ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Catania

INFN/AE-82/3
14 Giugno 1982

A. O. Barut, G. D. Maccarrone and E. Recami:

ON THE SHAPE OF TACHYONS.

N° 100 copie al:
Dr. E. RECAMI
CATANIA

Servizio Documentazione
dei Laboratori Nazionali di Frascati

Istituto Nazionale di Fisica Nucleare
Sezione di Catania

INFN/AE-82/3
14 Giugno 1982

ON THE SHAPE OF TACHYONS[x]

A.O. Barut
Department of Physics, University of Colorado, Boulder, Co. 80309 (USA)

G.D. Maccarrone and E. Recami
INFN, Sezione di Catania, Istituto di Fisica Teorica dell'Università di Catania, and
C.S.F.N. e S.d.M., Catania

ABSTRACT

We study some aspects of the experimental behaviour of tachyons, in particular by finding out their apparent shape. A Superluminal particle, which in its own rest-frame is spherical or ellipsoidal (and with an infinite life-time), would appear to a laboratory frame as occupying the whole region of space bound by a double cone and a two-sheeted hyperboloid. Such a structure (the tachyon "shape") rigidly travels with the speed of the tachyon. However, if the Superluminal particle has a finite life-time in its rest-frame, then in the laboratory frame it gets a finite space-extension. As a by-product, we are able to interpret physically the imaginary units entering - as wellknown - the transversal coordinates in the Superluminal Lorentz transformations. The various particular or limiting cases of the tachyon shape are thoroughly considered. Finally, some brief considerations concerning possible experiments to look for tachyons are added.

(x) - Work partially supported by M.P.I. and C.N.R.

1. - INTRODUCTION

Tachyons (or space-like states) are already known to exist as "internal states". Can they also exist as asymptotically free states? Here we shall address ourselves to this latter possibility.

In connection with the rôle of tachyons as intermediate states or exchanged objects, let us recall the following. If we consider a tachyon T emitted by body A and absorbed by body B, it is wellknown that suitable (subluminal) observers exist seeing T to have infinite speed, i.e. deeming A, B to be connected by a simultaneous, symmetric interaction[1]. Other observers, moreover, exist which describe that process as the exchange of an antitachyon $\overline{T}$ emitted by B and absorbed by A. Therefore, tachyons are actually quite fit to be the carriers of two-body *mutual* interactions[1,2]. Let us add that in the first one of Refs. (1) we showed that - when describing the elementary processes happening at A and at B during the tachyon exchange in the A and B rest-frame, respectively - one meets all the *four* kinematical possibilities: (a) "intrinsic emission" at A and "intrinsic absorption" at B; (b) "intrinsic absorption" at A and "intrinsic emission" at B; (c) "intrinsic emission" both at A and at B; (d) "intrinsic absorption" both at A and at B; where the last two situations are kinematically possible only in the case of tachyon exchange.

Let us also recall that it appeared convenient always to consider (in each frame) the tachyons together with their own source and detector.

For instance, from the classical point of view, typical tachyon sources are expected to be the black-holes, in the sense that only tachyons can be classically emitted by black-holes. By the "Reinterpretation procedure"[3] of *Extended Relativity*[1,3,4] it then follows that black-holes must also be suitable tachyon-absorbers. As a consequence, tachyonic matter should be possibly exchanged between black-holes[5,6], - where we mean a priori both gravitational black-holes and "strong black-holes" (= hadrons).

However, in this paper we want to deal - as already mentioned - with the problem how *free* tachyons would look like and how they are expected to behave experimentally.

That such a problem does deserve a careful investigation is suggested even by simple, preliminary considerations of the kind of the two following ones:

(i) Free bradyons always admit a particular class of subluminal reference-frames (the rest-frames) wherefrom they appear - in Minkowski space - as "points" in space extended in time along a line. On the contrary, free tachyons always admit a particular class of subluminal (with respect to us) reference-frames wherefrom thay appear with divergent speed ($V = \infty$), i.e. as "points" in time extended in space along a line[1]. Considerations of this kind correspond to the fact that the little groups of the time-like and space-like representations of the Poincaré group are SO(3) and SO(2,1), respectively[7].

(ii) When tachyons are seen by us by means of their electromagnetic emissions[8], they will generally appear as occupying *two positions* at the same time. Let us start by considering a macro-object (emitting spherical electromagnetic waves). When we see it travelling with Superluminal, constant velocity $\vec{V}$, because of the distortion due to the large relative speed $|\vec{V}| > c$ we shall observe the electromagnetic waves to be internally tangent to an enveloping (double) cone Γ having as axis the motion-line of body C (this cone has nothing to do with Cherenkov's ; see e.g. Ref. (9)). This is analogous to what happens with an airplane moving at a constant, supersonic speed in the air. A first observation is the following one. As we hear a sonic boom when we meet the initial sound-contact with the supersonic airplane, so we shall analogously see an optic boom when we first enter in radio-contact with body C, i.e. when we meet the Γ-cone surface. In fact, when C is seen by us under the angle α such that (see Fig. 1a):

$$V \cos \alpha = c \,, \tag{1}$$

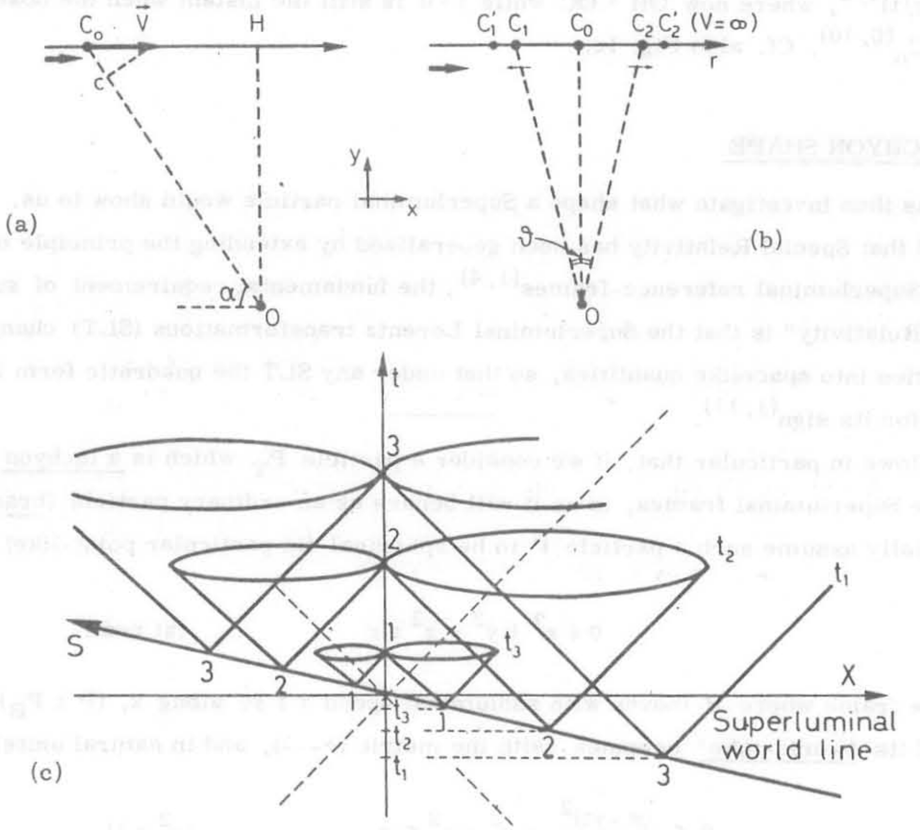


FIG. 1 - (a) When a source C of electromagnetic radiation approaches at constant Superluminal speed V, an "optic boom" will be seen by any observer O as soon as he enters in radio-contact with C, i.e. for $V \cos\alpha = c$. The latter condition is equivalent to say that O lies on the "retarded" half of the double-cone Γ (see the text at the end of Sect. 2) enveloping the spherical light-waves emitted by C ; (b) The same case, when $V \to \infty$. From Figs. a, b we can notice that, after the "optic boom", the tachyon source will appear to occupy simultaneously two positions. See the text ; (c) Representation of the same situation as in Fig. a, but in Minkowski space-time. Again we can notice that O will receive light simultaneously from two positions.

all the radiation emitted by C in a certain interval around its position C_o reach us simultaneously. Soon after the initial optic (or radio) contact with the emitting body C, we shall simultaneously receive the light emitted from suitable couples of points, one on the left and one on the right of C_o, respectively. We shall thus see the initial body, at C_o, to split in two luminous objects C_1, C_2 receeding from each other with the Superluminal (relative) speed U:

$$U = 2b^2 \frac{1 + d/bt}{\sqrt{1 + 2d/bt}} ; \quad b \equiv \frac{V}{\sqrt{V^2 - 1}} ; \quad c = 1 , \qquad (V^2 > 1) \tag{1'}$$

where $d \equiv \overline{OH}$ and $t = 0$ is just the time-instant when the observer enters in radio-contact with C, or rather sees C at C_o. In the simple case when C moves with almost infinite speed along r (see Fig. 1b), the apparent relative speed of C_1 and C_2 varies in the initial stage as $U = (2dc/t)^{1/2}$, where now $\overline{OH} = \overline{OC}$ while $t = 0$ is still the instant when the observer sees $C_1 \equiv C_2 \equiv C_o$[6, 10]. Cf. also Fig. 1c.

2. - ON TACHYON SHAPE

Let us then investigate what shape a Superluminal particle would show to us. Let us first recall that Special Relativity has been generalized by extending the principle of relativity also to Superluminal reference-frames[1, 4]; the fundamental requirement of such an "Extended Relativity" is that the Superluminal Lorentz transformations (SLT) change time-like quantities into spacelike quantities, so that under any SLT the quadratic form is invariant except for its sign[1, 11].

