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CHARGE CONJUGATION AND INTERNAL SPACE-TIME SYMMETRIES(x)

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ABSTRACT.

We adopt the relativistic framework in which fundamental particles are regarded as extended objects. Then, we show that the (geometrical) operation which reflects the internal space-time of a particle is equivalent to the operation C which invertes the sign of all its additive charges.

In the present paper we critically comment on the discrete transformations of Minkowski space-time, namely on the effects of space-reflection $\mathscr{P}$ and time-reversal $\mathscr{T}$, by exploiting some results contained in previous papers[1-3]. Our aim is to show the connection between the internal discrete transformations[2] and the charge-conjugation operator C. We assume fundamental particles to be extended objects, as many theoretical observations suggest to be the case, at least in relativistic theories[4].

First of all, let us recall that an inversion $\mathscr{I}_{ABC}(n)$ of the axes $x^A, x^B, x^C, \ldots\ldots$, in a n-dimensional space M_n is equivalent to an appropriate 180^o-rotation(+) $\mathscr{R}_{ABC\ldots}(m)$ in the hyperplane $(x^A, x^B, x^C, \ldots\ldots, x^n)$ of the m-dimensional space M_m with $m \geqslant n$. If the number

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(+) When M is Minkowskian, "rotation" will mean pseudo-rotation.

k of the inverted axes $x^A, x^B, x^C, \ldots$, is even, then it may be m = n; but if k is odd, then $m \geqslant n+1$. In particular, the total inversion (of all axes) in a n-dimensional space E_n corresponds to a rotation either in E_n (if n is even) or in a (n+1)-dimensional space E_{n+1} (if n+1 is even).

For instance, in the 2-dimensional plane the effect of the inversion $\mathscr{I}_x(2)$ (i.e., $x \to -x$, whilst $y \to y$) is equivalent to the effect of the 180°-rotation $\mathscr{R}_{xz}(3)$ in three dimensions around the y-axis:

$$\mathscr{I}_x(2) = \mathscr{R}_{xz}(3)\Big|_{E_2}, \tag{1}$$

where the subscript E_2 means that eq. (1) is true as far as we confine ourselves to the effect of its r.h.s. into the initial 2-dimensional space.

In Minkowski space there are the following discrete transformations(x) (we adopt the notation $x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (t, x, y, z)$):

space-reflection: $\mathscr{I}_1(4) \equiv \mathscr{P}$ (inversion of x^1);

time-reversal: $\mathscr{I}_0(4) \equiv \mathscr{T}$ (inversion of x^0).

The product $\mathscr{P}\mathscr{T} = \mathscr{T}\mathscr{P}$ is equivalent to the 180°-(pseudo) rotation in M_4:

$$\mathscr{T}\mathscr{P} \equiv \mathscr{I}_0(4)\,\mathscr{I}_1(4) = \mathscr{I}_{01}(4) = \mathscr{R}_{01}(4). \tag{2}$$

Though the product $\mathscr{P}\mathscr{T}$ can be considered as a rotation in M_4, of course neither $\mathscr{P}$ nor $\mathscr{T}$ alone can be replaced by any rotation in M_4. However, if instead of the 4-dimensional space M_4 we consider the 5-dimensional space M_5, so that an event e is described by the five coordinates

$$e: \quad x^A \equiv (x^0, x^1, x^2, x^3, x^4), \qquad e \in M_5,$$

then the effect in M_4 of the reflection $\mathscr{P}$ is equal to the effect in M_5 of the 180°-rotation around the space (x^0, x^2, x^3), i.e. of the 180°-rotation of M_5 "in the plane" (x^1, x^4):

$$\mathscr{P}x^\mu \equiv \mathscr{I}_1(4)x^\mu = \mathscr{R}_{14}(5)x^A\Big|_{M_4}; \qquad \begin{bmatrix} \mu = 0,1,2,3 \\ A = 0,1,2,3,4 \end{bmatrix} \tag{3}$$

and the effect of the time-reversal $\mathscr{T}$ is equal to the 180°-rotation of M_5 in the plane (x^0, x^4):

(x) The inversion in M_4 of one of the space-axes x^1, x^2 or x^3, e.g. $\mathscr{I}_1(4)$, is called space-reflection. Applying, after $\mathscr{I}_1(4)$, also the 180°-rotation in the plane (x^2, x^3) is equivalent to the inversion $\mathscr{I}_{123}(4)$ of all the three space-axes x^1, x^2 and x^3: $\mathscr{I}_{123}(4) = \mathscr{I}_1(4)\mathscr{R}_{23}(4)$.

$$\mathscr{T} x^{\mu} \equiv \mathscr{I}_0(4) x^{\mu} = \mathscr{R}_{04}(5) x^{A} \Big|_{M_4} . \tag{4}$$

The subscript M_4 means that, after having performed the rotation, we take account only of the events in $M_4 \subset M_5$.

