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ANTIPARTICLES FROM SPECIAL RELATIVITY WITH ORTHO-CHRONOUS AND ANTI-CHRONOUS LORENTZ TRANSFORMATIONS(x)

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ABSTRACT

Special Relativity can be based on the whole proper group of both ortho- and anti-chronous Lorentz transformations, and a clear physical meaning can be given also to antichronous (i.e., non-orthochronous) Lorentz transformations. From the active point of view, the latter require existence, for any particle, of its antiparticle within a purely relativistic, classical context. From the passive point of view, they give rise to frames "dual" of the ordinary ones, whose properties - here briefly discussed - are linked with the fact that in physics it is impossible to teach another, far observer (by transmitting only instructions, and no physical objects) our own conventions about the choices right/left, matter/antimatter, and positive/negative time direction. Interesting considerations follow, in particular, by considering - as it is the case - the CPT operation as an actual (even if antichronous) Lorentz transformation.

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1. - SPECIAL RELATIVITY WITH ORTHO- AND ANTI-CHRONOUS TRANSFORMATIONS.

From the standard postulates of Special Relativity[1] it follows that in such a theory - which refers to the class of Mechanical and Electromagnetic phenomena - the class of frames equivalent to a given inertial frame is obtained by means of transformations ("Lorentz transformations") which satisfy the following, sufficient requirements:

(a) to be linear (to transform inertial motion into inertial motion):

$$x^{\mu'} = \Lambda^{\mu'} x^{\nu} \; ; \tag{1}$$

(b) to preserve space isotropy (with respect to electromagnetic and mechanical phenomena);

(c) to form a group;

(d) to leave the quadratic form invariant:

$$\eta_{\mu\nu} x^{\mu} x^{\nu} = \eta_{\mu'\nu'} x^{\mu'}_{\nu} x^{\nu'} . \tag{2}$$

From condition (a), if we confine only to subluminal speeds, it follows that in eq. (2) it is:

$$\eta_{\mu'\nu'} = \mathrm{diag}(+1, -1, -1, -1) = \eta_{\mu\nu} . \tag{3}$$

Eqs. (1), (2), (3) imply that

$$\det \Lambda^2 = 1 ; \qquad (\Lambda_{o}^{o'})^2 \geq 1 .$$

The set of all (Lorentz) transformations satisfying all our conditions - as well known - consists in four pieces:

$$\begin{cases} \{\Lambda_{+}^{\uparrow}\} : \; \Lambda_{o}^{o'} \geq +1 ; \quad \det \Lambda = +1 ; & \text{(4.a)} \\ \{\Lambda_{+}^{\downarrow}\} : \; \Lambda_{o}^{o'} \leq -1 ; \quad \det \Lambda = +1 ; & \text{(4.b)} \\ \{\Lambda_{-}^{\uparrow}\} : \; \Lambda_{o}^{o'} \geq +1 ; \quad \det \Lambda = -1 ; & \text{(4.c)} \\ \{\Lambda_{-}^{\downarrow}\} : \; \Lambda_{o}^{o'} \leq -1 ; \quad \det \Lambda = -1 . & \text{(4.d)} \end{cases}$$

The whole set actually forms a non-compact, non-connected group (the Full Lorentz Group). It possesses, however, invariant subgroups; for instance the subgroup of the proper orthochronous transformations:

$$\mathscr{L}_{+} \equiv \{\Lambda_{+}^{\uparrow}\} , \tag{5}$$

and the subgroup of the proper (orthochronous and antichronous) transformations:

$$\mathscr{L}_+ \equiv \mathscr{L}_+^{\uparrow} \cup \mathscr{L}_+^{\downarrow} \equiv \{\Lambda_+^{\uparrow}\} \cup \{\Lambda_+^{\downarrow}\}. \tag{6}$$

For reasons that we shall see elsewhere[2], let us here rewrite $\mathscr{L}_+$ as follows:

$$\mathscr{L}_+ = \mathscr{L}_+^{\uparrow} \otimes Z(2); \qquad Z(2) \equiv \{\sqrt[2]{1}\} \equiv \{+1, -1\} \, . \tag{6'}$$

