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ON THE $\Phi \to K^0\overline{K^0}\gamma$ TRANSITION AT A Φ FACTORY *

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Abstract

We estimate the branching ratio of $\phi \to K^0\overline{K^0}\gamma$, by including a non-resonant smooth amplitude in addition to the scalar resonance poles. This transition is relevant to precision measurements of CP-violating parameters in the kaon system at a ϕ-factory, which have been recently proposed.

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The possibility of precision measurements of CP violation in correlated $K^0 - \overline{K^0}$ pairs from ϕ decays, to be copiously produced at a high-luminosity $e^+e^- \to \phi$ machine, has attracted much interest recently [1-4]. The point is that the p-wave $K^0 - \overline{K^0}$ complex from ϕ decay must be antisymmetric, therefore automatically pure $K_L - K_S$, so that cleaner determinations of the CP violating parameters should be allowed in principle.

Backgrounds to this configuration are due to events where the e^+e^- annihilate to an even-wave $K^0 - \overline{K^0}$ state, which must be symmetric, and therefore either $K_L - K_L$ or $K_S - K_S$. This issue has been investigated in details in Ref. [5], where the various possible sources of such contaminations have been assessed. According to that analysis the most important background should be represented by the radiative transition $\phi \to K^0\overline{K^0}\gamma$. In this mode the $K^0 - \overline{K^0}$ system is expected to be in s-wave due to the small Q-value, and indeed in the estimate of Ref. [5] the transition amplitude has been assumed to be dominated by the lowest-lying $S^*(975)$ and $\delta(980)$ scalar mesons, such that $\phi \to K^0\overline{K^0}\gamma$ should proceed via $\phi \to S^*/\delta\ \gamma$ followed by $S^*/\delta \to K^0\overline{K^0}$.

In this regard one could observe, however, that both the S^* and the δ lie somewhat below the $K - \overline{K}$ threshold, so that they decay into $K - \overline{K}$ through just the tail of their Breit-Wigner shapes, instead of exploiting their full widths. Thus the assumed scalar meson dominance may be not obvious a priori. Consequently, in such a situation, it should be sensible to look at the possible effect of a non-resonant, smooth amplitude, which in principle must be added to the resonance poles. Obviously this would lead to an improved theoretical estimate of the background to CP violation experiments at a ϕ factory. In addition to that, such a question is relevant to the attempts made to clarify the (as yet not quite established) nature of the S^* and of the δ, by the observation of $\phi \to S^*\ \gamma$ and $\phi \to \delta\ \gamma$, as recently proposed in Refs. [6] and [7].

With these motivations, we will try in what follows to supplement the calculation of Ref. [5] by an estimate of the non-resonant amplitude for the process $\phi \to K^0\overline{K^0}\gamma$. To this purpose we shall adopt the familiar framework of current algebra and PCAC, plus the vector meson dominance of the electromagnetic current coupled to the photon, as this scheme seems most suitable for the kind of calculation we wish to make here.

Thus, neglecting d-waves, we start by defining the transition amplitude for the process $\phi(p) \to K^0(k_1)\ \overline{K^0}(k_2)\ \gamma(q)$ as follows:

$$T(\phi \to K^0\overline{K^0}\gamma) = e\ \eta_\lambda(p)\epsilon_\sigma(q)\ \left[q_\lambda p_\sigma - (p\cdot q)\ \delta_{\lambda\sigma}\right]\ \mathcal{B}\left(\nu, \nu_B, k_1^2, k_2^2\right), \qquad (1)$$

where e is the electron charge, η and ϵ are the ϕ and the γ polarization vectors respectively, and

$$\nu = -\frac{(k_2 - k_1) \cdot (p+q)}{4\ M_\phi}$$
$$\nu_B = -\frac{(k_1 \cdot k_2)}{2\ M_\phi}. \tag{2}$$

For later purposes it is convenient to introduce also the $(K^0\overline{K^0})$ invariant mass squared:

$$t = -(k_1 + k_2)^2 = -(p-q)^2 = -k_1^2 - k_2^2 + 4M_\phi \nu_B = M_\phi^2 + 2(p \cdot q). \tag{3}$$

Clearly, at the kaon mass-shell $k_1^2 = k_2^2 = -m_K^2$. However, we let the amplitude B in eq.(1) to explicitly depend also on k_1^2 and k_2^2, as this allows the extrapolations in the kaon mass which are characteristic of the current algebra approach.