At this point let us stress that, if the considered space-time M_4 contains a particle a, we are going to assume that: (i) particle a is - as we already mentioned - an extended object[4], so that the interior of its world-tube is a finite portion of space-time; (ii) our operations $\mathscr{P}$, $\mathscr{T}$ are to be regarded as acting both on the external space-time and on the internal one ("internal" and "external" with respect to the particle world-tube). Since the ordinary parity and time-reversal act on the contrary only on the external space-time, to avoid possible confusion we shall call $P \equiv \mathscr{P}_E$ the ordinary space-reflection and $T \equiv \mathscr{T}_E$ the ordinary time-reversal (E = external)[2].

Then, we shall show - among the others - that the charge-conjugation C is equal to the product $\mathscr{P}_I \mathscr{T}_I$, where $\mathscr{P}_I$ is the internal space-reflection and $\mathscr{T}_I$ is the internal time-reversal (I = internal)[2]. So that $\mathscr{P}\mathscr{T} = CPT$.

Let us explicitly write:

$$\mathscr{P} = \mathscr{P}_E \mathscr{P}_I = \mathscr{P}_I \mathscr{P}_E ; \tag{5}$$

$$\mathscr{T} = \mathscr{T}_E \mathscr{T}_I = \mathscr{T}_I \mathscr{T}_E , \tag{6}$$

where $\mathscr{P}_I$ ($\mathscr{T}_I$) is the internal, $\mathscr{P}_E$ ($\mathscr{T}_E$) the external, and $\mathscr{P}$($\mathscr{T}$) the total space-reflection (time-reversal).

More precisely, the transformations $\mathscr{P}$, $\mathscr{P}_E$, $\mathscr{P}_I$, $\mathscr{T}$, $\mathscr{T}_E$ and $\mathscr{T}_I$ can be defined with the aid of the suitable rotations in M_5. The total space-reflection $\mathscr{P}$ is defined by eq. (3) and the total time-reversal by eq. (4). See Figs. 1, 2, where quantity s^A is chosen to be a space-like vector lying inside the particle world-tube(x) and orthogonal to the world-tube axis (specified by its unit-vector τ^A). The world-tube lies in the ordinary M_4.

The internal space reflection $\mathscr{P}_I$ can be defined as the 180°-rotation in M_5 of the particle world-tube around the space $\Sigma_p \equiv (x^0, x^2, x^3)$ orthogonal to the plane (x^1, x^4): See Figs. 1a. Notice that the space Σ_p around which one has to perform the rotation in M_5 contains the time-axis x^0. When the particle a is considered at rest, then the tube axis coincides of course with the time-axis; in such a particular case, therefore, Σ_p contains τ^A: See Figs. 1b.

(x) For simplicity, let us assume the particle a to be spherical (even if with a non-spherically-symmetric structure).

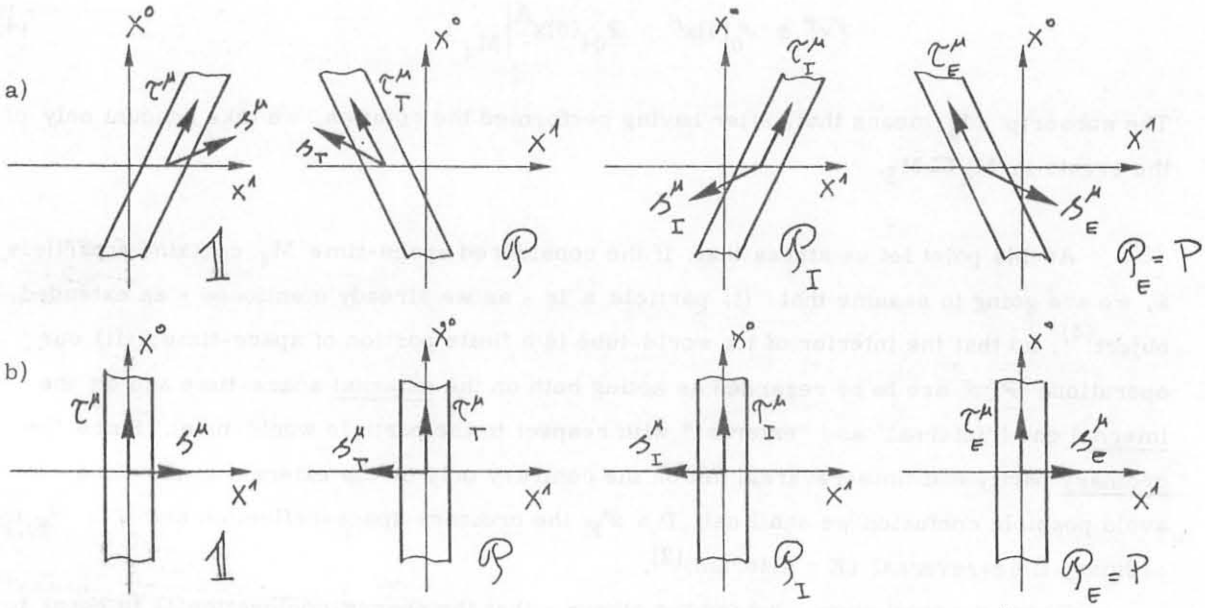

FIG. 1 - The effect of the total space reflection $\mathscr{P}$, the internal space reflection $\mathscr{P}_I$ and the external space reflection $\mathscr{P}_E$ on the world-tube of a particle. Fig. a refers to a moving particle, and b to the simpler case of a particle at rest. The world-tube is characterized by the time-like 4-vector τ^μ and the space-like 4-vector s^μ (see the text). The transformations $\mathscr{P}$, $\mathscr{P}_I$ and $\mathscr{P}_E$ change τ^μ into τ^μ_T, τ^μ_I and τ^μ_E, respectively; and analogously for s^μ.

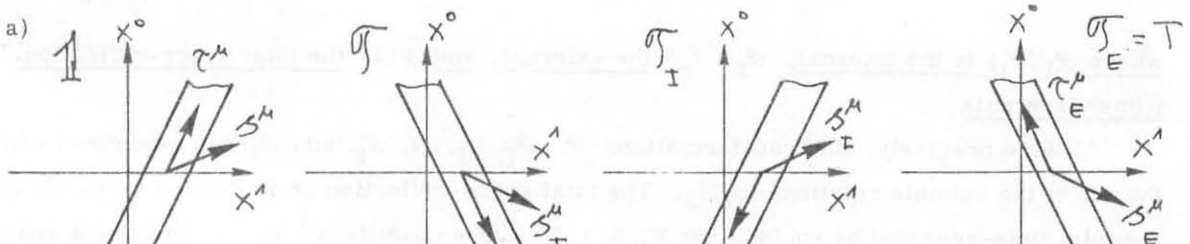

FIG. 2 - The effect of the total time reversal $\mathscr{T}$, the internal time reversal $\mathscr{T}_I$ and the external time reversal $\mathscr{T}_E$ on the world-tube of a particle. Again, Fig. a refers to a moving particle, and b to the simpler case of a particle at rest. As to τ^μ and s^μ, the same notations are used as in Fig. 1.

The internal time reversal $\mathcal{T}_I$ can be defined as the 180^o-rotation of the particle world--tube in M_5 around the space $\Sigma_T \equiv (x^1, x^2, x^3)$ orthogonal to the plane (x^0, x^4): See Figs. 2a. When the particle a is in particular at rest, s^A can be chosen so to coincide with the x^1-axis: See Figs. 2b.

The external space reflection $\mathcal{P}_E$ in M_4 affects a particle only by reflecting the world--line of its center-of-mass (the position of all other world-lines within the particle world-tube remaining unchanged relatively to the center-of-mass world-line). The external space reflection $\mathcal{P}_E$ is therefore nothing but the ordinary space-reflection P:

$$\mathcal{P}_E \equiv P . \tag{7}$$

The external time reversal $\mathcal{T}_E$ in M_4 is equivalent - with regard to a chosen particle a - to the operation transforming its velocity $\vec{v}$ into $-\vec{v}$ (Figs. 2a), without affecting its internal structure. The external time reversal $\mathcal{T}_E$ is therefore nothing but the ordinary time reversal T:

$$\mathcal{T}_E \equiv T . \tag{8}$$

We shall also generalize to the case of extended particles the Stückelberg-Feynman reinterpretation procedure[5].

Let us start by applying (from the active point of view) the total space-time reflection $\mathcal{PT}$ to the world-tube W of a particle a. We depict W as consisting in a sheaf of world-lines w which represent - say - its "constituents" (Fig. 3a); in Fig. 3 - besides the c. m. world--line - we show $w_1 \equiv A$; $w_2 \equiv B$. The operation $\mathcal{PT} \equiv \mathcal{P}_E \mathcal{T}_E \mathcal{P}_I \mathcal{T}_I$ will transform W into a new world-tube $\widetilde{W}$ consisting of the transformed world-lines $\widetilde{w}$ (Fig. 3b). The world-tube $\widetilde{W}$ differs from W in the fact taht its world-lines $\widetilde{w}$ point in the opposite time-direction and occupy - with respect to the center-of-mass world-line - the position symmetrical to the corresponding w.

By applying the Feynman procedure[5] each world-line $\widetilde{w}$ transforms into the corresponding world-line $\overline{w}$ (Fig. 3c). Each world-line $\overline{w}$ points in the positive time-direction, but represents an anti-"constituent". We now identify the sheaf $\overline{W}$ of the world-lines $\overline{w}$ of the "anti-constituents" with the antiparticle $\overline{a}$; and therefore $\overline{W}$ with the world-tube of $\overline{a}$. This identification corresponds to assume that the overall time-direction of a particle a (or $\overline{a}$) as a whole coincides with the time-direction of its "constituents". Such a procedure is an explicit generalization of Feynman procedure for extended particles.

A preliminary conclusion is that the antiparticle $\overline{a}$ of a can be regarded (from the chronotopical, geometrical point of view) as derived from the reflection of its internal space-time.

Let us repeat what precedes in a more rigorous way, and recall that the Stückelberg-

-Feynman reinterpretation procedure has been recently reformulated into one of the fundamental principles ("Third Postulate") of Special Relativity: See Refs. (3, 1, 2). Let us also recall that Special Relativity can be based(x) on the whole proper group $\mathscr{L}_+$ of both ortho- and anti-chrounous Lorentz transformations, $\mathscr{L}_+ = \mathscr{L}_+^\uparrow \cup \mathscr{L}_+^\downarrow$, since a clear physical meaning can be given also to antichronous (i. e. non-orthochronous) Lorentz transformations(3, 1). The central elements of $\mathscr{L}_+$ are $(+\mathbb{1}, -\mathbb{1})$, where $\mathbb{1}$ is the identity matrix in four-dimensions. That is to say, in such a formalization of Special Relativity the operation $-\mathbb{1}$ does represnt an actual (even if antichronous) Lorentz transformation, corresponding to the 180° space-time "rotation":

$$\overline{P}\,\overline{T} = -\mathbb{1}\,. \tag{9}$$

Notice explicitly that in eq. (9) the operators $\overline{P}$, $\overline{T}$ have a meaning different from the one of the ordinary space-parity P and time-reversal T. Namely, for the very fact that eq. (9) represents a Lorentz transformation, quantities $\overline{P}$, $\overline{T}$ and $\overline{P}\,\overline{T}$ will act not only on the chronotopical space, but also on the "dual" four-momentum space, etc. (This means that $\overline{T}$, in particular, when acting on a four-momentum vector, will change also the sign of energy). But let us go back to the mere chronotopical space.

Now, if we apply $\overline{P}\,\overline{T} = -\mathbb{1}$ from the active point of view to the world-tube W in Fig. 3a, we have to rotate it (by 180°, in four dimensions) into $\widetilde{W}$ (Fig. 3b). Such a rotation will effect also a reflection of the internal 3-space of a particle a, transforming it - among the others - into its mirror image. Analogously, from the passive point of view, if we apply $\overline{P}\,\overline{T}$ to the space-time in Fig. 3a, containing also W, we shall pass to a $\overline{P}\,\overline{T}$-ed frame whose space-time derives from the complete 180°-"rotation" of the initial space-time. Again, this will operate also the reflection of the internal space-time of particle a (relatively to the new observer).

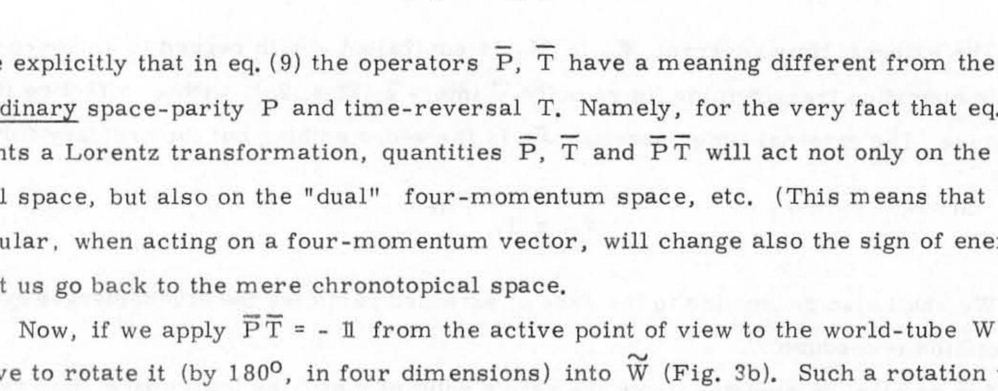

FIG. 3 - Given a world-tube (Fig. a), we show the effect of the (antichronous) Lorentz transformation $\overline{P}\,\overline{T} = -\mathbb{1}$ before (Fig. b) and after (Fig. c) the application of the "Reinterpretation Principle"(5, 1, 2). See the text.

(x) C. f. eq. (11) in the following.

Then, we extend the Reinterpretation Principle[3, 1, 2] to the case of extended objects, i. e. we apply it (e. g. within the active point of view) to the world-tube $\widetilde{W}$ of Fig. 3b. The world-tube $\widetilde{W}$ represents an (internally reflected) particle not only going backwards in time, but also carrying negative energy. Therefore applying the Reinterpretation Principle[6] will rigorously transform $\widetilde{W}$ into $\overline{W}$ (Fig. 3c), the anti-world-tube $\overline{W}$ representing the antiparticle $\overline{a}$.

In conclusion, as far as the chronotopical space is concerned, the (antichronous) Lorentz transformation $\overline{P}\,\overline{T} \equiv -\mathbb{1}$ can be considered as

$$-\mathbb{1} \equiv \overline{P}\,\overline{T} = \mathscr{P}_E \mathscr{T}_E \mathscr{P}_I \mathscr{T}_I = PT \mathscr{P}_I \mathscr{T}_I \,, \tag{10}$$

so that in particular:

$$\overline{P}\,\overline{T} = \mathscr{P}\mathscr{T} \,. \tag{10'}$$

At this point we have to recall that in Refs. (1, 3) we showed - by taking account also of the fourmomentum space and by applying the "Reinterpretation Principle" - that

$$\overline{P}\,\overline{T} = CPT \,, \tag{11}$$

where C represents the conjugation of all the additive charges[3, 1]. Let us add, going back to eq. (9), that all known (relativistic) equations and (relativistic) interactions are actually CPT-covariant. From eqs. (10), (11) it is immediate to derive that

$$\mathscr{P}_I \mathscr{T}_I = \mathscr{T}_I \mathscr{P}_I = C \,. \tag{12}$$

We have thus shown the (geometrical) operation of reflecting the internal space-time of the considered particle to be equivalent to the operation C which inverts the sign of all its additive charges.

We have also seen that the internal transformations $\mathscr{P}_I$, $\mathscr{T}_I$ do change the particle intrinsic state. If we convene to write $\mathscr{P}_I a_{++} = a_{+-}$; $\mathscr{T}_I a_{++} = a_{-+}$, then:

$$\mathscr{P}_I \mathscr{T}_I a_{++} = a_{--} \,, \tag{12'}$$

where the subscripts denote the internal parameters that transform under the action of $\mathscr{T}_I$ and $\mathscr{P}_I$, respectively; and where a_{--} represents the intrinsic (= internal) state of the antiparticle $\overline{a}$.

All what precedes can be applied also within the realm of quantum theories.

But let us here conclude by emphasizing that - in our opinion, and for the results in this paper and in Refs. (1-3) - we should advantageously substitute in theoretical physics the new oparations $\overline{P} \equiv \mathscr{P}$ and $\overline{T} \equiv \mathscr{T}$ for the ordinary operations P, T, which are merely external reflections (e. g., only the former do belong to the Full Lorentz Group).

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