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PAIR PRODUCTION IN HEAVY-ION COLLISION: SECOND-ORDER ADIABATIC APPROXIMATION

Pair production in heavy-ion collision:
second-order adiabatic approximation

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ABSTRACT

The electron-positron pair production in the collision of heavy ions show a rapid raise of the rate with the total nuclear charge ($\sim Z^{20}$). It has been shown that the gross features of the process are correctly reproduced by a first-order adiabatic approximation. In this paper we estimate the correction to the emission probability coming from a second-order iteration of the adiabatic treatement. The numerical results show that the correction both to the size and to the slope of the production rate is small, if compared to the present experimental uncertainties. The first-order description is therefore confirmed.

Quantum electrodynamics has been extensively and successfully tested in its perturbative sector for a long time.

Recently, experiments on heavy-ion collisions have thrown new light on the behaviour of QED in the non perturbative environement created by the strong fields surrounding the colliding ions. Two classes of new phenomena have stimulated the interest of physicists: the detection of narrow e^+e^- states and the strong dependence of the e^+e^- production rate on the total nuclear charge Z.

The appearance of e^+e^- peaks is a spectacular feature which have triggered a proliferation of unusual and sometimes very suggestive speculations. Contrary, the rapid raise of the e^+e^- production rate is usually explained with rather conventional tools.

This paper focuses on this latter phenomenon about which a long lasting investigation has been carried out [1-5]. For sake of completeness we find convenient to recall the relevant aspects of the formulation presented in ref. [1], on which we explicitly rely. The colliding ions are regarded as pointlike charges moving along classical Coulomb trajectories. Since the considered ions are not very fast a non relativistic dynamics is still allowed for them. The e^+e^- creation is treated in first quantization, a first order adiabatic approximation is used and it gives the following expression for the transition amplitude $A^{(1)}{}_{fi}$:

$$A_{fi}^{(1)} = -\int_{-\infty}^{+\infty} dt\, e^{i(E_f-E_i)t} \dot{R}_c(t) \, \langle \phi_f(R_c)| \frac{\partial}{\partial R_c}|\phi_i(R_c)\rangle \tag{1}$$

where $|\phi_i(R_c)\rangle$ and $|\phi_f(R_c)\rangle$ are eigenfunctions with lowest angular momentum of the Dirac Hamiltonian corresponding to a fixed internuclear distance R_c. $|\phi_f(R_c)\rangle$ is a positive energy state while $|\phi_i(R_c)\rangle$ is a negative energy one, so that they correspond to electron and positron states respectively. Since the analytic evaluation of eq. (1) is extremely difficult, one must resort to numerical methods. In order to reduce the amount of computations, the problem is further simplified and the true two-body potential is replaced by a blowed-up nucleus of radius $R = \frac{1}{2} R_c(t)$ and charge $Z = Z_1 + Z_2$ (Z_1 and Z_2 being the charges of the interacting ions). This approximation is usually referred to as the "monopole approximation" [3].

The resulting model actually reproduces the rapid raise of the e^+e^- production rate with the nuclear charge Z; the dependence on Z can be fitted by a function Z^n with n~20. Although the results show finally simple features, no straightforward analytical or heuristic derivation is yet available.

Various refinement of the model can be foreseen. One could try a better description of the potential of the two ions, which is really a non spherical potential. This line of investigation has been already followed [3,6]. One could consider, in a more or less standard way, radiative corrections coming from the dynamical degrees of freedom of the electromagnetic field. Published results [7-9] can be found also on this line which has ben followed in the attempt to explain the appearance of narrow states. One could finally improve the dynamical description, going beyond the first-order adiabatic approximation. We find interesting to consider this possibility and it is in fact the purpose of the present paper.

In the following we keep the monopole approximation together with the first quantized formalism, but we take a sight at the stability of the dynamical treatement under the iteration of the adiabatic approximation, possibly modified where it may be no longer reliable. Our aim is therefore to estimate the corrections that the second order induces on the transition rate as derived from eq. (1). The second-order contribution to the transition amplitude (see fig. 1) is given by the following expression:

$$A_{fi}^{(2)} = \int dE_v \, P\frac{1}{E_f-E_v} \, P\frac{1}{E_v-E_i} \int_{-\infty}^{+\infty} dt \, e^{i(E_f-E_v)t} \, \dot{R}(t) \, \langle\phi_f(R)|\frac{\partial H}{\partial R}|\phi_v(R)\rangle \times$$

$$\times \int_{-\infty}^{t} dt' \, e^{i(E_v-E_i)t'} \, \dot{R}(t') \, \langle\phi_v(R')|\frac{\partial H}{\partial R}|\phi_i(R')\rangle . \tag{2}$$

