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FORMAL AND PHYSICAL PROPERTIES OF THE GENERALIZED (SUB- AND SUPER-LUMINAL) LORENTZ TRANSFORMATIONS(x)

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ABSTRACT

We investigate the mathematical and physical properties of the Generalized Lorentz transformations (both sub- and Super-luminal). The form here adopted for the Superluminal Lorentz transformations is the one -recently introduced by us - which satisfies the requested group-theoretical properties. We clarify the rôle of the reinterpretation procedure also from the formal point of view, for both the "longitudinal" and "transverse" coordinates. Careful attention is devoted to define four-momentum and three-velocity for tachyons. At last, the shape of a tachyon - obtained by applying to an ordinary particle a generic Superluminal Lorentz transformation (without rotations) - is studied. As a simplifying tool, we make recourse also to the "light-cone coordinates" and to "dilation-invariant" coordinates.

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1. - INTRODUCTION

Recently the Generalized Lorents transformations[1] (GLT) [both subluminal (LT) and Superluminal (SLT)] have been rewritten in a form satisfying the requested group-theoretical properties[2], such a new form reducing to the previous one by Mignani and Recami in the particular case of collinear boosts.

Namely, in Refs. (2, 1) it has been shown that

$$\mathrm{SLT}(\vec{U}) = \pm S\left[\mathrm{LT}(\vec{u})\right] ; \qquad (\vec{u}//\vec{U};\ u \equiv c^2/U)$$
$$S \equiv i\mathbb{1} \tag{1}$$

so that the group $\mathbb{G}$ of all subluminal and Superluminal Lorentz transformations results to be

$$\mathbb{G} = \mathcal{L}_+^{\uparrow} \otimes \mathbb{Z}(4)$$
$$\mathbb{Z}(4) \equiv \left\{\sqrt[4]{1}\right\} \equiv \left\{+1, -1, +i, -i\right\} \tag{2}$$

where $\mathcal{L}_+^{\uparrow}$ represents the ordinary (proper, orthochronous) Lorentz group. The group $\mathbb{G}$ has, in particular, the properties[1-3]

$$\det \mathbb{G} = +1 , \qquad \forall\, G \in \mathbb{G} ; \tag{3a}$$
$$G \in \mathbb{G} \Longrightarrow -G \in \mathbb{G} , \qquad \forall\, G \in \mathbb{G} ; \tag{3b}$$
$$G \in \mathbb{G} \Longrightarrow iG \in \mathbb{G} , \qquad \forall\, G \in \mathbb{G} ; \tag{3c}$$

and of course

$$\pm S \in \mathbb{G} .$$

From eqs. (1) it follows that S is the "transcendent" SLT (i.e., it corresponds to $U \to \infty$).

Within the formalization adopted in Refs. (2, 1), the four-position x^{μ} is supposed to be a vector even with respect to $\mathbb{G}$ (i.e., to be a G-fourvector), so that the quadratic form $dx^{\mu} dx_{\mu}$ is a scalar under LT's and a pseudo-scalar under SLT's (in particular, under the transcendent transformation S). In other words, the GLT's are unimodular and special, and such that the LT's are orthogonal whilst the SLT's are anti-orthogonal:

$$G^{\mathrm{T}} G = +\mathbb{1} \qquad (\text{subluminal case} : u^2 < c^2) ; \tag{4a}$$
$$G^{\mathrm{T}} G = -\mathbb{1} \qquad (\text{Superluminal case} : u^2 > c^2) . \tag{4b}$$

Let us notice that $\mathbb{G}$ is non-compact, non-connected and with discontinuities on the light cone; moreover, its central elements are

$$\mathbb{C} \equiv (+\mathbb{1}, -\mathbb{1}, +i\mathbb{1}, -i\mathbb{1}) . \tag{5}$$

The new group $\mathbb{G}$ of the GLT's is the following extension of the group $\mathcal{L}_+^{\uparrow}$:

$$\mathbb{G} = E(\mathcal{L}_+^\uparrow, CPT, S) , \tag{2'}$$

since the operation $-\mathbb{1}$ has been shown[4, 1-3] to be equivalent to the ordinary CPT[1-4]

$$-\mathbb{1} \equiv \bar{P}\bar{T} = CPT .$$

Notice that our "new" GLT's, eqs. (1), do agree with our Refs. (1-4), but slightly disagree with the form adopted in Refs. (5).

In the particular case of boosts along x, the SLT's take the simple form (in natural units: $c = 1$):

$$\begin{aligned} t' &= \pm i \frac{t - ux}{\sqrt{1-u^2}} \equiv \mp i \frac{x - Ut}{\sqrt{U^2-1}} \\ x' &= \pm i \frac{x - ut}{\sqrt{1-u^2}} \equiv \mp i \frac{t - Ux}{\sqrt{U^2-1}} \qquad \left[\begin{array}{l}\text{Superluminal case} \\ U^2 > 1 ;\ u^2 < 1 ; \\ U \equiv 1/u\end{array}\right] \\ y' &= \pm iy ; \qquad z' = \pm iz ; \end{aligned} \tag{6}$$

the problem of interpreting eqs. (6) has been exploited in Refs. (2, 6). In Refs. (2) we also justified why we call "transformations" the eqs. (6). Here, let us only recall that in the 2-dimensional case the reinterpretation is straightforward, since the transcendent operation $S \equiv S_2 \equiv \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$ goes into $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ through the similarity transformation

$$\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} = T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T^\dagger \tag{7}$$

where T is a unitary transformation.

2. - THE GLT's BY DISCRETE SCALE TRANSFORMATIONS

What precedes can be rewritten in a more compact form by following the philosophy outlined in Refs. (7), i.e. by making recourse to the language of the discrete (real or imaginary) scale-transformations[7]:

$$ds'^2 = \varrho^2 ds^2 , \qquad \varrho^2 = \pm 1 . \tag{8}$$

For instance, the GLT's can be of course rewritten as follows:

$$\mathbb{G} = \mathcal{D} \circledast \mathcal{L}_+^\uparrow \tag{2"}$$

where $\mathcal{D}$ is the discrete group of the dilations D: $x'_\mu = \varrho x_\mu$; with $\varrho = \pm 1, \pm i$.

More formally, let us introduce the new, scale-invariant (or dilatation-invariant) coordinates

$$\eta^{\mu} \equiv \mathbf{k} x^{\mu}, \qquad (\mathbf{k} = \pm 1, \pm i) \tag{9}$$

where $\mathbf{k}$ is the intrinsic scale-factor of the considered object[7,8].

Notice that, under a dilation D, it is $\eta'_{\mu} = \eta_{\mu}$, with $\eta'_{\mu} \equiv \mathbf{k}' x'_{\mu}$; while $\mathbf{k}' = \varrho^{-1}\mathbf{k}$.

The important characteristic of the present formalism is that, under all the GLT's of the group $\mathbb{G}$, the quadratic form $d\sigma^2 \equiv d\eta_{\mu} d\eta^{\mu}$ is invariant(x):

$$d\sigma'^2 = d\sigma^2, \qquad \forall\, G \in \mathbb{G}. \tag{10}$$

Notice moreover that, under a generic proper orthochronous Lorentz transformation $L \in \mathcal{L}_+^{\uparrow}$, it holds $\eta'^{\mu} = L^{\mu}_{\nu}\eta^{\nu}$; $\mathbf{k}' = \mathbf{k}$.

It follows that - when going back from the coordinates η^{μ}, $\mathbf{k}$ to the ordinary coordinates x^{μ} - the generic GLT = G can be expressed as ($x' = Gx$)

$$\begin{aligned} &G = \mathbf{k}'^{-1} L \mathbf{k}, \qquad \left[\mathbf{k}, \varrho = \pm 1, \pm i;\ \mathbf{k}' = \varrho^{-1}\mathbf{k}\right] \\ &L \in \mathcal{L}_+^{\uparrow}. \end{aligned} \tag{11}$$

As it must be, the subluminal Lorentz transformations are the LT = $\pm$L and the Superluminal ones the SLT = $\pm$iL. In other words[7], bradyons (antibradyons) correspond to $\mathbf{k} = +1$ ($\mathbf{k} = -1$), whilst tachyons and antitachyons corresponds to $\mathbf{k} = \pm i$.

3. - GENERALIZED (SUB- AND SUPER-LUMINAL) BOOSTS B IN THE "LIGHT-CONE COORDINATES"

It is already known[9] that the ordinary subluminal boosts along x can be rewritten in a more symmetric, compact form in terms of the coordinates

$$\xi \equiv t - x; \qquad \zeta \equiv t + x; \qquad y; \qquad z, \tag{12}$$

where the first two coordinates refer to two new axes which are obtained from the axes t, x by means of an Euclidean, anti-clockwise 45°-rotation (See Fig. 1). We shall call ξ, ζ "light-cone coordinates" (even if they are sometimes, rather incorrectly, named "infinite-momentum-frame coordinates"). Namely, a proper orthochronous boost along x, with relative (subluminal) speed u, can be written

$$\xi' = \alpha\xi; \qquad \zeta' = \alpha^{-1}\zeta; \qquad y' = y; \qquad z' = z, \qquad (0 < \alpha < +\infty) \tag{13}$$

(x) - Its sign included.

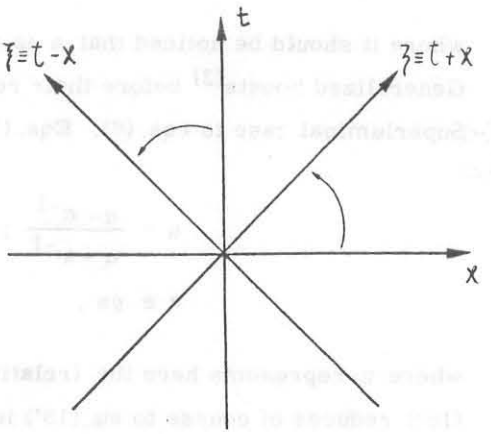

FIG. 1 - The "light-cone coordinates" $\xi \equiv t - x$ and $\zeta \equiv t + x$ correspond to two axes ξ, ζ which are obtained from the axes t, x through an Euclidean, anti-clockwise 45°-rotation.

$$\frac{\alpha - \alpha^{-1}}{\alpha + \alpha^{-1}} = u \; ; \qquad u^2 < 1 \; , \qquad (-1 < u < +1 \; ; \; u \equiv u_x) \tag{13'}$$

where parameter α is any real, positive number a.

It is interesting to notice that in the present formalism the Lorentz boosts along x correspond just to a dilation of the coordinates ξ, ζ (by the factor α and α^{-1}, respectively). In particular, the identity-transformation ($u = 0$) corresponds to $\alpha = +1$; the boosts along the positive x-direction correspond to $1 < \alpha < +\infty$; and the boosts along the negative x-direction correspond to $0 < \alpha < 1$. For $\alpha \to +\infty$ we have $u \to 1^-$; and for $\alpha \to 0^+$ we have $u \to -(1^-)$. It is apparent that

$$\alpha = e^R , \tag{14}$$

where R is the "rapidity".

By using this formalism, it is immediate to recognize that the proper antichronous (= non-orthochronous) subluminal Lorentz boosts along x will correspond to the negative (real) α-values: $-\infty < \alpha < 0$; together with $y' = -y$; $z' = -z$. Of course, the relative speed u will run again within the same range: $-1 < u < +1$, even in correspondence with the new (negative) values of α.

Similarly, the Superluminal boosts along x will corresponds - now - to the imaginary α-values; together with $y' = \pm iy$; $z' = \pm iz$.

More precisely, eqs. (13) can be extended to express in synthetic form all Generalized (both sub- and Super-luminal) Lorentz boosts along x, by means of the discrete scale parameter ϱ, as follows(+):

$$\xi' = \varrho a \xi \; ; \qquad \zeta' = \varrho a^{-1} \zeta \; ; \qquad y' = \varrho y ; \qquad z' = \varrho z \; ;$$
$$\varrho = \pm 1 \; , \; \pm i \; , \qquad (0 < a < +\infty \; ; \; u^2 = u_x^2 \lesseqgtr 1) \tag{15}$$

(+) - In this Section, for convenience, we shall represent by u the boost relative speeds both in the sub- and in the Super-luminal cases.

where it should be noticed that a is any real, positive number. Such eqs. (15) represent the Generalized boosts[2] before their reinterpretation; that is to say, they are equivalent in the Superluminal case to eqs. (6). Eqs. (13') must be generalized as follows:

$$u = \frac{\alpha - \alpha^{-1}}{\alpha + \alpha^{-1}} \; ; \qquad (u^2 = u_x^2 \gtrless 1 \; ; \quad 0 < a < +\infty) \tag{15'}$$
$$\alpha \equiv \varrho a \; ,$$

where u represents here the (relative) speed both of sub- and of Super-luminal boosts. Eq. (15') reduces of course to eq. (13') in the subluminal-boost case. In the Superluminal-boost cases, however, eq. (15') can be derived only after the reinterpretation of the first couple of eqs. (15) (i.e. after the interpretation of the meaning of $\varrho = \pm i$ in the first couple of eqs. (15)). For such a delicate question, see Refs. (2); soon we shall touch again this point. Here, let us anticipate that the reinterpretation procedure of the first couple of eqs. (15) - as given in Refs. (2) - is equivalent to rewrite them as follows:

$$\xi' = \varrho\alpha\xi \; ; \quad \zeta' = \varrho\alpha^{-1}\zeta \; ; \quad y' = \varrho y \; ; \quad z' = \varrho z \; ; \qquad (u^2 = u_x^2 \gtrless 1) \tag{15 bis}$$
$$\alpha \equiv \pm \varrho a \; ; \qquad \varrho = \pm 1, \pm i \; ; \qquad a \in (0, +\infty) \; ,$$

wherefrom eqs. (15') can be straighforwardly derived. See the following (eqs. (22)).

In conclusion, if B represents a generic boost along x, then all Generalized (sub- and Super-luminal) boosts can take the form (15) with $(\mathcal{L}_+ \equiv \mathcal{L}_+^{\uparrow} \cup \mathcal{L}_+^{\downarrow}; \; \alpha \equiv \varrho a \; ; \; 0 < a < +\infty)$:

$$\left.\begin{aligned} B \in \mathcal{L}_+^{\uparrow} : \quad & 0 < \alpha < +\infty \quad \Longleftrightarrow \quad \varrho = +1 \qquad (16a) \\ B \in \mathcal{L}_+^{\downarrow} : \quad & -\infty < \alpha < 0 \quad \Longleftrightarrow \quad \varrho = -1 \qquad (16b) \end{aligned}\right\} \; (u^2 = u_x^2 < 1)$$

$$B \in i\mathcal{L}_+ : \quad \alpha \equiv i\tilde{\alpha} \; ; \; -\infty < \tilde{\alpha} < +\infty \quad \Longleftrightarrow \quad \varrho = \pm i \; ; \qquad (u^2 = u_x^2 > 1) \; . \tag{16c}$$

In particular, it is immediate to check that in the case of Superluminal boosts ($\varrho = \pm i$) from eq. (15') it actually follows:

$$u = \frac{a + a^{-1}}{a - a^{-1}} > 1 \; . \qquad (\varrho = \pm i \; ; \; 0 < a < +\infty) \tag{17}$$

Of course, all Generalized x-boosts (eqs. (15)) preserve the quadratic form, except for its sign:

$$\xi'\zeta' - y'^2 - z'^2 = \varrho^2(\xi\zeta - y^2 - z^2) \; . \qquad (\varrho^2 = \pm 1) \tag{18}$$

Let us briefly come back to the problem of deriving eq. (15') in the case of Superluminal boosts, by observing that the change of the quadratic-form sign can be obtained either by writing down eqs. (15) and (18) with $\varrho = \pm i$, so as we did before, or by writing (instead of eqs. (15),

and only for the case of Superluminal boosts):

$$\xi' = a\xi\ ;\quad \zeta' = -a^{-1}\zeta\ ;\quad y' = -iy\ ;\quad z' = -iz\ ;\quad (u^2 = u_x^2 > 1\ ;\ 0 < a < +\infty) \tag{19}$$

where the real $a \in (0, +\infty)$; or rather - in more complete form[1] -:

$$\xi' = \tilde{\alpha}\xi\ ;\quad \zeta' = -\tilde{\alpha}^{-1}\zeta\ ;\quad y' = -i\frac{\tilde{\alpha}}{|\tilde{\alpha}|}y\ ;\quad z' = -i\frac{\tilde{\alpha}}{|\tilde{\alpha}|}z\ ,\quad (u^2 = u_x^2 > 1\ ;\ -\infty < \tilde{\alpha} < +\infty) \tag{19'}$$

now with real $\tilde{\alpha} \in (-\infty, +\infty)$. Eqs. (19') are the transcription of eqs. (6) in terms of the coordinates given by eqs. (12). It follows that in particular

$$t' = \frac{1}{2}(\tilde{\alpha} - \breve{\alpha}^{-1})t - \frac{1}{2}(\tilde{\alpha} + \breve{\alpha}^{-1})x\ ;\qquad x' = \frac{1}{2}(\tilde{\alpha} - \tilde{\alpha}^{-1})x - \frac{1}{2}(\tilde{\alpha} + \breve{\alpha}^{-1})t\ ,$$

so that for the relative boost-speed one obtains:

$$u = \frac{dx}{dt}\bigg|_{dx'=0} = \frac{\breve{\alpha} + \breve{\alpha}^{-1}}{\tilde{\alpha} - \breve{\alpha}^{-1}}\ ;\qquad u^2 > 1\ ; \tag{20}$$

where eq. (20) should be compared with eq. (13'). Notice explicitly that the procedure expressed by these eqs. (19)-(20) does correspond to our reinterpretation of the first couple of eqs. (15) given in Refs. (2) (i.e., eqs. (19') coincide with eqs. (15 bis) for the Superluminal case).

To formalize the whole matter (i.e. the previous reinterpretation-problem) let us take advantage - at this point - of the (discrete) scale-transformation language introduced in Sect. 2. That is to say, by substituting the dilation-invariant coordinates $\eta^\mu \equiv \mathbf{k}x^\mu$ for x^μ (and thus by "generalizing" definitions (12)), let us eventually define the following scale-invariant "light-cone coordinates":

$$\varphi \equiv \eta^0 - \eta^1\ ;\qquad \psi \equiv \eta^0 + \eta^1\ ,\qquad \eta^2\ ;\qquad \eta^3\ . \tag{21}$$

In terms of coordinates (21), the transformations (15) can be written

$$\varphi' = \alpha\varphi\ ;\qquad \psi' = \alpha^{-1}\psi\ ;\qquad \eta'^2 = \eta^2\ ;\qquad \eta'^3 = \eta^3\ ;\qquad (|u| \gtrless 1\ ;\ u = u_x) \tag{22}$$

$$\mathbf{k}' = \varrho^{-1}\mathbf{k}\ ;\qquad \alpha \equiv \varrho a\ ;\qquad \varrho = \pm 1, \pm i\ ;\qquad a \in (0, +\infty)\ ;$$

where, as usual, $\varrho = \pm 1$ yields the subluminal, and $\varrho = \pm i$ the Superluminal x-boosts. Now, all Generalized boosts (eqs. (22)) preserve the quadratic form, its sign included:

$$\varphi'\psi' - (\eta'^2)^2 - (\eta'^3)^2 = \varphi\psi - (\eta^2)^2 - (\eta^3)^2\ . \tag{23}$$

It is important to emphasize that eqs. (22) in the Superluminal case yield just eqs. (19'), that is to say they automatically include the reinterpretation of the first couple of eqs. (15) or (16), as given in Refs. (2). In particular, in the Superluminal-boost cases, eqs. (22) have the

advantage over eqs. (15) of yielding the correct Superluminal relative speed without any need of reinterpretation; actually, from eqs. (22) one derives exactly eq. (15'), for both subluminal and Superluminal boosts (without any explicit need of reinterpretation).

The more difficult problem of the generic velocity-composition-law will be considered in Sect. 6.

We want here to observe that our coordinates φ, ψ (or ξ, ζ) are so defined that u is subluminal whenever in eqs. (22) the quantities α and α^{-1} have the same sign: $\text{sign}(\alpha^{-1}) = \text{sign}(\alpha)$; and u is Superluminal whenever α and α^{-1} possess opposite signs: $\text{sign}(\alpha^{-1}) = -\text{sign}(\alpha)$.

In what follows we shall touch the question of interpreting the second couple of eqs. (15), or (6), or (22), following Refs. (6). The problem of geometrico-physically interpreting in the Superluminal case the second couple of eqs. (6), (15), (15 bis), (19'), (22) has been exploited in Refs. (6), but only for the case of Superluminal boosts along a space-axis (let us call it x): Cf. Fig. 2, and Refs. (6). Below, we shall extend those results.

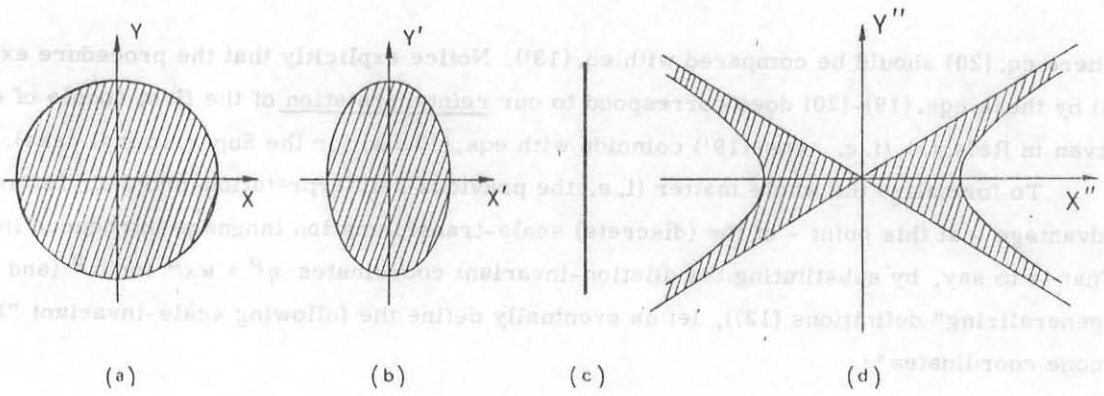

FIG. 2 - Let us consider a particle which is intrinsically spherical, i.e. that is a sphere in its rest-frame (Fig. a). Under a subluminal x-boost it appears - of course - as ellipsoidal (Fig. b). Under a Superluminal x-boost it will appear as in Fig. d. Fig. c refers to the limiting case when the boost relative speed $u \to c$. (It is understood that these figures refer to the solid objects got by rotating them around their axes of abscissas). Cf. also Refs. (6) and the text.

Another problem we shall deal with is generalizing eqs. (15'), (17), (20) for the case when the Superluminal velocity is composed with a non-zero initial velocity.

We are going to consider also some applications of the previous formalism.

4. - A SIMPLE APPLICATION

A first example to show the power of the present formalism is finding out how[9] a 4-dimensional (space-tima) sphere

$$t^2 + x^2 + y^2 + z^2 = A^2 , \tag{24}$$

that is to say

$$\frac{1}{2}\,\xi^2 + \frac{1}{2}\,\zeta^2 + y^2 + z^2 = A^2\,, \tag{24'}$$

deforms under a Lorentz transformation. Let us first consider a subluminal boost (eqs. (13)). Since the first two coordinates result to be merely scaled by the factor $a \in (0, \infty)$, we immediatly get that eq. (24') in terms of the new (primed) coordinates rewrites:

$$\frac{1}{2}\,a^{-2}\xi'^2 + \frac{1}{2}\,a^{+2}\,\zeta'^2 + y'^2 + z'^2 = A^2\,, \qquad \text{(subluminal case; } a \in (0, +\infty)) \tag{25a}$$

which in the new frame is a 4-dimensional ellipsoid.

In the case of a Superluminal boost [eqs. (19'), (15 bis)], eq. (24') can be rewritten - in terms of the new, primed coordinates - as

$$\frac{1}{2}\,a^{-2}\,\xi'^2 + \frac{1}{2}\,a^{2}\,\zeta'^2 - y'^2 - z'^2 = A^2\,, \qquad \text{(Superluminal case; } a \in (0, +\infty)) \tag{25b}$$

which in the new frame is a 4-dimensional hyperboloid.

Notice explicitly that this example (i.e., transforming under GLT's a 4-dimensional set of events) has nothing to do with what one performs usually (in fact, ordinarily, one considers a world-tube and then cuts it with different 3-dimensional hyperplanes).

5. - ON THE PHYSICAL INTERPRETATION OF SLT's

We would like to extend the whole reinterpretation-procedure[2, 6] (of the whole set of four equations constituting a SLT) to the case of Superluminal Lorentz transformation without rotations, i.e. of a Superluminal boost $L(\vec{U})$ along a generic motion-line ℓ. Let us first realize such an aim in terms of the ordinary coordinates x^μ. A Superluminal Lorentz transformation (without rotations) $L(\vec{U})$, according to eqs. (1), as a 4x4 matrix will write ($\vec{u} // \vec{U}$; $u \equiv 1/U$):

$$L(\vec{U}\,; x^\mu) = i\,L(\vec{u}; x^\mu) = \pm i \begin{pmatrix} \gamma & u\gamma n_s \\ -u\gamma n^r & \delta^r_s - (\gamma - 1) n^r n_s \end{pmatrix}\,; \tag{26}$$

$$\gamma \equiv (1 - u^2)^{-1/2}\,; \qquad (u^2 < 1\,;\; U^2 > 1) \tag{26'}$$

where $L(\vec{u})$ is the dual (subluminal) boost along the same (generic) direction ℓ. Quantity $\vec{n}$ is the unit-vector characterizing the boost motion-line ℓ: $n_r n^r = -1 = -|\vec{n}^2|$. The unit-vector n points in the (conventionally) positive direction along ℓ. Notice that u, U may be both positive and negative, and that $u n_s = u_s$.

Let us observe that eq. (26) expresses $L(\vec{U})$ in its "original" form, not yet reinterpreted. Of course, $L(\vec{U}; x^\mu)$ can be considered as obtained from the corresponding Superluminal

boost $L_o(x, U) \equiv B(x)$ along x through suitable rotations $\left[L_o(x, U) = iL_o(x, u)\right]$:

$$L(\vec{U}; x^\mu) = R^{-1}B(x)R\,; \qquad R \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & n_x & n_y & n_z \\ 0 & -n_y & 1 - An_y^2 & -An_yn_z \\ 0 & -n_z & -An_yn_z & 1 - An_z^2 \end{pmatrix} \tag{27}$$

$$A = (1 + n_x)^{-1}\,, \tag{27'}$$

where B(x) is given by eqs. (6).

It is important to underline that in Refs. (2, 6) we have been able to reinterpret the SLT's, eqs. (1), only in the case of Superluminal boosts along an axis (so as assumed in Sect. 3). Now, to reinterpret also the Superluminal transformation $L(\vec{U}; x^\mu)$ in eq. (26), let us compare L(U) with $\bar{L}(U)$:

$$\bar{L}(\vec{U}; x^\mu) = R^{-1}\bar{B}(x)R\,, \tag{28}$$

where $\bar{B}(x)$ is now the (partially) reinterpreted version[2, 1] of eqs. (6) for the Superluminal case[1, 2, 6]:

$$t' = \pm\frac{x - ut}{\sqrt{1 - u^2}} = \mp\frac{t - Ux}{\sqrt{U^2 - 1}}\,;$$

$$x' = \pm\frac{t - ux}{\sqrt{1 - u^2}} = \mp\frac{x - Ut}{\sqrt{U^2 - 1}}\,; \qquad \text{(Superluminal case} \quad u^2 > 1\,;\ U^2 < 1\,;\ U \equiv 1/u) \tag{6 bis}$$

$$y' = \pm iy\,; \qquad z' = \pm iz\,.$$

We adopt eqs. (6 bis) even if only the first couple of them appears as actually "reinterpreted" in real terms, since in Refs. (6) we already showed how to interpret the imaginary units appearing in the last couple of eqs. (6 bis) (at least in some relevant cases); we shall take account of that in the following.

In connection with the (partially reinterpreted) eqs. (6 bis), let us recall[1, 6] - incidentally - that the Generalized Lorentz boosts, both sub- and Super-luminal, can be written down in a compact form and in terms of a continuous parameter $\theta \in \left[0, 2\pi\right]$ as follows:

$$x' = \Omega\gamma_o(x - t\,\mathrm{tg}\,\theta)\,; \quad t' = \Omega\gamma_o(t - x\,\mathrm{tg}\,\theta)\,; \qquad (u^2 \lesseqgtr 1\,;\ 0 \le \theta \le 2\pi)$$

$$y' = -\Omega\delta y\,; \qquad z' = -\Omega\delta z\,,$$

with

$$u \equiv \mathrm{tg}\,\theta\,; \qquad \Omega = \Omega(\theta) \equiv \frac{\cos\theta}{|\cos\theta|}\,\delta^2\,; \qquad \delta \equiv +\sqrt{\frac{1 - \mathrm{tg}^2\,\theta}{|1 - \mathrm{tg}^2\,\theta|}}\,;$$

$$y_o \equiv +\left(|1 - \mathrm{tg}^2\,\theta|\right)^{-1/2}\,; \qquad 0 \le \theta \le 2\pi\,;$$

such a form[1] of the GLT's shows explicitly how the various (positive or negative) signs in front of x' and t' and the various (real or imaginary, positive or negative) "signs" of y' and z' do succeed one another[1, 6] as functions of u, or rather of θ. (Notice that in this last paragraph $u^2 \lesseqgtr 1$).

From eqs. (28) and (6 bis) we get for the Superluminal transformation $\bar{L}$:

$$\bar{L}(\vec{U}; x^\mu) = \pm \begin{pmatrix} -u\gamma & -\gamma n_s \\ \gamma n^r & i\delta^r_s + (i+u\gamma)n^r n_s \end{pmatrix} ; \qquad (u^2 < 1 ; \ r, s = 1, 2, 3) \qquad (29a)$$

where γ is defined in eq. (26'), with $|u| < 1$. Eq. (29a) can however be written also as follows

$$\bar{L}(\vec{U}; x^\mu) = \pm \begin{pmatrix} -\bar{\gamma} & -U\bar{\gamma} n_s \\ U\bar{\gamma} n^r & i\delta^r_s + (i+\bar{\gamma})n^r n_s \end{pmatrix} ; \qquad (U \equiv 1/u ; \ U^2 > 1) \qquad (29b)$$

where now $\bar{\gamma} \equiv (U^2 - 1)^{-1/2}$, with $U \equiv 1/u$; $u^2 < 1$; $U^2 > 1$. Notice explicitly that, even if the SLT's in their original mathematical form are always purely imaginary, the SLT's in their "(partially) reinterpreted" form appears to contain on the contrary complex quantities: But this is not a problem, because the origin of those "complex quantities" is evident and we know - of course - how to interpret them.

We have just to compare the matrices (29a) or (29b) with the matrix in eq. (26), including in it its imaginary coefficient, in order to get an interpretation of eq. (26) analogous to the one forwarded in Refs. (1, 2, 6) for the Superluminal boosts along x. Namely, the reinterpretation will procede - as usual - in two steps: The first step consists (cf. also Sect. 3) in reinterpreting the space-coordinate along the motion-line ℓ and the time-coordinate; the second step consists in interpreting[6] the imaginaries entering the transverse space-coordinates.

For instance, let us compare eq. (26) with eq. (29a), apart from their double signs:

$$t' = i\gamma t + i\gamma u n_s x^s ; \qquad x'^r = -iu\gamma n^r t + i\delta^r_s x^s - i(\gamma - 1)n^r n_s x^s ; \qquad (26)$$

$$t' = -u\gamma t - \gamma n_s x^s ; \qquad x'^r = \gamma n^r t + i\delta^r_s x^s + (u\gamma + i)n^r n_s x^s . \qquad (29a)$$

* * *

First Step - To reinterpret (in terms of real quantities only) the time-coordinate and the space-coordinate along the motion-line, one has to adopt the following recipe (notice that $r_{\parallel} \equiv \vec{r} \cdot \vec{n} = -n_s x^s$):

You can eliminate the imaginary unit in all addenda containing γ as a multiplier, provided that you substitute t for $r_{\parallel} \equiv -n_s x^s$ and $-n_s x^s$ for t.

Let us emphasize (following Refs. (2)) that - when dealing with a chain of GLT's - such a rein

terpretation-rule has to be applied, if necessary, only at the end of the chain(+).

Second Step - In the second ones of eqs. (26) and (29a), if we put $\vec{r} \equiv \vec{x} \equiv (x, y, z)$ and $\vec{r}' \equiv \vec{x}' \equiv (x', y', z')$, we can write $\vec{r} = \vec{r}_{\parallel} + \vec{r}_{\perp}$, where $\vec{r}_{\parallel} \equiv (r_{\parallel})\vec{n}$ and $\vec{r}_{\perp} \equiv \vec{r} - r_{\parallel}\vec{n} \equiv \vec{r} - (\vec{r}\cdot\vec{n})\vec{n}$. Then, eq. (29a), e. g., can be written:

$$\vec{r}' = \vec{r}'_{\parallel} + \vec{r}'_{\perp} = \gamma(t - ur_{\parallel})n + i\,r_{\perp} . \tag{29c}$$

After having applied the "first-step Recipe", we are left only with the following relation

$$\vec{r}'_{\perp} = i\vec{r}_{\perp} \tag{30}$$

to be reinterpreted yet, i. e. only with the imaginary terms (not containing γ as a multiplier):

$$i(\vec{r}_{\perp})^r \equiv i(\delta^r_s + n^r n_s)x^s , \tag{30'}$$

which enter only x'^r. Of course, $\vec{r}_{\perp}$ is a space-vector lying on the plane orthogonal to the boost motion-line, and therefore corresponds to two further coordinates only.

