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1. INTRODUCTION

We will investigate in this report the situation arising when a linearly polarized laser light beam traverses a region where an almost static pseudoscalar field $\vec{E}_a$ is present. This field is defined by

$$\vec{E}_a = \text{grad}\Phi_a, \tag{1}$$

where Φ_a is the pseudoscalar potential; one possible origin of such a potential will be discussed below. The interaction Lagrangian between the pseudoscalar potential and the electromagnetic field is expressed by the term[*]

$$L = \frac{1}{M}\vec{E}\cdot\vec{B}\,\Phi_a, \tag{2}$$

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[*] Throughout this report we will use normalized Heaviside units.

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors of the light wave and M is a parameter ($1/M=g_{a\gamma\gamma}$) which characterizes the strength of the photon-photon-pseudoscalar field coupling. We will restrict ourselves to the case of a light beam propagating for a distance ℓ in a static transverse pseudoscalar field $\vec{E}_a$, and determine the changes that will occur, due to the interaction term (2), in the light polarization state and in the light propagation vector.

The potential Φ_a in equation (1) can be interpreted as being due to the existence of light pseudoscalar particles such as the axion [1,2,3], the arion [4,5] or the maioron [6]. Present limits for the mass m_a of these hypothetical particles range around $m_a \leq 10^{-3}$ eV.

Physical effects due to the real presence of a pseudoscalar field could in practice arise in experimental apparati realized to detect small changes in the polarization state of a light beam in presence of a strong static magnetic field B_0 caused by vacuum fluctuation phenomena in Q.E.D. [7,8] or by an interaction term of the type shown in equation (2) [9]. In these experiments a strong transverse magnetic field, throught which the light beam is sent, is generally provided by a coil surrounded by a soft iron yoke for the purpose of field containment. A magnetization is therefore induced in the soft iron, and the oriented electron spins could be, according to ref. [10], a source for the pseudoscalar field. In the following we will assume the mass m_a of the pseudoscalar field sufficiently small and apply the formalism of ref. [10] to have a real quantitative example of the field $\vec{E}_a$ generated in the mentioned experimental set-up.

2. DEDUCTION OF THE RELEVANT EXPRESSIONS

An arion type field $\vec{E}_a$, if it exists can have as source charged fermions. In particular the interaction of the arion potential Φ_a with electrons is given by (in the non relativistic limit) [10] :

$$L_e = \frac{1}{\mathfrak{M}} \vec{\nabla}(\psi^* \vec{\sigma}\psi)\Phi_a \tag{3}$$

where ψ is the two component electron wave function and $\mathfrak{M}$ is a parameter ($1/\mathfrak{M}=g_{aee}$) characterizing the arion-electron-electron coupling constant.

From the Lagrangian in eq.(3) considering a distribution $\vec{S}(\vec{r})$ of spins due to polarized electrons, the pseudoscalar potential Φ_a induced by the spin density $\psi^* \vec{\sigma}\psi$ is given by [10] :

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$$\Phi_a(\vec{r}) = \frac{1}{4\pi\mathfrak{M}} \int \frac{\vec{S}(\vec{r}')\bullet(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \, dr'^3 \qquad (4)$$

Before deriving an approximate expression for $\vec{S}(\vec{r})$, we will calculate, using Maxwell's equations, the effect of such a field $\vec{E}_a$ on light polarization state

From eq.(2) we have

$$\vec{D} = \vec{E} + \frac{\Phi_a}{M}\vec{B} \qquad (5a)$$

$$\vec{H} = \vec{B} - \frac{\Phi_a}{M}\vec{E} . \qquad (5b)$$

In absence of charges and currents Maxwell's equations now become

$$\operatorname{div}\vec{B} = 0 \qquad (6a)$$

$$\operatorname{div}\vec{D} = 0 \qquad (6b)$$

$$\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (6c)$$

$$\operatorname{rot}\vec{H} = +\frac{\partial\vec{D}}{\partial t} \qquad (6d)$$

from which $\qquad\qquad \operatorname{div}\vec{E} = -\frac{1}{M}\vec{E}_a\bullet\vec{B} .\qquad\qquad (7)$

Assuming a plane wave solution for the electric field in eq.(6) we find

$$\operatorname{div}\vec{E} = -i\vec{k}\bullet\vec{E} = -\frac{\vec{B}\bullet\vec{E}_a}{M} \qquad (8a)$$

$$\operatorname{rot}\vec{B} = -i\vec{k}\times\vec{B} = \frac{1}{M}\vec{E}_a\times\vec{E} + i\omega\vec{E} \qquad (8b)$$

whence

$$\vec{B} = \frac{1}{\omega}(\vec{k}\times\vec{E}) \qquad (9)$$

Thus eq (8b) becomes

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$$\text{rot } \vec{B} = -i\frac{\vec{k}}{\omega} \times (\vec{k} \times \vec{E}) = \frac{1}{M} \vec{E}_a \times \vec{E} + i\omega\vec{E} \tag{10}$$

or

$$-i\frac{\vec{k}}{\omega}(\vec{k} \cdot \vec{E}) + i\frac{k^2}{\omega}\vec{E} = \frac{1}{M} \vec{E}_a \times \vec{E} + i\omega\vec{E} \tag{11}$$

taking the scalar product with $\vec{E}$ of both sides eq.(11) yields

$$-(\vec{k} \cdot \vec{E})^2 + k^2 E^2 = \omega^2 E^2, \tag{12}$$

from which using eq.(8a) we can write

$$(\omega^2 E^2)E^2 = \left(\frac{\vec{B} \cdot \vec{E}_a}{M}\right)^2 \tag{13}$$

The refractive index n, assuming (n-1) « 1, can be evaluated from

$$(\omega^2 - k^2) = (1+n)(1-n)k_0^2 = 2(1-n)k_0^2 \tag{14}$$

where k_0 is the wave number in vacuum.
Using eq.(14) in eq.(13) we obtain

$$(n-1) = -\frac{1}{2}\left(\frac{\vec{B} \cdot \vec{E}_a}{k_0 E M}\right)^2 \tag{15}$$

Let $\vec{u}_b$ be the unit vector along the direction of $\vec{B}$, then eq(15) can be expressed as

$$(n-1) = -\frac{1}{2}\left(\frac{\vec{u}_b \cdot \vec{E}_a}{k_0 M}\right)^2 \tag{16}$$

From eq.(16) we can see that n will be different for light beams with the polarization vector $\vec{E}$ parallel or orthogonal to $\vec{E}_a$. The region of space where the arion field is present will therefore behave as a birefringent medium.

Furthermore a light beam travelling in a medium is deflected if the refractive index varies as a function of coordinates [11]. Following ref.11 in the geometrical optics approximation we can write the fields of the light wave in a general form

$$\vec{E} = \vec{e}(\vec{r})\, e^{ik_0 S(\vec{r})}, \quad \vec{H} = \vec{h}(\vec{r})\, e^{ik_0 S(\vec{r})},$$

where $S(\vec{r})$ is a real scalar function of position, and $\vec{e}(\vec{r})$ and $\vec{h}(\vec{r})$ are vector function of position which may in general be complex. The function S is often called the eikonal. The surface $S = $ constant is usually called geometrical wavefront.

The behaviour of the light rays defined as the orthogonal trajectories to the geometrical wavefront is governed by eq.(17) [11]

$$\frac{d}{ds}\left(n(\vec{r}) \frac{d\vec{r}}{ds} \right) = \text{grad } n(\vec{r}) \qquad (17)$$

where $\vec{r}$ is a position vector of a typical point on a ray and s the length of the ray measured from a fixed point on it.

If the refractive index is constant equation (17) can be solved to give the parametric equation of a straight line. If $|\text{grad } n| \ll 1$ equation (17) becomes

$$d\theta = d\ell\, \frac{|\text{grad } n(\vec{r})|}{n} \qquad (18)$$

where ℓ is the length of the light path in the medium and θ is the deflection angle.
In our particular case eq.(18) yields

$$d\theta = d\ell\, \frac{\left| \text{grad}\left(1 - \dfrac{1}{2}\dfrac{(\vec{u}_b \cdot \vec{E}_a)^2}{k_0^2 M^2} \right) \right|}{1 - \dfrac{1}{2}\dfrac{(\vec{u}_b \cdot \vec{E}_a)^2}{k_0^2 M^2}} \qquad (19)$$

where we have used eq.(16).

At the first order in $(n-1)$ eq.(19) can be approximated by

$$d\theta = d\ell\, \frac{(\vec{u}_b \cdot \vec{E}_a)}{k_0^2 M^2} \sqrt{ \left(\frac{\partial(\vec{u}_b \cdot \vec{E}_a)}{\partial x} \right)^2 + \left(\frac{\partial(\vec{u}_b \cdot \vec{E}_a)}{\partial y} \right)^2 + \left(\frac{\partial(\vec{u}_b \cdot \vec{E}_a)}{\partial z} \right)^2 } \qquad (20)$$

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This last eq. shows that a light beam traversing a region of space where a non uniform arion field is present will be deflected by an angle which depend both on the arion field and on the light polarization state.

3. APPLICATION

We will now apply the above results to the special case of a long dipole magnet and calculate an expression for Φ_a. For the sake of simplicity the yoke of the magnet is assumed to be a soft iron cylinder. Near the magnet axis the magnetic field $\vec{B}$ and according to eq.(4) an arion field $\vec{E}_a$ are present; both generated by the magnetization density due to the oriented electrons (fig. 1).in the iron yoke. As we have already said in the introduction we are interested only in the effect of $\vec{E}_a$.
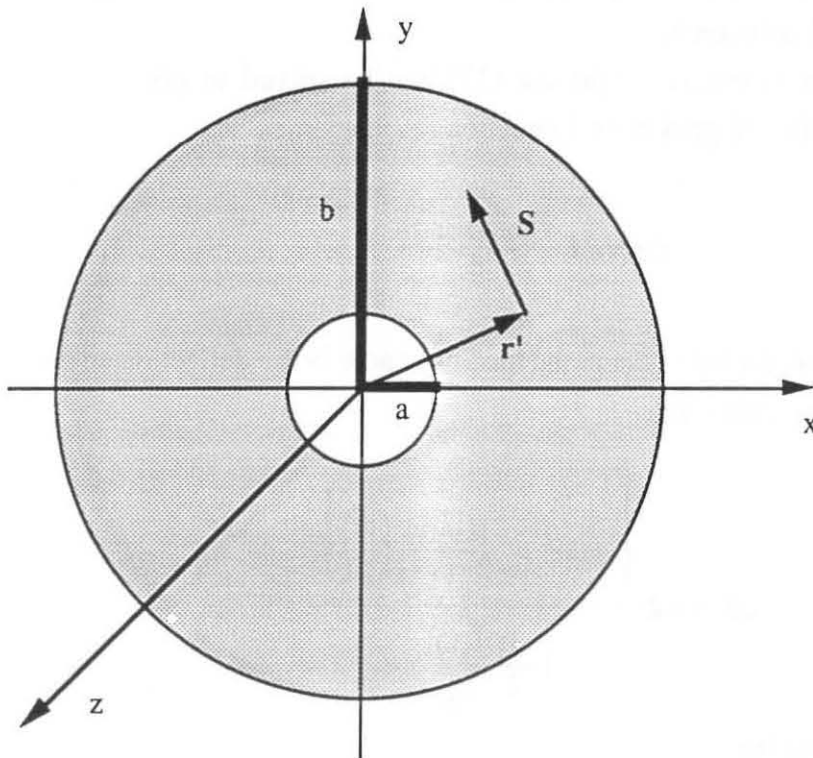

Fig. 1 - Schematic drawing of the magnet iron yoke with the coordinate axis.
(Lenght of the magnet = d).

To evaluated Φ_a we assume that $S = |\vec{S}|$ is constant over the region of interest and that $S_y(x<0) = S_y(x>0)$, $S_x(x<0) = -S_x(x>0)$ and $S_z = 0$. This is a good approssimation of the spin density distribution when the magnetic field is intense enough to saturate the magnetization. As shown in appendix (A) in the neighbourhood of $(x,y) = (0, 0)$ the arion potential can be written as

$$\Phi_a(x,y) = \frac{S}{\pi \mathfrak{M}} \left[2y Log\frac{a}{b} + \left(\frac{1}{a^2} - \frac{1}{b^2}\right)yx^2 \right] \tag{21}$$

Saturated soft iron has 2 oriented electrons per atom [12]. Thus we have

$$S = 2\frac{\rho}{W} N_0 \tag{22}$$

where ρ is the iron density, W its atomic weight and N_0 is Avogadro's number. From eq.(22) we can estimate $S \approx 1.5 \cdot 10^{23}$ spin/cm^3. Assuming that $a = 5$ cm and $b = 20$ cm (fig 1), we can finally write from eq.(1) and (21).

$$E_{ax}\left(\sqrt{\frac{erg}{cm^3}}\right) = \frac{0.1}{\mathfrak{M}(GeV)} yx \tag{23a}$$

$$E_{ay}\left(\sqrt{\frac{erg}{cm^3}}\right) = -\frac{7.4}{\mathfrak{M}(GeV)} (1 - 0.01\, x^2) \tag{23b}$$

We can now apply eq.(16) to obtain the value of $(n - 1)$ in the two cases of light polarized along x axis or along y axis. At the first order in the coordinates we have for $\vec{E}$ along y:

$$n-1 = 0 \tag{24a}$$

for $\vec{E}$ along x:

$$(n-1) = \frac{1}{2k_0^2} \left(\frac{7.4}{M(GeV)\mathfrak{M}(GeV)}\right)^2 \tag{24b}$$

and, taking $k_0 \approx 10^5$ cm^{-1},

for $\vec{E}$ along y: $\qquad\qquad\qquad n-1 = 0 \tag{25a}$

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for $\vec{E}$ along x:

$$(n-1) = 3.0 \ 10^{-20} \left(\frac{1}{M(GeV)\mathfrak{M}(GeV)} \right)^2 \tag{25b}$$

A light beam linearly polarized at 45° with respect to the x axis after travelling a distance ℓ in the arion field will acquire an ellipticity given by

$$\Psi = \pi \frac{\ell}{\lambda} \Delta n \tag{26}$$

where Δn is the difference between the refractive index along x and y axis, ℓ is the length of the optical path and λ is the light wavelength. Taking $\ell = 10^6$ cm and $\lambda = 6.3 \ 10^{-5}$ cm we have

$$\Psi = 1.5 \ 10^{-9} \left(\frac{1}{M(GeV)\mathfrak{M}(GeV)} \right)^2 \tag{27}$$

Considering now finally the effect of the non uniformity of the arion field $\vec{E}_a$ we can use eq.(20) to evaluate the angle by which the light beam will be deflected after traversing a distance dl in the field region. Let us assume that the beam enters this region at y = 0. From eq.(23 a,b) we see that in this case the arion field is directed along y axis and depends on x. Eq.(20) then yield

$$dq = - 2.1 \ 10^{-24} x \ d\ell \left(\frac{1}{M(GeV)\mathfrak{M}(GeV)} \right)^2 \tag{28}$$

CONCLUSIONS

We have shown that any present laboratory experiment devoted to the production of pseudoscalars by the interaction of light with a strong magnetic field [13] will not be pratically affected by the interaction of light with the pseudoscalar field produced by the spin density in the magnet yoke. Infact if we consider the today accepted laboratory limits [13] (M(GeV), $\mathfrak{M}$ (GeV) > 10^6) the effects are many order of magnitude below the sensitivity. Actually the effects are so small that it seems very unlikely that they will be in the sensitivity range of any future proposed experiment.

APPENDIX A

We have already pointed out that in the static limit, the arion potential $\Phi_a(\vec{r})$ induced by the spin density $\psi^* \vec{\sigma} \psi$ can be written

$$\Phi_a(\vec{r}) = \frac{1}{4\pi\mathfrak{M}} \int \frac{\vec{S}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \, dr'^3 \tag{4}$$

In figure 1 we show the reference coordinate system. Let's take S constant over the region of interest with $S_y(x<0) = S_y(x>0)$, $S_x(x<0) = -S_x(x>0)$ and $S_z = 0$. Therefore we define in the region $(x > 0)$ $S_x = S \sin\theta$, $S_y = -S \cos\theta$, $S_z = 0$ and in the region $(x < 0)$ $S_x = -S \sin\theta$, $S_y = -S \cos\theta$, $S_z = 0$ where θ is the angle between the position vector $\vec{r}'$ and the x axis. Eq.(4) becomes

$$\Phi_a(\vec{r}) = \frac{S}{4\pi\mathfrak{M}} \left\{ \left[\int_{-\pi/2}^{\pi/2} \int_a^b \int_{-d/2}^{d/2} \frac{sen\theta(x-r'cos\theta)-cos\theta(y-r'sen\theta)}{\left(\sqrt{(x-r'cos\theta)^2 + (y-r'sen\theta)^2 + (z-z')^2}\right)^3} \, r' \, dr' \, d\theta \, dz' \right] + \right.$$

$$\tag{A.1}$$

$$\left. - \left[\int_{\pi/2}^{3\pi/2} \int_a^b \int_{-d/2}^{d/2} \frac{sen\theta(x-r'cos\theta)-cos\theta(y-r'sen\theta)}{\left(\sqrt{(x-r'cos\theta)^2 + (y-r'sen\theta)^2 + (z-z')^2}\right)^3} \, r' \, dr' \, d\theta \, dz' \right] \right\}$$

Integrating over z' with d->∞ we obtain a simpler formula that using the properties of the trigonometric functions becomes

$$\Phi_a = \frac{S}{2\pi\mathfrak{M}} \left[\int_{-\pi/2}^{\pi/2} \int_a^b \frac{(xSin\theta - yCos\theta)}{(x - r' Cos\theta)^2 + (y - r' Sin\theta)^2} \, r' \, dr' \, d\theta \right] \tag{A.2}$$

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Eq. (A2) yields

$$\Phi_a = \frac{S}{4\pi\mathfrak{M}} \left[-2x\,\text{ArcTan}\!\left(\frac{a-y}{x}\right) + 2x\,\text{ArcTan}\!\left(\frac{b-y}{x}\right) + 2x\,\text{ArcTan}\!\left(\frac{a+y}{x}\right) - 2x\,\text{ArcTan}\!\left(\frac{b+y}{x}\right) + \right.$$

$$+ y\,\text{Log}\,(a^2+x^2-2ay+y^2) + y\,\text{Log}\,(a^2+x^2+2ay+y^2) +$$

$$- y\,\text{Log}\,(b^2+x^2-2by+y^2) - y\,\text{Log}\,(b^2+x^2+2by+y^2) + \qquad\text{(A.3)}$$

$$- a\,\text{Log}\,(2a^2+2x^2-4ay+2y^2) + a\,\text{Log}\,(2a^2+2x^2+4ay+2y^2) +$$

$$\left.+ b\,\text{Log}\,(2b^2+2x^2-4by+2y^2) - b\,\text{Log}\,(2b^2+2x^2+4by+2y^2)\right]$$

Eq.(21) derives from eq.(A3) by a standard power series expansion about origin.

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