high energies [10]. This situation motivated an investigation of possible intermittency effects in e^+e^- annihilation at the Z^0 peak. This analysis [1] was carried out by making use of data collected with the DELPHI detector [11] at the e^+e^- storage ring LEP during its first runs at the Z^0 resonance. The results of the DELPHI analysis [1] were essentially that the presence of intermittency, defined as in (2), was questionable when searched in the rapidity distribution of final-state hadrons, evident when searched in a 2-dimensional projection of the phase space, but in any case the results were compatible with the predictions of parton shower models. After that, CELLO [12] reported an agreement with Parton Shower models at the same energy as TASSO, and OPAL [13] and ALEPH [14] at the same energy as DELPHI.

Intermittency has motivated during the last years many theoretical hypotheses and experimental investigations, for which exhaustive reviews have been recently published [15]. Often, theoretical works hypothesize new physics for explaining the phenomenon; many authors pointed out however that self-similar cascading mechanisms [2, 16, 17] or models in which short-range correlations derived from the data are introduced *ad hoc* [18] can reproduce the effect.

²In a vague definition of intermittent are put signals for which a traditional statistical description through probability density function, mean value, root mean square, etc., fails.

3. Event selection

The sample of events used in the analysis was collected by the DELPHI detector at the LEP e^+e^- collider during 1990. A description of the DELPHI detector can be found elsewhere [1]. Features of the apparatus relevant for the analysis of multihadronic final states (with emphasis on the detection of charged particles) are outlined in Ref. [19], as well as the cuts used for the selection of hadronic events.

Only charged tracks reconstructed by means of the central detectors (with polar angle between 25° and 155°) were used in the present analysis. A total of 80,000 events accumulated during 1990 satisfied the cuts. Events due to beam-gas scattering, to $\gamma\gamma$ interactions and to decays of the Z^0 into $\tau^+\tau^-$ have been estimated to be less than 0.3% of the selected sample.

The resolving power of the detector was tested by means of Monte Carlo, and by studying the two-track density as a function of the rapidity y and of the area in the (y, ϕ) space³ in the experimental data. We obtain indications that the limits of our experimental investigations (see below) are well above the experimental resolution for two-track particle density. We did not find any suitable indicator of the goodness of n -track resolution, and thus we assumed this quantity to scale as \sqrt{n} .

The Monte Carlo simulation DELSIM [20] was used to correct the data for the geometrical acceptance, kinematical cuts, resolution, particle interaction with the detector material and other detector imperfections. A sample of Z^0 decays, approximately equal in size to the sample of real events for reasons stated below, was generated with JETSET 6.3 PS [5], and followed through this detailed simulation of the detector. From the samples of accepted and generated events, correction factors

$$C(\delta y) = \frac{F(\delta y)_{\text{generated}}}{F(\delta y)_{\text{accepted}}}$$

were computed. These factors were then used to correct the quantities calculated from the real data. The generated event sample contained all final state particles with a lifetime above 10^{-9} s before any tracking was done through the detector. The accepted event sample contained all final state particles observed after tracking the fully simulated events through the DELPHI detector. Simulated raw data were then processed through the same reconstruction and analysis chain as the real data.

3. Analysis and results

In the following, factorial moments of projections of the phase space of hadrons originated from the decay of the Z^0 are compared with the predictions of QCD-based Monte Carlo programs.

The comparison is mainly done with JETSET PS, that has proven, after one year of activity of LEP, to well reproduce the hadronic final states from the decay of

³The rapidity y and the azimuth ϕ are defined, where not explicitly stated, with respect to the sphericity axis.

the Z^0 , both from the point of view of the description of shape variables [19, 21, 22] and, more important for our study, of multiplicity (global and in restricted intervals of rapidity [23]). Parameters of JETSET PS have been eventually retuned according to Ref. [21]. In some cases, comparisons were made also to other Monte Carlos, based on parton cascades or on the exact second order QCD matrix element followed by string fragmentation. In particular:

1. the ARIADNE [24] Monte Carlo, with parameters optimized as in Ref. [21];
2. the JETSET 7.2 Monte Carlo with a matrix element calculation up to $O(\alpha_s^2)$, and the optimization of parameters described in [25] (JETSET ME retuned).

The essential features of these Monte Carlos are summarized in Ref. [10].

3.1 Projections of Phase Space Onto 1 Dimension

First, we extended the previous DELPHI work to the present statistics, by plotting the factorial moments of the rapidity distribution between -2 and +2 and comparing with the prediction by JETSET PS default, JETSET PS retuned, JETSET ME retuned, ARIADNE. The small deviations with respect to JETSET PS default, for which an indication was existing in the previous paper (i.e., a larger value of the factorial moments with respect to the values predicted by the Monte Carlo), become now statistically significant. The discrepancy does not however go in the direction of a spectacular power-law rise of factorial moments, not reproduced in the Monte Carlo. Data in Fig. 1 are corrected via Monte Carlo for the resolution of the detector; the correction factors differ from 1.0 by less than 5% on average, as one can see in the upper part of the figures. Errors keep into account the effects of an uncertainty of $\pm 2\%$ in the probability of γ conversions in the detector.

The retuning of the fragmentation parameters causes a drastic change both in the magnitude of the factorial moments and in the slope of their distribution for small values of δy . However, even after retuning, the ME model fails to reproduce quantitatively the data. For all ranks, the logarithms of the factorial moments appear to grow as the number of subdivisions increases. A slope for this growth cannot be uniquely defined.

Expression (1) is in principle a biased estimator of the "true" value of the factorial moments. For this reason, the amount of simulated data in Fig. 1 was chosen to be of approximately the same size as the amount of real data. However, the effect of bias has been checked by comparing the factorial moments of a Monte Carlo sample (JETSET PS at generator level) of the size of $\simeq 10,000$ events with a ten times larger sample, and a sample of real data from 10,000 events with a three times larger sample. Results are in good agreement, making us confident that at the Z^0 energy a statistics of $\simeq 10,000$ events is adequate to make the bias negligible up to the factorial moment of rank 5 (at least for projections of the phase space onto one dimension, and maximum number of subdivisions used in this note).

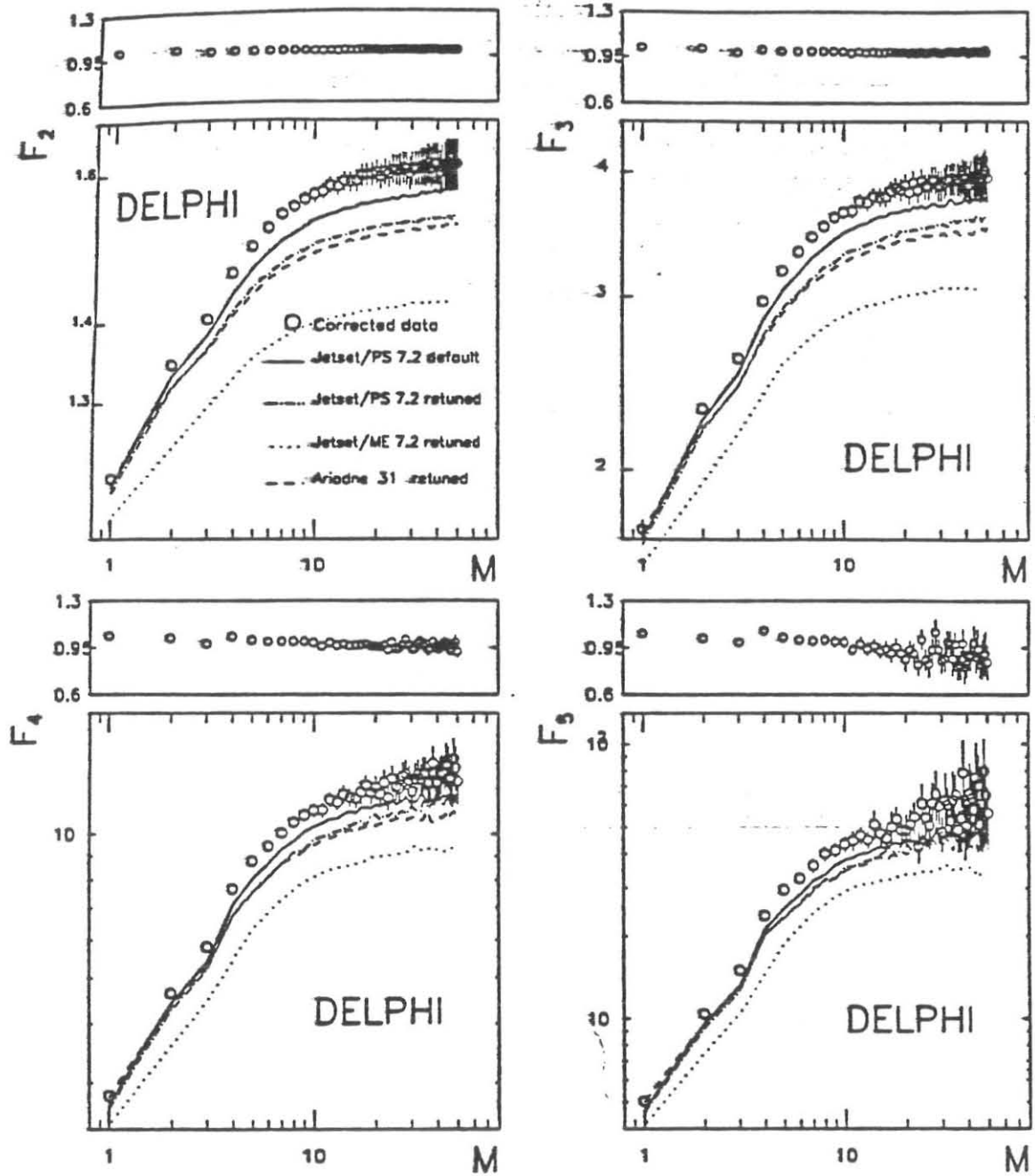


Figure 1: Dependence of the factorial moments of rank 2 (a), 3 (b), 4 (c) and 5 (d) on the number M of subdivisions of the rapidity interval, compared to several Monte Carlos.

The central region of the rapidity interval was chosen because, in this region, the density of particles is almost uniform. One thus avoids by this choice the problem of non-uniform population of the phase space, that can fake the signal from anomalous clusters we are looking for. The price to pay is that one excludes from the study the particles closer to the core of the jet. To overcome this problem, it was recently suggested by Ochs [26] and by Bialas and Gazdzicki [27] to study a distribution \bar{y} that is the y distribution rebinned in such a way that the population is, in average, uniform. Using self-similar models for the hadronization, the authors find in their work that factorial moments defined with respect to this variable follow more closely the power law (2).

The study of factorial moments in the \bar{y} distribution corresponding to the y region between -5 and +5 did not show results qualitatively different with respect to the study of factorial moments of the y distribution.

In the following, unless otherwise stated, we always apply the flattening procedure analyzing the p_t , the multiplicity and the jet topology dependence of factorial moments, working in the rapidity region between -5 and +5: this guarantees a good way to compare the results, being independent of how the choice of the cut can modify the average y distribution.

3.1.1 Dependence on p_t

The NA22 collaboration has recently reported on a striking disagreements between data and hadronic Monte Carlos for the track at low p_t [29]. Although p_t has of course a different physics meaning in hadronic and e^+e^- collisions, we tried the same exercise, mainly:

- To evidence if the description of the p_t sector in JETSET PS is satisfactory.
- Motivated by the fact that the low- p_t region is almost free from effects related to hard gluon radiation, that has been demonstrated [14] to be the cause of the most relevant part of the behavior of factorial moments.

We divided the full p_t range into 3 regions, chosen in such a way that the average number of particles for each region is the same, to remove a possible contamination of the result from mathematical properties of factorial moments. The 3 regions were respectively $p_t < 0.255$, $0.255 < p_t < 0.532$, $0.532 < p_t < 2$. GeV/c.

When dividing the final state particles into p_t slices, it becomes of primary importance, due to the correlation between p_t and y , to apply to the variable y a transformation that flattens the distribution. In each p_t interval the y distribution is in fact far from being flat.

The results obtained are plotted in Fig. 2, and they evidence discrepancies from JETSET PS more pronounced than in the case of no slicing in p_t . This suggests that the treatment of correlations in the p_t sector is not adequate in JETSET PS. The effects of an uncertainty of $\pm 2\%$ in the probability of γ conversions in the detector

are larger in the low- p_t spectrum, as expected from the average small momentum of electrons produced. We find that, at low p_t , the factorial moments follow a power law behaviour in the $\delta\bar{y} < 0.2$ range. The high p_t region presents larger fluctuations, but they saturate at low $\delta\bar{y}$, being compatible with the fact that the first rise is due to the spikes caused by the presence of the hard gluon jets, and fluctuations are then milder when digging inside the 3rd jet.

3.1.2 Dependence on Multiplicity

In the case of the dependence on the multiplicity of the event, it is more difficult to draw physics conclusions, due to the fact that the mathematical effect due to the use of a different number of tracks for computing factorial moments makes the result fuzzy.

The results on factorial moments of different intervals in the observed multiplicity N ($N \leq 15$, $15 < N \leq 20$, $N > 20$) are displayed in Fig. 3 for uncorrected data. The discrepancies are concentrated mainly in the region of low multiplicities, where the contamination from non-hadronic events is larger. Discrepancies are milder when the y distribution, instead than the \bar{y} distribution, is studied.

3.1.3 Dependence on Jet Topology

Jets were defined according to the JADE/E0 algorithm [30], with a y_{cut} value of 0.04. Factorial moments with respect to \bar{y} are plotted in Fig. 4, for 2-jet and 3-jet events. The fact that factorial moments are higher in 3-jet events is compatible with being due to the spike in rapidity caused by hard gluon jets. We do not observe, in any case, striking disagreements with respect to JETSET PS.

It can be of interest to investigate more deeply in 2-jet events, to see if the structure of fluctuations inside a jet is correctly reproduced by JETSET PS. Results from 2-jet events selected by means of a y_{cut} value of 0.01 do not differ qualitatively from the previous ones.

In 3-jet events, selected with $y_{cut} = 0.04$, we calculated also factorial moments for the tracks belonging to jet 1, jet 2 and jet 3, ordered by energy. We calculate the jet energy:

1. by defining for each jet the jet direction as the direction of the sum of the momenta of the tracks clustered in the jet
2. by balancing the projections into the event plane, that is the one identified by the first and the second eigenvectors of the sphericity tensor, and assuming that the total energy is the center of mass energy.

The results, for data and Monte Carlo, are plotted in Fig. 5. It is well known that the less energetic jet has a greater probability to be the gluon jet than the other two: it is our purpose to determine if there are differences between gluon and

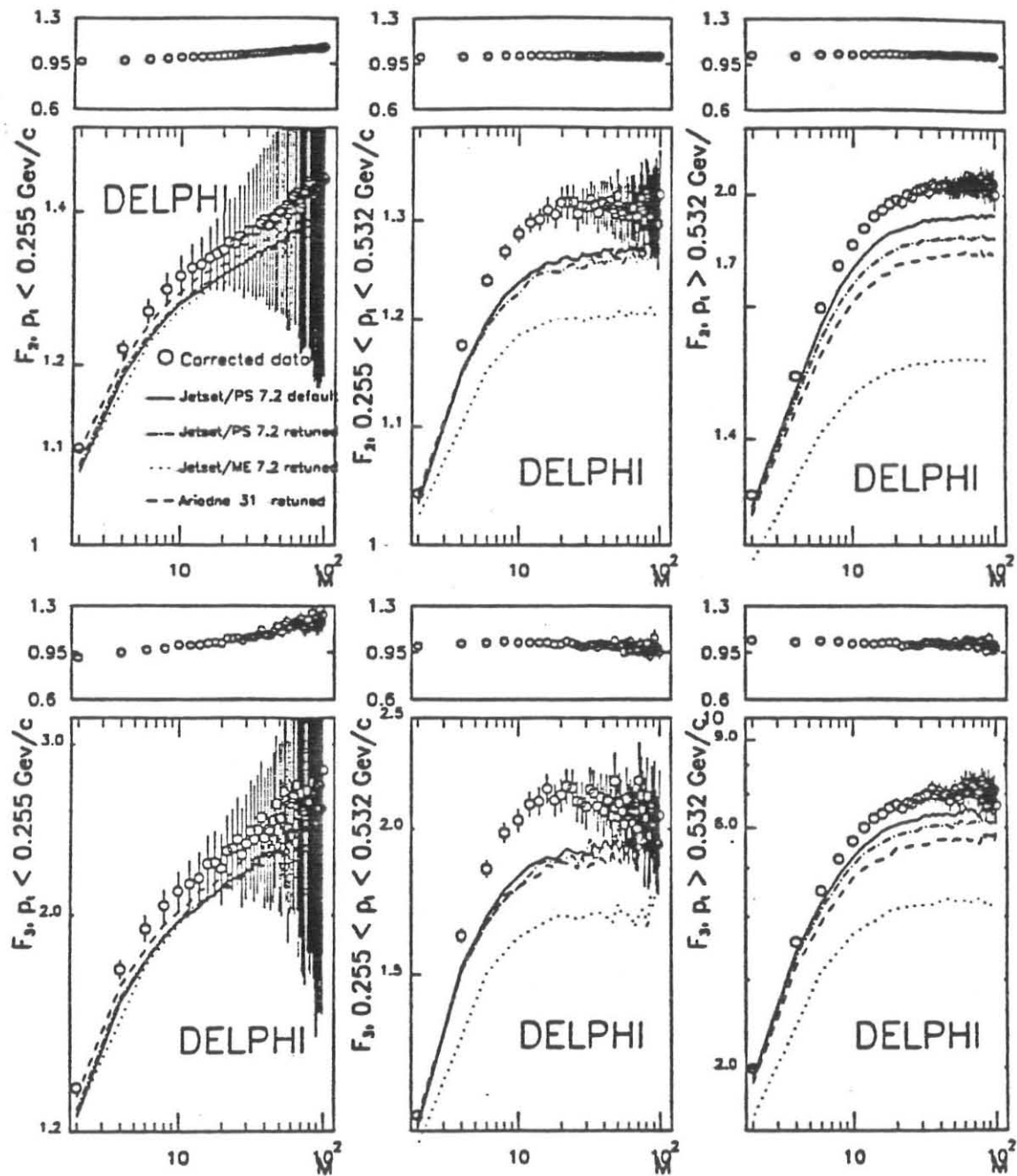


Figure 2: Factorial moments of rank 2 (a) and 3 (b) of the \bar{y} distribution for the three intervals of p_t (see text). DELPHI corrected data (white circles) compared with JETSET PS default (solid line).

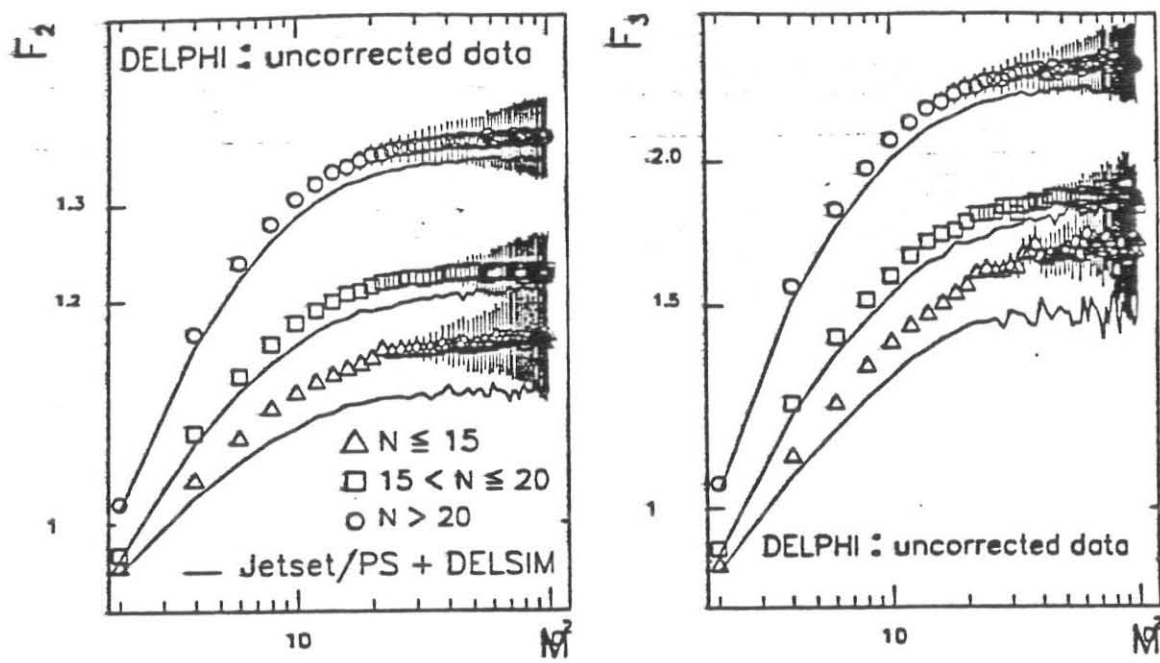


Figure 3: Factorial moments of rank 2 (a) and 3 (b) of the \bar{y} distribution for three ranges of multiplicity (see text). DELPHI uncorrected data (white circles) compared with JETSET PS 6.3 default + DELSIM (solid line).

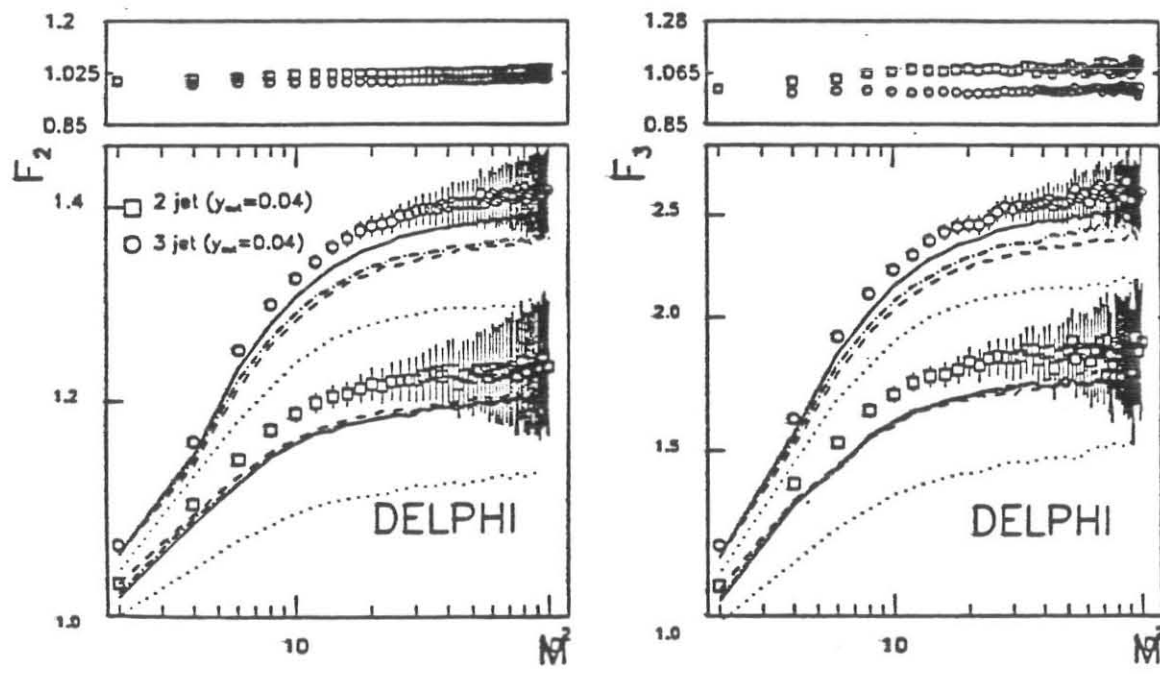


Figure 4: Factorial moments of rank 2 (a) and 3 (b) of the \bar{y} distribution for 2-jet and 3-jet events (y_{cut} of 0.04). DELPHI corrected data (white circles) compared to models as in Fig. 2.

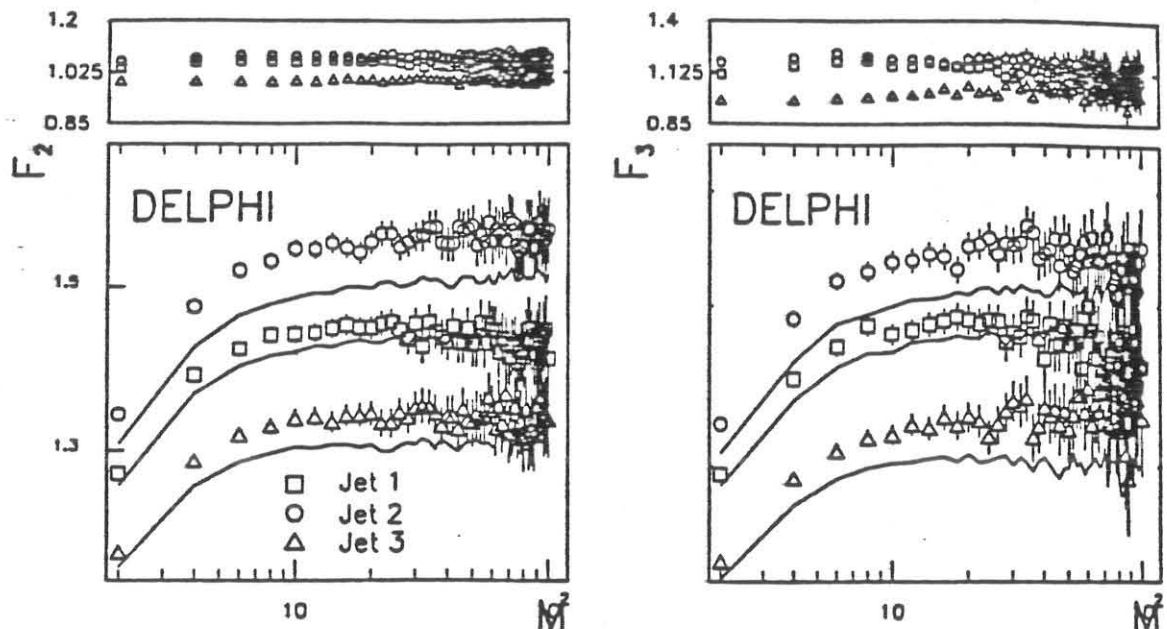


Figure 5: Factorial moments of rank 2 (a) and 3 (b) of the first, second and third jet, ordered by energy. DELPHI corrected data (open symbols) compared with JETSET PS default (solid line).

quark jets. We find that jet 3 presents weaker fluctuations. A disagreement with JETSET PS is visible in few standard deviations for jet 2 and jet 3.

3.1.4 Effects of Dalitz Pairs and of Bose-Einstein Correlations

We verified by a Monte Carlo study based on 50,000 events that switching on and off the Dalitz decay of the π^0 does not change the values of factorial moments out of the errors.

The study of the Bose-Einstein effect is more subtle, due to the lack of a satisfactory description in the Monte Carlos. On one hand, a test of comparison of factorial moments for same sign particles with random sign particles did not show differences [1]; on the other hand, this test cannot be conclusive since it does not keep into account the fact that non Bose-Einstein correlations are different for the two sets of particles.

To reproduce inside JETSET PS our results on Bose-Einstein correlations [31], we need to adjust the "strength parameter" λ to an unphysical value around 2.5. In addition, this adjustment affects the single particle inclusive distributions (rapidity in particular), causing them not to reproduce our data. We thus cannot thrust a Monte Carlo comparison for studying the effect of Bose-Einstein correlations.

In conclusion, we did not find any satisfactory probe to disentangle the effect of Bose-Einstein correlations on factorial moments.

3.2 Projections of Phase Space Onto Higher Dimensions

Recent theoretical works [28] suggest that saturation of factorial moments at $\delta y \simeq 0.1$ is an effect of smearing of fluctuations due to the projection of phase space into a one-dimensional subspace. Stronger intermittency effects should then appear in two-dimensional and three-dimensional spectra.

When intermittency is investigated with respect to the 2-dimensional (y, ϕ) distribution, the existence of a positive slope according to Eq. 2 becomes in fact no more questionable. Also in this case, however, the predictions of JETSET PS are in good agreement with the data [1].

When increasing the dimension of the phase-space in which one is studying fluctuations, the problem of the non-uniformity of the distribution in the phase space becomes of primary importance. One could naively think that, due to the fact that the ϕ distribution is not correlated to the y distribution and, in average, flat, the distribution of (\bar{y}, ϕ) is flat. Unfortunately, on an event by event basis, the ϕ distribution is nonuniform, being peaked at the azimuth ϕ_{s2} of the second eigenvector of the momentum tensor⁴.

To solve this problem, we considered in all the events the values of the azimuth starting from ϕ_{s2} , and we applied a transformation 'a la Bialas-Gazdzicki'. After this transformation, the population of the (\bar{y}, ϕ) space is flat. In the same way, for the 3-dimensional distribution, we calculated a flat bin-per-bin (\bar{y}, ϕ, p_t) distribution over the full phase space.

The results on factorial moments of the 2- and 3- dimensional projections of phase space are plotted in Fig. 6, and tabulated in Table 1. Errors keep into account the effects of an uncertainty of $\pm 2\%$ in the probability of γ conversions in the detector.

Factorial moments in three dimensions display intermittent behavior in the full phase space available. This means that, in the full phase space, the fluctuations related to the presence of the hard gluon jets as seen "from far" have the same behavior with respect to factorial moments as the fluctuations of the hadrons inside a jet.

3.3 Factorial Moments and 2-particle Correlations

Carruthers and Sarcevitv [18] suggested that the behavior of factorial moments can be understood on the basis of conventional short-range correlations for the 2-particle correlations, plus the Linked Pair 'ansatz' (LPA, requiring one free parameter for each order of the factorial moment) to go from the 2-particle correlations to higher-order correlations.

In the literature, 2-particle correlations are usually fitted in the central rapidity

⁴This was made evident by plotting factorial moments of the ϕ distribution and comparing to factorial moments of a $\bar{\phi}$ distribution. Factorial moments are higher in the first than in the second case.

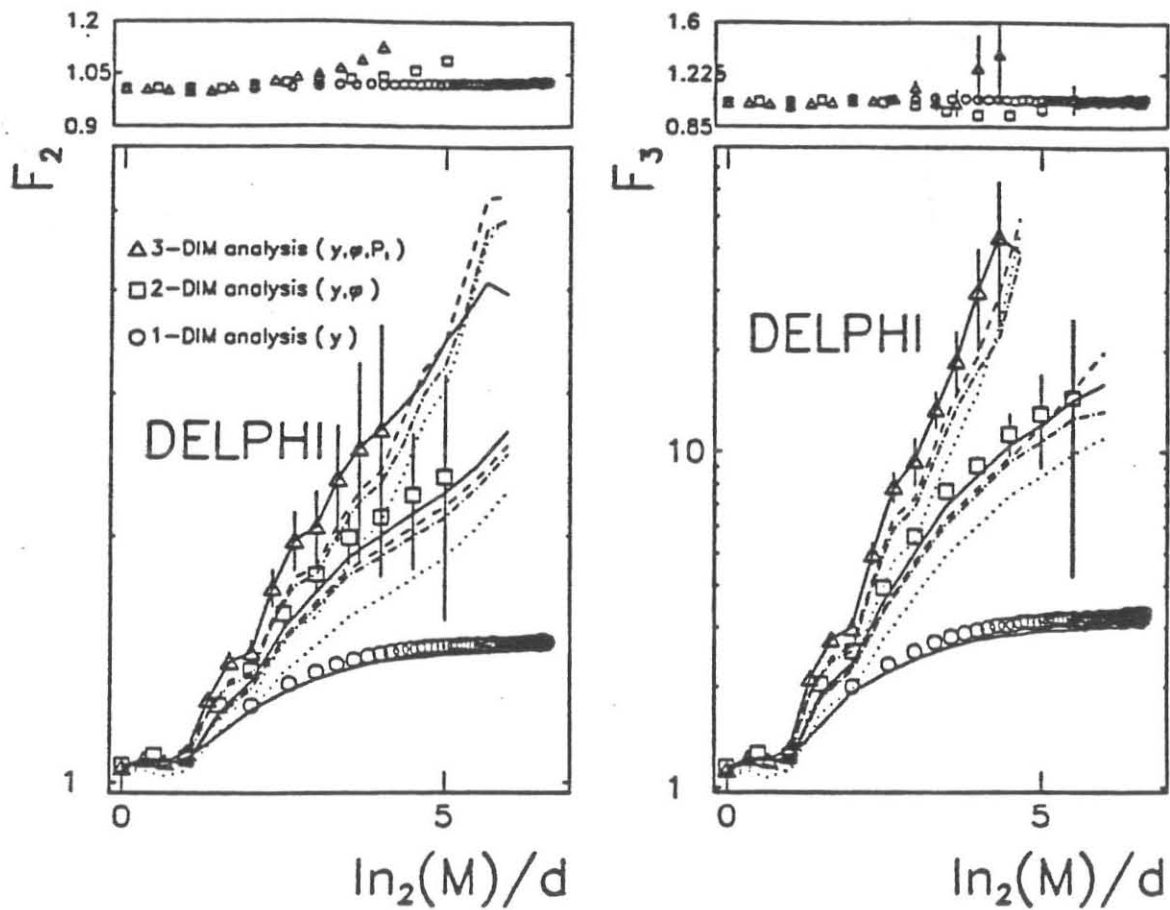


Figure 6: Factorial moments of rank 2 (a) and 3 (b) of the \bar{y} , $(\bar{y}, \bar{\phi})$ and $(\bar{y}, \bar{\phi}, \bar{p}_i)$ distributions. DELPHI corrected data (open symbols) compared to models as in Fig. 2.

$\ln_2 \frac{M}{d}$	Rank 2		
	1-d	2-d	3-d
0		1.051 ± 0.001 ± 0.009	1.043 ± 0.001 ± 0.007
1	1.081 ± 0.002 ± 0.002	1.071 ± 0.001 ± 0.011	1.090 ± 0.002 ± 0.004
2	1.245 ± 0.003 ± 0.003	1.376 ± 0.004 ± 0.036	1.449 ± 0.006 ± 0.049
3	1.369 ± 0.004 ± 0.004	1.803 ± 0.008 ± 0.090	2.06 ± 0.02 ± 0.21
4	1.441 ± 0.004 ± 0.008	2.12 ± 0.01 ± 0.24	2.71 ± 0.05 ± 0.92
5	1.471 ± 0.005 ± 0.015	2.36 ± 0.03 ± 0.79	
6	1.479 ± 0.006 ± 0.025		

$\ln_2 \frac{M}{d}$	Rank 3		
	1-d	2-d	3-d
0		1.159 ± 0.003 ± 0.027	1.136 ± 0.003 ± 0.021
1	1.274 ± 0.004 ± 0.006	1.249 ± 0.005 ± 0.037	1.323 ± 0.007 ± 0.012
2	2.008 ± 0.014 ± 0.001	2.56 ± 0.02 ± 0.15	3.01 ± 0.05 ± 0.18
3	2.56 ± 0.02 ± 0.02	5.62 ± 0.09 ± 0.28	9.4 ± 0.5 ± 1.4
4	2.96 ± 0.03 ± 0.03	9.16 ± 0.28 ± 0.49	30.1 ± 6.4 ± 7.7
5	3.14 ± 0.04 ± 0.07	13.0 ± 0.9 ± 4.0	
6	3.23 ± 0.05 ± 0.13		

Table 1: Factorial moments of rank 2 and 3 of the \bar{y} , (\bar{y}, ϕ) and (\bar{y}, ϕ, p_t) distributions. Statistical errors and systematics from γ conversions separated.

region to the formula

$$\tau_2(\Delta y) = 1 + \gamma_2 e^{-\Delta y/\xi} \quad (3)$$

where τ_2 is the ratio of the two-particle density to the product of the single-particle densities for a couple of particles, Δy is the absolute value of the difference of the rapidities, and γ_2 and ξ are the parameters of the fit. Our two-particles correlations data in the central rapidity region (with the cuts $|y_{1,2}| < 2$ and $|y_1 + y_2| < 2$) display disagreement to expression (3) in the low- δy region.

The best fit to expression (3) gives $\gamma_2 = 0.968 \pm 0.004$, $1/\xi = 0.748 \pm 0.003$. When applying the formulae of Ref. [18] using the values of γ_2 and ξ fitted to the two-particle correlations, we obtain a not satisfactory agreement to the data, also for F_2 . We remind that that F_2 should in principle, if approximation (3) and the assumption of translational invariance in the central rapidity region were true, contain the same information as the two fit parameters.

If we proceed in the opposite way, and we use the expressions of Ref. [18] to parametrize factorial moments, the fitted values γ_2 and ξ fitted from F_2 disagree with the ones previously determined ($\hat{\gamma}_2 = 0.640 \pm 0.001$, $1/\hat{\xi} = 0.724 \pm 0.001$). However, we find that also this representation of correlations is not accurate enough to describe factorial moments of ranks higher than three.

This is an evidence of non-trivial $n > 2$ -particle correlations in e^+e^- annihilations.

4. Conclusions

Confirming our previous results [1], a more detailed analysis, based on a statistics ten times larger, evidenced that present Monte Carlos based on Parton Showers provide a reasonably good description of fluctuations in the phase space of hadrons produced in the decay of the Z^0 . Discrepancies observed are of the same order as the effect of retuning parameters.

The presence of intermittency, questionable in 1-dimensional projections of phase space, becomes evident in the full phase space, showing a similarity of the fluctuations that give rise to jets with the fluctuations of hadrons inside jets.

We have observed the evidence of nontrivial higher($n > 2$)-order particle correlations in e^+e^- annihilations.

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