While for the general procedure we refer to the appendix, some further explanations are necessary. The operator $\frac{\partial H}{\partial R}$ takes in coordinate representation, within the monopole approximation, the form

$$\langle r|\frac{\partial H}{\partial R}|r'\rangle = Z\alpha \, \frac{1}{R^2(t)} \, \theta(R(t)-r) \, \delta(r-r') . \tag{3}$$

Eq. (3) expresses, in particular, the fact that outside the charged shell the potential remains constant. With the aid of eq. (3) the matrix elements in eq. (2) can be written as:

$$\langle\phi_{f,i}(R)|\frac{\partial H}{\partial R}|\phi_v(R)\rangle = \frac{Z\alpha}{R^2(t)} \int d\Omega \int_0^{R(t)} dr \, r^2 \, \phi^*_{f,i}(r;R) \, \phi_v(r;R) . \tag{4}$$

The radial wave functions are proportional to the Bessel functions in the domain of integration $r<R(t)$ and to the usual Coulomb waves in the region $r>R(t)$.

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E_v ⎯⎯⎯⎯⎯⎯⎯⎯⎯ ....

E_f

E'_v ....

$E=0$ — — — — — — + — — — — —

E_i ⎯⎯⎯⎯⎯⎯⎯⎯⎯

a)

E_f ⎯⎯⎯⎯⎯⎯⎯⎯⎯

$E=0$ — — — — — — + — — — — —

E_v ....

E_i ⎯⎯⎯⎯⎯⎯⎯⎯⎯

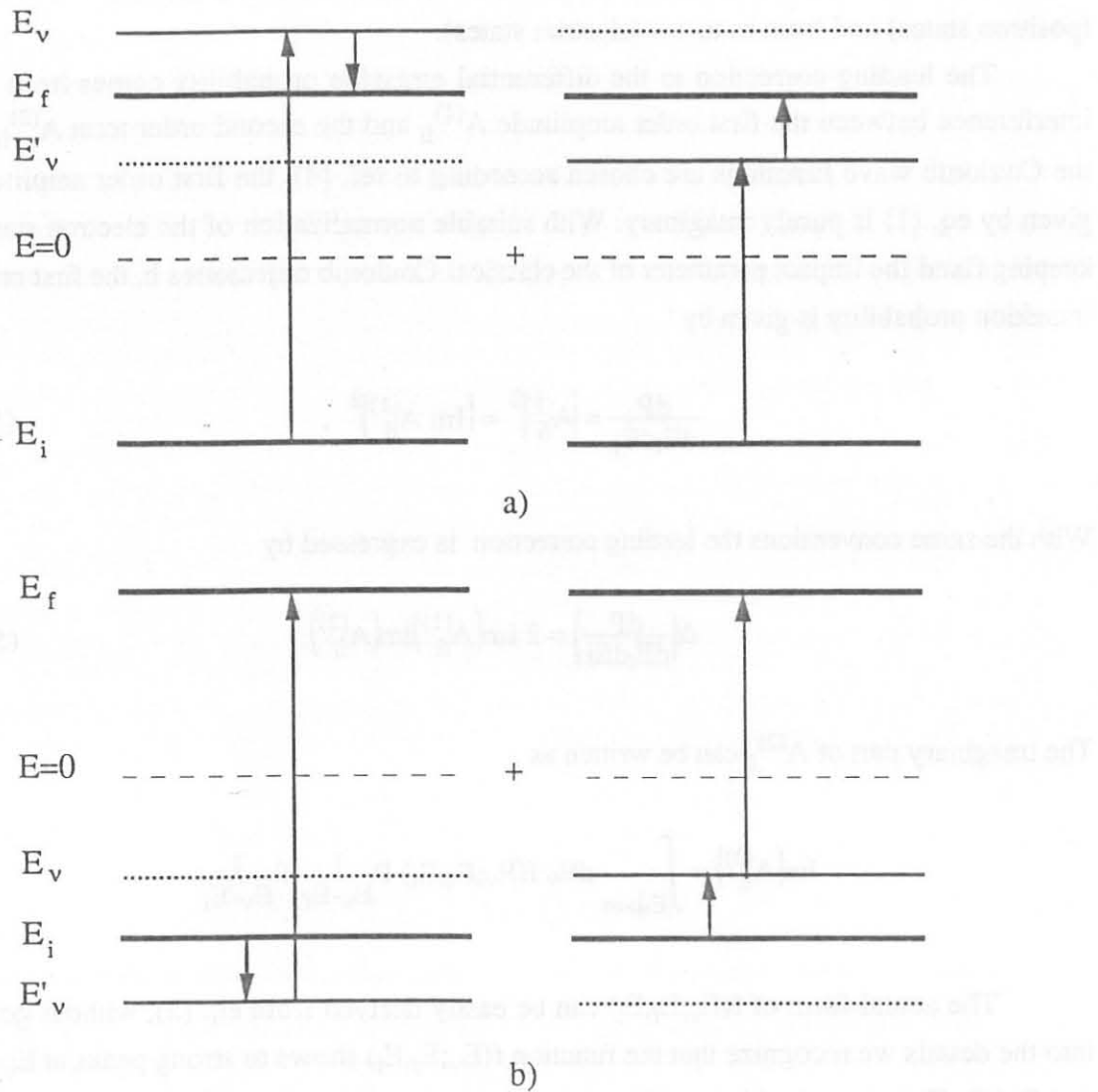E'_v ⎯⎯⎯⎯⎯⎯⎯⎯⎯ ....