It follows in particular that, if we consider a particle P_T which is a tachyon with respect to the Superluminal frames, to us it will behave as an ordinary particle (bradyon). Let us initially assume such a particle P to be spherical (in particular point-like) when at rest:

$$0 \leq x^2 + y^2 + z^2 \leq r \qquad \text{(at rest).} \tag{2}$$

In the frame where P moves with subluminal speed $v \equiv \beta c$ along x, ($P \equiv P_B$), the equation of its "world-tube" becomes (with the metric (+---), and in natural units):

$$0 \leq \frac{(x - vt)^2}{1 - v^2} + y^2 + z^2 \leq r \qquad (v^2 < 1) \tag{3}$$

which in Lorentz-invariant form reads (cf. Fig. 2):

$$0 \leq \frac{\left[(x_\mu - c_\mu) u^\mu\right]^2}{u_\mu u^\mu} - (x_\mu - c_\mu)(x^\mu - c^\mu) \leq r^2, \qquad (v^2 < 1) \tag{4}$$

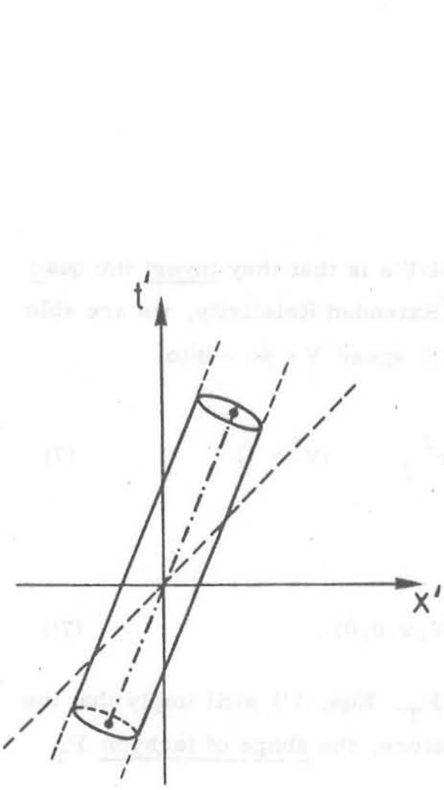


FIG. 2 - The "word-tube" of an ordinary (bradyonic) particle $P \equiv P_B$, assumed to be spherical - or ellipsoidal - in its rest-frame. For simplicity, it is assumed particle P to move along the x-axis and the world-line of the center C of P to pass through the space-time origin, so that $C \equiv 0$ for $t = 0$. Notice however that eqs. (4), (6), (7), (9) of the text have been written down for the most general case.

where $x_\mu \equiv (t, x, y, z)$; the coordinates c_μ refer to the center C of P_B ; and[1,4] the four-velocity u_μ is defined $u_\mu \equiv dx_\mu / d\tau_o$ (see Appendix A). Eq. (4) reduces to eq. (3) in the special case when the world-line of C passes through the space-time origin, and moreover

$$x_\mu \equiv (t, x, y, z) \; ; \qquad c_\mu \equiv (t, vt, 0, 0) \; . \tag{4'}$$

In the more general case when P_B has in its own rest-frame an ellipsoidal shape with semiaxes $x_o \equiv r, y_o, z_o$, then eq. (3) becomes

$$0 \leq \frac{(x - vt)^2}{x_o^2 (1 - v^2)} + \frac{y^2}{y_o^2} + \frac{z^2}{z_o^2} \leq 1 \qquad (v^2 < 1 \; ; \; x_o \equiv r) \tag{5}$$

and, in Lorentz-invariant form, instead of eq. (4) we get :

$$0 \leq \frac{\left[\Delta X_\mu u^\mu\right]^2}{u_\mu u^\mu} - \Delta X_\mu \Delta X^\mu \leq 1 \, , \qquad (v^2 < 1) \tag{6}$$

with $\Delta X_\mu \equiv X_\mu - c_\mu$; $X_\mu \equiv (t, x/x_o, y/y_o, z/z_o)$. Eq. (6) reduces to eq. (5) if :

$$\begin{cases} \Delta X_\mu \equiv X_\mu - c_\mu \; ; \qquad X_\mu \equiv (t, x/x_o, y/y_o, z/z_o) \; ; \\ c_\mu \equiv (t, vt/x_o, 0, 0) \; . \end{cases} \tag{6'}$$

We have now to consider the same object P endowed however with Superluminal speed V along x ; i.e. a tachyonic particle, $P \equiv P_T$, under the condition that it is a sphere (or an ellipsoid) when seen in its rest-frame. In order to get the shape that P assumes with respect to us when it is faster-than-light, we have merely to apply a SLT to eq. (4), or to eq. (6).

Actually, the only characteristic we need to know about the SLT's is that they <u>invert</u> the quadratic-form sign(4, 1). Once we borrow this information from Extended Relativity, we are able to state that eq. (4) transforms - for a <u>tachyon</u> P_T moving with speed $V \equiv \beta c$ - into

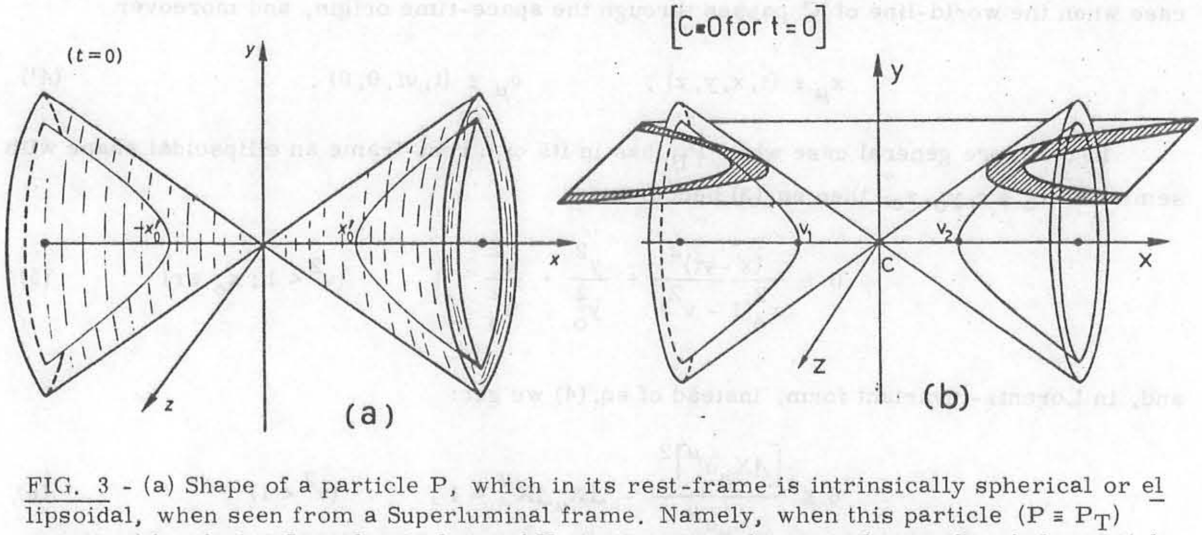
$$0 \leqslant (x_\mu - c_\mu)(x^\mu - c^\mu) - \frac{\left[(x_\mu - c_\mu)u^\mu\right]^2}{u_\mu u^\mu} \leqslant r^2 , \qquad (V^2 > 1) \tag{7}$$

where for a motion along x we have

$$x_\mu \equiv (t, x, y, z) ; \qquad c_\mu \equiv (x/V, x, 0, 0) . \tag{7'}$$

Again, quantities c_μ are the coordinates of the <u>center</u> C of P_T. Eqs. (7') still imply that the world-line of C passes through the space-time origin. Therefore, the <u>shape of tachyon</u> P_T is given by (cf. Figs. 3a and 3b)

$$0 \geqslant - \frac{(x - Vt)^2}{V^2 - 1} + y^2 + z^2 \geqslant - r^2 ; \qquad (V^2 > 1) \tag{8}$$



<u>FIG. 3</u> - (a) Shape of a particle P, which in its rest-frame is intrinsically spherical or ellipsoidal, when seen from a Superluminal frame. Namely, when this particle ($P \equiv P_T$) moves with relative Superluminal speed V, it appears to be spread over the whole spatial region delimited by a double-cone and by a two-sheeted hyperboloid asymptotic to the cone. The whole structure moves of course with the speed V of P_T. If P is intrinsically spherical, then the cone semi-angle is $\mathrm{tg}\,\alpha = (V^2 - 1)^{-1/2}$. Notice that P_T is infinitely extended in space only if P_B was supposed to be infinitely extended in time: See the following. (b) The same structure already depicted, for t = 0, in Fig. a. We cut it with a plane $\mathscr{Q}$ parallel to the motion-line x, and in particular orthogonal to the y-axis. The intersection is a (two-fold) "hyperbolic annulus", delimited by the (four) branches of two hyperbolas. Such intersections rigidly travel with the same speed and direction of P_T. The intersections with planes $\mathscr{P}$ orthogonal, on the contrary, to the motion-line are shown in Fig. 6.