Indeed, by the application of standard soft-meson techniques [8], one can derive the following low-energy theorem:

$$\frac{m_K^4\ F_K^2\ T(\phi(p) \to K^0(k_1)\overline{K^0}(k_2)\gamma(q))}{(m_K^2 + k_1^2)\ (m_K^2 + k_2^2)} = k_{1_\mu} k_{2_\nu} M_{\mu\nu} - \frac{e}{2}(m_s + m_d)\Sigma. \tag{4}$$

In eq.(4) F_K is the kaon leptonic decay constant ($F_K \simeq 1.2\ F_\pi$ with $F_\pi = 93\ MeV$), and

$$M_{\mu\nu} = i^2 \epsilon_\sigma(q) \int d^4x d^4y \exp\left(-i(k_1 \cdot x + k_2 \cdot y)\right)$$
$$< 0| T\left(A_\mu^{(K^0)}(x) A_\nu^{(\overline{K^0})}(y) V_\sigma^{e.m.}(0)\right) |\phi(p,\eta)>, \tag{5}$$

where $V_\sigma^{e.m.}$ denotes the electromagnetic hadronic current, and $A_\mu^{(K^0)}$, $A_\nu^{(\overline{K^0})}$ are the axial vector currents, whose divergences interpolate the K^0 and $\overline{K^0}$ fields:

$$A_\mu^{(K^0)} = i\bar{s}\gamma_\mu\gamma_5 d,$$
$$A_\nu^{(\overline{K^0})} = i\bar{d}\gamma_\nu\gamma_5 s. \tag{6}$$

Moreover, m_s and m_d in eq.(4) are current quark masses, and

$$\Sigma = i\epsilon_\sigma(q) \int d^4x \exp\left(-i(q \cdot x)\right) < 0|T\left(S(0) V_\sigma^{e.m.}(x)\right)|\phi(p,\eta)>, \tag{7}$$

with $S(x)$ the scalar quark density:

$$S = \bar{d}(x)d(x) + \bar{s}(x)s(x). \tag{8}$$

Using the SU(3) notation, we can rewrite eq.(8) as:

$$S = 2\sqrt{\frac{2}{3}} S_0 - S_3 - \sqrt{\frac{1}{3}} S_8, \tag{9}$$

where

$$S_i = Tr \overline{\psi} \frac{\lambda_i}{2} \psi, \tag{10}$$

with ψ the basic u, d and s quark (flavor) triplet. Analogously:

$$V_\sigma^{e.m.} = V_{\sigma,3} + \sqrt{\frac{1}{3}} V_{\sigma,8}. \tag{11}$$

Eq.(4) directly follows from well known, model independent current algebra Ward identities and equal-time commutation relations, plus the PCAC principle.

The next step is to expand Σ in eq.(7), which is reminiscent of the familiar σ-term of pion-nucleon scattering, in a gauge invariant form analogous to eq.(1):

$$\Sigma(p,q) = \eta_\lambda(p)\epsilon_\sigma(q)\ \left[q_\lambda p_\sigma - (p \cdot q)\ \delta_{\lambda\sigma}\right]\ \mathcal{H}\left(t, q^2 = 0\right), \tag{12}$$

where we directly put the photon on its mass-shell $q^2 = 0$.

From eq.(4) we can then derive the following two conditions on the amplitude B, rigorously valid at the soft-kaon points $k_1 \to 0$, $k_2^2 = 0$ and $k_1^2 = 0$, $k_2 \to 0$ respectively:

$$\mathcal{B}\left(\nu_0, 0, 0, 0\right) = -\frac{1}{2\ F_K^2}(m_s + m_d)\mathcal{H}\left(t = 0, q^2 = 0\right), \tag{13}$$

$$\mathcal{B}\left(-\nu_0, 0, 0, 0\right) = -\frac{1}{2\ F_K^2}(m_s + m_d)\mathcal{H}\left(t = 0, q^2 = 0\right), \tag{14}$$

with $\nu_0 = \frac{1}{4} M_\phi$.

Two more conditions follow from the constraint that the amplitude B should have Adler zeros [9], at the points $k_1 \to 0$, $k_2^2 = -m_K^2$ and $k_1^2 = -m_K^2$, $k_2 \to 0$ respectively:

$$\mathcal{B}\left(\nu_0, 0, 0, -m_K^2\right) = 0, \tag{15}$$

$$\mathcal{B}\left(-\nu_0, 0, -m_K^2, 0\right) = 0. \tag{16}$$

Obviously, conditions (13)-(16) are symmetric under the exchange $K^0 \leftrightarrow \overline{K^0}$, as implied by the s-wave character of eq.(1).

In order to turn eqs.(13)-(16) into a prediction for physical $\left(k_1^2 = k_2^2 = -m_K^2\right)$ kaons, we try a linear extrapolation in the range $0 \geq k_{1,2}^2 \geq -m_K^2$ and ν and ν_B in the neighborhood of the physical region, which should be quite reasonable for the non-resonant, smooth transition amplitude. To this we add the resonant amplitude, determined by the lowest lying s-wave poles (S^* and δ) coupled to $K^0\overline{K^0}$. Accordingly we write the amplitude B of eq.(1) in the form:

$$\mathcal{B}\left(\nu,\nu_B,k_1^2,k_2^2\right) = A + B_1 k_1^2 + B_2 k_2^2 + C\nu + \sum_i \frac{G_i}{M_i^2 - t}, \tag{17}$$

where $G_i = G_{\phi i\gamma} G_{iK^0\overline{K^0}}$ represent the residues of the poles ($i = S^*, \delta$). By imposing the constraints (13)-(16) on eq.(17), we obtain for the kaon on-shell amplitude:

$$\begin{aligned} \mathcal{B}\left(\nu,\nu_B,-m_K^2,-m_K^2\right) =& \frac{1}{2\,F_K^2}(m_s + m_d)\mathcal{H}\left(t=0,q^2=0\right) \\ &+ \sum_i G_i\left[\frac{1}{M_i^2} - \frac{2}{M_i^2 - m_K^2} + \frac{1}{M_i^2 - t}\right]. \end{aligned} \tag{18}$$

One important point to notice is that the procedure followed to arrive at eq.(18) has allowed the extrapolation of the on-shell amplitude to the soft-kaon limit without forcing $q \to p$ and thus $q^2 \to p^2 = -M_\phi^2$ [10], so that the photon is kept on its mass-shell $q^2 = 0$ all the way down. Also, we have tacitly assumed in the parametrization (17) that the dependence of the amplitude on the $K^0\overline{K^0}$ invariant mass t is entirely generated by the poles. Regarding these, we shall account for finite width effects in actual calculations by adding, as usual, imaginary parts $-iM_i\Gamma_i$ to the denominators in eqs.(17) and(18). As a matter of fact we should observe from eq.(18) that, besides providing the non-resonant amplitude, the current algebra conditions somewhat modify the form of the resonant amplitude from the simplest scalar pole dominance expression. As we shall see, this can have appreciable effects on the final numerical results for the rate of $\phi \to K^0\overline{K^0}\gamma$.

We must now estimate $\mathcal{H}\left(t=q^2=0\right)$, which appears in eq.(18). This can be done by vector meson dominating the current $V_\sigma^{e.m.}$ in eq.(7). To this purpose we write Σ for convenience as:

$$\Sigma = \eta_\lambda(p)\epsilon_\sigma(q)\mathcal{H}_{\lambda\sigma}, \tag{19}$$

where for $q^2 = 0$ gauge invariance requires that

$$\mathcal{H}_{\lambda\sigma} = \mathcal{H}_1(t,0)\delta_{\lambda\sigma} + \mathcal{H}_2(t,0)q_\lambda p_\sigma \tag{20}$$

with

$$\mathcal{H}_1(t,0) + (p\cdot q)\mathcal{H}_2(t,0) = 0, \tag{21}$$

giving eq.(12) with $\mathcal{H}\left(t, q^2=0\right) = \mathcal{H}_2\left(t, q^2=0\right)$. Here $(p \cdot q) = -\frac{1}{2}\left(M_\phi^2 - t\right)$. By vector meson dominating $\mathcal{H}_1(0,t)$ we have (only the ϕ contributes for an ideally mixed vector nonet):

$$\mathcal{H}_1\left(0, q^2\right) = \frac{C_S\ F_\phi}{M_\phi^2 + q^2}, \tag{22}$$

where C_S is the s-wave coupling for

$$< \phi|S(0)|\phi >= C_S\ (\eta \cdot \epsilon) \tag{23}$$

and F_ϕ is defined by

$$< 0|V_\sigma^{e.m.}|\phi(q,\epsilon) >= F_\phi\ \epsilon_\sigma(q) \tag{24}$$

with $F_\phi = 0.08\ GeV^2$ from the measured $\phi \to e^+e^-$ width. Using eq.(21) at $t = 0$, for which $(p \cdot q) = -\frac{1}{2}M_\phi^2$, we obtain:

$$\mathcal{H}(0,0) \equiv \mathcal{H}_2(0,0) = \frac{2\ \mathcal{H}_1(0,0)}{M_\phi^2} = \frac{2\ C_S\ F_\phi}{M_\phi^4}. \tag{25}$$

We may notice that the extrapolation in the photon mass from $q^2 = -M_\phi^2$ to $q^2 = 0$, which is implicit in the vector dominance approach, has been done here in an explicitly gauge invariant way.