Given a transformation $\bar{\Lambda}_+^{\downarrow}$, always a transformation $\bar{\Lambda}_+^{\uparrow} \in \mathscr{L}_+^{\uparrow}$ exists such that

$$\bar{\Lambda}_+^{\downarrow} = (-1) \cdot \bar{\Lambda}_+^{\uparrow} , \qquad \forall \bar{\Lambda}_+^{\downarrow} \in \mathscr{L}_+^{\downarrow} , \tag{7}$$

and vice-versa. Such a one-to-one correspondence allows us to write formally that

$$\mathscr{L}_+^{\downarrow} = - \mathscr{L}_+^{\uparrow} . \tag{7'}$$

It follows in particular that the central elements of $\mathscr{L}_+$, eq. (6), are:

$$C = (+\mathbb{1}, -\mathbb{1}) . \tag{8}$$

Since we want to confine ourselves to space-time "rotations", we release the pieces (4.c) and (4.d).

Usually, also the piece (4.b) is released. Our aim is to show - on the contrary - how a physical meaning can be attributed also to the transformations (4.b), both in the passive and in the active sense (Cf. Appendix A).

In other words, we are going here to show: (i) that all the transformations of the group (6) can be attributed the meaning of actual transformations between two physical (inertial) frames; and: (ii) that the theory of Special Relativity (SR), once based on the whole proper Lorentz group (6), and not only on its orthochronous part, will describe a Minkowski space-time populated by both Matter and Anti-matter.

Such results will be got on the basis of the assumption that, for any observer, <u>no negative energy objects (travelling forward in time) exist.</u> To such an assumption can be given tha status of a fundamental postulate[1] of SR ("Third Postulate"). Cf. Refs. (1).

Before going on, let us explicitly recall that the ordinary relativistic laws (of mechanics and electromagnetism) are actually covariant under the whole proper group $\mathscr{L}_+$, since they are known to be CPT-symmetric besides covariant under the orthochronous proper Lorentz group $\mathscr{L}_+^{\uparrow}$ (see the following).

2. - THE ACTIVE POINT OF VIEW. ANTIMATTER FROM SR.

Let us start with the second (active) point of view, and consider an object (particle) P endowed with positive energy and motion forward in time. An antichronous Lorentz transformation $\Lambda_+^{\downarrow}$ will change sign (among the others) to the time - components of all the four-vectors associated with the considered particle P. In particular, it will transform P into a particle P' endowed with negative energy and motion backwards in time. Such an object P' can[1] be reinterpreted as the antiparticle $\overline{P}$, orthodoxically endowed now with positive energy and motion forward in time.

This reinterpretation[3](x) not only can, but must be performed, since relativistic observers (macro-objects) can do nothing but explore space-time along the positive time--direction. As a consequence,

$$P' \equiv \overline{P} \tag{9}$$

in the sense that in our case any $\Lambda_+^{\downarrow}$ has the same kinematical effect of its dual transformation $\Lambda_+^{\uparrow}$, defined through eq. (7), except for the fact[1] that it moreover transforms[(o)] P into its antiparticle $\overline{P}$.

We may also write

$$-\mathbb{1} \equiv \overline{P}\,\overline{T}\,, \tag{10}$$

where the symmetry-operations $\overline{P}$, $\overline{T}$ are to be understood in their strong sense[1]: $\overline{P} \equiv$ $\equiv$ strong parity; $\overline{T} \equiv$ strong time-reversal. Since it has been shown elsewhere[1,3] that $\overline{P}\,\overline{T} \equiv CPT$, we may write eq. (7') as:

$$\mathscr{L}_+^{\downarrow} = (\overline{P}\,\overline{T}) \cdot \mathscr{L}_+^{\uparrow} \equiv (CPT) \cdot \mathscr{L}_+^{\uparrow}. \tag{7''}$$

More generally, due to eqs. (7), (7''), when $\Lambda_+^{\downarrow}$ acts on a phenomenon ph, the transformed phenomenon ph' (deformed because of the antichronous Lorentz transformation) will have its initial and final states interchanged with respect to those of ph, and will moreover be constituted by the antiparticles of the original particles. In particular, the operation $\overline{\Lambda}_+^{\downarrow} = -\mathbb{1} = \overline{P}\,\overline{T} = CPT$ transforms a process $a+b \rightarrow c+d$ into the process $\overline{d}+\overline{c} \rightarrow \overline{b}+\overline{a}$, without any change in the velocities; and the chain of processes $a+b \rightarrow c+d \rightarrow e+f$ into the chain $\overline{f}+\overline{e} \rightarrow \overline{d}+\overline{c} \rightarrow \overline{b}+\overline{a}$. See also Ref. (3).