Since those terms (eq. (30')) refer to the space-coordinates orthogonal to the boost direction, their imaginary "sign" has to be interpreted so as we did in Refs. (6) for the transverse coordinates y', z' in the case of Supeluminal x-boosts (Cf. Fig. 2).

This means that, if the considered SLT is applied to a body P_B initially at rest (B = bradyonic = slower-than-light; for simplicity, let it be spherical in its rest-frame), we shall finally obtain a body P_T (T = tachyonic) moving along the boost motion-line ℓ with Superluminal speed $V = U$, such a body P_T - however - being no more spherical or ellipsoidal in shape: The tachyon P_T will appear, on the contrary, as occupying the spatial region confined between a two-sheeted hyperboloid and a double cone, both having as symmetry-axis the boost motion-line ℓ. See Fig. 3 and Refs. (6). More precisely, let us consider the vector $\vec{r}'_{\perp}$ in eq. (30), once eliminated its imaginary "sign" (i. e., the vector $\vec{r}_{\perp}$); since $\vec{r}_{\perp}$ lies on the plane orthogonal to ℓ, it can be described by the two coordinates $r^1_{\perp} \equiv Y$, $r^2_{\perp} \equiv Z$ such that

$$Y' = iY ; \qquad Z' = iZ ; \tag{30''}$$

and the coordinates $|Y'| = Y'/i = Y$ and $|Z'| = Z'/i = Z$ express the fundamental sizes of the "fundamental rectangles"(6) which individuate the double-cone shape, i. e. the fundamental asymptotes of the two-sheeted hyperboloid (see also the following). In other words, quantities Y'/i and Z'/i (together with quantity $\Delta X = \overline{V_1 V_2}$: cf. Fig. 3) allows us to determine the shape of the tachyon. Fig. 3 refers to the simple case when P_B is intrinsically spherical: More in

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(+) - Let us recall also that, after the reinterpretation, the SLT's loose their group-theoretical properties(2).

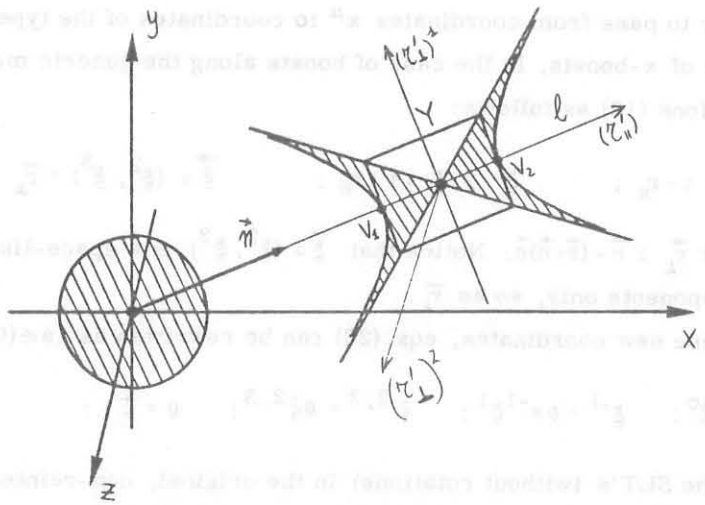

FIG. 3 - If we start again from a spherical particle P_B as in Fig. 2a, then - after a generic SLT without rotations, i.e. under a Superluminal boost along a generic motion-line ℓ - we get what represented in this figure. In this case, the tachyon P_T occupies the spatial region confined between a two-sheeted hyperboloid and a double cone, both having as symmetry-axis the boost motion-line ℓ. Such a structure (the "tachyon shape") travels of course along ℓ with the speed V = U of the Superluminal ℓ-boost. Notice that, if P_B is not intrinsically spherical (but e.g. ellipsoidal, in its rest-frame), then the tachyon-shape axis will not coincide with ℓ, and its position will depend on the speed V of the tachyon itself. For the cases when the space-extension of the tachyon is finite, see Refs. (6).

general, the axis of the tachyon-shape will not coincide with ℓ (but will depend on the tachyon speed V = U). The double-cone semi-angle α is given[6] in our present case by the relation $\mathrm{tg}\,\alpha = (V^2 - 1)^{-1/2}$.

To clarify the above (second) step of our reinterpretation, it is necessary to add some comments: (i) We do not aim to consider - and reinterpret - the GLT's when they are applied to a vacuum point: In fact, the main teaching of Special Relativity is that each observer has a right to consider the vacuum (i.e. the space, or the ether if you like) as at rest with respect to himself[10]; (ii) We do apply - and reinterpret - the GLT's (in particular the SLT's) only to transform the space-time regions associated with physical objects; where we assume the existing objects to be essentially extended (as required by the relativistic theories[11, 6, 4]), so to consider the point-like situation only as a limiting case; (iii) When considering an extended-type physical object, we adopt the symplifying convenction of referring the frame-axes to its symmetry center.

* * *

We have finally to pass from coordinates x^μ to coordinates of the type (12). We defined eqs. (12) for the case of x-boosts. In the case of boosts along the generic motion-line ℓ, let us generalize definitions (12) as follows:

$$\xi^o \equiv \xi \equiv t - r_{\parallel} \; ; \qquad \xi^1 \equiv \zeta \equiv t + r_{\parallel} \; ; \qquad \vec{\xi} \equiv (\xi^2, \xi^3) = \vec{r}_{\perp} \; , \tag{31}$$

where $r_{\parallel} \equiv \vec{r} \cdot \vec{n}$; and $\vec{r}_{\perp} \equiv \vec{r} - (\vec{r} \cdot \vec{n})\vec{n}$. Notice that $\vec{\xi} \equiv (\xi^2, \xi^3)$ is a space-like vector characterizable by two components only, so as $\vec{r}_{\perp}$.

In terms of these new coordinates, eqs. (26) can be rewritten as ($a \in (0, +\infty)$)

$$\xi'^o = \varrho a \xi^o \; ; \qquad \xi'^1 = \varrho a^{-1} \xi^1 \; ; \qquad \xi'^{2,3} = \varrho \xi^{2,3} \; ; \qquad \varrho = \pm i \; ; \tag{32}$$

eqs. (32) represent the SLT's (without rotations) in the original, non-reinterpreted form. To reinterpret the first couple of eqs. (32) it is enough to remember defs. (31) and apply the rule in the previous recipe, i.e.

$$\text{omit } i \; ; \qquad \text{and} \qquad t \rightleftarrows r_{\parallel} \; .$$

The consequences in eqs. (32) are:

$$\xi'^o = \pm(-a\xi^o) \; ; \qquad \xi'^1 = \pm a^{-1} \xi^1 \; ; \qquad \xi'^{2,3} = \pm i \xi^{2,3} \; , \tag{33}$$

where ξ'^o, ξ'^1 are now real but nevertheless correspond to Superluminal relative motion, due to the change of sign in ξ'^o (see Sect. 3).

Such a reinterpretation can be easily formalized (i.e. "automatized") by making recourse to dilation-invariant light-cone coordinates, and proceding in analogy to eqs. (21)-(22).

As to the imaginary "signs" of $\xi'^{2,3}$, the interpretation procedure is just the same as for y', z' above.

6. - THE VELOCITY-COMPOSITION PROBLEM

In Sect. 3 we left open the velocity-composition problem.

First of all, let us observe that dx^μ/ds does not represent a G-fourvector (since dx^μ is a G-fourvector but ds^2 is a pseudo-scalar under SLT's); therefore, the four-velocity and the four-momentum, in order to be G-fourvectors, are to be defined:

$$u^\mu \equiv \frac{dx^\mu}{d\tau_o} \; ; \qquad p^\mu \equiv m_o u^\mu \tag{34}$$

for both tachyons and bradyons; where $d\tau_o$ is the (G-invariant) proper-time element[1,2].

Now, from our SLT's in their original (not yet reinterpreted) from (26), it is immediate to obtain that, if $u^\mu u_\mu = +1$, then

$$u'_\mu u'^\mu = -1 \; ,$$

so that we expect a SLT to transform bradyons B into tachyons T, and vice-versa. However, since the velocity (in particular the 3-velocity) does refer to the already interpreted situation, it is better to start from the reinterpreted form (29) of the SLT's. (In any case, when starting from eqs. (26) one has soon after to apply the reinterpretation-rule contained in the "first-step recipe", Sect. 5).

Let us for instance start from SLT's (without rotations) in their form (29b), and apply a SLT along a generic motion-line ℓ with Superluminal speed $U \equiv 1/u$ ($U^2 > 1$; $u^2 < 1$), to the case of bradyon P_B with (initial) four-velocity u^μ and (initial) velocity $\vec{v}$. For the purpose of generality, it is essential that $\vec{v}$ and $\vec{U}$ are not parallel. We get ($r, s = 1, 2, 3$)(*) :

$$\begin{aligned} u'^0 &= -\bar{\gamma}(u^0 + U u^s n_s) \equiv -\bar{\gamma}(u^0 - U u_{\parallel}) \; ; \qquad (U \equiv 1/u \; ; \; U^2 > 1 \; ; \; u^2 < 1) \\ u'^r &= +\bar{\gamma}(U u^0 + u^s n_s) n^r + i(u^r + u^s n_s n^r) \equiv -\bar{\gamma}(u_{\parallel} - U u^0) n^r + i u_{\perp}^r \, , \end{aligned} \tag{35}$$

where $u_{\parallel} \equiv -u^s n_s$; $u_{\perp}^r \equiv u^r + u^s n_s u^r \equiv u^r - u_{\parallel} n^r$, where $\vec{n}$ is still the unit vector along ℓ, and $\bar{\gamma} \equiv (U^2 - 1)^{-1/2}$ so as in eq. (29b). Let us observe that u'^0 is real, and that the second one of eqs. (35) rewrites :

$$u'_{\parallel} \equiv -u'^s n_s = -\bar{\gamma}(u_{\parallel} - U u^0) = -\frac{u_{\parallel} - U u^0}{\sqrt{U^2 - 1}} \; ; \quad u'_{\perp} \equiv i u_{\perp} \, , \qquad (U^2 > 1) \tag{36}$$

where $u'_{\parallel}$ is real too and only $u'_{\perp}$ is purely imaginary. Notice that $u_{\parallel}$, $u'_{\parallel}$ ($u_{\perp}$, $u'_{\perp}$) are the longitudinal (transverse) components of the space-part of the object four-velocity, with respect to the boost motion-line ℓ.

At this point, let us define the 3-velocity $\vec{V}'$ for tachyons in terms of their 4-velocity u'^μ as follows(1, 2) :

$$u'^j \equiv \frac{V'^j}{\sqrt{V'^2 - 1}} \; ; \qquad u'^0 \equiv \frac{1}{\sqrt{V'^2 - 1}} \, . \qquad (j = 1, 2, 3) \tag{37}$$

From eqs. (36), (37) it follows :

$$V'_{\parallel} = \frac{U - v_{\parallel}}{U v_{\parallel} - 1} = \frac{1 - u v_{\parallel}}{v_{\parallel} - u} \; ; \quad V'_{\perp} = i \frac{v_{\perp}\sqrt{U^2 - 1}}{U v_{\parallel} - 1} = \frac{i v_{\perp}\sqrt{1 - u^2}}{v_{\parallel} - u} \tag{38}$$

$$(U^2 > 1 \; ; \quad u^2 < 1 \; ; \quad u \equiv 1/U)$$

where once more $\parallel$ and $\perp$ mean parallel and orthogonal, respectively, to the boost motion-line ℓ. It should be noticed that

$$V'_{\parallel} = \frac{1}{\tilde{v}_{\parallel}} \; ; \qquad V'_{\perp} = i \frac{\tilde{v}_{\perp}}{\tilde{v}_{\parallel}} \tag{38'}$$

(*) - One should pay attention to not confuse the boost speeds u, U with the 4-velocity components u^μ of the considered object.

where $\tilde{\vec{v}}$ is the transform of $\vec{v}$ under the dual (subluminal) Lorentz transformation $L(\vec{u})$, with $u \equiv 1/U$; $\vec{u} // \vec{U}$.

Again, $V'_{\parallel}$ is real and $V'_{\perp}$ is purely imaginary. However V'^2 is always positive, so that $|V'|$ is real; actually, from eqs. (38') it follows:

$$V'^2 = V'^2_{\parallel} + V'^2_{\perp} = V'^2_{\parallel} - |V'^2_{\perp}| > 1.$$

More in general, from eqs. (38) one derives for the magnitudes the "Terletsky relation":

$$1 - V'^2 = \frac{(1 - v^2)(1 - U^2)}{(1 - \vec{U} \cdot \vec{v})^2}, \qquad (v^2 < 1; \quad U^2, V'^2 > 1) \tag{39}$$

which - incidentally - has been shown elsewhere[1, 2] to have general validity and to be G-covariant (i.e. to hold for any values, sub- or Super-luminal, of v, U, V').

It is worthwhile to recall explicitly that eqs. (35), (36) and (38), since they have been derived from the (partially) reinterpreted form of the SLT's, do not possess any more[2] their group-theoretical properties. For instance, eqs. (38) cannot be applied when transforming (under a SLT) a speed initially Superluminal[2].

We do not pass here to the light-cone coordinates, since nothing would essentially change.

Eq. (39) shows that, under a Superluminal Lorentz transformation ($U^2 > 1$), a bradyonic speed v goes into a tachyonic speed V'. But we have still to discuss the presence of imaginary units in the components of the tachyon 3-velocity transverse to the SLT motion-line (cf. the second one of eqs. (38)). To such an aim, we have to remember what said in Sect. 5 for the transverse coordinates, in connection with the "second step" of the reinterpretation procedure: See eq. (30") and the comments following it [we are going to work under the same conditions (i)-(iii)].

Under those circumstances and conditions, we can interpret $V'^1_{\perp}$ and $V'^2_{\perp}$ in analogy with Y', Z', or rather in analogy with p'_y, p'_z.

Namely: Let us consider, in its center-of-mass frame, an initial, spherical object with center at 0 whose external surface expands however in time for $t \geq 0$; that is to say, let us consider in the initial frame the following "symmetrically-exploding spherical bomb":

$$0 \leq x^2 + y^2 + z^2 \leq (R + vt)^2, \qquad (t \geq 0) \tag{40}$$

where the initial ($t = 0$) radius R of the "bomb" and the speed v of the "spherical explosion" are fixed, constant quantities. Let us now pass to a second observer, moving e.g. along the x-axis with Superluminal relative speed -U. The first limiting-equality in eq. (40) gives rise - as we already know - to a double cone with the x-axis as its symmatry-axis, and moving with speed $V = U$ along the axis $x \equiv x'$. The second inequality in eq. (40), when expressing it in terms of the Superluminal-frame (primed) coordinates, transforms into

$$(1 - v^2V^2)x'^2 - (V^2 - 1)(y'^2 + z'^2) - 2t'V(1 - v^2)x' - 2RvV\sqrt{V^2 - 1}\,x' \leqslant$$
$$\leqslant (V^2 - 1)R^2 - 2t'Rv\sqrt{V^2 - 1} - (V^2 - v^2)t'^2 \,. \qquad (x' \geqslant t'/V) \tag{41}$$

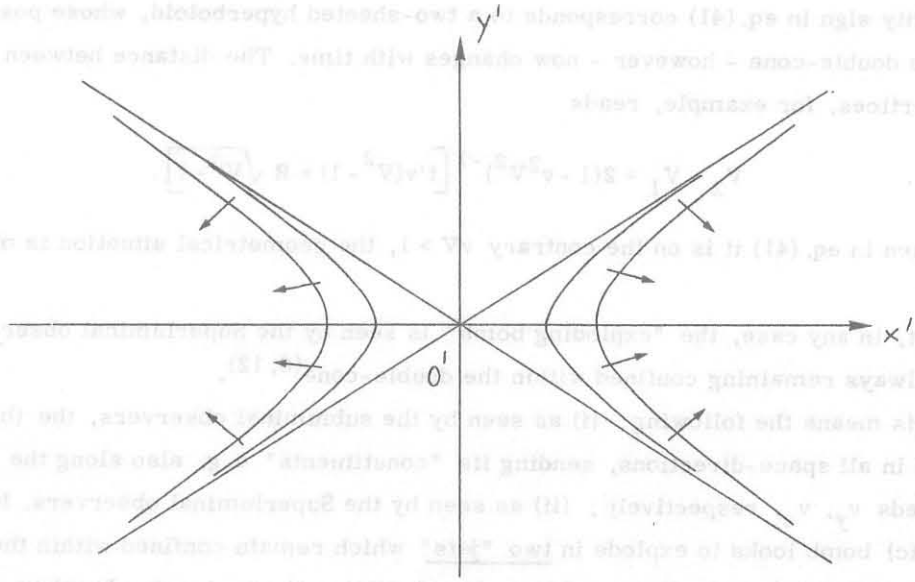

FIG. 4 - Let us start from a spherically symmetric object P_B whose radius, however, for $t \geq 0$ changes with time: $r = R + vt$. In its rest-frame, P_B remains always spherically symmetric. Under a Superluminal x-boost we get a tachyon P_T with a complicated shape and time-evolution. This figure refers to the case when $vV < c^2$, quantity V being the tachyon speed (i.e., the relative speed of the Superluminal boost). It actually depicts the simple case when $v \ll c^2/V$. In all cases, however, the initial (bradyonic) "exploding bomb" P_B transforms into a final (tachyonic) "bomb" P_T which "explodes" in two jets that remain confined within the double-cone. Notice that the limitation $x' \geq t'/V$ should be added to these pictures. The arrows in this figure indicate velocities that are slower-than-light with respect to 0'; that is to say, the vertex 0' of the double-cone travels (of course) with the Superluminal speed V, but the hyperboloid sheets move with subluminal speed with respect to 0'.

The same results may be obtained, more elegantly, expressing eq. (40) - or, rather, the equation of the "bomb" world-cone - in Lorentz-invariant form (for the subluminal observers):

$$(x_\mu + b_\mu)(x^\mu + b^\mu) \leqslant \frac{\left[(x^\mu + b^\mu)u_\mu\right]^2}{u_\mu u^\mu} \leqslant (1 - v^2)^{-1}(x_\mu + b_\mu)(x^\mu + b^\mu) \,; \tag{40'}$$
$$x^\mu u_\mu \geqslant 0 \,,$$

and then passing to the Superluminal observers just remembering that the SLT's invert the quadratic-form sign. Eqs. (40), (40') refer, actually, to a truncated "world-cone". In eq. (40'), quantity $x^\mu \equiv (t, x, y, z)$ is the generic event-vector inside the world-cone, vector u^μ is the

four-velocity of the "bomb" center-of-mass, and $b^{\mu} \equiv u^{\mu} R/v$.

When in eq. (41) it is

$$vV < 1 , \tag{41'}$$

the equality sign in eq. (41) corresponds to a two-sheeted hyperboloid, whose position relatively to the double-cone - however - now changes with time. The distance between the two hyperboloid vertices, for example, reads

$$V_2 - V_1 = 2(1 - v^2V^2)^{-1} \left[t'v(V^2 - 1) + R\sqrt{V^2 - 1} \right] . \tag{42}$$

When in eq. (41) it is on the contrary $vV > 1$, the geometrical situation is more complicated.

But, in any case, the "exploding bomb" is seen by the Superluminal observers to "explode" always remaining confined within the double-cone[6, 12].

This means the following: (i) as seen by the subluminal observers, the (bradyonic) bomb explodes in all space-directions, sending its "constituents" e. g. also along the y and z-axes, with speeds v_y, v_z, respectively; (ii) as seen by the Superluminal observers, however, the (tachyonic) bomb looks to explode in two "jets" which remain confined within the double-cone, in such a way that no constituents of its move along the y' or z'-axis: In other words, the speeds V'_y, V'_z of the tachyonic bomb constituents "moving" along the y', z' axes, respectively, would result to be imaginary[6, 12].

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