One can immediately notice that (if the world-tube of P_B was supposed to be unlimited, i. e. if P_B was supposed to be indefinitely extended in time) the tachyon P_T appears as accupying the whole space bound by the double, unlimited cone $y^2 + z^2 = (x-Vt)^2/(V^2-1)$ and the (two-

-sheeted, rotation) hyperboloid $y^2 + z^2 = (x-Vt)^2/(V^2-1) - r^2$, where the latter is asymptotic to the former. Notice that the cone semi-angle α is (Fig. 3a)

$$\mathrm{tg}\,\alpha = 1/\sqrt{V^2 - 1}\,. \tag{8'}$$

More generally, for the ellipsoidal case, eq. (6) transforms in the case of tachyons into

$$0 \geq \frac{\left|\Delta X_\mu u^\mu\right|^2}{u_\mu u^\mu} - \Delta X_\mu \Delta X^\mu \geqslant -1\,, \qquad (V^2 > 1) \tag{9}$$

where, for (Superluminal) motion along x, if $C \equiv 0$ for $t = 0$,

$$\Delta X_\mu \equiv X_\mu - c_\mu\,; \qquad X_\mu \equiv (t/x_o, x, y/y_o, z/z_o)\,; \qquad c_\mu \equiv (x/Vx_o, x, 0, 0)\,. \tag{9'}$$

We conclude that the shape of tachyon P_T is given by

$$0 \geqslant -\frac{(x-Vt)^2}{x_o^2(V^2-1)} + \frac{y^2}{y_o^2} + \frac{z^2}{z_o^2} \geqslant -1\,. \qquad (V^2 > 1) \tag{10}$$

It follows that (if P_B was supposed to be indefinitely extended in time) a generic tachyon P_T will appear to be spread over the whole space confined between the double, unlimited cone $\mathscr{C}$

$$\frac{y^2}{y_o^2} + \frac{z^2}{z_o^2} = \frac{(x-Vt)^2}{x_o^2(V^2-1)} \qquad (V^2 > 1) \tag{11a}$$

and the two-sheeted hyperboloid $\mathscr{H}$

$$\frac{y^2}{y_o^2} + \frac{z^2}{z_o^2} = \frac{(x-Vt)^2}{x_o^2(V^2-1)} - 1\,. \qquad (V^2 > 1) \tag{11b}$$

The hyperboloid $\mathscr{H}$ is asymptotic to the cone $\mathscr{C}$ (Cf. Figs. 3a, 3b). For $t = 0$, the cone vertex C coincides with the space-origin 0 and the coordinates of the vertices V_1, V_2 of $\mathscr{H}$ are:

$$V_{1,2} = \mp r\sqrt{V^2 - 1}\,. \qquad (V^2 > 1) \tag{12}$$

Eq. (12) needs of course to be suitably modified if $C \neq 0$ when $t = 0$; if the coordinate x_C of C is $x_C \equiv \varrho$ for $t = 0$, then as time elapses:

$$V_{1,2} = \varrho + Vt \mp r\sqrt{V^2 - 1}\,. \tag{12'}$$

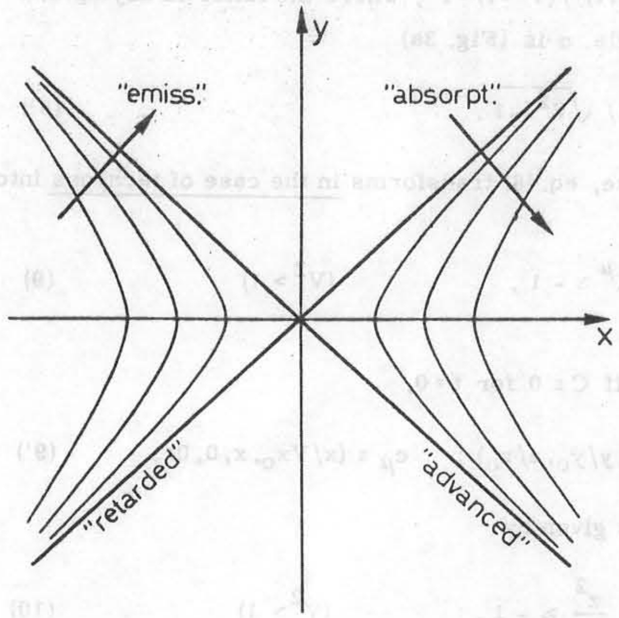


FIG. 4 - The equipotential surfaces of the electrostatic field of a charged particle P, assume the form of two-sheeded hyperboloids when the particle (P_T) travels with Superluminal, constant speed (e.g., along x). Such is the result of the physical "distortion" due to the very high relative speed. The asymptotic cone has nothing to do - of course - with Cherenkov's, since no actual radiation-energy is emitted by P_T during its inertial (Superluminal) motion: In a sense, one might say that the apparent emission associated to the "retarded" cone is exactly counterbalanced by the apparent absorption associated to the "advanced" cone. The asymptotic cone, of course, is the one associated to the shape of P_T (i.e. is the same as in Figs. 3).

The whole "structure" $\mathscr{C} + \mathscr{H}$ rigidly moves, with the speed V of tachyon P_T, along x. We can also notice that the central point $(x - vt)^2/\left[x_o^2(1 - v^2)\right] + y^2/y_o^2 + z^2/z_o^2 = 0$ of the bradyonic ellipsoid (cf. eqs. (5)) goes into the cone (11a): i.e. the center of the bradyon-ellipsoid (internal boundary, in a sense) goes into $\mathscr{C}$, that might be regarded as the "external boundary" of the tachyon. Vice-versa, the external ellipsoidal surface of the bradyon goes - under the considered SLT - into the hyperboloid (11b), that might be regarded as the "internal boundary" of the tachyon.

Also the equipotential surfaces associated with the electrostatic field of a charged tachyon will of course appear in the shape of two-sheeted hyperboloids (Fig. 4).

The problem is quite different, however, when P_B is emitting electromanetic waves, i.e. when the emitted spherical-waves propagate with the light-speed c. In such a case, the waves will appear as spherical also to Superluminal observers(4), even if enveloped now by a (double) cone Γ in partial analogy with what is known to happen for a supersonic sound-source in the air.

But let us go back to the abovementioned fact that also the equipotential surfaces associated with the electrostatic field of a (charged) tachyon P_T will appear to us shaped like the (hyperboloidal) surface of particle P_T itself. In other words, the equipotential surfaces of the electrostatic field of a (charged) tachyon - moving with constant velocity $\vec{V}$ - will be two-sheeted hyperboloids just asymptotic to the double cone $\mathscr{C}$(4). We thus re-obtain previous results(9, 12) which showed that: (i) the cone $\mathscr{C}$ has nothing to do - of course - with Cherenkov's, and (ii) it is associated with no radiation emission, as required by the static character of the initial field and by the tachyon inertial motion (since, in a sense, the apparent emis

(x) - The semi-angle α of cone Γ is still given by $\mathrm{tg}\,\alpha = (V^2 - 1)^{-1/2}$.

sion of the "retarded" cone is compensated by the apparent absorption of the "advanced" cone). See Fig. 4. If P_B is point-like and charged, then the charge of P_T a priori will be suitably distributed over the whole cone $\mathscr{C}$; an interesting mathematical problem would be to find out explicitly those non-radiating solutions of the Maxwell equations for tachyons (in Mignani-Recami's[8] or in Corben's[11] form) corresponding to such a charge-distribution. But we are confining ourselves here to the tachyon-shape problem.

3. - TACHYON LOCALIZABILITY IN TIME AND SPACE : VARIOUS POSSIBLE CASES

Let us explicitly notice that, if the "world-tube" of P_B was not supposed to be unlimited in time but on the contrary has a finite life-time, then also the space-time extension of tachyon P_T gets constrained. For instance, let us start by considering the case when the particle P in its rest-frame: (i) is spherical and with its center situated at the space-origin; (ii) is created at time $\overline{t'_1}$, and (iii) is absorbed at time $\overline{t'_2}$. Then, when P_T is endowed with speed V along x, we shall see (instead of the whole structure in Fig. 3, for $-\infty < x < +\infty$) only that part of $\mathscr{C}+\mathscr{H}$ confined between the 2-dimensional planes:

$$x = \overline{t'_1}\,\sqrt{1-v^2} + vt \equiv x_1(t)\ ; \quad x = \overline{t'_2}\,\sqrt{1-v^2} + vt \equiv x_2(t)\ ; \quad v \equiv \frac{1}{V}, \tag{13}$$

such a couple of planes $x = x_1$, $x = x_2$ shifting however in space along the tachyon direction with the "dual" (subluminal) speed $v = 1/V$. Since the tachyon travels on the contrary with the (Superluminal) speed V, even the tachyon finite shape - i.e. the portion of $\mathscr{C}+\mathscr{H}$ confined between that couple of planes - changes in time. Chosen any fixed plane $x = \bar{x}$, the considered (finite) tachyon will be crossing it during the finite time-interval

$$t_1(\bar{x}) \equiv \bar{x}V - \overline{t'_2}\,\sqrt{V^2-1} \leq t \leq \bar{x}V - \overline{t'_1}\,\sqrt{V^2-1} \equiv t_2(\bar{x})\,, \qquad (x = \bar{x}) \tag{14}$$

i.e. for a time-duration Δt independent of $\bar{x}$:

$$\Delta t = (\overline{t'_2} - \overline{t'_1})\,\sqrt{V^2-1} = \overline{\Delta t'}\,\sqrt{V^2-1}\,. \qquad (x = \text{const.}) \tag{14 bis}$$

Cf. Appendix B.

In other words, we expect the tachyon P_T to be a double unlimited structure $\mathscr{C}+\mathscr{H}$, infinitely extended in space, only when the corresponding bradyon P_B exists for $-\infty < t' < +\infty$, i.e. is infinitely extended in time. On the contrary, if the life-time of P_B is finite, the space-extension of P_T is finite too.

To go on, let us first restrict ourselves - for simplicity - to the case of a point-like P_B. Then, one finds that the vertex C of cone $\mathscr{C}$ will be visible (i.e. the actually existing portion of $\mathscr{C}$ will contain C) in the range $\bar{x}_1 \leq x \leq \bar{x}_2$, where $\bar{x}_1$, $\bar{x}_2$ are the fixed positions

$$\bar{x}_1 \equiv \overline{t'_1}\,V\,\sqrt{V^2-1}\ ; \qquad \bar{x}_2 \equiv \overline{t'_2}\,V\,\sqrt{V^2-1}\ , \tag{15}$$

corresponding to the time-range $\overline{t}_1 \leq t \leq \overline{t}_2$, where $\overline{t}_1$, $\overline{t}_2$ are the fixed time-instants

$$\overline{t}_1 \equiv \overline{t}'_1 / \sqrt{V^2 - 1} \; ; \qquad \overline{t}_2 \equiv \overline{t}'_2 / \sqrt{V^2 - 1} . \qquad (x' = y' = z' = 0) \qquad (16)$$

Notice from eqs. (13) that also the <u>mobile</u> space-interval

$$\Delta x = x_2(t) - x_1(t) = \overline{\Delta t'} \sqrt{1 - v^2} \qquad (v \equiv 1/V) \qquad (13\text{ bis})$$

is actually independent of time; it is always smaller than the fixed space-interval given by eqs. (15)

$$\overline{\Delta x} = \overline{x}_2 - \overline{x}_1 = \overline{\Delta t'} / \sqrt{1 - v^2} \; ; \qquad (v \equiv 1/V) \qquad (15\text{ bis})$$

that is to say:

$$\overline{\Delta x} \geq \Delta x ,$$

the equality-sign holding only when the tachyon P_T has divergent speed $V = \infty$. [On the contrary, the time-duration Δt in eq. (14 bis) can be smaller or larger than the time-duration $\overline{\Delta t}$ given by eqs. (16):

$$\overline{\Delta t} \equiv \overline{t}_2 - \overline{t}_1 = \overline{\Delta t'} / \sqrt{V^2 - 1} , \qquad (16\text{ bis})$$

depending on whether $V \lessgtr \sqrt{2}$]. As depicetd in Fig. 5a, in the fixed space-range $\overline{x}_1 < x < \overline{x}_2$ we have:

(i) during the time-interval $\overline{t}_1$ to $\overline{t}_2$, a finite double cone (with the x-axis as symmetry axis and bound by the two planes $x = \overline{x}_1$ and $x = \overline{x}_2$) moving with speed V, so that its vertex C moves from $\overline{x}_1$ to $\overline{x}_2$;

(ii) at time $\overline{t}_1$, a finite single cone with vertex at $\overline{x}_1$ and base on the plane $x = \overline{x}_2$;

(iii) at time $\overline{t}_2$, a finite single cone with base on the plane $x = \overline{x}_1$ and vertex at $\overline{x}_2$,

even if we shall not see simultaneously the whole pattern, because of eqs. (13). [We spent some time on this description, expressed in eqs. (15)-(16 bis), also since for various reasons it <u>may</u> be convenient to assume the (finite) tachyon P_T to exist only within the global time-interval $\overline{t}_1 < t < \overline{t}_2$ given by eqs. (16)].

If P_B is not point-like, such a description has to be suitably modified (we still assume that particle P_B is instantaneously born at time $\overline{t}'_1$ and instantaneously disappears at time $\overline{t}'_2$ <u>in its rest-frame</u>). When $r \neq 0$, the vertex C of $\mathscr{C}$ has to be substituted[4] by the vertices V_1, V_2 of $\mathscr{H}$; and eqs. (16) become

$$\overline{t}_1 = (\overline{t}'_1 - Vr) / \sqrt{V^2 - 1} \; ; \qquad \overline{t}_2 = (\overline{t}'_2 + Vr) / \sqrt{V^2 - 1} . \qquad (16')$$

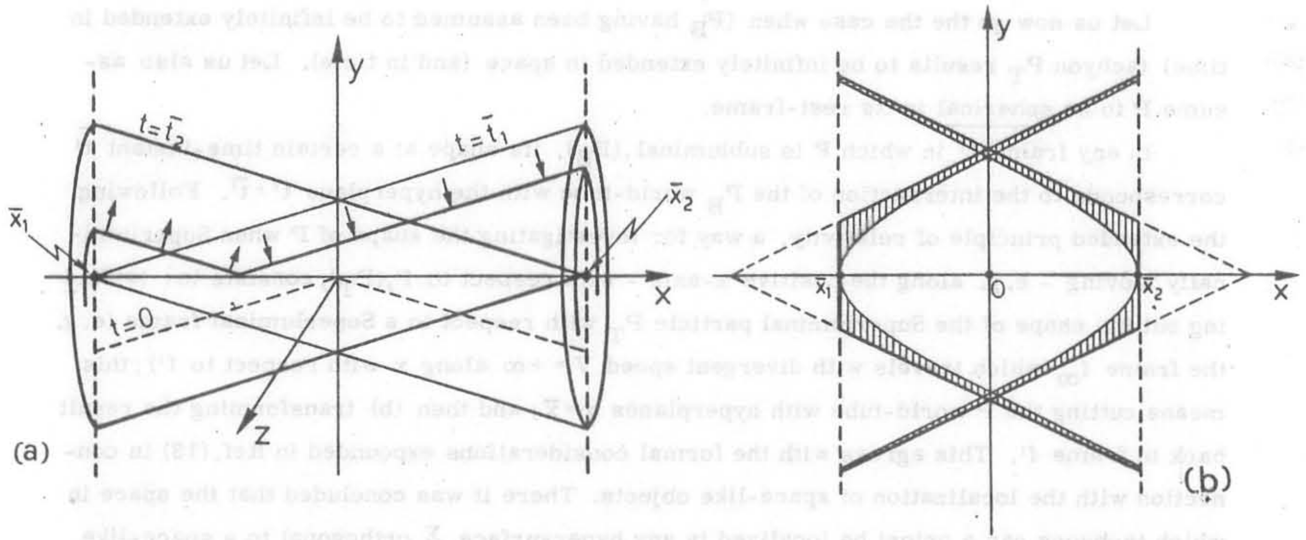


FIG. 5 - (a) If a particle P_B has a finite time-extension in its rest-frame (e.g., is created at time $\bar{t}'_1$, and absorbed at time $\bar{t}'_2$), then P_T will appear to possess a finite space-extension. Namely, instead of the whole structure in Figs. 3, we obtain that P_T will consist only in that part of $\mathscr{C}+\mathscr{H}$ confined between the spatial, 2-dimensional planes $x=x_1(t)$ and $x=x_2(t)$ given by eqs. (13) in the text; such a couple of limiting planes travelling rigidly in space along the tachyon direction with the dual (subluminal) speed $v=1/V$. This figure refers to the case of a point-like P_B. It shows the (fixed) range $\bar{x}_1 \leq x \leq \bar{x}_2$ within which the cone-vertex C is visible, i.e. in which the actually existing portion of $\mathscr{C}$ happens to contain C. Of course, we shall not see the whole pattern at the same time, due to limitations (13); (b) Here it is shown the same picture as in fig. a, for a particle P intrinsically spherical (not point-like). See the text.

In correspondence with these time-instants, for the vertices of $\mathscr{H}$ it is $V_1 = \bar{x}_1$ for $t = \bar{t}_1$, and $V_2 = \bar{x}_2$ for $t = \bar{t}_2$, with(x) (see Fig. 5b)

$$\bar{x}_1 = (\bar{t}'_1 V - r)/\sqrt{V^2 - 1} \; ; \qquad \bar{x}_2 = (\bar{t}'_2 V + r)/\sqrt{V^2 - 1} \; ; \tag{15'}$$

while the cone-sections with both the planes $x = \bar{x}_1$ and $x = \bar{x}_2$ have radius $r_{ext} = r$. In correspondence with the same time-instants $\bar{t}_1$, $\bar{t}_2$ the coordinate of the cone-vertex C is

$$C_1 = \bar{t}_1 V \; ; \qquad C_2 = \bar{t}_2 V \, , \tag{15"}$$

respectively; which in particular yield that now $C_1 < \bar{x}_1$; $C_2 > \bar{x}_2$. Fig. 5b depicts the general pattern in the fixed space-range $\bar{x}_1 < x < \bar{x}_2$ (for time ranging in the interval $\bar{t}_1 < t < \bar{t}_2$) of the "finite" tachyon P_T, when endowed with speed V along the positive x-axis and under the conditions specified in eqs. (13). [Again, it may be convenient for various reasons to assume the global time-extension of tachyon P_T to be confined within the range $\bar{t}_1 < t < \bar{t}_2$].

(x) - For simplicity's sake, we disregard the double sign enetring the Generalized Lorentz transformations(1, 4).

Let us now go the the case when (P_B having been assumed to be infinitely extended in time) tachyon P_T results to be infinitely extended in space (and in time). Let us also assume P to be spherical in its rest-frame.

In any frame f' in which P is subluminal,(P_B), its shape at a certain time-instant $\bar{t}'$ corresponds to the intersection of the P_B world-tube with the hyperplane $t' = \bar{t}'$. Following the extended principle of relativity, a way for investigating the shape of P when Superluminally moving - e.g. along the positive x-axis - with respect to f',(P_T), consists in: (a) finding out the shape of the Superluminal particle P_T with respect to a Superluminal frame (e.g. the frame f_∞ which travels with divergent speed $V = +\infty$ along x with respect to f'); this means cutting the P world-tube with hyperplanes $x = \bar{x}$; and then (b) transforming the result back to frame f'. This agrees with the formal considerations expounded in Ref. (13) in connection with the localization of space-like objects. There it was concluded that the space in which tachyons can a priori be localized is any hypersurface Σ orthogonal to a space-like line; for example, perpendicular to the vector (0, x, 0, 0): in this case we reduce ourselves to the abovementioned hyperplanes $x = \bar{x}$. Any hypersurface Σ has of course two space-like and one time-like orthogonal basis-vectors.

Let us analyse what we shall see in any such space Σ when observing a tachyon P_T. To us a hyperplane $x = \bar{x}$, e.g., is nothing but the "world-space" described as time elapses by the 2-dimensional space-plane parallel to (y, z) at $x = \bar{x}$; so that we have to investigate the evolution in time of the intersection of tachyon P_T with a given spatial plane $\mathscr{P}$ parallel to (y, z). Inserting $x = \bar{x}$ in eq. (8) we get:

$$\frac{(\bar{x} - Vt)^2}{V^2 - 1} \geqslant y^2 + z^2 \geqslant \frac{(\bar{x} - Vt)^2}{V^2 - 1} - r^2 \; ; \qquad (x = \bar{x}; \; V^2 > 1)$$

which means that, in the plane $\mathscr{P}$, tachyon P_T occupies the circular ring $\mathscr{R}$

$$\left\{ \begin{array}{ll} 0 > y^2 + z^2 - \dfrac{(\bar{x} - Vt)^2}{V^2 - 1} \; ; & \qquad (17a) \\ & (V^2 > 1; \; \bar{x} = \text{const.}) \\ y^2 + z^2 - \dfrac{(\bar{x} - Vt)^2}{V^2 - 1} > -r^2 \; , & \qquad (17b) \end{array} \right.$$

where $C \equiv 0$ for $t = 0$, and where eqs. (17a), (17b) individuate the external circumference (= cone intersection), with radius

$$r_{ext} = |Vt - \bar{x}| / \sqrt{V^2 - 1} \; , \qquad (18a)$$

and the internal circumference (= hyperboloid intersection), with radius

$$r_{int} = \sqrt{r^2_{ext} - r^2} \; , \qquad (18b)$$

respectively, that vary with time. Of course, if P_B is point-like, than P_T corresponds only to the cone $\mathscr{C}$ and - on the plane $\mathscr{P}$ - we obtain only the circumference in eq. (17a). On the contrary, if P_B is an ellipsoid, then the circular ring $\mathscr{R}$ becomes an "elliptical ring".

The circular ring $\mathscr{R}$ appears, on each plane $\mathscr{P}$, first to move inwardly (untill when it reduces to a point), and then outwardly (cf. Fig. 6) ; in such a way that the external radius r_{ext} varies with constant speed

$$|\dot{r}_{ext}| = \frac{V}{\sqrt{V^2 - 1}} , \qquad (V^2 > 1) \tag{19a}$$

while the internal radius r_{int} possesses the speed (varying with time)

$$\dot{r}_{int} = \dot{r}_{ext} \left[1 - (\frac{r}{r_{ext}})^2 \right]^{-1/2} , \qquad (x = \bar{x} = \text{const.} ; \; V^2 > 1) \tag{19b}$$

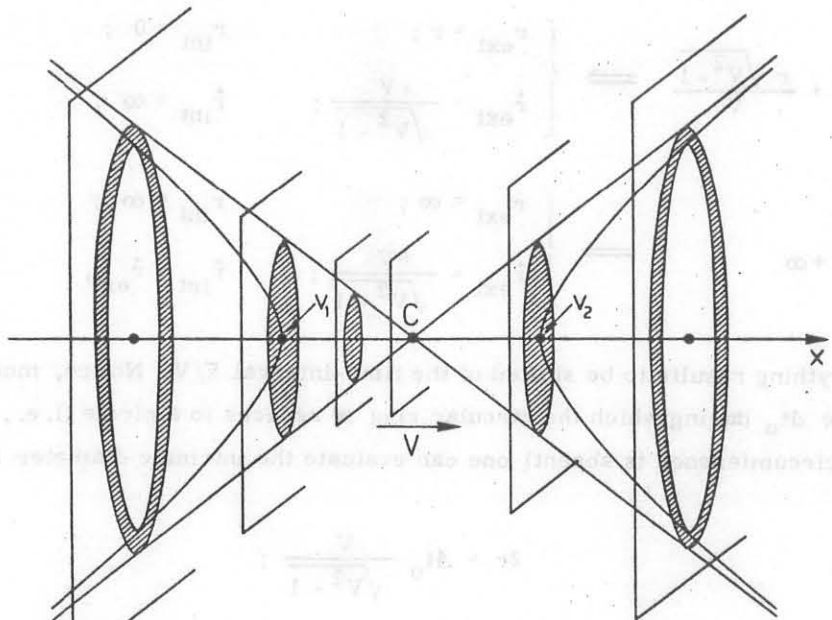


FIG. 6 - Here we show the intersections of tachyons P_T with (2-dimensional, spatial) planes $\mathscr{P}$ orthogonal to the tachyon motion-line, the x-axis, in the same case considered in Figs. 3. For simplicity, we assume P to be spherical in its rest-frame, and $C \equiv 0$ for $t = 0$. Such intersections evolve in time, so that the same pattern reproduces on a second plane - shifted by Δx - after the time $\Delta t = \Delta x/V$. On each plane, as time elapses, the intersection is a circular ring which, for negative times, goes on shrinking till it reduces to a circle and then to a point (for $t = 0$) ; afterwards, such a point becomes again a circle and then a circular ring that goes on broadening. If P_B is supposed to have a finite life-time, then also the above pattern exists and evolves for a finite time on each plane $\mathscr{P}$. Shape and time-evolution of those intersections are of course relevant to any possible experiments (the main problem still open being: What ordinary "material" would reveal the intersection with a tachyon?).

In the simple case $\bar{x} = 0$, we have, as time elapses:

$$\text{for } t = -\infty \Longrightarrow \begin{cases} r_{ext} = \infty ; & r_{int} = \infty ; \\ \dot{r}_{ext} = \dfrac{-V}{\sqrt{V^2 - 1}} ; & \dot{r}_{int} = \dot{r}_{ext} ; \end{cases} \tag{20a}$$

$$\text{for } t = -\frac{r\sqrt{V^2-1}}{V} \Longrightarrow \begin{cases} r_{ext} = r ; & r_{int} = 0 ; \\ \dot{r}_{ext} = \dfrac{-V}{\sqrt{V^2 - 1}} ; & \dot{r}_{int} = \infty ; \end{cases} \tag{20b}$$

$$\text{for } t = 0^{\mp} \Longrightarrow \begin{cases} r_{ext} = 0 ; & \text{———} \\ \dot{r}_{ext} = \mp\dfrac{V}{\sqrt{V^2 - 1}} ; & \text{———} \end{cases} \tag{20c}$$

$$\text{for } t = +\frac{r\sqrt{V^2-1}}{V} \Longrightarrow \begin{cases} r_{ext} = r ; & r_{int} = 0 ; \\ \dot{r}_{ext} = \dfrac{+V}{\sqrt{V^2 - 1}} ; & \dot{r}_{int} = \infty ; \end{cases} \tag{20d}$$

$$\text{for } t = +\infty \Longrightarrow \begin{cases} r_{ext} = \infty ; & r_{int} = \infty ; \\ \dot{r}_{ext} = \dfrac{+V}{\sqrt{V^2 - 1}} ; & \dot{r}_{int} = \dot{r}_{ext}. \end{cases} \tag{20e}$$

If $\bar{x} \neq 0$, everything results to be shifted of the time-interval $\bar{x}/V$. Notice, moreover, that from the time Δt_o during which the circular ring $\mathscr{R}$ reduces to a circle (i.e., during which the internal circumference is absent) one can evaluate the intrinsic diameter 2r of our tachyon P_T:

$$2r = \Delta t_o \frac{V}{\sqrt{V^2 - 1}} ;$$

and that the above equations slightly simplify by the substitution $\sqrt{V^2-1}/V \equiv \sqrt{1 - v^2}$, with $v \equiv 1/V$.

In summary, when both $t_1', t_2' \to \infty$ and $r \neq 0$, we have

$$-\infty < t < +\infty ; \qquad -\infty < x < +\infty ; \tag{21}$$

$$-\infty < x_C < +\infty , \tag{22}$$

where eq. (22) refers to the position of the cone vertex C.

In the limiting case when $r \to 0$, the tachyon shape - depicted in Figs. 3, 6 - reduces only to the mere cone $\mathscr{C}$.

As to the tachyon speed, we have for instance ($\bar{x} = 0$; and $C \equiv 0$ for $t = 0$) :

$$\text{if } V = c\sqrt{2} \implies \begin{cases} r_{ext} = Vt \; ; & r_{int} = \sqrt{V^2t^2 - r^2} \; ; \\ |\dot{r}_{ext}| = V \; ; & |\dot{r}| = V \dfrac{r_{ext}}{r_{int}} \; ; \\ V_{1,2} = \pm r \; ; & \alpha = 45^o \; ; \end{cases} \tag{23a}$$

$$\text{if } V \to c^+ \implies \begin{cases} r_{ext} \to \infty \; ; & r_{int} \to \infty \; ; \\ |\dot{r}_{ext}| \to \infty \; ; & |\dot{r}| \to \infty \; ; \\ V_{1,2} \to \infty \; ; & \alpha \to 90^o \; , \end{cases} \tag{23b}$$

where also the positions V_1, V_2 of the hyperboloid vertices (cf. eqs. (12)) and the value of the cone semi-angle α (cf. eq. (8')) are given.

On the contrary, when P_B is finite in time, that is to say $\bar{t}_1' < t < \bar{t}_2'$ with $\bar{t}_1'$, $\bar{t}_2'$ finite, then eqs. (14), (14bis) tell us that the aboveseen pattern on each plane $\mathscr{P}$ will last a finite time $\Delta t = \overline{\Delta t'} \sqrt{V^2 - 1}$. In other words, the intersection of the (finite) tachyon P_T with any plane $x = \bar{x}$ (see Fig. 6) will be existing only in the finite time interval $t_1(\bar{x}) < t < t_2(\bar{x})$: See eq. (14).

4. - THE INFINITE-SPEED CASES

The case $V \to \infty$, when the cone semi-angle $\alpha \to 0$ requires a detailed analysis (since, e. g., a cone with $\alpha \to 0$ whose vertex C goes to infinity will appear as a cylinder). Still we are assuming $C \equiv 0$ for $t = 0$.

First, let us consider $\bar{t}_1'$, $\bar{t}_2'$ to be finite, as well as r.

It is essential to remember (cf. eqs. (13bis), (15bis)) that when $V \to \infty$ the mobile space--interval Δx does coincide with the fixed space-interval $\overline{\Delta x}$, so that the mobile space-interval (within which the finite tachyon P_T is actually confined, as time elapses) becomes fixed. And, instead of eqs. (13), (14), we can make recourse to eqs. (15), (16). Or, more generally, since $\Delta x = \overline{\Delta x}$ also for $r \neq 0$ when $V = \infty$, instead of eqs. (13), (14) we can make recourse to eqs. (15'), (16').

Then, from eqs. (16'), (15') and (15"), we get for $V \to \infty$; $\alpha \to 0$:

$$\bar{t}_1 \equiv -\frac{r}{c} \leqslant t \leqslant +\frac{r}{c} \equiv \bar{t}_2 \; ; \qquad (V \to \infty ; \; \alpha \to 0) \tag{24}$$

$$\bar{x}_1 \equiv c\bar{t}_1' \leqslant x \leqslant c\bar{t}_2' \equiv \bar{x}_2 \; ; \qquad (V \to \infty ; \; \alpha \to 0) \tag{25}$$

$$-\infty < x_C < +\infty \; ; \quad |x_C| = \infty \quad \text{for } t \neq 0 \, , \qquad (V \to \infty ; \; \alpha \to 0) \tag{26}$$

respectively. The shape of the tachyon P_T in this situation is illustrated in Fig. 7a. The tachyon would appear as a solid cylinder, finite in space and time, with varying radius (when $C \equiv 0$ for $t = 0$):

$$|r_{ext}| \to ct \,, \qquad (V \to \infty)$$

whose speed is:

$$|\dot{r}_{ext}| \to c^+ \,. \qquad (V \to \infty)$$

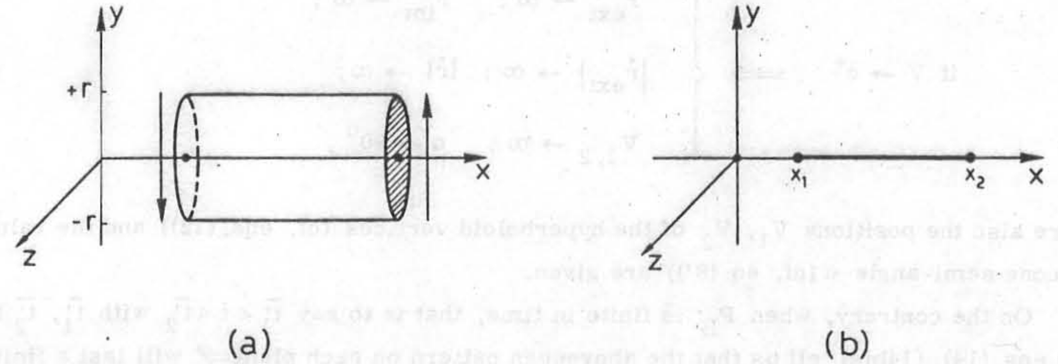


FIG. 7 - (a) A particle P (which is spherical, with $r \neq 0$, and exists for a finite time in its rest-frame) appears, when travelling along x with divergent speed $V \to \infty$, as a solid cylinder finite both in space and time. The cylinder-radius shrinks with constant speed c, reducing from the initial value r to zero (in which case the cylinder becomes a segment), and then increases again - with the same speed - to the value r; (b) Under the previous hypotheses, in the particular case when P_B is point-like in space, tachyon P_T appears as a mere segment on the x-axis, existing only for an instant (i.e., as an "instantaneous segment"). See also Fig. 8.

Initially, as well as at the end, the cylinder radius is just $|r_{ext}| = r$; and at $t = 0$ it is $r_{ext} = 0$. For instance, the (cylinder-generating) straight-segment which in particular is at $y = +r$ for $t = \bar{t}_1$, will be at $y = 0$ for $t = 0$ and at $y = -r$ for $t = \bar{t}_2$. In order to understand the present results, one ought intuitively to think that, since $V \to \infty$, during the infinitesimal time interval $\delta t = [-\varepsilon, \varepsilon]$ the vertex coordinate x_C travels from $x_C \to -\infty$ to $x_C \to +\infty$; so that for all times outside the infinitesimal interval δt the cone-vertex C lies at infinity:

$$x_C(-\frac{r}{c} \leq t < 0) = -\infty \,; \qquad x_C(0 < t \leq \frac{r}{c}) = +\infty \,.$$

Notice that r_{int} exists only at the initial and final time-instants $\bar{t}_1$, $\bar{t}_2$ when $r_{int} = 0$ (for $\bar{t}_1 < t < \bar{t}_2$ it becomes imaginary). In any case, the tachyon P_T must be thought to occupy at each time $\bar{t}_1 \leq t \leq \bar{t}_2$ the whole interior of the cylinder existing at that time.

In the limiting case $r = 0$, we merely get (for $V \to \infty$; $\alpha \to 0$; and for finite $\bar{t}'_1$, $\bar{t}'_2$) the linear segment $\bar{x}_1 \longmapsto \bar{x}_2$, existing only at time $t = 0$ (under our hypotheses): See Fig. 7b. In fact, eqs. (15), (16) yield:

$$\bar{t}_1 = 0 \ ; \qquad \bar{t}_2 = 0 \ ; \tag{24'}$$

$$\bar{x}_1 = c\,\bar{t}'_1 \ ; \qquad \bar{x}_2 = c\,\bar{t}'_2 \ , \tag{25'}$$

as it can be verified also by starting from a point-like object, existing (at $x' = y' = z' = 0$) in the time interval $\bar{t}'_1$ to $\bar{t}'_2$, and then applying a "transcendent Lorentz transformation"(1, 4). Cf. also Fig. 8. This means that, under our hypotheses, the infinite-speed tachyon appears to us as an instantaneous line-segment confined between $\bar{x}_1$ and $\bar{x}_2$ (in agreement with the heuristical considerations at point (i) in Section 1).

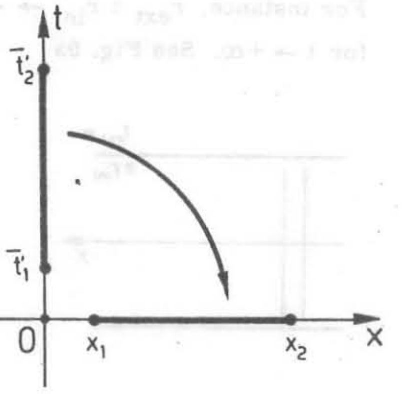


FIG. 8 - The same case as in Fig. 7(b). Namely, here it is schematically shown that, under a transcendent Lorentz boost along $x^{(1,4)}$, a point-like particle P_B characterized in its rest-frame by the finite life-time $\Delta t'$ transforms into the "instantaneous segment" $\Delta x = c\Delta t'$ lying on the x-axis. In other words, a bradyon P_B at rest (living for a finite time, and point-like in space) is transformed by a transcendent Lorentz boost into an infinite-speed tachyon P_T (extended over a finite segment, and point-like in time).

Before going, on, let us recall that in relativity the point-like case appears to be meaningful only as a limiting case(14); such consideration does actually inspire our present analysis.

As the second case, we have to consider not only $V \to \infty$, but also both $\bar{t}_1, \bar{t}_2 \to \infty$ (with $r \neq 0$). Let us for simplicity assume $\bar{t}'_2 = -\bar{t}'_1 = \bar{t}'$; then, when $\bar{t}' \to \infty$, let us call

$$\xi \equiv \frac{1}{c} \lim_{V,\, t' \to \infty} \frac{V}{\bar{t}'} .$$

We get:

$$-\frac{1+\xi r/c}{\xi} \leq t \leq +\frac{1+\xi r/c}{\xi} \ ; \tag{27}$$

$$-\infty < x < +\infty \ ; \tag{28}$$

$$-\infty < x_C < +\infty \ , \tag{29}$$

where actually eqs. (28), (29) do not depend on V. With regard to eq. (27), we have to distinguish the following subcases:

$$\text{if} \quad \xi = 0 \qquad \Longrightarrow \qquad -\infty < t < \infty \ ; \tag{27a}$$

$$\text{if} \quad \xi = \text{finite} \quad \Longrightarrow \quad -\bar{\Delta} \leq t \leq \bar{\Delta} \; ; \qquad (\bar{\Delta} > \frac{r}{c}) \tag{27b}$$

$$\text{if} \quad \xi = \pm \infty \quad \Longrightarrow \quad -\frac{r}{c} \leq t \leq +\frac{r}{c} \, . \tag{27c}$$

In the limiting case for $r \to 0$, one has that (when $V, t' \to \infty$; $r \to 0$):

a) Eqs. (27a), (28), (29), for $\xi = 0$, yield that the tachyon P_T appears as a non-static cylinder infinitely extended both in space and time with:

$$r_{ext} \equiv r_{int} = ct \; ; \qquad \left|\dot{r}_{ext}\right| \equiv \left|\dot{r}_{int}\right| = c^{+} \, . \tag{30}$$

For instance, $r_{ext} \equiv r_{int} \to -\infty$ for $t \to -\infty$; $r_{ext} \equiv r_{int} = 0$ for $t = 0$; $r_{ext} \equiv r_{int} \to +\infty$ for $t \to +\infty$. See Fig. 9a.

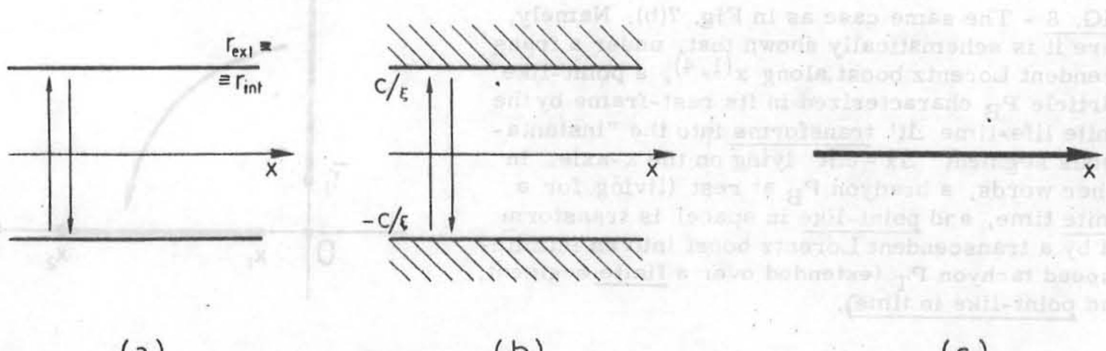


FIG. 9 - When the life-time of P_B diverges ($\Delta t' \to \infty$) together with the speed of tachyon P_T along x ($V \to \infty$), the tachyon shape depends on the value $c\xi$ of the limit of $V/\Delta t'$ for $V, \Delta t' \to \infty$. For instance, in the case when furtherly $r \to 0$, we observe (if $C \equiv 0$ when $t = 0$): (a) for $\xi = 0$, a non-static cylinder having the x-axis as symmetry-axis and infinitely extended both in space and time. The cylinder-radius varies for negative times from $-\infty$ to zero (at $t = 0$), and from zero to $+\infty$ for positive times; (b) for $\xi \neq 0$ and finite, a non-static cylinder similar to the previous one (i.e. infinitely extended in space), but existing only for a finite time. The cylinder-radius contracts (with constant speed c) from the finite value c/ξ to zero (at $t = 0$) and afterwards increases from zero again to c/ξ; (c) for $\xi = \pm\infty$, a mere straight-line, infinitely extended along the x-axis but existing only for a time-instant.

b) Eqs. (27b), (28), (29), for ξ finite, yield that tachyon P_T appears as a cylinder infinitely extended in space but existing only for a finite time:

$$-\frac{1}{\xi} \leq t \leq \frac{1}{\xi} \; ; \tag{31}$$

for instance, if $\xi = 1 \, s^{-1}$, one gets (in seconds):

$$-1 \leq t \leq 1 \, .$$

Also this cylinder is non-static, in the sense that:

$$r_{ext} \equiv r_{int} = ct \quad (\text{with } -1 \leq t \leq 1) \; ; \qquad \left|\dot{r}_{ext}\right| \equiv \left|\dot{r}_{int}\right| = c \; ,$$

so that (see Fig. 9b):

$$-c \leq r_{int} \equiv r_{ext} \leq +c . \tag{32}$$

c) Eqs. (27c), (28), (29), for $\xi = \pm\infty$ yield that tachyon P_T appears as a straight-line, infinitely extended along the x-axis, but existing only for a time-instant (i. e. as an object "point-like in time"):

$$0 \leq t \leq 0 . \tag{33}$$

Moreover:

$$r_{ext} \equiv r_{int} = 0 . \tag{34}$$

See Fig. 9c.

In connection with Figs. 7, 9, let us add that charged tachyons appear to attract each other when they possess equal charges; and, vice-versa, they appear to repell each other when endowed with opposite charges; this is actually reminiscent of the behaviour of ordinary electric-current wires. (Let us recall, in fact, that approach-motion transforms into departure-motion - and vice-versa - under a Superluminal Lorentz transformation, as it can be got from direct inspection in Minkowski space-time).

5. - FURTHER REMARKS

Let us consider again the generic case of a tachyon P_T moving with Superluminal speed V along x, assuming P_B to be infinitely extended in time (Section 2). However, instead of cutting the (moving) structure $\mathscr{C} + \mathscr{H}$, depicted for t = 0 in Fig. 3a, with planes $\mathscr{P}$ orthogonal to the motion-line, let us now consider its intersections with space-planes $\mathscr{Q}$ parallel to the motion-line (e. g., orthogonal to y, or to z). Then instead of the pattern in Fig. 6 we get the pattern already shown in Fig. 3b. Namely, the general shape of the intersection $\mathscr{I}$ of tachyon P_T with a plane $\mathscr{Q}$ parallel to the tachyon motion-direction is a (two-fold) "hyperbolic annulus"; in the sense that the intersection $\mathscr{I}$ results to be the portion of $\mathscr{Q}$ delimited by the branches of two hyperbolas (such a portion consisting, of course, of two disconnected parts $\mathscr{I}_1$ and $\mathscr{I}_2$). On any such a plane $\mathscr{Q}$, as the tachyon moves, the intersections $\mathscr{I}_1$ and $\mathscr{I}_2$ rigidly move with the same speed and direction of P_T. See Fig. 3b. In the limiting case $(r \to 0)$ of a point-like P_B, when the P_T shape degenerates into the cone $\mathscr{C}$, the intersections of P_T with planes orthogonal to y and to z are shown in the Figs. 3a, 3b, respectively, of Ref. (15). In relation to the present remark, and in particular to those Figs. 3(15) we like to recall here the following.

As a by-product of all what derived above on the shape of tachyons, we are now able to geometrico-physically interpret - at least in some relevant cases - the rôle and meaning of the imaginary units entering, as wellknown, the Superluminal Lorentz transformations for the transverse space-coordinates. Such a point has been exploited in Refs. (4).

This paper constitutes, moreover, a first step in the direction of suggesting - eventually - "sensible" experiments with the object of detecting tachyons. We consider any such experiment to be sensible only when based on a definite, developed theoretical framework for tachyons (even if not yet complete in its background and/or in its applications). A preliminary problem still to be carefully investigated is finding out how a (charged) tachyon interacts electromagnetically with an ordinary-matter electron, by using - again - Maxwell equations for tachyons either in Mignani-Recami's[8] or in Corben's[11] form.

Here, let us merely stress once more how unconventional the behaviour of tachyons can be - even if tachyons and bradyons are particles "relativistically dual", - because of the physical "distortion" due to the very high relative speed. This is just shown by the present paper. To add something more, we should like to submit the following consideration, taken out from Ref. (16). It is known that, in a gravitational field, tachyons are subjected to a repulsive force, since[1]

$$F^{\mu} = \pm m_o \Gamma^{\mu}_{\varrho\sigma} \frac{dx^{\varrho}}{ds} \frac{dx^{\sigma}}{ds}, \qquad (V^2 \gtrless 1)$$

such that they absorb (emit) gravitons, while in the same situations bradyons would emit (absorb) gravitons. Let us assume that an analogous behaviour holds also for the electromagnetic field, i.e., when a charged tachyon interacts with ordinary matter. In the case of interaction between a charged ordinary particle and matter, energy is released by the particle to the medium: For instance, in a bubble chamber one encounters along the track of high-energy particles a series of overheated spots. Contrariwise, along the track of a tachyon, we may expect to encounter a series of underheated spots, with all its consequences...

We shall deal with these questions in another paper.

ACKNOWLEDGEMENTS

The present article first appeared, in preliminary form, as a Catania University preprint, PP/637 (March, 1980). Two of the authors (G.D.M. and E.R.) thank M. Pavšič for his kind collaboration. One of them (E.R.) acknowledges stimulating discussions with A. Agodi, M. Baldo, R. Casalbuoni, H.C. Corben, G. Iernetti, P.-O. Löwdin, B. Milewski, H.B. Olesen, G. Parisi, D. Sciama, K.T. Shan, E.C.G. Sudarshan, D. Wilkinson and the collaboration of L.R. Baldini. Thanks are finally due to Mr. F. Arriva for skilful help in the drawings.

APPENDIX A

Let us first remind that in Extended Relativity[4, 1, 3] any Superluminal Lorentz Transformation (SLT) does invert the sign of the quadratic forms, so that fourvector products like $x_\mu x^\mu$ or $p_\mu p^\mu$ behave as "pseudo-scalars" under the SLT's, and in particular under the discrete "Superluminal operation" $S: x_\mu \rightarrow i x_\mu$ (where S represents the "transcendent" SLT : see e.g. Refs. (4)). The SLT's, together with the subluminal Lorentz transformations, have been shown[4] to form a new group G.

Moreover, let us recall that quantities so as $\bar{u}_\mu \equiv dx_\mu / ds$ are Lorentz fourvectors, but are not G-fourvectors, due to the fact that $ds \rightarrow \pm i\, ds$ under any SLT. So that, in order to get a G-fourvector, we defined[1] the four-velocity as follows :

$$u_\mu \equiv dx_\mu / d\tau_o \ , \tag{A1}$$

$d\tau_o$ being the (G-invariant) proper-time element. From eq. (A1) we got[1, 4] e.g. that $p_\mu = m_o u_\mu$ also for tachyons. In the bradyonic case $u_\mu \equiv \bar{u}_\mu$; but in the tachyonic case it is $u_\mu = \pm i \bar{u}_\mu$. For further details, cf. Refs. (4).

APPENDIX B

Let us consider the subluminal particle P_B, intrinsically spherical and subjected to the conditions set at the beginning of Sect. 3 : That is to say, P_B is suddenly created at time $\bar{t}'_1$ in its rest-frame, and is suddenly absorbed at time $\bar{t}'_2$ in its rest-frame. In Minkowski space--time, therefore, the world-tube of P_B is to be confined between two suitable hyperplanes. The generic equation of any such hyperplane (cf. Sect. 2) is $(x_\mu - c_\mu)u^\mu = \text{const.}$ In the particular case when the world-line of C passes through the space-time origin, one may simply write $x_\mu u^\mu = \text{const.}$ In any case, since the fourvector products are scalar under subluminal Lorentz transformations, the equation of the P_B world-tube, i. e. $0 \leq (x_\mu u^\mu)^2 / u_\mu u^\mu - x_\mu x^\mu \leq r^2$, must be associated with the further constraint $\bar{t}'_1 \leq x_\mu u^\mu \leq \bar{t}'_2$. Passing to the Superluminal Lorentz transformation case, the fourvector products are still invariant (except for the sign, since they behave as pseudo-scalars under SLT's[4, 1]; the essential point is that in the tachyon case u_μ is space-like and no more time-like, so that the limiting hypersurfaces are no more space-like, but are referred to 2 spatial and 1 temporal basis-vectors. In the tachyonic case, therefore, one has to associate, with the world-tube transformed equation, the additional contraint : $-\bar{t}'_2 \sqrt{V^2 - 1} + xV \leq t \leq -\bar{t}'_1 \sqrt{V^2 - 1} + xV$, which yields as well : $\bar{t}'_1 \sqrt{V^2 - 1}/V + t/V \leq x \leq \bar{t}'_2 \sqrt{V^2 - 1}/V + t/V$. Notice that, in the case when $y = z = 0$, the last relation becomes $\bar{t}'_1 V / \sqrt{V^2 - 1} \leq x \leq \bar{t}'_2 V / \sqrt{V^2 - 1}$.

REFERENCES

(1) - G. D. Maccarrone and E. Recami, Nuovo Cimento 57A, 85 (1980) ; E. Recami and R. Mignani, Riv. Nuovo Cimento 4, 209 and 398 (1974), and references therein ; E. Recami (editor), "Tachyons, Monopoles, and Related Topics"(North-Holland, Amsterdam, 1978), and references therein ; E. Recami, in "Centenario di Einstein : Astrofisica e Cosmologia, Gravitazione, Quanti e Relatività . . . ", ed. by M. Pantaleo (Giunti-Barbera, Firenze, 1979), chapt. 18, p. 1021-1097 ; E. Recami, in "Albert Einstein 1879-1979 : Relativity, Quanta and Cosmology in the Development of the Scientific thought of A. Einstein", ed. by F. De Finis (Johnson Rep. Co., New York, 1979), vol. 2, chapt. 16, p. 537-597.

(2) - See e. g. P. Castorina and E. Recami, Lett. Nuovo Cimento 22, 195 (1978), and references (especially refs. (1, 4, 5)) therein ; R. Mignani and E. Recami, Phys. Letters B65, 148 (1976) ; E. Recami, in "Centenario di Einstein : Astrofisica e Cosmologia, Gravitazione, Quanti e Relatività . . . ", ed. by M. Pantaleo (Giunti-Barbera, Firenze, 1979), chapt. 18, p. 1071 folls. ; E. Recami, in "Relativity, Quanta and Cosmology . . . ", ed. by F. De Finis (Johnson Rep. Co., New York, 1979), vol. 2, chapt. 16, p. 575 folls. ; E. Recami, in "Tachyons, Monopoles, and Related Topics", ed. by E. Recami (North-Holland, Amsterdam, 1978), p. 17 ; P. Castorina and E. Recami, Lett. Nuovo Cimento 22, 195 (1978) ; M. Pavšič and E. Recami, Nuovo Cimento 36A, 171 (1976), particularly footnotes (17, 21, 32) ; E. Recami, R. Mignani and G. Ziino, in "Recent Developments in Relativistic Quantum Field Theory and its Application", ed. by W. Karwowski (Wroclaw, 1976), vol. 2, p. 269 ; E. Recami and R. Mignani, Phys. Letters 62B, 41 (1976), p. 43 ; R. Mignani and E. Recami, Phys. Letters 65B, 149 (1976), footnotes at p. 149 ; Nuovo Cimento 30A, 533 (1975), Sect. 4, p. 538-539 ; M. Baldo and E. Recami, Lett. Nuovo Cimento 2, 643 (1969), p. 646 ; V. S. Olkhovsky and E. Recami, Nuovo Cimento 63A, 814 (1969), p. 122 ; E. Recami, Giornale di Fisica 10, 195 (1969), p. 203-205 ; "Possible . . . and Comments on Tachyons, Virtual Particles, Resonances", Report IFUM-088/S. M., University of Milano (1968), p. 4-8.

(3) - See e. g. E. Recami, Found. of Phys. 8, 329 (1978), and references therein ; E. Recami and R. Mignani, Ref. (1) ; P. Caldirola and E. Recami, in "Italian Studies in the Phisophy of Science", ed. by M. Dalla Chiara (Reidel, Boston, 1980), p. 249-298 ; E. Recami and W. A. Rodrigues, Found. of Phys. 12, 709 (1982) ; E. Recami, in "Annuario '73, Enciclopedia EST-Mondadori" (Mondadori, Milano, 1973), p. 85. See also E. C. G. Sudarshan et al., Am. Journ. Phys. 30, 718 (1962).

(4) - G. D. Maccarrone and E. Recami, Lett. Nuovo Cimento 34, 251 (1982); E. Recami and G. D. Maccarrone, Lett. Nuovo Cimento 28, 151 (1980) ; P. Caldirola, G. D. Maccarrone and E. Recami, Lett. Nuovo Cimento 29, 241 (1980) ; G. D. Maccarrone and E. Recami, to be submitted to Nuovo Cimento ; G. D. Maccarrone, M. Pavšič and E. Recami, to be submitted for publication.

(5) - See e. g. E. Recami, in "Tachyons, Monopoles, and Related Topics", ed. by E. Recami (North-Holland, Amsterdam, 1978), p. 16 ; V. DeSabbata, M. Pavšič and E. Recami, Lett. Nuovo Cimento 19, 441 (1977) ; E. Recami and K. T. Shan, Lett. Nuovo Cimento 24, 115 (1979).

(6) - This point might a priori be interesting even with respect to some astrophysical observations ; cf. M. H. Cohen et al., Nature 268, 405 (1977).

(7) - A. O. Barut, in "Tachyons, Monopoles, and Related Topics", ed. by E. Recami (North-Holland, Amsterdam, 1978), p. 148.

(8) - Cfr. e. g. R. Mignani and E. Recami, Nuovo Cimento 30A, 533 (1975) ; Phys. Letters 62B, 41 (1976) ; E. Recami, in "Tachyons, Monopoles, and Related Topics", ed. by E. Recami (North-Holland, Amsterdam, 1978), p. 3 ; E. Recami and R. Mignani, in "The Uncertainty Principle and Foundations of Quantum Mechanics", ed. by C. Price and S. S. Chissick (J. Wiley, London, 1977), p. 321.

(9) - R. Mignani and E. Recami, Lett. Nuovo Cimento 7, 388 (1973).

(10) - It is interesting enough that a similar behaviour, associated to solitonic solutions of non-linear equations, was already considered from the mathematical viewpoint in A. O. Barut, Phys. Letters 67A, 257 (1978). See also Ø. Grøn, Lett. Nuovo Cimento 23, 97 (1978).

(11) - E. Recami and R. Mignani, Lett. Nuovo Cimento 4, 144 (1972) ; L. Parker, Phys. Rev. 188, 2287 (1969). See also V. S. Olkhovsky and E. Recami, Visnik Kiivskogo Universi tetu, Seria Fiziki (Kiev) 11, 58 (1970) ; Preprint ITF/70 (Ukrainian Ac. Sc., Kiev, 1970) ; E. Recami and R. Mignani, Ref. (3) ; H. C. Corben, Nuovo Cimento A29, 415 (1975) ; Int. Journ. Theor. Phys. 15, 703 (1976).

(12) - See e. g. C. C. Chang, private communication ; V. A. Gladkikh, Fizika (Is. Tomsk Univ.) no. 6, p. 69 (1978) ; no. 6, p. 130 (1978) ; no. 12, p. 52 (1978) ; Ya. P. Terletsky, in "Tachyons, Monopoles, and Related Topics", ed. by E. Recami (North-Holland, Amsterdam, 1978), p. 47 ; H. C. Corben, Lett. Nuovo Cimento 11, 533 (1974) ; Nuovo Cimento A29, 415 (1975). See also N. Fleury, J. Leite-Lopes and G. Oberlechner, Acta Phys. Austriaca 38, 113 (1973) ; J. R. Gott III, Nuovo Cimento B22, 49 (1974). For the different case of the spherical electromagnetic waves actually emitted by a source, cf. Fig. 1 in E. Recami and G. D. Maccarrone, Ref. (4).

(13) - A. O. Barut, in "Tachyons, Monopoles, and Related Topics", ed. by E. Recami (North-Holland, Amsterdam, 1978), p. 143.

(14) - See e. g. J. M. Jauch, "Foundations of Quantum Mechanics" (London, 1968) ; A. J. Kàlnay, in "Tachyons, Monopoles, and Related Topics", ed. by E. Recami (North-Holland, Amsterdam, 1978), p. 53, and references therein ; Phys. Rev. D7, 1707 (1973) ; J. C. Gallardo, A. J. Kàlnay, B. A. Stec and B. P. Toledo, Nuovo Cimento A48, 1008 (1967) ; A. J. Kàlnay and B. P. Toledo, Nuovo Cimento A48, 997 (1967) ; V. S. Olkhovsky and E. Recami, Lett. Nuovo Cimento (1^{st} series) 4, 1165 (1970) ; E. Recami, in "Quarks and the Nucleus" (Progress in Particle and Nuclear Physics, Vol. 8), ed. by D. Wilkinson (Pergamon, