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CONFINEMENT AND HADRON-HADRON INTERACTIONS BY GENERAL RELATIVISTIC METHODS

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ABSTRACT.

By postulating covariance of physical laws under global dilations, one can describe gravitational and strong interactions in a unified way. Namely, in terms of the new discrete dilational degree of freedom, our cosmos and hadrons can be regarded as finite, similar systems. And a discrete hierarchy of finite "universes" may be defined, which are governed by fields with strengths inversally proportional to their radii; in each universe an Equivalence Principle holds, so that the relevant field can be there geometrized.

Scaled-down Einstein equations - with cosmological term - are assumed to hold inside hadrons (= strong micro-cosmoses); and they yield in a natural way classical confinement, as well as "asymptotic freedom", of the hadron constituents. In other words, the association of strong micro-universes of Friedmann type with hadrons (i. e., applying the methods of General Relativity to subnuclear particle physics) allows avoiding recourse to phenomenological models so as the Bag Model. Inside hadrons we have to deal with a tensorial field (= strong gravity), and hadron constituents are supposed to exchange spin-2 "gluons".

Our approach allows us also to write down a tensorial, bi-scale field theory of hadron-hadron interactions, based on modified Einstein-type equations here proposed for strong interactions in our space. We obtain in particular: (i) the correct Yukawa behaviour of the strong scalar potential at the static limit and for $r \gtrsim 1$ fm; (ii) the value of hadron radii.

As a byproduct, we derive a whole "numerology", connecting our gravitational cosmos with the strong micro-cosmoses (hadrons), such that it does imply no variation of G with the epoch.

Finally, since a structure of the "micro-universe" type seems to be characteristic even of leptons, a hope for the future is including also weak interactions in our classical unification of the fundamental forces.

PART A : HEURISTICS

1. - Introduction

We purpose to check how far one can go in describing hadron structure and strong interactions by making recourse to the classical methods of General Relativity. In so doing, we shall try to realize the old idea by Riemann (and later Clifford) that the very appearance of matter-particles is due to a strong local curvature of space. In other words, we shall attempt a unified classical approach to gravitational and strong interactions, the latter too considered to be tensorial.

One of our immediate aims refers to the following context. The standard theory of strong interactions (QCD, quantumchromodynamics) is known to be unable to explain the apparent confinement of hadron constituents - the so-called infrared-slavery - so that one has to make recourse to ad-hoc models like the Bag Model in the MIT or SLAC versions. One aim of ours, actually, is avoiding to use phenomenological models, getting on the contrary the confinement by the association of suitable "strong micro-universes"(1) of Friedmann type with hadrons.

This theory is due to P. Caldirola, P. Castorina, G. D. Maccarrone, M. Pavšič, besides the present author, so as it appears from Refs. (2, 3, 4, 5).

Our results are ofter similar to those ones of the "strong gravity theory" by Abdus Salam and coworkers(6), even if the starting points are very different.

2. - Hadrons as Micro-Universes

Our starting point is the empiric observation that the ratio R/r between the Hubble radius $R \simeq 10^{26}$ m of our cosmos (gravitational cosmos) and the characteristic radius $r \simeq \simeq 10^{-15}$ m of subnuclear particles is roughly equal to the ratio S/s between the strength S of the nuclear field and the strength s of the gravitational field. For details see Refs. (5) and (4). This suggests the existence of a possible similarity between macro-cosmos and hadrons (conceived as strong micro-universes). To fix our ideas, let us assume for a moment the naive model of "Newtonian balls" in 3-dimensional space for both hadrons and cosmos; later on we shall adopt more sensible models, of Friedmann type.

We shall therefore assume cosmos and hadrons - both regarded as finite objects - to be systems similar in a geometric-physical sense: That is to say, to be systems governed by laws similar and differing only for a global scale-transformation which carries R into r and the gravitational field into the strong one(2-5). Let us recall, at this point, that the symmetries of the most important classical equations have not been fully exploited by the ordinary relativistic theories. In fact, Maxwell equations are covariant also under conformal transformations and, in particular, under dilations (when charges are present, such covariance holds provided that even electric charges are suitably scaled). Moreover, also Einstein gravitational equations are covariant under dilations (provided that, when matter and cosmological term are present, even masses and cosmological constant are scaled on the basis of correct dimensional considerations). On the other hand, covariance under global scale transformations is equivalent to nothing but covariance under unit transformations, and therefore is an obvious, necessary requirement. We shall take advantage, for instance, of the fact that Einstein Equations do not contain any inbuilt fundamental length, so that they can be related to the space-time of cosmoses with any size.

"Hierarchical" theories have a long story, starting perhaps with Democritus of Abdera. Democritus, basing himself on his Indifference Principle, found no reasons why atoms should have a particular shape or size. As a consequence, he believed even atoms with the size of a cosmos to exist, thus reversing our analogy.

Before going on, let us quote a passage by Einstein himself[7] taken from his last scientific writing, i. e. from his Preface to the volume Cinquant'anni di Relatività[7]. On April 4, 1955, at Princeton, with regard to his last unitary theory of the asymmetric field, Einstein wrote: "... From the field equations one can immediately derive what follows: If $g_{\mu\nu}(x)$ is a solution of the field equations, then also $g_{\mu\nu}(x/\alpha)$ is a solution, where α is a positive constant ('similar solutions'). Let us for instance suppose system $g_{\mu\nu}$ to represent a finite-sized crystal embedded in a flat space. We could then have a second "universe" with another crystal, exactly similar to the previous one, but dilated to have its linear sizes α times as big. As far as we confine ourselves to a universe containing nothing but a unique crystal, we do not meet any difficulties. We realise only that the size of such a crystal ("standard of length") is not fixed by the field equations ..." (our translation from German).

What precedes leads us to require explicitely that physical laws are covariant also under the global (space-time) dilations

$$x'_\mu = \varrho x_\mu , \qquad (\mu = 0, 1, 2, 3) \tag{1}$$

where - however - ϱ is supposed to assume only a set of discrete values[8]. Such an enlargement of the ordinary covariance basic groups by the inclusion (as a first step) also of discrete dilations can be justified even in the following way. When physicists took into due account the electromagnetic phenomena (besides the mechanical ones), it was necessary to give up Galilean Relativity in favour of Einstein's. Since we are now confronted with nuclear and subnuclear phenomena (in particular with strong forces), looking for new, enlarged Relativity theories may be in order. Let us remember, in any case, that within our present approach the gravitational Einstein equations can be used as a basis for deducing cosmological models which describe not only our cosmos, but possibly also other suitably "contracted" micro-cosmoses (or even other

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suitably "dilated" super-cosmoses).

3. - A Hierarchy of "Universes"

By recalling the beginning of the previous section, we evaluate[(3-5)]:

$$s/S \equiv (Gm^2/\hbar c)/(Ng^2/\hbar c) \simeq 10^{-41}; \tag{2a}$$

$$r/R \simeq (10^{-15})/(10^{26}) = 10^{-41}; \tag{2b}$$

and set:

$$\varrho \equiv r/R \equiv s/S \simeq 10^{-41}, \tag{2c}$$

where: (i) G and N are the gravitational and strong universal constants in vacuum, respectively; (ii) quantities m and g represent gravitational charge (= mass) and strong charge (= "strong mass"), respectively[(2-5)], of one and the same hadron; E. g., nucleon, or pion. In order to get evaluations correct in general within a factor 2, we shall adopt the pion as "reference hadron": $m = m_\pi$; $Ng^2/\hbar c \simeq 3 \div 15$. From the dimensional point of view, we have a twofold possibility. If we conventionally choose to put $m = g$, then the "strong universal constant" N becomes

$$N = \varrho^{-1} G \simeq hc/m_\pi^2. \tag{3}$$

On the contrary, if we adopt units such that $[N] = [G]$ and moreover $N = G = 1$, then we immediately obtain:

$$g = m/\sqrt{\varrho} \simeq \sqrt{\hbar c/G} = \text{Planck-mass}, \tag{4}$$

which tells us that the Planck-mass $\sqrt{\hbar c/G} \simeq m\sqrt{\varrho^{-1}}$ is nothing but - in suitable units - the strong charge[(2-5)] of the typical hadron. Or rather, when we borrow from experience the information that quarks appear to be the true carriers of the strong charge and we define $g = ng_o$ $(n = 2, 3)$, quantity g_o being the average magnitude of the quark strong charges g_i $(g_i = s_i g_o;\ \Sigma s_i = 0;\ \Sigma g_i = 0;\ g_o = |g_i|)$, eq. (4) happens more precisely to hold for g_o. Consequently, the Planck-mass seems to be nothing but the magnitude of quark strong charge, in suitable units. The known fact that gravitational forces become as strong as the "strong" ones for masses as high as the Planck-mass merely means, in our approach, that the strong-gravity created in the strong-micro-universes (i. e., inside hadrons) by quarks - which possess a strong-mass equal to Planck-mass - is just the nuclear strong field. Such preliminary dimensional consideration allows us moreover to expect that the "small black-holes" predicted by some authors to have the Planck-mass as their mass could be simply identified with hadrons (or rather quarks), that have the Planck-mass as their strong charge - or strong mass - in suitable units.

PART B : THEORY FORMALIZATION

4. - Hadron Structure

Briefly, on the basis of the last two sections, we postulate :

a) Inside our cosmos (gravitational universe), the Einstein equations with attractive[1] cosmological constant Λ :

$$\left[G=1\right] \qquad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - \Lambda g_{\mu\nu} = - \frac{8\pi}{c^4} T_{\mu\nu} . \tag{5}$$

b) Inside hadrons (strong universes), the scaled-down Einstein equations :

$$\left[N=G=1\right] \qquad \tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\varrho}_{\varrho} - H\tilde{g}_{\mu\nu} = - \frac{8}{c^4} S_{\mu\nu} . \tag{6}$$

Simple dimensional considerations (within the present "dilation covariant" relativity) tell us at once that

$$H \equiv \varrho^{-2}\Lambda \simeq 10^{82}\Lambda ; \qquad S_{\mu\nu} \equiv \varrho^{-1} T_{\mu\nu} \simeq 10^{41} T_{\mu\nu} ,$$

so that :

$$\Lambda \simeq 10^{-56} \text{ cm}^{-2} ; \qquad H^{-1} \simeq 10^{-25} \text{ cm}^2 \simeq 0.1 \text{ barn} , \tag{7}$$

and $\hbar\sqrt{2\Lambda}/c \equiv m_G \approx 10^{-68}$ Kg; $\hbar\sqrt{2H}/c \equiv m_S \simeq m_\pi$.

In eq. (5) the sign of Λ was chosen as follows. In QFT language, the vacuum polarization when (only) gravitational fields are present is expected to act so that different vacuum regions attract - very weakly - one another. One can classically reproduce that effect, in a general covariant way, just by choosing an attractive cosmological term. Once Λ is attractive, we are moreover sure to get cosmological models subject to expansion/recontraction cycles.

As a byproduct, our elementary hypotheses yield the systematic derivation of Large Numbers relations of the same type than those ones heuristically uncovered by Weyl, Eddington, Dirac, etc. Our numerology, however, connects the gravitational interactions with the strong ones, and not with the electromagnetic ones (as suggested, on the contrary, by Dirac). For instance, we proved that, if M and m are the cosmos and pion masses, respectively, then :

$$M = \varrho^{-2} m \simeq 10^{54} \text{Kg}; \qquad m = \varrho^2 M \simeq 10^{-28} \text{Kg}.$$

Before going on, let us specify that we are going to adopt Friedmann models both for the cosmos and for hadrons, taking advantage also of the fact that they : (i) are compatible[9] with Mach principle ; and (ii) are embeddable in five dimensions[10]. We shall thus be able to extend to the hadronic universe the Mach principle, in the sense that the inertia of every hadron constituent (parton) will coincide with its strong charge (and not with its gravitationa charge!). In such a way, an Equivalence Principle will hold inside hadrons, justifying the present geometrization of the strong field (first of all inside hadrons and then - as we shall see - even in their surroundings).

5. - Confinement

Let us now find out an exact solution of eqs. (6), for a spherically symmetric distribution g' of strong charge. The geodesic equation in vacuum for a (small) test-costituent g" yields in the radial case

$$[N=1] \qquad d^2r/dt^2 = -\frac{1}{2}c(1-2g'/c^2r + Hr^2/3)(2g'/cr^2 + 2Hr/3) \qquad (8)$$

where g' can be e.g. identified with a quark.

In the case of large values of r, from eq. (8) we get a confining radial force proportional to $-r$ $[N=1\ ;\ [N]=G]$:

$$[r \lesssim 1\ \text{fm}] \qquad F \simeq -g''c^2Hr/3 \propto -r\,; \qquad (9)$$

in other words, by applying the methods of grneral relativity to hadron structure, we got in a natural way a confining radial potential $V \propto r^2$.

Our completely defined (radial) potential for constituent-constituent interaction inside hadrons, eq. (8), would deserve further attention.

In the case of small values of r, if we attribute to g" an angular momentum J relative to g', i.e. if we take account of a "kinetic energy term", with the choice of eq. (3) for the units, we shall have of course to deal with the total potential

$$[r \ll 1\ \text{fm}] \qquad V \simeq (J/g'')^2/r^2 - Ng/r\,, \qquad (10)$$

which simply accounts for the asymptotic freedom of hadron constituents. By extrapolating to the case $|g''| \simeq |g'|$, one would obtain $V \simeq 0$ for $r \simeq 10 \times J^2/(Ng^3)$; moreover, if we attribute a speed $v \simeq c$ to the moving quark considered, then $J \simeq \hbar$ and it results $V \simeq 0$ for $r \simeq 0.01$ fm. Conversely, if we suppose - for instance in the case of baryons, with $N \simeq 10^{40}G$ and $g \equiv m =$ = nucleon-mass - that the "stability radius" r_o is of the order of $r = 0.02\ Nm/c^2 \simeq 0.01$ fm, then we obtain the Regge-type relation

$$J/\hbar \simeq m^2\,,$$

where m is now expressed in GeV/c^2.

The introduction of micro-universes, therefore, allows to avoid making recourse to phenomenological models so as the Bag Model.

6. - In the Surrounding of a Hadron (in our space)

We may regard the spatial parts of cosmos and hadrons (time aside) as embedded in a four-dimensional flat space E^4. Let us choose, for both cosmos and hadrons, Friedmann models whose spatial parts are merely the hypersurface of a 4-dimensional hypersphere. The problem of the strong interactions between two hadrons (e. g., two nucleons) would require using the fiber bundle techniques[(5)]. On an intuitive ground, however, we can solve our problem by considering in E^4 what heuristically we can call the "intersections" of the space-part of hadrons with the space-part of our cosmos : such intersections being 2-dimensional spherical surfaces, that we shall just call "hadrons" tout court in the following (these "intersections" will moreover evolve in time, e. g. by undergoing expansion/recontraction cycles which take $\Delta t = \varrho 10^{18}\,s \simeq 10^{-23}\,s$ if $10^{18}\,s$ is the time taken by our cosmos : For this reason in our theory G does not depend on the epoch. Cf. Ref. (4)). Our present aim is describing the strong interaction between the abovementioned "intersections". For the surroundings of a hadron, in our space, we need therefore introducing a bi-scale theory for studying the motion of a hadronic test particle which possesses charges both gravitational and strong. In other words, we must modify the Einstein gravitational equations by introducing, in the micro-neighbourhood of the abovementioned "hadrons", a strong metric-deformation $s_{\mu\nu}$ which affects only the objects with strong charge (i. e. with scale-factor $k = \varrho \simeq 10^{-41}$) and not the objects with gravitational-charge only (i. e. with scale-factor $k = 1$). The simplest choice for our two tensorial fields $f_{\mu\nu}$ (the ordinary, gravitational one) and $s_{\mu\nu}$ (the new, strong one) is

$$g_{\mu\nu} = f_{\mu\nu} + s_{\mu\nu} , \tag{11a}$$

where $g_{\mu\nu}$ is the total metric tensor. The components of the strong metric tensor $s_{\mu\nu}$ must vanish for $r \gg 1$ fm. Far from a hadron, $g_{\mu\nu} = f_{\mu\nu}$ and all particles - both with and without strong charge - will feel only the gravitational metric tensor $f_{\mu\nu}$. In the surroundings of a hadron, on the contrary, the non-hadronic particles will go on feeling only $f_{\mu\nu}$, whilst the hadronic particles will feel both $f_{\mu\nu}$ and $s_{\mu\nu}$ (in the last case, one will disregard $f_{\mu\nu}$ with respect to $s_{\mu\nu}$, of course). Any "hadron", as an object belonging to our cosmos, possesses also a gravitational charge m ; but it is so small that - in suitable coordinates - we can assume

$$f_{\mu\nu} \simeq \eta_{\mu\nu} ,$$

so that eq. (11a) writes

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu} ; \quad s_{\mu\nu} \simeq g_{\mu\nu} - \eta_{\mu\nu} ; \tag{11b}$$

since $\eta_{\mu\nu}$ is not a tensor, in eq. (11b) we lost the general covariance, in the sense that eq. (11b) holds now in suitable coordinates.

If we confine ourselves to the relevant problem of the motion in the surrounding of a hadron of a test particle endowed with charges both gravitational and strong, the simplest field equations - with our two tensorial fields - would be (as proposed in Ref. (3)):

$$R_{\mu\nu} + \Lambda g_{\mu\nu} + Hs_{\mu\nu} = -\frac{8\pi}{c^4}(S_{\mu\nu} + GT_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho} - \frac{1}{2} g_{\mu\nu} GT^{\varrho}_{\varrho}),$$

where $S_{\mu\nu} = NT_{\mu\nu}$; $N = \varrho^{-1}G$. By disregarding the terms negligible, we end up with:

$$\left[s_{\mu\nu} \simeq g_{\mu\nu} - \eta_{\mu\nu}\right] \qquad R_{\mu\nu} + Hs_{\mu\nu} = -\frac{8\pi}{c^4}(S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho}), \tag{12}$$

which are our new field equations. Notice that in eqs. (12) the "strong cosmological term" with the hadronic constant H takes account of the geometric properties of the strong field in the neibourhood of the "source hadron": For instance, if we had $Hg_{\mu\nu}$ instead of $Hs_{\mu\nu}$, then eqs. (12) would reduce to eqs. (6), and our space would shrink down to hadronic size; as we shall see, the term $Hs_{\mu\nu}$ on the contrary acts in such a way that only $s_{\mu\nu}$ is correctly confined within a distance of the order of a few fm from the considered hadron.

As usual, eqs. (12) can read

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - Hg_{\mu\nu} = -\frac{8\pi}{c^4} S'_{\mu\nu};$$

$$S'_{\mu} \equiv S_{\mu} - c^4 H(\eta_{\mu} - \frac{1}{2} g^{\alpha\beta} \eta_{\alpha\beta} g_{\mu\nu})/8\pi, \tag{13}$$

where the last addendum has the meaning of interference term between the two tensorial fields.

In the weak field approximation, we can linearize with respect to the flat metric; from eqs. (12) then the following eqs. can be derived:

$$\left[r \gtrsim 1\ \text{fm}\right] \qquad \partial^{\mu}\partial_{\mu} s_{\alpha\beta} + 2Hs_{\alpha\beta} \simeq \frac{16\pi}{c^4}(S_{\alpha\beta} - \eta_{\alpha\beta} S^{\varrho}_{\varrho}); \tag{14}$$

such equations (Lorentz covariant, and with "(cosmological) hadronic term") refer to a massive tensorial field.

Notice that as strong-field tensor it should be taken not exactly $s_{\mu\nu}$ but quantity Φ defined as follows

$$\Phi_{\mu\nu}/g' \equiv \frac{1}{2} s_{\mu\nu} - \frac{1}{2}(g_{\mu\nu} - \eta_{\mu\nu}).$$

Let us add a comment. At the beginning of this section we had no idea about the structure of $s_{\mu\nu}$ and of $\Phi_{\mu\nu}$; our considerations expressed by eqs. (11a), (11b) have been enough - however - to completely define our new field-equations (12), that we rewrite here as follows

$$R_{\mu\nu} + H(g_{\mu\nu} - \eta_{\mu\nu}) \simeq - \frac{8\pi}{c^4}(S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho}) . \tag{12'}$$

Moreover, the "external" equations (12)-(12') have been derived independently of the "internal" equations (6). A link is however represented by the following relations, which are to be associated to eqs. (12)-(12'):

$$H \equiv \varrho^{-2} \Lambda \; ; \qquad N \equiv \varrho^{-1} G \; ; \qquad S_{\mu\nu} \equiv N T_{\mu\nu} . \tag{12''}$$

7. - Yukawa Behaviour

Let us go back to the linearized equations (14), derived in the weak field approximation, which corresponds - as already mentioned - to a massive tensorial field(1). At the static limit, eqs. (14) yield for the scalar potential $V \equiv \frac{1}{2} c^2 s_{oo}$ the equation

$$s_{oo} - 2 H s_{oo} \simeq 8 \pi N \gamma / c^4 ,$$

quantity $N\gamma$ being the strong-charge magnitude density associated with the considered hadron (and γ being the ordinary mass density associated to the hadron). A spherically symmetric solution of the last equation, for a point-like particle at rest at the origin, endowed with "strong charge" g(2-5), is finally:

$$\left[g_{oo} = 1 + s_{oo} \right] \qquad 2V/c^2 \equiv s_{oo} = - \frac{2g}{c^2 r} \exp\left[- r \sqrt{2H} \right] . \tag{15}$$

By identifying $\sqrt{2H} \equiv m_S c/\hbar$, we find at once for the field mass the value

$$m_S = \hbar \sqrt{2H}/c \simeq m_\pi .$$

In the nucleon case, it is $Ng^2/\hbar c \simeq 15$, corresponding to the $pp\pi$ coupling-constant square (see eqs. (2)). In conclusion, for "weak" field and test-particle low speeds, we eventually - and actually - obtained a scalar field with the correct Yukawa behaviour

$$V_{ext} \equiv V = - \frac{g}{r} \exp\left[- r m_\pi c/\hbar \right] . \tag{15'}$$

8. - A New Schwartzschild Problem. Strong Black-holes ?

Let us now turn to investigate the Schwartzschild-type problem for our new field equations (12)-(12'), in our space. Notice again explicitely that eqs. (12) are new field-equations, very different from Einstein equations. They refer to two tensorial fields ("bi-scale" theory), even if they were written in a simplified form to be valid in the neibourhood of a hadron for a hadronic test-particle. In such a form (12), it is $f_{\mu\nu} \simeq \eta_{\mu\nu}$ and the essential field becomes $s_{\mu\nu}$.

We want now to solve the spherically symmetric problem in connection with our new eqs. (12). In other words, we look now for "Black-Hole-type" solutions (in our space) associated to the strong-gravity field $s_{\mu\nu} \equiv 2\Phi_{\mu\nu}/g'$.

In our space, therefore, let us associate with ordinary "hadrons" spherically symmetric sources of the strong-gravity field. However, we cannot take advantage of the linearized eqs. (14) since we are no more in the weak field approximation. Eqs. (13) may be written

$$R_{\mu\nu} = -\frac{8\pi}{c^4}(\overline{S}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\overline{S}^{\varrho}_{\varrho}) ; \qquad \overline{S}_{\mu\nu} \equiv S_{\mu\nu} + t_{\mu\nu} ;$$
$$t_{\mu\nu} \equiv -c^4 H (g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g^{\alpha\beta}\eta_{\alpha\beta} g_{\mu\nu})/8\pi , \tag{16}$$

with $|t_{\mu\nu}| \ll 1$ for $r \gg 1$ fm. Let us recall that - as well known - eqs. (12)-(12') can be put in the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - H(g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g^{\alpha\beta}\eta_{\alpha\beta} g_{\mu\nu}) = -\frac{8\pi}{c^4} S_{\mu\nu} . \tag{13'}$$

For simplicity, let us reduce ourselves to the static limit. Due to the spherical symmetry, it will be in eqs. (16)

$$t_{oo} = \frac{1}{2}(g_{oo} - 1) \cdot u(r) ,$$

where the structure of our eqs. (16) suggests for the function u(r), in analogy to what one does in the electromagnetic field case (formal analogy)[5,11]:

$$u(r) = \left[|\vec{\nabla}\Phi|^2 + \tilde{\mu}^2 |\Phi|^2 \right] / 8\pi g'^2 = \left[|\vec{\nabla}(g_{oo} - 1)|^2 + \tilde{\mu}^2 |g_{oo} - 1|^2 \right] / 32\pi , \tag{17}$$

with $\Phi \equiv \Phi_{oo}$; $\tilde{\mu} \equiv m_S c/\hbar \simeq m_\pi c/\hbar$. At this point we can adopt an iterative procedure; for the first iteration, we can use the expression given by eq. (15): $\frac{1}{2}(g_{oo} - 1) \simeq -(g/r)\cdot \exp[-\tilde{\mu} r]$, then obtaining $u(r) \simeq (1/8\pi r^2)\cdot \exp[2\tilde{\mu} r] \cdot (r^{-2} + 2r^{-1} + 2\tilde{\mu}^2)$. Now, if we write down the "strong" metric as follows

$$ds^2 = \exp[\nu(r)]\, c^2 dt^2 - \exp[\lambda(r)]\, dr^2 - r^2(d\theta^2 + \sin^2\theta \cdot d\Phi^2) ,$$

our procedure yields eventually the equation (among the others):

$$-8\pi g^2 u(r)/m'^2 c^2 \simeq \exp[-\lambda(r)] \cdot (r^{-2} - \frac{1}{2} d\lambda(r)/dr) - r^{-2} , \tag{18}$$

where $Ng^2/\hbar c \simeq 15$, and m' is the hadronic test-particle mass (we can e. g. choose m' =

$= m_q$ = quark average mass, the "test-quark" being a priori considered as initially outside the possible horizon). The exact solution of eq. (18) is[11, 5]:

$$\exp\left[-\lambda(r)\right] = 1 - 2\ell/r + (\tilde{\mu}k/r + k/r^2)\cdot\exp\left[-2\tilde{\mu}r\right],$$

where $k \equiv g^4/c^4m'^2$ and where $\ell = g^2m/c^2m'^2$ is an integration constant, quantity m being the mass of the considered hadron (e. g. , of the nucleon).

The strong Schwartzschild-type horizon-radius will be individuated by the equations

$$\exp\left[-\lambda(r)\right] = 0 \;; \qquad \exp\left[\nu(r)\right] = 0 . \tag{19}$$

The first eq. (19), for the strong Schwartzschild-type radius, yields approximately the relation

$$r_s^2 - 2\ell r_s + k \simeq 0 ,$$

that is to say:

$$r_s \simeq \ell \pm \sqrt{\ell^2 - k} . \tag{20}$$

In the case of nucleons, eq. (20) gives e. g. the values

$$r_1 \simeq 10^{-15}\ \text{cm} \;; \qquad r_2 \simeq 0.8\ \text{fm} ,$$

where the second value is in good agreement with the radius shown by nucleons in strong interactions (whilst many alternative interpretations might be suggested for the first value). We have still to verify, however, that also the second eq. (19) is verified for the same r_s values.

Such a task is computationally hard; one can however approximately check that in the present case it is actually[11]

$$\exp\left[\nu(r)\right] \approx 1/\exp\left[\lambda(r)\right] \approx 1 - 2g^2m/rc^2m'^2 .$$

As a consequence, ordinary hadrons (in our space) can be probably associated with the strong black hole-type solutions of our new field equations (12). Such solutions, however, have nothing to do with the ordinary Black Holes since eqs. (12) are very different from Einstein equations. Let us recall, e. g. , that - in the surroundings of the "strong black-holes" found from our eqs. (12) - at the static limit and for intermediate distances the "strong" scalar potential has Yukawa behaviour. Further details can be found in Refs. (2-5).

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