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TWO-BODY INTERACTIONS BY TACHYON EXCHANGE

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ABSTRACT.

Due to its relevance for the possible applications to particle physics and for causality problems, we thoroughly analyse in this paper the kinematics of (classical) tachyon-exchange between two bodies A, B, for all possible relative velocities. In particular, the two cases $\vec{u} \cdot \vec{v} \leq c^2$ are carefully investigated, where u , v are the body B and tachyon speeds relative to A, respectively.

PART I. - INTRODUCTORY PART.

I. 1. - INTRODUCTION.

It is known since long that, when investigating tachyon dynamics, it is always necessary to take into proper account the tachyon together with its emitter A and absorber B⁽¹⁾.

Let us recall that, if two particles or bodies A, B exchange a tachyon T, then suitable subluminal observer always exist, which see the intermediate tachyon T with divergent speed, i. e. which judge the tachyon exchange as an instantaneous symmetrical interaction⁽¹⁾. Moreover other observers always exist, which see an antitachyon \bar{T} flying from B to A, with the exchange of the emission and absorption rôles. The very "Reintepretation procedure"^(1, 2) loses its meaning, therefore, if we cannot refer our T(\bar{T}) to some interaction regions.

In other words, tachyons (even when macro-objects) are typical carriers of - mutual and symmetric - interaction between A and B^(x). It seems thus probable that in our cosmos tachyons have a rôle as "interaction carriers" rather than as stable "asymptotical" objects⁽¹⁾. Frequently, attention has been called⁽⁴⁾ to the possible connections, for instance, between tachyons and internal lines of the quantum-relativistic processes. It is wellknown⁽⁵⁾ that black-holes, in classical physics, can emit only tachyons, therefore constituting a typical kind of tachyonic sources; from "Extended Relativity"⁽¹⁾ black-holes then follow to be also absorbers of tachyons: This means that tachyonic matter can a priori be exchanged - for instance - between black-holes, where we mean both "gravitational black-holes" (the ordinary ones) and possibly "strong black-holes" (hadrons).

For what precedes, it follows to be in any case quite important studying in detail the kinematics of tachyon exchange between two (micro- or macro-) bodies.

The kinematics of tachyon exchanges between two bodies A, B has been already investigated in part, but the previous results appeared scattered in a series of papers⁽⁶⁾. Due to that fragmentary "spreading" of the past results, increased by the presence of a couple of "errata"⁽⁷⁾, we deem useful to expound again - in improved, completed, and more organic form - the whole question.

However, before dealing with the tachyon exchange, let us premise an analysis of the emission and absorption of a tachyon T from a body (or particle) A.

I. 2. - TACHYON EMISSION (Description of "intrinsic emission", as seen in the rest-frame and in generic frames).

Let us consider first - in its rest-frame - a body C, with rest-mass M, emitting towards a second body D a tachyon (or antitachyon) T, endowed with (real) rest-mass m and fourmomentum $p \equiv (E_T, \vec{p})$, which travels with speed V for instance in the x-direction.

The fourmomentum conservation requires, in natural units, that^(1, 6):

$$M = \sqrt{\vec{p}^2 - m^2} + \sqrt{\vec{p}^2 + M'^2} \quad (\text{rest-frame}) \quad (1)$$

i. e.

$$2M|\vec{p}| = \sqrt{[m^2 + (M'^2 - M^2)]^2 + 4m^2M^2}, \quad (1')$$

wherefrom it follows that a body (or particle) C cannot emit in its rest-frame any tachyon T (whatever its rest-mass m is), unless the rest-mass M of C jumps (classically) to a lower

(x) It is not without meaning that Wheeler and Feynman⁽³⁾ were able to construct, even for the limiting case of photons, a theory (equivalent to the usual electromagnetism) where sources emit photons only if their detectors are, in a sense, (already) ready to absorb them.

value M' , such that⁽¹⁾ $\left[E_T \equiv \sqrt{\vec{p}^2 - m^2} \right]$:

$$\Delta \equiv M'^2 - M^2 = -m^2 - 2ME_T, \quad (\text{emission}) \quad (2)$$

so that:

$$-M^2 < \Delta \leq -\vec{p}^2 \leq -m^2. \quad (\text{emission}) \quad (3)$$

Eq. (1') can read⁽¹⁾:

$$V = \sqrt{1 + 4m^2M^2/(m^2 + \Delta)^2}. \quad (1'')$$

In particular, since infinite-speed T's carry zero energy but non-zero impulse $|\vec{p}| = mc$, then C cannot emit any transcendent tachyon without lowering its rest-mass; in fact, in the case of infinite-speed T emission, i. e. when $E_T = 0$ (in the rest-frame of C), eq. (2) yields⁽¹⁾

$$\Delta = -m^2. \quad (V = \infty; E_T = 0) \quad (4)$$

Since emission of transcendent tachyons (antitachyons) is equivalent to absorption of transcendent antitachyons (tachyons), we shall again get eq. (4) also as limiting case of tachyon absorption (cf. eq. (10)).

Notice that Δ is, off course, an invariant quantity. In fact eq. (2) can be read, in a generic frame f :

$$\Delta = -m^2 - 2p_\mu P^\mu, \quad (5)$$

where P^μ is now the fourmomentum of body C in the generic frame. Still $-M^2 < \Delta \leq -m^2$. The word "emission" in eq. (3) aims to indicate an intrinsic, proper behaviour, in the sense that it refers to "emission (as seen) in the rest-frame of the emitting body or particle". In suitably moving frames f , such an "emission" can even appear as an absorption^(1,2). (Conversely, other (suitably moving) frames f' can observe a T-emission from C (in flight) which does not satisfy inequation (3) since it corresponds in the rest-frame of C to an (intrinsic) absorption). However, if - in the moving frame f - the inequation (3) appears to be satisfied, this implies that in the rest-frame of C the process under exam is a tachyon emission, both when f observes an emission and when it observes an absorption. Let us anticipate that, in the case of "intrinsic absorption", relation (8') will be shown to hold, instead of relation (3). Before going on, let us add here only the following observation: Since the (invariant) quantity Δ in relation (8') can assume also positive values (contrary to the invariant quantity Δ in eqs. (2), (3)), if an observer f sees body A to increase its rest-mass in the process, then the "proper description" of the process can be nothing but an (intrinsic) absorption: see the following.

When Δ in eqs. (2-5) can assume only known, discrete, values (so as in elementary particle physics), then - once M is fixed - eq. (2) imposes a link between m and E_T , i. e.

between m and $|\vec{p}|$.

Let us repeat, at last, that the body C, when in flight, can appear to emit (suitable) tachyons without lowering (or even changing) its rest-mass. In particular, a particle in flight can a priori emit a suitable tachyon t transforming into itself (but, in such cases, if we go to the rest-frame of the initial particle, then the "emitted" tachyon will appear as an absorbed antitachyon \bar{t})⁽¹⁾

I. 3. - TACHYON ABSORPTION.

Secondly, let us consider our body C, with rest-mass M , now absorbing in its rest-frame a tachyon (or antitachyon) T' endowed with (real) rest-mass m , fourmomentum $p \equiv (E_T, \vec{p})$, emitted by a second body D, and travelling with speed V (for instance along the x -direction).

The fourmomentum conservation requires, in natural units, that^(1,6)

$$M + \sqrt{\vec{p}^2 - m^2} = \sqrt{\vec{p}^2 + M^2}, \quad (\text{rest-frame}) \quad (6)$$

wherefrom it follows that a body (or particle) C at rest can a priori absorb (suitable) tachyons both when increasing or lowering its rest-mass, and when conserving it. More precisely, eq. (6) yields^(6,1):

$$|\vec{p}| = \frac{1}{2M} \sqrt{(m^2 + \Delta)^2 + 4m^2M^2}, \quad (\text{rest-frame}) \quad (7)$$

which corresponds to:

$$\Delta = -m^2 + 2ME_T \quad (8)$$

so that

$$-m^2 \leq \Delta < \infty. \quad (\text{absorption}) \quad (8')$$

Eq. (7) tells us that body C (in its rest-frame) can absorb the tachyon (or antitachyon) T , emitted by the second body D, only when the tachyon speed V is^(6,1):

$$V = \sqrt{1 + 4m^2M^2/(m^2 + \Delta)^2}. \quad (9)$$

It should be explicitly noticed that eq. (8) differs from eq. (2). On the contrary, eqs. (7), (9) formally coincide with eqs. (1'), (1''), respectively; but they refer to different domains of Δ ; in fact e. g. in eq. (1'') we have $\Delta < -m^2$, whilst in eq. (9) we have $\Delta > -m^2$.

In particular, from eq. (9) one observes that C can absorb (in its rest-frame) infinite-speed tachyons only when $m^2 + \Delta = 0$, i. e.

$$V = \infty \longleftrightarrow \Delta = -m^2, \quad (\text{rest-frame}) \quad (10)$$

in agreement with eq. (4).

Quantity Δ is, of course, invariant. Namely, eq. (8) can be written, in a generic frame $f^{(1,6)}$,

$$\Delta = -m^2 + 2p_\mu P^\mu, \tag{11}$$

where P^μ is now the fourmomentum of body C in the generic frame f. Still $\Delta \geq -m^2$. Notice that the word absorption in eq. (8') means "intrinsic absorption", since it refers to "absorption (as seen) in the rest-frame of the absorbing body or particle". This means that, if a moving observer f sees relation (8') to be satisfied, the "intrinsic" description of the process (in the rest-frame of C) is a tachyon-absorption, both when f observes an absorption and when it observes an emission. In the particular case $\Delta = 0$, we'd simply get:

$$2ME_T = m^2. \tag{\Delta = 0}$$

When Δ in eqs. (7;11) can assume only known, discrete values (so as in elementary particle physics), then - once M is fixed - eqs. (7;11) state a link between m and E_T (or $|\vec{p}|$, or V).

For further considerations, cf. the end of Sect. I. 2.

I. 4. - SOME REMARKS.

In view of describing the tachyon-exchange between two bodies (or particles) A and B, let us thoroughly write down the implications of the fourmomentum conservation at A and at B. In order to do that, we need choosing a unique frame for describing the processes both at A and at B. Let us choose to these purposes the rest-frame of A.

Before going on, let us since now explicitly mention the important fact that, when bodies A and B exchange one tachyon T, the tachyon kinematics⁽¹⁾ is such that the "intrinsic description" of the process at A and at B (where the process at A is described in the rest-frame of A, and the process at B is described in the rest-frame of B) can a priori be of the following four types⁽¹⁾:

- (i) emission-absorption ;
 - (ii) absorption-emission ;
 - (iii) emission-emission ;
 - (iv) absorption-absorption.
- (12)

It is noticeable that the possible cases are not only (i) and (ii). Case (iii) can happen when the tachyon-exchange takes place in the recession phase (i. e. while bodies A, B are receding one from the other); case (iv) can happen when the tachyon-exchange takes place in the approaching phase (i. e. when A, B are approaching one another). For instance, let us consider

an elastic scattering between two (different) particles a, b. In the c. m. s., as wellknown, a and b exchange momentum but not energy. An infinite-speed tachyon T can therefore be a suitable carrier of that interaction (T will appear as a finite-speed tachyon in the rest-frames of a, b).

However, if a, b have to retain their rest-mass during the process, then tachyon-exchange can describe that elastic process only when we have "intrinsic absorption" both at a and at b (this can happen only when a, b are approaching one another).

Notice that the descriptions (i-iv) above do not refer to one and same observer since they on the contrary add together the "local" descriptions of observers A and B.

PART II. - TACHYON EXCHANGE WHEN $\vec{u} \cdot \vec{V} < c^2$.

Let \vec{V}, \vec{u} be the tachyon and body B velocities respectively, in the rest-frame of A.

Let us now consider A, B to exchange a tachyon (or antitachyon) T when $\vec{u} \cdot \vec{V} < c^2$.

In the rest-frame of A, we can have either intrinsic emission or intrinsic absorption from body A.

II. 1. - CASE OF "INTRINSIC EMISSION" AT A.

In the case when one observes, in the rest-frame of A, an (intrinsic) tachyon emission from A, both A and B will see the exchanged tachyon to be emitted by A and absorbed by B. In fact, given a tachyon T with speed \vec{V} in the frame A, a moving observer B endowed with speed \vec{u} will see an antitachyon \bar{T} (travelling the opposite way, according to the re-interpretation principle^(1, 2)) only when $\vec{u} \cdot \vec{V} > c^2$, whilst in the present case $\vec{u} \cdot \vec{V} < c^2$. Cf. refs. (1, 2, 6).

Imposing the four-momentum conservation at A, we get (in the A rest-frame) from eqs. (1), (2):

$$\Delta_A \equiv M_A'^2 - M_A^2 = -m^2 - 2M_A E_T; \quad (\text{rest-frame}) \quad (13)$$

$$2M_A |\vec{p}| = \left[(m^2 + \Delta_A)^2 + 4m^2 M_A^2 \right]^{1/2}; \quad (13')$$

$$v = \left[1 + 4m^2 M_A^2 / (m^2 + \Delta_A)^2 \right]^{1/2} \quad (13'')$$

(o) For instance, this includes tachyon exchanges in the "approaching phase" (for intrinsic T-emission at A), and in the "receding phase" (for intrinsic T-absorption at A).

where now we called M_A, M'_A the initial and final rest-mass of body A, respectively. According to eq. (5), in a generic frame f , quantity Δ_A can be written in explicitly covariant form as follows:

$$\Delta_A = -m^2 - 2p_\mu P_A^\mu, \quad (14)$$

wherefrom:

$$-M_A^2 < \Delta_A \leq -m^2, \quad (\text{intrinsic emission}) \quad (14')$$

where now p_μ and P_A^μ are tachyon T and body A four-momenta, respectively, in the frame f . Remember that, whatever the process description be in f , eq. (14') holds iff the process "intrinsic description" in A is a (tachyon) emission. Remember also inequation (3).

Let us remain in the rest-frame of A, and now study the kinematical conditions under which the tachyon T emitted by A can be absorbed by a second body B moving - in general - with speed u along a generic direction (with respect to A).

Let M_B and $P_B \equiv (E_B, \vec{P}_B)$ be rest-mass and four-momentum of body B, respectively. If T must be absorbed by B, then⁽⁶⁾:

$$\sqrt{\vec{P}_B^2 + M_B^2} + \sqrt{\vec{p}^2 - m^2} = \sqrt{(\vec{P}_B + \vec{p})^2 + M_B'^2}, \quad (15)$$

where M_B' is the final rest-mass of B.

Let us define:

$$\Delta_B \equiv M_B'^2 - M_B^2,$$

which reads^(1, 6):

$$\Delta_B = -m^2 + 2\tilde{m}\tilde{M}_B(1 - uV \cos \alpha), \quad (16)$$

where $\tilde{m} \equiv E_T$ and $\tilde{M}_B \equiv E_B \equiv \sqrt{\vec{P}_B^2 + M_B^2}$ are the relativistic masses of T and B, respectively, and $\alpha \equiv \hat{u} \cdot \hat{V}$ is the angle between \vec{u} and \vec{V} . The invariant quantity Δ_B , in a generic frame f , can be written^(6, 1)

$$\Delta_B = -m^2 + 2p_\mu P_B^\mu \quad (17)$$

where now p_μ, P_B^μ are T and B four-momenta in the generic frame f . Differently from the (intrinsic) emission case, Δ_B can a priori assume both negative and positive, or null, values:

$$-m^2 \leq \Delta_B < \infty. \quad (\text{intrinsic absorption}) \quad (18)$$

Notice that, if in the generic frame f relation (18) is verified, then (whatever be the description of the process at B given by f) the process will appear in the rest-frame of B as an (in-

trinsic) absorption. Of course, the kinematics connected with eq. (15) is such that Δ_B can even be smaller than $-m^2$: cf. eq. (16); but such a case ($uV \cos \alpha > 1$) would correspond to intrinsic emission (and no more to intrinsic absorption).

In conclusion the tachyon exchange, in the case of "intrinsic emission" at A with $\vec{u} \cdot \vec{V} < c^2$ in the A rest-frame, is kinematically allowed when the following equations are simultaneously satisfied:

$$\begin{aligned} \Delta_A &= -m^2 - 2M_A E_T; & (-M_A^2 < \Delta_A \leq -m^2) \\ \Delta_B &= -m^2 + 2E_T E_B (1 - \vec{u} \cdot \vec{V}). & (\Delta_B \geq -m^2) \end{aligned} \quad (19)$$

In the particular case when B moves along the direction-line of tachyon T (in the negative or positive x-direction, let us say), so that $\vec{P}_B // (\pm \vec{p})$, then the second of eqs. (19) can also be written^(6,1):

$$2M_B^2 |\vec{p}| = E_B \sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} \pm (m^2 + \Delta_B) |\vec{P}_B|. \quad (\vec{P}_B // (\pm \vec{p})) \quad (20)$$

When B is at rest with respect to A (i. e., when $\vec{P}_B = 0$) we are back to Sect. I, 3 and recover eqs. (7), (8), (9).

Finally, let us add the consideration that in this case ("intrinsic absorption" at B) quantity Δ_B can a priori vanish, - differently from quantity Δ_A which has always to be negative (cf. eq. (3)). In the case when $\Delta_B = 0$, the second of eqs. (19) simplifies into

$$2E_T E_B (1 - \vec{u} \cdot \vec{V}) = m^2; \quad (\Delta_B = 0)$$

and eqs. (20) become:

$$|\vec{p}| = \frac{m}{2M_B^2} (E_B \sqrt{m^2 + 4M_B^2} \pm m |\vec{P}_B|). \quad (\vec{P}_B // (\pm \vec{p}); \Delta_B = 0) \quad (21)$$

In the very particular case when both $\vec{P}_B = 0$ and $\Delta_B = 0$, eqs. (20), (21) yield^(6,1) (cf. eq. (9)):

$$V = \sqrt{1 + 4M_B^2/m^2}. \quad (\vec{P}_B = 0; \Delta_B = 0) \quad (22)$$

II. 2. - CASE OF "INTRINSIC ABSORPTION" AT A.

Let us consider tachyon-exchanges such that the process at A appears, in the A rest-frame, as an (intrinsic) absorption. Observer A will see the (exchanged) tachyon T to be emitted by B. The condition $\vec{u} \cdot \vec{V} < c^2$ implies^(1, 2, 6) in this case that body B appears to emit tachyon T also in its rest-frame.

The present case, therefore, is just the symmetrical of the previous one in Sect. II. 1.

The only difference is that now we are in the rest-frame of A, i. e. of the absorbing body.

For the process at A we have

$$M_A + \sqrt{\vec{p}^2 - m^2} = \sqrt{\vec{p}^2 + M_A'^2} \quad (\text{A-rest frame}) \quad (23)$$

where now :

$$\Delta_A \equiv M_A'^2 - M_A^2 = -m^2 + 2M_A E_T. \quad (\text{intrinsic absorption}) \quad (24)$$

In a generic frame f, the invariant quantity Δ_A can read

$$\Delta_A = -m^2 + 2p_\mu P_A^\mu, \quad (25)$$

wherefrom :

$$-m^2 \leq \Delta_A < \infty, \quad (\text{intrinsic absorption}) \quad (26)$$

where now p_μ, P_A^μ are the four-momenta of T and A, respectively, in the generic frame f.

Let us recall that f will see relation (26) to be satisfied iff it refers to a process (at A)

which is "intrinsically" a tachyon absorption, whatever his description from f be.

Let us recall that, in the particular case $\Delta_A = 0$, we get: $2M_A E_T = m^2$.

For the process at B, in the rest-frame of A, we have:

$$\sqrt{P_B^2 + M_B^2} = \sqrt{\vec{p}^2 - m^2} + \sqrt{(\vec{P}_B - \vec{p})^2 + M_B'^2}, \quad (27)$$

where (cf., for the symbols, eq. (16)):

$$\Delta_B \equiv M_B'^2 - M_B^2 = -m^2 - 2\tilde{m}\tilde{M}_B(1 - \vec{u} \cdot \vec{V}). \quad (28)$$

In a generic frame f, the invariant quantity Δ_B can be written (\vec{p}_μ, \vec{P}_B now being the four-momenta of T and B, respectively, in the generic frame f):

$$\Delta_B = -m^2 - 2p_\mu P_B^\mu, \quad (29)$$

wherefrom

$$-M_B^2 < \Delta_B \leq -m^2. \quad (\text{intrinsic emission}) \quad (30)$$

Let us observe that, in a frame f, relation (30) holds iff the process at B (no matter how it may appear to f) is "intrinsically" - i. e. , in the B-rest frame - a tachyon emission. We have already seen that it would be an "intrinsic absorption" only if we had $\Delta_B \geq -m^2$; that is to say, in general:

$$(\vec{u} \cdot \vec{v} \leq c^2) \therefore \Delta_B = \begin{cases} -m^2 + 2p_\mu P_B^\mu \geq -m^2 \implies \text{intrinsic absorption;} \\ -m^2 - p_\mu P_B^\mu \leq -m^2 \implies \text{intrinsic emission.} \end{cases} \quad (31)$$

For clarity's sake, let us explicitly repeat that: Necessary condition in order that the tachyon (or antitachyon) T, seen by A to be absorbed by B, can be seen in the rest-frame of B as an antitachyon (or tachyon) \bar{T} actually emitted by B, is that during the process B lowers its rest-mass (invariant statement!) in such a way that $-M_B^2 < \Delta_B \leq -m^2$.

In conclusion the tachyon exchange, in the case of "intrinsic" absorption at A and $\vec{u} \cdot \vec{v} < c^2$ (in the rest-frame of A), is kinematically allowed when the following eqs. are simultaneously satisfied:

$$\begin{aligned} \Delta_A &= -m^2 + 2M_A E_T; & (\Delta_A \geq -m^2) \\ \Delta_B &= -m^2 - 2E_T E_B (1 - \vec{u} \cdot \vec{v}). & (-M_B^2 < \Delta_B \leq -m^2) \end{aligned} \quad (32)$$

In the particular case when B moves along the direction line of tachyon T (in the x-direction, let us say), so that $\vec{P}_B // (\vec{1} \vec{p})$, then the second of eqs. (32) can be written:

$$2M_B^2 |\vec{p}| = E_B \sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} \mp (m^2 + \Delta_B) |\vec{P}_B|; \quad (\vec{P}_B // (\vec{1} \vec{p})) \quad (33)$$

where attention should be paid to the fact that the signs in the r. h. s. of eq. (33) are opposite to the ones entering eq. (20), as it should be also for self-evident symmetry reasons.

When B is at rest with respect to A, so that $\vec{P}_B = 0$, the second one of eqs. (33) transforms into:

$$|\vec{p}| = \frac{1}{2M_B} \sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} \quad (\vec{P}_B = 0) \quad (34)$$

in obvious agreement with eq. (1'). And so on: Cf. eq. (1''), (2). Let us repeat the observation that, in the present case of intrinsic emission, eq. (34) corresponds to values of Δ_B in the range $-M_B^2 < \Delta_B \leq -m^2$; whilst eq. (7), which holds in the opposite case of intrinsic absorption, corresponds to $\Delta_B \geq -m^2$.

PART III - TACHYON EXCHANGE WHEN $\vec{u} \cdot \vec{V} \geq c^2$.

Still in the rest-frame of A, let us now consider A, B to exchange a tachyon (or anti tachyon) T when $\vec{u} \cdot \vec{V} > c^2$. Under the present condition, again we can have either "intrinsic emission" or "intrinsic absorption" by body A.

The present cases differ from the ones previously considered in Part II for the fact that now - due to the reinterpretation procedure^(1,2) - the T-emission (at A) and T-absorption (at B) are described in the rest frame of B as a \bar{T} -absorption (at A) and \bar{T} -emission (at B), respectively⁽⁶⁾.

III. 1. - CASE OF "INTRINSIC EMISSION" AT A.

If, in the rest-frame of A, we observe body A (intrinsically) emit tachyon T, then in the B rest-frame we would observe an antitachyon \bar{T} absorbed by A, - due to the present condition $\vec{u} \cdot \vec{V} > c^2$, and to the reinterpretation procedure^(1,2,6).

Necessary condition for this case to exist is that A, B are receding one from the other (i. e. are in the "recession phase").

In any case, in the A rest-frame, we get for the process at A the same kinematics already expounded in Sect. II.1. Here we confine ourselves, therefore, to quote eqs. (1+5), or rather eqs. (13), (14).

As to the process at B, in the A rest-frame body B is observed to absorb a tachyon T:

$$\sqrt{\vec{P}_B^2 + M_B^2} + \sqrt{\vec{p}^2 - m^2} = \sqrt{(\vec{P}_B + \vec{p})^2 + M_B^2} \quad (15)$$

In the B rest-frame, however, one would observe an ("intrinsic") \bar{T} -emission, so that what we stated between eqs. (31) and (32) is here in order. Namely, relation (30) has to hold in this case (even if it is now associated to eq. (15) and not to eq. (27), in the A rest-frame). Notice that, when passing from the A rest-frame to the B rest-frame (and applying the reinterpretation procedure^(1,2)), in eq. (15) one has that: (i) Quantity E_T changes its sign, so that quantity $\sqrt{\vec{p}^2 - m^2}$ appears added to the r. h. s. (and now more l. h. s.); (ii) The tachyon three-momentum \vec{p} changes its sign (since we go from a tachyon T with impulse \vec{p} to its antitachyon \bar{T} with impulse $-\vec{p}$).

In any case, from eq. (15) with the condition $\vec{u} \cdot \vec{V} \geq c^2$ it directly follows

$$\Delta_B = -m^2 + 2\tilde{m}\tilde{M}_B(1 - \vec{u} \cdot \vec{V}) \leq -m^2, \quad (35)$$

with can read

$$\Delta_B = -m^2 + 2p_\mu P_B^\mu \leq -m^2. \quad (36)$$

In other words, eq. (15) with $\vec{u} \cdot \vec{V} \geq c^2$ yields

$$-M_B^2 < \Delta_B \leq -m^2. \quad (\text{intrinsic emission}) \quad (37)$$

In conclusion, the tachyon exchange, in the case of "intrinsic emission" at A and $\vec{u} \cdot \vec{V} \geq c^2$ (in the A rest-frame), is kinematically allowed when the following eqs. are simultaneously satisfied:

$$\begin{aligned} \Delta_A &= -m^2 - 2M_A E_T; & (\Delta_A \leq -m^2) \\ \Delta_B &= -m^2 + 2E_T E_B (1 - \vec{u} \cdot \vec{V}). & (\Delta_B \leq -m^2) \end{aligned} \quad (38)$$

In the particular case when \vec{P}_B and \vec{p} are collinear, we can have only $\vec{P}_B // \vec{p}$ ("recession phase") and the second of eqs. (38) can be written:

$$2M_B^2 |\vec{p}| = E_B \sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} + (m^2 + \Delta_B) |\vec{P}_B|; \quad (\vec{P}_B // \vec{p}) \quad (39)$$

eq. (39) is formally identical to part of eq. (20), but refers to values of Δ_B in the range $-M_B^2 < \Delta_B \leq -m^2$. It refers, therefore, to the same range of Δ_B values of eq. (33), but its r. h. s. contains a sign which is at variance with eq. (33).

III. 2. - CASE OF "INTRINSIC ABSORPTION" AT A.

Due to the present condition $\vec{u} \cdot \vec{V} \geq c^2$ and to the reinterpretation procedure^(1, 2), if we observe in the A rest-frame body A (intrinsically) to absorb a tachyon T, then in the B rest-frame we'd observe an antitachyon \bar{T} emitted by A.

Necessary condition for this case to exist is that A, B are approaching each other (i. e. are in the "approaching phase").

In any case, in the rest-frame of A, we get (for the process at A) the same kinematics already expounded in Sect. II. 2. Here we confine ourselves, therefore, to quote eqs. (6÷11), or rather eqs. (23÷26).

As to the process at B in the A rest-frame body B is observed to emit a tachyon T:

$$\sqrt{\vec{P}_B^2 + M_B^2} = \sqrt{\vec{p}^2 - m^2} + \sqrt{(\vec{P}_B - \vec{p})^2 + M_B'^2}. \quad (27)$$

In the B rest-frame, however, one would observe an ("intrinsic") \bar{T} -absorption, so that it must be

$$-m^2 \leq \Delta_B < \infty. \quad (\text{intrinsic absorption}) \quad (40)$$

In fact, at variance with eqs. (31), in the case $\vec{u} \cdot \vec{V} \geq c^2$ we have:

$$(\vec{u} \cdot \vec{V} > c^2) \therefore \Delta_B = \begin{cases} -m^2 - 2p_\mu P_B^\mu \geq -m^2 \implies \text{intrinsic absorption;} \\ -m^2 + 2p_\mu P_B^\mu \leq -m^2 \implies \text{intrinsic emission.} \end{cases} \quad (41)$$

Namely, from eq. (27) with condition $\vec{u} \cdot \vec{V} \gg c^2$; it directly follows:

$$\Delta_B = -m^2 - 2\tilde{m}\tilde{M}_B(1 - \vec{u} \cdot \vec{V}) \geq -m^2, \quad (42)$$

which can read

$$\Delta_B = -m^2 - 2p_\mu P_B^\mu \geq -m^2. \quad (43)$$

In other words, eq. (27) with the condition $\vec{u} \cdot \vec{V} \geq c^2$ yields:

$$-m^2 \leq \Delta_B < \infty. \quad (\text{intrinsic absorption}) \quad (44)$$

In conclusion the tachyon exchange, in the case of "intrinsic absorption" at A and $\vec{u} \cdot \vec{V} \geq c^2$ (in the A rest-frame), is kinematically allowed when the following eqs. are simultaneously verified:

$$\begin{aligned} \Delta_A &= -m^2 + 2M_A E_T; & (\Delta_A \geq -m^2) \\ \Delta_B &= -m^2 - 2E_T E_B (1 - \vec{u} \cdot \vec{V}). & (\Delta_B \geq -m^2) \end{aligned} \quad (45)$$

In the particular case when \vec{P}_B and \vec{p} are collinear, we can have only $\vec{P}_B // \vec{p}$ ("approaching phase"), and the second of eqs. (45) can be written:

$$2M_B^2 |\vec{p}| = E_B \sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} - (m^2 + \Delta_B) |\vec{P}_B|; \quad (\vec{P}_B // \vec{p}) \quad (46)$$

with $-m^2 \leq \Delta_B < \infty$.

Finally, let us recall that in the present case ("intrinsic absorptions" at B and at A) both quantities Δ_A, Δ_B can vanish. When $\Delta_A = 0$, we simply get: $2M_A E_T = m^2$. In the particular case when $\Delta_B = 0$, we would get:

$$2E_T E_B (\vec{u} \cdot \vec{V} - 1) = m^2, \quad (\Delta_B = 0)$$

and eqs. (46) become (when $\vec{P}_B // \vec{p}$):

$$|\vec{p}| = \frac{m}{2M_B^2} \left[E_B \sqrt{m^2 + 4M_B^2} - m |\vec{P}_B| \right]. \quad (\vec{P}_B // \vec{p}; \Delta_B = 0) \quad (47)$$

At this point let us remember that, when elementary interactions are considered to be mediated by the strong field quanta, no ("realistic") ordinary particles can actually be the carriers of the transferred energy-momentum⁽⁸⁾. On the contrary, tachyons (instead of the so-called virtual particles) can a priori work as the actual carriers of the strong interactions^{(8, 9)(*)}.

For instance, let us recall that any elastic scattering can be considered as "realistically" (classically) mediated by a suitable tachyon-exchange during the approaching phase of the two bodies. In such a case eqs. (45) write (always in the A rest-frame):

$$E_T = m^2 / (2M_A); \quad E_B = M_A / (\vec{u} \cdot \vec{V} - 1); \quad (\Delta_A = \Delta_B = 0) \quad (48)$$

we are neglecting the angular momentum conservation.

In the c. m. s., for instance, we would have $|\vec{P}_A| = |\vec{P}_B| = |\vec{P}|$, and

$$\cos \check{\theta}_{C.M.} = 1 - \frac{m^2}{2|P|^2}, \quad (\text{elastic scattering}) \quad (49)$$

so that (once $|\vec{P}|$ is fixed), for each tachyon-mass m , we'd get one particular $\theta_{C.M.}$; if m assumes only discrete values (according to the Duality Principle^(1, 2), then $\theta_{C.M.}$ results to be (classically) "quantized"^(8, 9), except for a cylindrical symmetry. More in general, for each discrete value of the tachyon-mass m , quantity $\theta_{C.M.}$ assumes too a discrete value, which is merely a function of $|\vec{P}|$. Such naive considerations are neglecting the mass-width of the tachyonic ("mesonic") resonances^(8, 9). Let us recall that in the c. m. s. any elastic scattering appears classically as mediated by an infinite-speed tachyon having $p_\mu \equiv (0, \vec{p})$,

(*) For instance, let us consider the vertex $\Delta_{33} \rightarrow p + \pi_T$ of a suitable one-particle-exchange diagram, and suppose the exchanged particle π_T ("internal line") to be a tachyon pion (instead of a virtual object). Then, from eqs. (2), (13) we'd get:

$$(1232)^2 - (938)^2 = (140)^2 + 2 \times 1232 \times \sqrt{c^2 |\vec{p}|^2 - (140)^2}$$

and therefore

$$|\vec{p}|_{\pi_T} = 287 \text{ MeV}/c; \quad E_{\pi_T} = 251 \text{ MeV},$$

so that, in the c. m. s. of the $\Delta(1232)$, the total energy of the tachyonic pion - under the present hypotheses - should be centered around 251 MeV.

Again, let us consider the decay $\pi \rightarrow \mu + \nu_T$ under the hypothesis that ν_T be a tachyon-neutrino (with $m_\nu \gtrsim 0$; $v_\nu \gtrsim c$). It has been shown e. g. by R. G. Cawley (Lett. Nuovo Cimento 3, 523 (1972)) that this hypothesis is not inconsistent with the experimental data and implies for the muon-neutrino $m_\nu \lesssim 1.7 \text{ MeV}$. In the two limiting cases ($m_\nu = 0$, and $m_\nu = 1.7 \text{ MeV}$), from eqs. (2), (13) one gets in the c. m. s. of the pion:

$$\begin{aligned} m_\nu = 0 &\implies |\vec{p}|_\nu = 29.79 \text{ MeV}/c; & (v = c) \\ m_\nu = 1.7 &\implies |\vec{p}|_\nu = 29.83 \text{ MeV}/c, & (v \approx 1.0016 c) \end{aligned}$$

where the first result coincides, of course, with the standard one.

where $|\vec{p}| = m$. Moreover, eqs. (48) impose a link between m and the direction of \vec{p} , i. e. between m and $\alpha \equiv \widehat{\vec{p}} \cdot \vec{P}$ (where e. g. we can choose $\vec{P} = \vec{P}_B$; remember that $\vec{P}_B = -\vec{P}_A$):

$$\cos \alpha = \frac{m}{2|\vec{P}|}; \quad (50)$$

again we find that (once $|\vec{P}|$ is fixed), if the tachyon-meson masses are discrete, then also the exchanged three-momentum results to be (classically) "quantized" in both its magnitude and direction.

This means again that, for each discrete value of m , also the exchanged three-momentum assumes one discrete direction (except for a cylindrical symmetry), which is a function only of $|\vec{P}|$. Notice that such a result cannot be obtained at the classical level when confining ourselves only to bradyons, since ordinary particles cannot, from the kinematical viewpoint, be the interaction carriers.

Of course, also non-elastic scattering can be considered as mediated by suitable tachyon exchanges⁽⁸⁾.

III. 3. - FINAL CONSIDERATIONS.

Roughly speaking, we can summarize what precedes by saying that:

(a) in the case of "intrinsic emission" at A:

$$\vec{u} \cdot \vec{V} \leq c^2 \implies \Delta_B \gtrsim -m^2; \quad (51)$$

(b) in the case of "intrinsic absorption" at A:

$$\vec{u} \cdot \vec{V} \leq c^2 \implies \Delta_B \lesssim -m^2. \quad (52)$$

At this point, let us recall^(1, 2) that no causal problems arise in tachyon micro-physics, some problems remaining possibly open only in tachyon macro-physics.

More precisely, when $\vec{u} \cdot \vec{V} \leq c^2$ no causality problem arises even in tachyon macro-physics^(6, 10). Only when $\vec{u} \cdot \vec{V} > c^2$ it seems that some interesting causal problems remain to be exploited in tachyon macro-physics⁽¹⁰⁾ (but not in micro-physics), whose discussion apparently requires taking into account different subjects which may range from the peculiar behaviour of tachyon sources and detectors, to the spontaneous tachyon-emission properties of matter, to information theory, and even to the question whether Minkowski space-time is enough for allocating the "free-will" behaviour.

In any case, let us warn once more that the correct procedure for getting physically realizable processes among tachyons is: (i) to start from any possible processes among bradyons; (ii) to apply to them a Superluminal Lorentz transformation (thus obtaining any "real" interactions among tachyons).

As to the possible applications of the present work, here let us confine ourselves to refer - besides to what already sketched in Sect. III, 2 - to the hints contained in refs. (8,9,1).

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