b)

Fig. 1. Second-order transition from positive to negative energy states $|E_i\rangle$ and $|E_f\rangle$. The intermediate states are both of electron and positron type, fig. 1a and fig. 1b respectively. In the neighbourhood of E_f and E_i we sum the amplitudes corresponding to the symmetrically arranged levels E_v and E'_v in order to reconstruct the principal value prescription of eq. (2).

The product of the Bessel functions can be integrated analytically, their time dependent normalization is obtained by connecting smoothly the wave functions at r=R(t) [4].

In eq. (2) there is an integration over the energy of the intermediate states; more precisely also a sum over the electron bound states should appear. Those configurations are, however, poorly represented in the monopole approximation, moreover they are expected to be not very relevant if we are interested in final states not too near the

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threshold, so their contribution is omitted. The integration over E_v runs from $-\infty$ to $-m$ (positron states) and from m to $+\infty$ (electron states).

The leading correction to the differential emission probability comes from the interference between the first order amplitude $A^{(1)}_{fi}$ and the second order term $A^{(2)}_{fi}$. If the Coulomb wave functions are chosen according to ref. [4], the first order amplitude given by eq. (1) is purely imaginary. With suitable normalization of the electron states, keeping fixed the impact parameter of the classical Coulomb trajectories b, the first order transition probability is given by

$$\frac{dP}{dE_f dE_i} = \left|A^{(1)}_{fi}\right|^2 = \left(\text{Im } A^{(1)}_{fi}\right)^2 . \tag{5,a}$$

With the same conventions the leading correction is expressed by

$$\Delta\left(\frac{dP}{dE_f dEi}\right) = 2 \text{ Im}\left(A^{(1)}_{fi}\right)\text{Im}\left(A^{(2)}_{fi}\right) \tag{5,b}$$

The imaginary part of $A^{(2)}_{fi}$ can be written as

$$\text{Im}\left(A^{(2)}_{fi}\right) = \int_{|E_v|>m} dE_v \ f(E_v; E_f, E_i) \ P\frac{1}{E_v-E_f} \ P\frac{1}{E_v-E_i} \tag{6}$$

The actual form of $f(E_v; E_f, E_i)$ can be easily derived from eq. (2), without going into the details we recognize that the function $f(E_v; E_f, E_i)$ shows to strong peaks at $E_v=E_f$ and $E_v=E_i$. The numerical integration over the intermediate energy E_v is then performed by sampling the function $f(E_v; E_f, E_i)$ around the position of these two peaks. The factors $\frac{1}{E_v-E_f}$ and $\frac{1}{E_v-E_i}$ improve the convergence for large values of E_v.

The contribution coming from the positron states, $E_v<-m$, is small as compared to the electron states contribution, $E_v>m$. In order to explain this circumstance let us go back to expression (4) for the matrix elements appearing in eq. (2) . Since the integration involved in eq. (4) is over a finite range surrounding the nuclei, the result is strongly sensitive to the size of the wave function near the origin; in particular when $E_v<-m$ the repulsive potential depresses the intermediate wave function in the relevant r-range, while the function is enhanced when it feels an attractive potential, i. e. for $E_v>m$.

In order to compute the correction to the spectrum of the emitted particles, one should integrate the function $\frac{dP}{dE_f dE_i}$ over the impact parameter b, moreover to get the total

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cross section one should perform a further integration over the energies E_f and E_i. However, since our aim is simply to estimate the size of the second order corrections, we restrict to single values of b,E_f and E_i. We choose b=3fm, since, as shown in ref. [1], this value is expected to give the major contribution to the total cross section. From reference [1] we also learn that the positron spectrum is peaked at -E_i~2m, while the electron spectrum is monotonically decreasing (obviously starting from E_f=m). Since our treatement may be not reliable very near the threshold, we think that the choice -E_i=2m for the positron energy and E_f=1.7m for the electron energy is suitable for our purposes.
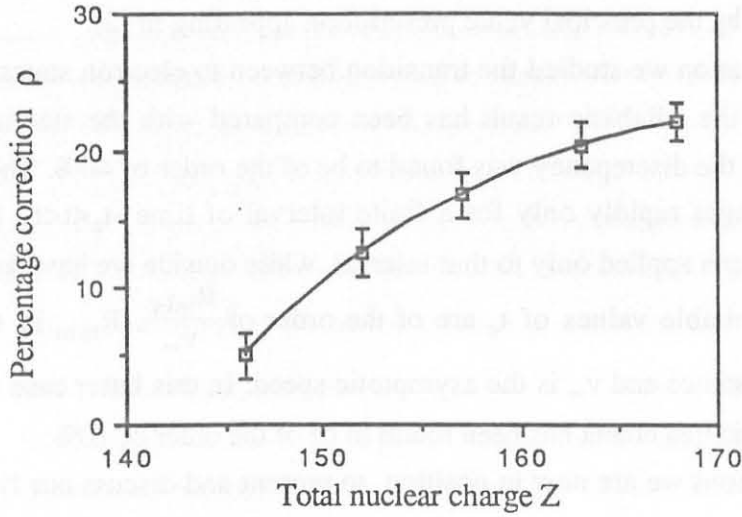

Fig. 2. Percentage correction to the first-order differential probability emission as a function of the total nuclear charge Z at fixed impact parameter b=3fm. The plot corresponds to E_{e+}=2m_e and E_{e-}=1.7m_e, m_e being the electron mass. The distance of closest approach between the ions is also fixed at R_{min}=16fm. The uncertainty bars are a rough estimate of the errors involved in the overall numerical procedure.

Our quantitative results are presented in the form of a ratio between the correction and the first-order transition probability as given by eq. (5a,b). In figure (2) we show this ratio $\rho = 2\dfrac{Im(A^{(2)}_{fi})}{Im(A^{(1)}_{fi})}$ as a function of the total nuclear charge Z. We choose the field of variation of Z to be 146<Z<168. The upper bound is suggested by the consideration that, for higher values, the decay of the neutral vacuum to the charged one is expected to play a very relevant role; below the lower bound both the first order rate and the corrections become very small.

Before drawing the final conclusions we find useful to justify the iteration of the adiabatic approximation up to the second order.

Eq. (1) states that the first-order transition amplitude is obtained by taking the time Fourier transform of $\langle \phi_f(R)|\frac{\partial H}{\partial R}|\phi_i(R)\rangle$ with frequency $\omega=(E_f-E_i)$ which is the total energy of the produced particles; if it is possible to define a "time scale" τ for this matrix element, then the adiabatic treatement is reliable if $\tau \gg \frac{1}{E_f-E_i}$. The iteration of eq. (1) involves transitions from the intermediate energy E_v to the final energy E_f; since one has to integrate over E_v, the condition $\tau \gg \frac{1}{E_v-E_f}$ is eventually violated and we have no reason, in principle, to believe to the adiabatic approximation when $E_v \sim E_f$.

The contribution to the amplitude coming from these dangerous states is, however, highly suppressed by the principal value prescription appearing in (2).

For a further clarification we studied the transition between to electron states of nearby energies. Formerly the adiabatic result has been compared with the standard perturbative calculation and the discrepancy was found to be of the order of 40%. Then, since the Hamiltonian changes rapidly only for a finite interval of time $-t_c<t<t_c$, the perturbative approach has been applied only to that interval, while outside we have kept the adiabatic evolution. Suitable values of t_c are of the order of $\frac{R_{min}}{v_\infty}$, R_{min} is the minimum of the two-ion distance and v_∞ is the asymptotic speed. In this latter case the discrepancy with the adiabatic treatement has been found to be of the order of 10%.

With these justifications we are now in position to present and discuss our final results.

The percentage corrections, shown in fig. (2), are never very big, they are in fact less than 20% for almost all the considered points. This result is welcome since the first-order describes correctly the gross features of the available experimental data [10-15], which refer to different kinematical configurations of the produced particles. The calculations confirm also the already stated prevision that the contribution coming from the intermediate positron states is much smaller, actually less than 1%, as compared to the electron states contribution. As far as the sign of the correction is concerned we find it to be positive for our choice of the kinematical parameters. We foresee some situation where the sign may become opposite: according to previous considerations, eq. (6) can be approximated as :

$$Im\left(A_{fi}^{(2)}\right) = \int_{E_v>m} dE_v\ f(E_v;E_f,E_i)\ P\frac{1}{E_v-E_f}\ P\frac{1}{E_v-E_i} \tag{7}$$

Because of the presence of the factor $\frac{1}{E_f - E_v}$ the contribution from $m < E_v < E_f$ and the contribution from $E_f < E_v$ have opposite sign. If $E_f \sim 2m$, as in our choice, the former contribution is the dominant one and dictates the sign of the whole expression; as $E_f \to m$ the range $m < E_v < E_f$ shrinks and so the sign of the whole expression changes, consequently the sign of the correction is reversed. In this case, however, the missing contribution of the electron bound states, which are hardly accomodated within the model, may affect heavily this conclusion.

As expected the relative correction increases with Z but besides being small it is also weakly dependent on the charge, in fact if we tried to fit it as a power of Z we would get an exponent less than one. Also in this respect, therefore, we conclude that the first-order calculation is quite stable with respect to the corrections. Nevertheless they should be taken into account if we were performing a precision test of non perturbative QED in a heavy-ion collision experiment.

Appendix

Since our discussion is centred on the next-to-the first approximation in the adiabatic formalism we find convenient to recall a compact presentation of this formalism which would enable us to derive all the orders.

We start from a Schrödinger equation with a Hamiltonian which depends explicitly on time :

$$i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle \tag{A.1}$$

and we associate to it a "stationary" equation where t is considered as an arbitrary parameter

$$H(t)|\phi_n(t)\rangle = E_n(t)|\phi_n(t)\rangle \tag{A.2}$$

where, evidently, the $|\phi_n(t)\rangle$ do not satisfy the evolution equation (A.1)

We introduce a time dependent change of representation by means of a unitary operator $\Lambda(t)$:

$$|\psi_a(t)\rangle = \Lambda^{-1}(t)\,|\psi(t)\rangle \tag{A.3}$$

so the evolution equation for $|\psi_a(t)\rangle$ corresponding to the original one is known to be :

$$i\frac{d}{dt}|\psi_a(t)\rangle = \mathcal{H}(t)|\psi_a(t)\rangle$$

with $\mathcal{H}(t) = \Lambda^{-1}(t)\,H(t)\,\Lambda(t) - i\,\Lambda^{-1}(t)\,\dot{\Lambda}(t)$ and the usual evolution operator is

$$U_a(t,t_0) = P\,\exp\left\{-i\int_{t_0}^t \mathcal{H}(\tau)d\tau\right\} \tag{A.4}$$

where P is the standard chronological ordering.

The adiabatic treatement is obtained by choosing the unitary operators Λ as :

$$\Lambda(t,t_0)\,|\phi_n(t_0)\rangle = \exp\left\{-i\int_{t_0}^t d\tau\,E_n(\tau)\right\}|\phi_n(t)\rangle$$

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t_0 denotes the instant at which the representations coincide.

After computing explicitly $\hat{\Lambda}$ the matrix elements of $\mathcal{H}$ are found to be

$$\langle\phi_f(t_0)|\mathcal{H}(t)|\phi_i(t_0)\rangle = -i \exp\left\{i\int_{t_0}^t [E_f(\tau)-E_i(\tau)]d\tau\right\} \langle\phi_f(t)|\frac{d}{dt}|\phi_i(t)\rangle \qquad (A.5)$$

With this result we see that the expression of the evolution operator $U_a(t,t_0)$, see (A.4), gives rise to a usual perturbative series which, however, corresponds term by term to the iteration of the adiabatic approximation [16] for eq. (A.1).

In the original representation, the transition amplitude from the state $|\phi_i(t_0)\rangle$ to the

state $\exp\left\{-i\int_{t_0}^t d\tau\, E_f(\tau)\right\} |\phi_f(t)\rangle$ is found to be to the first order:

$$A_{fi}^{(1)} = -\int_{t_0}^t d\tau \exp\left\{i\int_{t_0}^\tau d\tau'[E_f(\tau')-E_i(\tau')]\right\} \langle\phi_f(\tau)|\frac{d}{d\tau}|\phi_i(\tau)\rangle \qquad (A.6)$$

Usually, the Hamiltonian H and thus the states $|\phi_n(t)\rangle$ depend on time through some parameter $\gamma(t)$; moreover the matrix element appearing in (A.6) is known to be expressible [16] through the matrix elements of the Hamiltonian H(t). We can recast therefore the transition amplitude into the expression :

$$A_{fi}^{(1)} = -\int_{t_0}^t d\tau \exp\left\{i\int_{t_0}^\tau d\tau'\ [E_f(\tau')-E_i(\tau')]\right\}\left(\frac{\dot{\gamma}}{E_i-E_f}\langle\phi_f|\frac{\partial H}{\partial\gamma}|\phi_i\rangle\right)_\tau \qquad (A.6')$$

With the same procedure the second-order contribution to the transition amplitude takes on the form:

$$A_{fi}^{(2)} = \sum_n \int_{t_0}^t d\tau \exp\left\{i\int_{t_0}^\tau d\tau'\ [E_f(\tau')-E_n(\tau')]\right\}\left(\frac{\dot{\gamma}}{E_n-E_f}\langle\phi_f|\frac{\partial H}{\partial\gamma}|\phi_n\rangle\right)_\tau \times$$

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$$\times \int_{t_0}^{\tau} dt' \, \exp\left\{i\int_{t_0}^{t'} d\tau'[E_n(\tau')-E_i(\tau')]\right\}\left(\frac{\dot{\gamma}}{E_i-E_n}\langle\phi_n|\frac{\partial H}{\partial\gamma'}|\phi_i\rangle\right)_{t'} \qquad (A.7)$$

In most situations the spectrum of H contains a continuum part so that the summation must be supplemented by an integration; this integration goes also through the points $E_v=E_i$ and $E_v=E_f$ where it seems to be not well defined. This problem arises when the particles are allowed to fly to infinity. To answer to this problem we go back to the matrix element appearing in (A.6) and we write in coordinate representation:

$$\langle\phi_f|\frac{\partial}{\partial\gamma}|\phi_i\rangle = \int_0^\infty dr \, \phi^*_f(r)\frac{\partial}{\partial\gamma}\phi_i(r) \qquad (A.8)$$

Then, noting that no singularity may arise by integrating over a finite interval we focus our attention to the region $r\to\infty$, here we substitute the wave functions with their asymptotic forms:

$$\phi_f(r) \to \phi_A(r)=\sin(kr+\delta(\gamma)+\delta_c)$$
$$\phi_i(r) \to \phi'_A(r)=\sin(k'r+\delta'(\gamma)+\delta'_c)$$

δ_c and δ'_c are the possible pure Coulomb phases. The asymptotic expression of (A.8) after introduction of a radial cutoff $e^{-\eta r}$ is :

$$\int_0^\infty dr \, e^{-\eta r}\sin(kr+\Delta)\frac{\partial}{\partial\gamma}\sin(k'r+\Delta') =$$

$$=\frac{\partial\Delta'}{\partial\gamma}\left(\frac{i}{4}\right)\left\{\frac{e^{i(\Delta+\Delta')}}{(k+k')+i\eta}+\frac{e^{i(\Delta-\Delta')}}{(k-k')+i\eta}+\frac{e^{-i(\Delta-\Delta')}}{(k-k')-i\eta}+\frac{e^{-i(\Delta+\Delta')}}{(k+k')-i\eta}\right\}$$

with $\Delta=\delta+\delta_c$ $(\Delta'=\delta'+\delta'_c)$.

Some terms are quite regular in the limit $k\to k'$ and $\eta\to 0$, there are some more dangerous terms which take the explicit form:

$$\cos(\Delta-\Delta')\left\{\frac{1}{k-k'+i\eta}+\frac{1}{k-k'-i\eta}\right\} = \cos(\Delta-\Delta')\, P\frac{1}{k-k'}$$

Out of this result we learn finally the principal value prescription $P\frac{1}{E_v-E_{f,i}}$ for $\frac{1}{E_v-E_{f,i}}$ in eq. (A.7).

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