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MULTI-PARTON INTERACTIONS AND INELASTIC CROSS SECTION
IN HIGH ENERGY HADRONIC COLLISIONS[†]

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ABSTRACT

We discuss the leading contribution of multiple semi-hard partonic interactions to the inelastic cross section in high energy hadronic collisions. Two parton correlations are explicitly taken into account and shown to give a negligible contribution to the integrated cross section.

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In high energy hadronic reactions one observes an increasingly large hard component in the interaction¹. One evidence is the large size of the cross section for production of mini-jets that has been observed at CERN $p\bar{p}$ Collider². Estimates of the rate of production of mini-jets at higher energies show that inclusive cross sections much larger than the total hadronic cross section have to be expected³. An inclusive cross section larger than the total one is not inconsistent, since the inclusive cross section counts the multiplicity⁴, the indication is therefore that the multiplicity of mini-jets is going to increase much at large energies. One has then to keep into account the possibility of several partonic interactions with exchange of a relatively large momentum in each inelastic event⁵. While all the multiple partonic collisions cancel when evaluating the inclusive cross section⁶, the complexity of the interaction shows up when looking to different physical observables⁷.

In the present note we want to discuss the simplest quantity that is sensible to the presence of multiple partonic collisions, namely the semi-hard cross section σ_H . One will define as semi-hard cross section the cross section for the events where there is at least one partonic interaction with momentum transfer larger than a given cut-off, the cross section is therefore obtained summing all possible multiple partonic collisions. To discuss σ_H one needs to introduce multiple parton collisions, the problem is that they depend also on multiple parton distributions that are quantities independent of the single parton distributions appearing in the QCD-parton-model. We will show, however, that, although multiple parton distributions are independent of the single parton distributions, since they contain the correlation term, this last one is not seen when looking at the leading contribution to the semi-hard cross section.

Multiple parton collisions are of two qualitatively different kinds. The first kind consists of the disconnected processes: several pairs of partons scatter indepen-

dently, the collisions being localized in different points in the transverse plane. The second kind is represented by parton rescatterings, namely a parton in the projectile can interact with exchange of relatively large transverse momentum with more than one parton of the target. The two possibilities can then combine. The process is obviously incoherent in the first case, while in the second case the situation is more complex since the two collisions are localized at the same transverse coordinate. The rescattering contribution to the integrated cross section is obtained summing over all possible discontinuities of the rescattering diagrams. As it has been shown in Ref.8, when looking to the integrated cross section, all possible cuts in the rescattering diagram are, at the leading order, proportional each other, the different weights been given by the AGK rules⁹. The sum of all the contributions will, as a consequence, amount to the introduction of an absorptive correction that is given by the iteration of the single scattering term. Introducing the probability for a parton i of the projectile to have a semi-hard interaction with a parton j of the target $\hat{\sigma}_{i,j}$ the absorptive correction to the semi-hard cross section can therefore be estimated consistently. We will then approach the problem of the semi-hard cross section in a probabilistic way.

To deal with multiple parton distributions we will use the technique of the generating functional, often used in the context of inclusive reactions¹⁰.

We start from the exclusive n -parton distribution $W_n(u_1 \dots u_n)$, where u_i represents the variables (b_i, x_i) , being b the transverse partonic coordinate and x the corresponding fractional momentum. The scale for the distributions is given by the cut off p_t^{min} that defines the separation between soft and hard collisions. The distributions are symmetric in the variables u_i and their generating functional is defined as:

$$\mathcal{Z}[J] = \sum_n \frac{1}{n!} \int J(u_1) \dots J(u_n) W_n(u_1 \dots u_n) du_1 \dots du_n. \quad (1)$$

The conservation of the probability yields the overall normalization condition $\mathcal{Z}[1] = 1$. The logarithm of the same functional is also useful: $\mathcal{Z}[J] = e^{\mathcal{F}[J]}$, with the normalization $\mathcal{F}[1] = 0$. The many body densities, i.e. the inclusive distributions, can then be introduced in the following way:

$$\begin{aligned} D_1(u) &= W_1(u) + \int W_2(u, u') du' + \frac{1}{2} \int W_3(u, u', u'') du' du'' + \dots \\ &= \left. \frac{\delta \mathcal{Z}}{\delta J(u)} \right|_{J=1} = \left. \frac{\delta \mathcal{F}}{\delta J(u)} \right|_{J=1}, \\ D_2(u_1, u_2) &= W_2(u_1, u_2) + \int W_3(u_1, u_2, u') du' + \frac{1}{2} \int W_4(u_1, u_2, u', u'') du' du'' \dots \\ &= \left. \frac{\delta^2 \mathcal{Z}}{\delta J(u_1) \delta J(u_2)} \right|_{J=1} = \left. \frac{\delta^2 \mathcal{F}}{\delta J(u_1) \delta J(u_2)} \right|_{J=1} + \left. \frac{\delta \mathcal{F}}{\delta J(u_1)} \frac{\delta \mathcal{F}}{\delta J(u_2)} \right|_{J=1} \end{aligned} \quad (2)$$

and so on. The expansion of the functional \mathcal{F} around $J = 1$ gives the correlations $C_n(u_1 \dots u_n)$, describing how much the distribution deviates from a Poisson distribution, for which in fact $C_n \equiv 0, n \geq 2$:

$$\mathcal{F}[J] = \int D_1(u) [J(u) - 1] du + \sum_{n=2}^{\infty} \frac{1}{n!} \int C_n(u_1 \dots u_n) [J(u_1) - 1] \dots \dots [J(u_n) - 1] du_1 \dots du_n \quad (3)$$

Given the general expressions for the multiparton distributions one can write down an expression for the semi-hard cross section that will take into account all multiple partonic interactions:

$$\begin{aligned}
\sigma_H = & \int d\beta \int \sum_n \frac{1}{n!} \frac{\delta}{\delta J(u_1)} \cdots \frac{\delta}{\delta J(u_n)} \mathcal{Z}_A[J] \\
& \times \sum_m \frac{1}{m!} \frac{\delta}{\delta J'(u'_1 - \beta)} \cdots \frac{\delta}{\delta J'(u'_m - \beta)} \mathcal{Z}_B[J'] \\
& \times \left\{ 1 - \prod_{i=1}^n \prod_{j=1}^m [1 - \hat{\sigma}_{i,j}(u, u')] \right\} \prod dud u' \Big|_{J=J'=0}
\end{aligned} \tag{4}$$

where β is the impact parameter between the two interacting hadrons A and B and $\hat{\sigma}_{i,j}$ is the probability for parton i (of A) to have a hard interaction with parton j (of B). The expression for σ_H is obtained summing over all partonic configurations of the two hadrons and for each configuration requiring at least one hard interaction (the factor in curly brackets). In fact here one takes into account all multiparton interactions since any given configuration with n partons of one hadron can interact with any configuration with m partons of the other hadron. Not all multiple collisions are equally likely, however, since the elementary interaction probability $\hat{\sigma}$ is small. We will then work out of the general expression for the cross section the contribution of disconnected partonic collisions. We can therefore require that each parton interacts at most once, but, even with this simplification, the general form of the multiparton correlations makes the treatment exceedingly complicated. We will then consider the case of two body correlations only, so that in Eq.(3) $\mathcal{F}[J]$ is given by the first two terms of the expansion. In order to simplify the notation we will avoid in the following writing explicitly the variables u and u' (a part of a few cases where it will be convenient for a better understanding of the argument) and the integration on the impact parameter β . We will also represent the functional derivative $\frac{\delta}{\delta J(u_i)}$ with ∂_i . The semi-hard cross section σ_H is therefore expressed as:

$$\sigma_H = \sum_n \frac{1}{n!} \partial_1 \dots \partial_n \sum_m \frac{1}{m!} \partial'_1 \dots \partial'_m \left\{ 1 - \prod_{i,j}^{n,m} [1 - \hat{\sigma}_{ij}] \right\} \mathcal{Z}_A[J] \mathcal{Z}_B[J'] \Big|_{J=J'=0} \quad (5)$$

We now write the term in curly brackets in Eq.(5) as:

$$S \equiv 1 - \exp \sum_{ij} \ln(1 - \hat{\sigma}_{ij}) = 1 - \exp \left[- \sum_{ij} \left(\hat{\sigma}_{ij} + \frac{1}{2} \hat{\sigma}_{ij} \hat{\sigma}_{ij} + \dots \right) \right] \quad (6)$$

and, in order to eliminate all partonic re-interactions we perform the substitutions:

$$S \Rightarrow 1 - \exp \sum_{ij} (\hat{\sigma}_{ij}) \Rightarrow \sum_{ij} \hat{\sigma}_{ij} - \frac{1}{2} \sum_{ij} \sum_{k \neq i, l \neq j} \hat{\sigma}_{ij} \hat{\sigma}_{kl} \dots$$

Keeping into account the symmetry of the derivative operators in Eq.(5) one can carry out all the sums. The expression for σ_H then becomes:

$$\begin{aligned} \sigma_H &= \exp(\partial) \cdot \exp(\partial') \left[1 - \exp(-\partial \cdot \hat{\sigma} \cdot \partial') \right] \mathcal{Z}_A[J] \mathcal{Z}_B[J'] \Big|_{J=J'=0} \\ &= \left[1 - \exp(-\partial \cdot \hat{\sigma} \cdot \partial') \right] \mathcal{Z}_A[J+1] \mathcal{Z}_B[J'+1] \Big|_{J=J'=0} \end{aligned} \quad (7)$$

With the help of the logarithmic functional \mathcal{F} one can introduce explicitly the partonic correlations. We will limit ourselves, as already anticipated, to the case where all correlation functions C_n with $n > 2$ can be neglected. Eq.(7) is then given by:

$$\begin{aligned} \sigma_H &= \left[1 - \exp \left\{ - \int dudu' \frac{\delta}{\delta J(u)} \hat{\sigma}(u, u') \frac{\delta}{\delta J(u')} \right\} \right] \cdot \\ &\quad \cdot \exp \left\{ \int D_A(u) J(u) du + \frac{1}{2} \int C_A(u, v) J(u) J(v) dudv \right\} \cdot \\ &\quad \cdot \exp \left\{ \int D_B(u) J(u) du + \frac{1}{2} \int C_B(u, v) J(u) J(v) dudv \right\} \Big|_{J=J'=0} \end{aligned} \quad (8)$$

Eq.(8) can be expressed in a compact way by introducing the matrices:

$$\varphi(u) \equiv \begin{pmatrix} J(u) \\ J(u) \end{pmatrix}, \quad \eta(u) \equiv \begin{pmatrix} D_A(u) \\ D_B(u) \end{pmatrix}$$

$$\mathbf{M}(u, u') \equiv \hat{\sigma}(u, u') \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{C}(u, v) \equiv - \begin{pmatrix} C_A(u, v) & 0 \\ 0 & C_B(u, v) \end{pmatrix}$$

so that one can write

$$\sigma_H = \left[1 - \exp \left\{ -\frac{1}{2} \frac{\delta}{\delta \varphi}{}^T \mathbf{M} \frac{\delta}{\delta \varphi} \right\} \right] \cdot \exp \left\{ \eta^T \varphi - \frac{1}{2} \varphi^T \mathbf{C} \varphi \right\} \Big|_{\varphi=0} \quad (9)$$

where the upper T is for transposed. One can now eliminate the term linear in φ in the argument of the exponential by means of the shift

$$\varphi \rightarrow \chi + \mathbf{C}^{-1} \eta,$$

in such a way that the operator in square parentheses in Eq.(9) will act on

$$\mathcal{R} \equiv \exp \left\{ -\frac{1}{2} \chi^T \mathbf{C} \chi \right\} \cdot \exp \left\{ \frac{1}{2} \eta^T \mathbf{C}^{-1} \eta \right\}.$$

It is useful to express \mathcal{R} by means of a functional Fourier transform, in this way σ_H is given by:

$$\sigma_H = \left[1 - \exp \left\{ -\frac{1}{2} \frac{\delta}{\delta \chi}{}^T \mathbf{M} \frac{\delta}{\delta \chi} \right\} \right] \cdot \frac{(2\pi)^{-\nu}}{\sqrt{\det \mathbf{C}}} \int e^{i\lambda^T \chi} \cdot \exp \left\{ -\frac{1}{2} \lambda^T \mathbf{C}^{-1} \lambda \right\} d\lambda \cdot \exp \left\{ \frac{1}{2} \eta^T \mathbf{C}^{-1} \eta \right\} \Big|_{\chi = -\mathbf{C}^{-1} \eta} \quad (10)$$

where the index ν is symbolic, it remembers that the functional integral is treated as an integral over a space of finite (2ν) dimensions. The \det (and the tr symbol in the following expressions) must be understood to operate both on the 2×2 matrices and in the functional u -space. Acting with the operator in square parentheses one

introduces, in the integral representation, the factor $1 - \exp\left(\frac{1}{2}\lambda^T \mathbf{M} \lambda\right)$. The cross section is therefore represented with a Gaussian functional integral. Performing the functional integral one obtains:

$$\sigma_H = 1 - \frac{\exp\left[-\frac{1}{2}\eta^T \mathbf{C}^{-\frac{1}{2}} \left(1 - \mathbf{C}^{\frac{1}{2}} \mathbf{M} \mathbf{C}^{\frac{1}{2}}\right)^{-1} \mathbf{C}^{-\frac{1}{2}} \eta + \frac{1}{2}\eta^T \mathbf{C}^{-1} \eta\right]}{\sqrt{\det(1 - \mathbf{C}^{\frac{1}{2}} \mathbf{M} \mathbf{C}^{\frac{1}{2}})}} \quad (11)$$

Eq.(11) is a closed expression for the semi-hard cross section, it represents the sum of all the disconnected partonic processes (disconnected from the side of the semi-hard partonic interaction). One can analyze the physical content of Eq.(11) explicitating the dependence on the parton averages D and on the correlations C . Let us look first to the exponent:

$$\begin{aligned} & -\frac{1}{2}\eta^T \mathbf{C}^{-\frac{1}{2}} \left[1 - \left(1 - \mathbf{C}^{\frac{1}{2}} \mathbf{M} \mathbf{C}^{\frac{1}{2}}\right)^{-1}\right] \mathbf{C}^{-\frac{1}{2}} \eta = \\ & = -\frac{1}{2}\eta^T \left[\mathbf{M} + \mathbf{M} \mathbf{C} \mathbf{M} + \mathbf{M} \mathbf{C} \mathbf{M} \mathbf{C} \mathbf{M} + \dots\right] \eta \\ & = -\frac{1}{2} \left[2D_A \hat{\sigma} D_B - D_A \hat{\sigma} C_B \hat{\sigma} D_A - D_B \hat{\sigma} C_A \hat{\sigma} D_B + \dots\right] \end{aligned} \quad (12)$$

where all the products have to be understood as convolutions. Also the root of the determinant can be written as an exponential:

$$\frac{1}{\sqrt{\det(1 - \mathbf{C}^{\frac{1}{2}} \mathbf{M} \mathbf{C}^{\frac{1}{2}})}} = \exp\left[-\frac{1}{2} \text{tr} \left(\mathbf{M} \mathbf{C} + \frac{1}{2} \mathbf{M} \mathbf{C} \mathbf{M} \mathbf{C} + \dots\right)\right] \quad (13)$$

where all the traces with an odd power of $\mathbf{M} \mathbf{C}$ are zero. The first term different from zero in the exponent in Eq.(13) is:

$$\text{tr}(\mathbf{M} \mathbf{C} \mathbf{M} \mathbf{C}) = \int \hat{\sigma}(u_1, u'_1) C_A(u_1, u_2) \hat{\sigma}(u_2, u'_2) C_B(u'_1, u'_2) \prod_{i=1}^2 du_i du'_i + A \leftrightarrow B. \quad (14)$$

One has then two possible structures for the connected parton collisions, the first has the form:

$$\int D_A(u_1)\hat{\sigma}(u_1, u'_1)C_B(u'_1, u'_2)\hat{\sigma}(u'_2, u_2)C_A(u_2, u_3)\dots \dots \hat{\sigma}(u_n, u'_n)D_B(u'_n) \prod_{i=1}^n du_i du'_i \quad (15)$$

while the second is:

$$\int C_A(u, u_1)\hat{\sigma}(u_1, u'_1)C_B(u'_1, u'_2)\hat{\sigma}(u'_2, u_2)C_A(u_2, u_3)\dots \dots \hat{\sigma}(u_n, u'_n)C_B(u'_n, u')\hat{\sigma}(u', u)dudu' \prod_{i=1}^n du_i du'_i \quad (16)$$

As shown in the figure (case *b*) Eq.(16) corresponds to a 'closed' graph while Eq.(15) is an 'open' graph (case *a*). One can 'open' a 'closed' graph replacing one C with a DD pair. A 'closed' graph of order $\hat{\sigma}^{2n}$ can be 'opened' in $2n$ ways, so that for each 'closed' graph of order $\hat{\sigma}^{2n}$ there are $2n$ open graphs of the same order, this explains the $\frac{1}{n}$ in the argument on the exponential coming from the expansion of the square root of the determinant in Eq.(13). All disconnected scatterings can be obtained expanding the exponential *.

One will notice that, when no correlations are present, everything becomes trivial and one is left only with the term $D_A\hat{\sigma}D_B$ in the exponent. Switching on two body correlations corresponds to take into account all possible structures of the kind represented in Eq.(15) and in Eq.(16). One then expects that the

* *We remark that the whole machinery of the functional integration has been merely used as a device to deal with the combinatorial properties of Eq.(9). For this reason the calculations were carried out as if we knew that the eigenvalues of C are definite positive. If this is not the case we must perform a sort of continuation (in C_A, C_B) in Eq.(12) and (13)*

problem of including more than two body correlations will be a very difficult one to handle. On the other hand, in the limit of a large number of partonic interactions, the hadron becomes 'black' so that keeping terms of order $\hat{\sigma}^2$ in the exponent in Eq.(11) is not going to be of much importance in estimating σ_H . One then expects that σ_H can be approximated well by:

$$\sigma_H = \int d\beta \left\{ 1 - \exp \left[- \int D_1^A(u - \beta) D_1^B(u') \hat{\sigma}(u, u') du du' \right] \right\} \quad (17)$$

where no correlation is present any more, the reason being not that partonic correlations are a priori small, but rather that they appear with higher powers in the elementary interaction probability $\hat{\sigma}$. In writing Eq.(17), one is actually neglecting terms of kind $D_A \hat{\sigma} C_B \hat{\sigma} D_A$ in comparison with terms of kind $D_A \hat{\sigma} D_B D_B \hat{\sigma} D_A$. That is not reasonable when correlations are important, however this attitude can be justified if the first term in the exponent, namely $D_A \hat{\sigma} D_B$, is already large enough to make the exponential close to zero. That will happen when the typical number of partonic collisions is large and the impact parameter is small. For large impact parameters the number of collisions becomes small, and neglecting terms of order $\hat{\sigma}^2$ is then reasonable. As a consequence σ_H is represented well by Eq.(17) both for small values of β and for large ones. In the eikonal models for high energy hadronic interaction¹¹ the contribution from semi-hard partonic interactions is included writing σ_H as in Eq.(17).

One nice feature of the expression for σ_H , even in the simplest case as given by Eq.(17), we want to point out is that, being obtained keeping unitarity into account, it is infrared safe: when the cut-off p_t^{min} becomes smaller and smaller, while the argument of the exponent becomes very large, σ_H , as given by Eq.(17), will rather tend to a finite limiting value. The reason is that, even if the cut off and the c.m. energy are such that the average number of partons D is very large,

still the argument of the exponential depends on the hadronic impact parameter β . When β is larger than some typical transverse hadronic scale R , the argument of the exponent becomes zero since there is no overlap between the matter distributions of the two interacting hadrons. σ_H is therefore a regular function of the cut off p_t^{min} for $p_t^{min} \rightarrow 0$, its limiting value being πR^2 .

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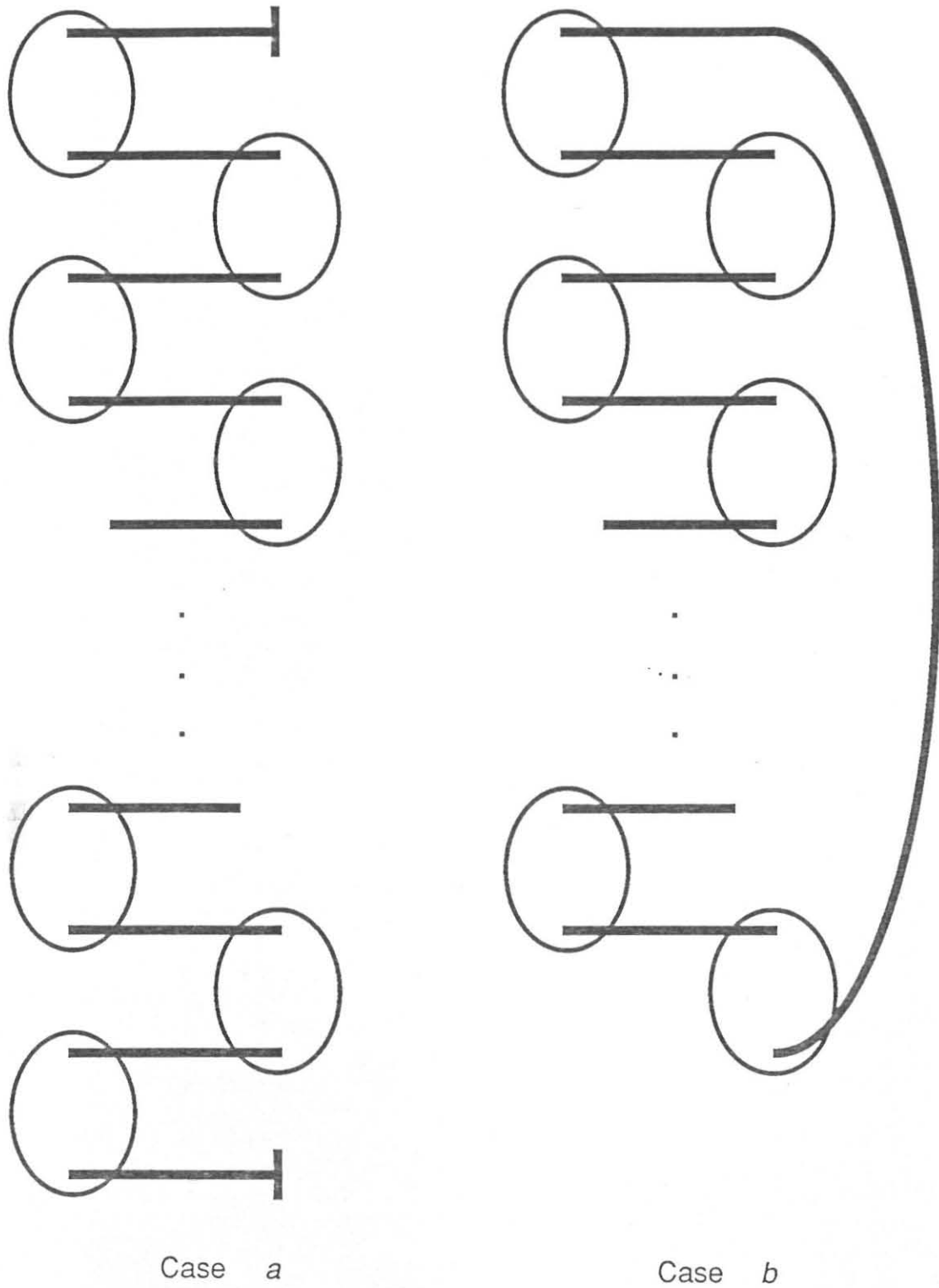


Figure: Graphical representation of equations 15 and 16 in the text. The vertical lines in 'case a' indicate the average number of partons D , the bubbles the two parton correlation C and the lines connecting the bubbles represent the elementary interaction probability $\hat{\sigma}$.