To estimate C_S in eqs.(23) and (25), we recall the familiar form of the chiral $SU(3) \times SU(3)$ symmetry breaking Lagrangian density:

$$\mathcal{L}_{\mathrm{SB}} = \sum_{i=u,d,s} m_i \bar{q}_i q_i = \sqrt{6} m_0 S_0 + \frac{2}{\sqrt{3}}(\overline{m} - m_s) S_8, \tag{26}$$

where $m_0 = \frac{1}{3}(m_u + m_d + m_s)$ and $\overline{m} = \frac{1}{2}(m_u + m_d)$. With $|\phi>\equiv|s\bar{s}>= \sqrt{\frac{1}{3}}V_0 - \sqrt{\frac{2}{3}}V_8$, the OZI rule suggests:

$$< \phi|S_0|\phi >= -\sqrt{\frac{1}{2}} < \phi|S_8|\phi >, \tag{27}$$

$$< \phi|S_3|\phi >= 0, \tag{28}$$

so that, in eqs.(9) and (23):

$$< \phi|S(0)|\phi >= -\sqrt{3} < \phi|S_8(0)|\phi > . \tag{29}$$

Using

$$< V_i|S_8|V_j >= d_{8ij}\ \mathcal{D}, \tag{30}$$

with $i, j = 0, \ldots, 8$ as implied by the nonet symmetry of eqs.(27)and (28), we can relate the reduced matrix element $\mathcal{D}$ to the mass splitting among the vector meson nonet induced by S_8. In this way one finds:

$$C_S = 2\,\frac{M_\phi^2 - M_{K^*}^2}{m_s - \overline{m}}. \tag{31}$$

Replacing eq.(31) into eq.(25), and then into eq.(18), we can write the complete $\phi \to K^0 \overline{K^0} \gamma$ amplitude defined in eq.(1), in the following final form:

$$\begin{aligned}\mathcal{B}\left(\nu, \nu_B, -m_K^2, -m_K^2\right) =& \frac{2}{F_K^2} \frac{F_\phi}{M_\phi^2} \frac{M_\phi^2 - M_{K^*}^2}{M_\phi^2} \frac{m_s + \overline{m}}{m_s - \overline{m}} \\ &+ \sum_{i=S^*,\delta} G_i \left[\frac{1}{M_i^2} - \frac{2}{M_i^2 - m_K^2} + \frac{1}{M_i^2 - t}\right].\end{aligned} \tag{32}$$

The rate is obtained by integrating from $4m_K^2$ to M_ϕ^2 the differential spectrum in the $(K^0\overline{K^0})$ invariant mass, which in terms of the amplitude B can be expressed as:

$$\frac{d\Gamma(\phi \to K^0\overline{K^0}\gamma)}{dt} = \frac{\alpha}{128\pi^2 M_\phi^3}\,\frac{1}{3}\left(M_\phi^2 - t\right)^3 \left(1 - \frac{4m_K^2}{t}\right)^{\frac{1}{2}} |\mathcal{B}|^2. \tag{33}$$

Numerically the non-resonant (n.r.) amplitude, which corresponds to taking $G_i \equiv 0$ in eq.(32), is $\mathcal{B}_{n.r.} \simeq 3\ GeV^{-2}$, irrespective of the value of $m_s \gg \overline{m}$. This would give, using eq.(33):

$$\frac{\Gamma(\phi \to K^0\overline{K^0}\gamma)_{n.r.}}{\Gamma(\phi \to K^0\overline{K^0})} \simeq 1.4\ 10^{-9}. \tag{34}$$

We should emphasize that the estimate of the non-resonant amplitude presented here is model independent to a large extent, as it relies on some general, well established principles which underlie the theoretical description of low energy hadronic interactions.

To complete our calculation we need the values of the constants G_i in eq.(32), which cannot be fixed a priori, and therefore in principle should be taken from experiment. This is not possible, because the couplings $G_{\phi S^*\gamma}$, $G_{\phi\delta\gamma}$ and $G_{\delta K\overline{K}}$ are not measured, while only $G_{S^* K\overline{K}}$ is roughly known, so that one has to rely on theoretical predictions. However, the structure itself of the scalar mesons is not quite understood. Consequently, theoretical estimates of these constants dramatically depend on the assumed nature of S^* and δ ($^3P_0\ q\overline{q}$ quarkonium, glueball, $qq\overline{q}\overline{q}$ four-quark states, $K\overline{K}$ molecules).

To cover a range of interesting possibilities [5,7], we consider the values $G_{S^*} = -G_\delta = 0.6$, and $G_{S^*} = -G_\delta = 2.4$ as significant ones. The former choice is representative of the $q\overline{q}$ model for S^* and δ, and would give in the simplest scalar meson pole dominance (p.d.) approach (i.e. without any non-resonant amplitude):

$$\frac{\Gamma(\phi \to K^0\overline{K^0}\gamma)_{p.d.}}{\Gamma(\phi \to K^0\overline{K^0})} = 2\ 10^{-9}. \tag{35}$$

The latter choice is representative of the description of S^* and δ as $K\overline{K}$ molecules, and gives in the same approximation of eq.(35):

$$\frac{\Gamma(\phi \to K^0\overline{K^0}\gamma)_{p.d.}}{\Gamma(\phi \to K^0\overline{K^0})} = 3.7\ 10^{-8}. \tag{36}$$

We have taken opposite signs for G_{S^*} and G_δ because negative interference between S^* and δ (as well as positive interference in the mode $\phi \to K^+K^-\gamma$) seems to be a general fact [5,7]. Actually this would exactly occur, in general, for S^* and δ belonging to an ideally mixed nonet of scalar mesons, since $G_{S^*K^0\overline{K^0}} = -G_{\delta K^0\overline{K^0}}$ in that case, leading to a cancellation, modulo differences in the total widths.

Replacing the values of G_i reported above into the complete expression of the amplitude (non-resonant plus poles) in eq.(32), we obtain the predictions:

$$\frac{\Gamma(\phi \to K^0\overline{K^0}\gamma)}{\Gamma(\phi \to K^0\overline{K^0})} = (0.7 - 6.3)\ 10^{-9} \tag{37}$$

for the $q\overline{q}$ model of S^* and δ, and

$$\frac{\Gamma(\phi \to K^0\overline{K^0}\gamma)}{\Gamma(\phi \to K^0\overline{K^0})} = (2.8 - 5.1)\ 10^{-8} \tag{38}$$

in the case in which the S^* and δ were $K\overline{K}$ molecules. Two values are reported in each of eqs.(37) and (38), because we have allowed for more generality an unknown phase between the current algebra and the polar amplitudes in eq.(32), which we have varied between the extreme values 0 and 180 degrees. This seems the best determination we can offer, at present, of the process $\phi \to K^0\overline{K^0}\gamma$.

Comparing eqs.(37) and (38) with (35) and (36) respectively, we see that for $q\overline{q}$ scalar mesons the presence of the current algebra determined non-resonant amplitude makes a large effect, as its size is comparable to that of the purely polar term (see eq. (34)). In the case of the interpretation of δ and S^* as $K\overline{K}$ molecules, the effect of the non-resonant amplitude is reduced, but is still quite appreciable. This is due in large part to the modification of the polar term by the current algebra constraints, mentioned following eq.(18). Finally, had we assumed equal signs for G_{S^*} and G_δ (thus positive interference instead of negative interference between these two resonances), the numerical results reported in eqs.(35)-(38) would have increased by about one order of magnitude. However, as previously mentioned, such a possibility should be considered as unlikely.

In conclusion, our estimates of the branching ratio for $\phi \to K^0\overline{K^0}\gamma$ indicate it to be of the order of $10^{-9} - 2\ 10^{-8}$, depending on the structure of the $S^*(975)$ and the $\delta(980)$, and assuming in general partial cancellation among these resonances, as it is verified in explicit calculations. This result has relevance for using a ϕ-factory as a facility for precise measurements of the CP violating parameters in the kaon system at the ϕ resonance. Indeed, although confirming the process studied here as the dominant potential source of contamination to $\phi \to K_L K_S$, it indicates that the size of this CP-conserving background should not reach the level of significantly affecting the feasibility of CP-violation measurements at the precision planned at a ϕ factory.

Our study of $\phi \to K^0\overline{K^0}\gamma$ shows that in principle it may also provide a selective test of the structure of the lightest scalar mesons, if the ϕ-factory luminosity were high enough. If not, the process $\phi \to K^+K^-\gamma$ would be more promising in that regard, owing to the expected positive interference between S^* and δ, provided one could distinguish experimentally very low energy "direct emission" photons from bremsstrahlung ones.

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