(x) Let us mention that such a reinterpretation procedure has been shown in ref. (3) to be equivalent to the application of the chirality operation γ_5.

(o) Notice that, under the "Stong Reflection" (or "Total Inversion"), the 3-velocity does not change sign. For the explication of the reason why on the contrary the 3-momentum does change sign, see refs. (3, 1).

3. - THE PASSIVE POINT OF VIEW. THE "DUAL" FRAMES.

Let us pass now to the passive point of view. Eqs. (7), (7") and what procedes tell us that any $\bar{\Lambda}_+^{\downarrow}$ acts kinematically so as its dual transformation $\bar{\Lambda}_+^{\uparrow}$ defined via eq. (7), but with the important difference that the new frame: (i) is made of antimatter, instead of matter (due to the action of C); (ii) has the opposite space-parity (due to the action of P); (iii) has chosen as positive the opposite time-direction (due to the action of T). By definition, we shall call such a frame $\tilde{f}$, got from the initial frame by action of $\bar{\Lambda}_+^{\downarrow}$, the dual of the frame f got from the initial frame by action of the "dual" transformation $\bar{\Lambda}_+^{\uparrow}$ (see eq. (7)).

By the elements of the group $\mathscr{L}_+$, therefore, we are able to effect transition from a given frame f(M, R, +t), - chosen e. g. to be made of matter M, and to possess right space-parity R and the standard direction +t for the time-axis, - not only to all possible frames f'(M, R, +t) in relative straight motion, but also to all possible dual frames $\tilde{f}'(\bar{M}, L, -t)$ in relative straight motion (where the dual frames $\tilde{f}'$ result to be made of antimatter $\bar{M}$, to possess left space-parity L, and to use as conventionally positive the opposite time-direction - t).

Relativistic physical laws will appear the same (i. e. covariant) both from inertial frames f and from the dual (inertial) frames $\tilde{f}$, as implied - incidentelly - by the CPT ≡ ≡ $\bar{P}\bar{T}$ covariance when interpreted in the passive sense. Actually, due to the covariance of the relativistic laws under $\bar{P}\bar{T}$ ≡ CPT, two observers that exchange only "informations"(x) and no physical objects, cannot teach each other how to build up two frames of the same type (e. g., f_1 and f_2); in the sense that the second frame - just following the instructions of the first frame f_1 - could build up a ("dual") frame $\tilde{f}_2$ without realizing any difference. In other words, it is not possible - making reference to the relativistic physical laws - to teach how to use the same conventions in assigning the names of matter/antimatter, right/ /left space parity, and positive/negative time-direction.

It is interesting that, if an observer f sees the chain of processes $a+b \to c+d \to e+f$, then tha dual observer $\tilde{f}$ would see the "inverted" chain of processes $\bar{f}+\bar{e} \to \bar{d}+\bar{c} \to \bar{b}+\bar{a}$.

In the present case - differently from the usual ones - it is the active point of view to be surely realized in nature, whilst a priori frames realizing the passive point of view might not to exist. However, if we like to have both points of view coinciding, then it seems necessary to assume the actual existence even of the dual frames $\tilde{f}(\bar{M}, L, -t)$.

(x) Regarding (relativistic) micro-physical processes, at least.

4. - AND IN THERMODYNAMICS ?

When passing from elementary objects to large collections of particles, the facts mentioned at the end of Sects. 2 and 3 arise some interesting issues.

Let us first of all suppose that also thermodynamics can be aventually written down in $\mathscr{L}_+$-covariant form. We expect such a Relativistic Thermodynamics to state that every observer - within an isolated system made of the same kind of matter (i. e., any f inside a "world" of matter, and any $\tilde{f}$ inside a "world" of antimatter) - would see his world to evolve thermodynamically along his own positive time-direction.

A possible conclusion is that (from the passive point of view) relativistic thermodynamics, - if it assigns a certain time-arrow to the physical evolution of a macroscopic isolated system (e. g. a cosmos) made of matter, when described by an observer made of matter as well, - is required apparently by Relativity to assign it the opposite time-arrow when it is described by an observer made of antimatter.

In other words (going back to the active point of view), this would mean that relativistic thermodynamics is required by Relativity to assign a certain time-arrow to isolated macroscopic systems (e. g. a whole cosmos) made of matter, and the opposite time-arrow to macro-systems made of antimatter, when both are described by one and the same observer.

A different conclusion, however, derives from another starting assumption. We can expect the ($\mathscr{L}_+$-covariant) Relativistic Thermodynamics to imply that we would observe the same, ordinary evolution (e. g., $A \to B \to C$; $\bar{A} \to \bar{B} \to \bar{C}$) both in a world W of matter and in a world $\overline{W}$ of antimatter. Then, any CPT-ed observer would see(x) the opposite evolution ($\bar{C} \to \bar{B} \to \bar{A}$; $C \to B \to A$) both in $\overline{W}$ and W. But of course all observers (the initial and the CPT-ed ones) must - and will - see the same relativistic, $\mathscr{L}_+$-covariant laws to be effective in W and $\overline{W}$.

This discussion is here incomplete, since we neglected the essential question of the initial conditions (cf. e. g. the second and third of Refs. (3)).

In any case, even if observers $\tilde{f}$ should not exist, what shown above has its intrinsic validity in exploiting the symmetries imposed on nature by the theory of Relativity.

(x) In order to be actually able to see the world, the CPT-ed observer should apply the Wheeler-Feynman procedure the other way round, so to make eventually recourse to those photons which were considered as "advanced" by the initial observers (cf. e. g. the third and second one of Refs. (3)).

APPENDIX A.

One way to define the passive and active aspects of a group of symmetry transformations is this.

Following Refs. (4), let us consider a set {ph} of phenomena ph, one observer O and a theory T developed by O to describe the set {ph}. Moreover, let be $\psi_A \in \{\psi_A\}$, $(A = 1, 2, \ldots, N)$, the mathematical objects which the theory T deals with.

By definition, a kinematically possible trajectory (kpt) is any choice of a set of possible values $\{\psi_A\}$ for the N quantities ψ_A. However, not any kpt will be actually realizable within the set {ph}; we shall call dynamically possible trajectories (dpt) the kpt's corresponding to real phenomena ph's, i. e. such that the physical laws of theory T are satisfied:

$$f_i(\{\overline{\psi}_A\}, \partial_\mu^{(p)}\{\overline{\psi}_A\}, \ldots, \partial^{(p)}_{\mu\ldots\nu}\{\overline{\psi}_A\}, x) = 0 \qquad (i = 1, 2, \ldots, M) \tag{A.1}$$

Our form (A.1) for the physical laws, even if not general, holds however for all local theories T which use the space-time framework as basic background. In such theories, space-time is usually supposed to be a differentiable manifold V_4, and quantities ψ_A are geometrical objects defined on V_4 (4, 5).

Now, let G be a group. We shall say that the theory T is G-covariant if and only if:

(a) each $\psi_A \in \{\psi_A\}$ is the basis of a faithful realization of G;

(b) each such realization does associate to any dpt other dpt's.

If space-time is a differentiable manifold, then it is natural to assume the manifold mapping group (MMG) to be a covariance group (as above defined) of theory T. Then, under a transformation $x' = \Lambda x$ belonging to MMG, if $d \equiv \{\psi_A(x)\}$ is a dpt, we have that also $d' \equiv \{\overline{\psi}'_A(x')\}$ is a dpt, where $\psi'_A(x') = F(\psi_A(x), x'(x))$; it follows that

$$f_i(\{\overline{\psi}'_A\}, \partial_\mu^{(1)}\{\overline{\psi}'_A\}, \ldots\ldots, \partial^{(p)}_{\mu\ldots\nu}\{\overline{\psi}'_A\}, x') = 0 , \tag{A.2}$$

and we shall say that the physical laws (motion equations) of theory T are covariant under MMG.

Any set $d \equiv \{\overline{\psi}_A(x)\}$ satisfying eq. (A.1) is, as known, the dpt characterizing a particular $ph \in \{ph\}$, as described by the observer O when using a local chart $\{x(e)\}$, where e is the generic world-event: $e \in V_4$.

Now, the interpretation of the sets $d' \equiv \{\overline{\psi}'_A(x')\}$ satisfying eq. (A.2), is multiple, depending on the so called passive and active points of view.

Passive point of view. According to it (Fig. 1), the set $d' \equiv \{\overline{\psi}'_A(x')\}$ is the dpt of the same phenomenon ph as described by the new observer O' that uses the transformed

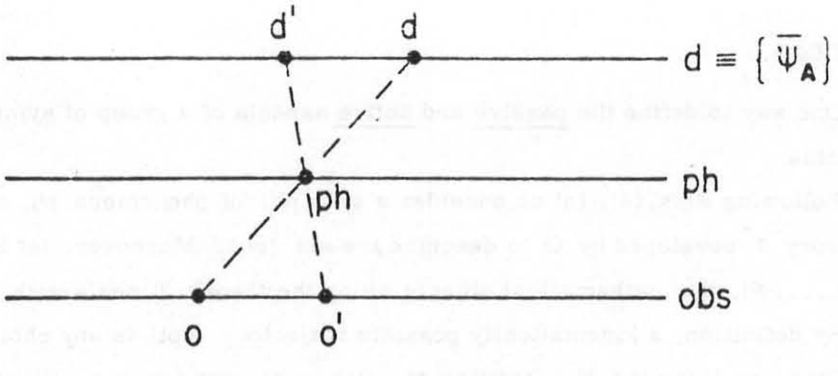

FIG. 1 - The passive point of view. See the text and the last one of Refs. (4).
It is : $ph' \equiv ph; \quad O' = \Lambda O \Rightarrow d' = \Lambda d.$

local chart $\{x'(e)\}$. In this case, the transformation acts on the reference frames. However, $\{\overline{\Psi}'_A(x')\}$ satisfies eq. (A.2), but with boundary conditions different from those of eq. (A.1).

Intermediate point of view. According to it (Fig. 2), the set $d' = \{\overline{\Psi}'_A(x')\}$ is the dpt of a new phenomenon ph' which is described by O', that uses the local chart $\{x'(e)\}$, so as ph was described by O. In other words, the descriptions $d \equiv \{\overline{\Psi}_A(x)\}$ and $d' \equiv \{\overline{\Psi}'_A(x')\}$ do coincide.

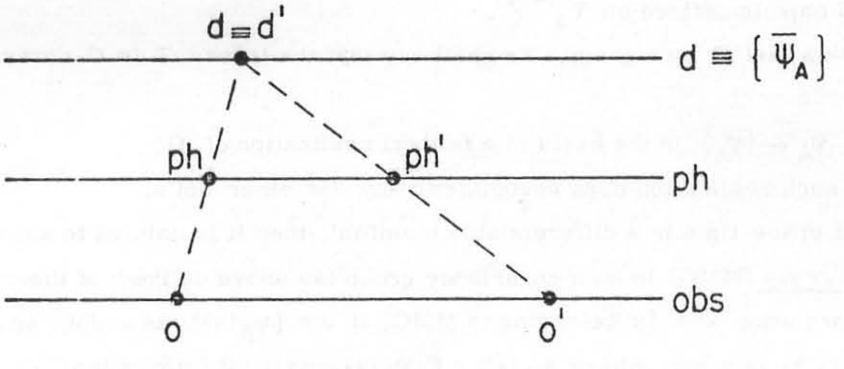

FIG. 2 - The intermediate point of view. It is :
$ph' = \Lambda ph; \quad d \equiv d' \Rightarrow O' = \Lambda O.$

Active point of view. According to it (Fig. 3), the set $\{\overline{\Psi}'_A(x')\}$ is first of all transformed into the set $\{\overline{\Psi}'_A(x)\}$ by the known transformation $x \rightarrow x'$. Then, the set $d_2 \equiv \{\overline{\Psi}'_A(x)\}$ is interpreted as the dpt of a new phenomenon ph' (the transformed phenomenon) as still described by the same observer O which used the local chart $\{e(x)\}$. In this case, the transformation acts on the phenomena. The set here considered, $d_2 \equiv \{\overline{\Psi}'_A(x)\}$, satisfies eq. (A.1), however with boundary conditions different from those for $\{\overline{\Psi}_A(x)\}$.

It is possible to divide the class of all the dpt's in disjoint sets C of equivalent dpt's. Two dpt's will be said to be equivalent when one can be transformed into the other by an ele

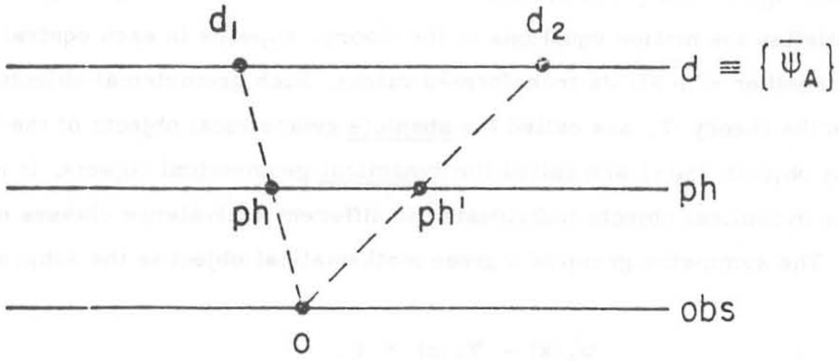

FIG. 3 - The active point of view. It is:

$$O_2 \equiv O_1 ; \qquad ph' = \Lambda ph \Rightarrow d_2 = \Lambda d_1 .$$

ment Λ of the covariance group MMG, according to either the passive or active points of view.

It is obvious that, according to the passive point of view, all dpt's belonging to a given "equivalence class" C_p are descriptions of one and the same phenomenon ph as seen by different observers transformed one into the other by the MMG (see Fig. 1).

According to the active point of view, on the contrary, the dpt's of a given equivalence class C_a represent the descriptions by one and the same observer O of different phenomena, transformed one into the other by the MMG (see Fig. 3).

<u>The passive and active points of view will be equivalent, by definition, if the union of all C_p's is equal to the union of all C_a's.</u> In our case, the passive and active points of view are equivalent, since the set $\mathscr{D}$ of the descriptions $\{d, d', \dots\}_p$ is equal to the set $\tilde{\mathscr{D}}$ of the descriptions $\{d_1, d_2, \dots\}_a$, because

$$\begin{aligned} \mathscr{D} &: \; d' = \Lambda^{-1} d , \qquad && \forall \Lambda \in G ; \\ \tilde{\mathscr{D}} &: \; d_2 = \tilde{\Lambda}^{-1} d_1 , \qquad && \forall \tilde{\Lambda} \in G , \end{aligned} \tag{A.3}$$

provided that the phenomenon ph of Fig. 1 belongs to the class of phenomena in Fig. 3, and that the observer O in Fig. 3 belongs to the class of observers in Fig. 1.

* * * * *

Let us now add some considerations on the concept of "symmetry group" (or "<u>invariance</u> group") of a theory which is G-covariant. To this aim, let us divide our set $\{\psi_A\}$ of geometrical objects in two pieces:

$$\{\psi_A(x)\} = \{A_\alpha(x)\} \cup \{D_\beta(x)\} , \qquad (\alpha + \beta = N) ,$$

such that $A_\alpha(x)$ satisfy the condition (i) above and the following: Any value $\bar{A}_\alpha(x)$ of $A_\alpha(x)$, that satisfies the motion equations of the theory, appears in each equivalence class C of the dpt's, together with all its transformed values. Such geometrical objects $A_\alpha(x)$, if they exist in the theory T, are called the absolute geometrical objects of the theory. The remaining objects $D_\beta(x)$ are called the dynamical geometrical objects. It is interesting that only the dynamical objects individuate the different equivalence classes of dpt's.

The symmetry group of a given mathematical object is the subgroup of G = MMG such that

$$\psi'_A(x) - \psi_A(x) = 0 .$$

The symmetry group of a theory T that is G-covariant is then the largest subgroup of G which is simultaneously the symmetry group of all the absolute objects of T.

Let us recall that, e. g. , the absolute objects of Special Relativity are the metric tensor $\eta_{\mu\nu}$ and the Casimir invariants of the Poincarè group, m_o^2 and $m_o^2 s(s+1)$.

Usually it is called covariance group what we called here symmetry (or invariance) group. In Special Relativity, when one restricts himself only to inertial frames, the two groups coincide.

From what precedes we see that any theory T can be formalized so to be covariant under the manifold mapping group (i. e. , under all the C^∞-transformations). However, in general this is got only at the price of introducing many absolute objects which hardly describe physical phenomena (this is the case, e. g. , of Newton theory when written down in General Covariant way). It is well-known, on the contrary, that General Relativity has the exceptional character of possessing no absolute objects. As a consequence, even for General Relativity the covariance and invariance groups coincide.

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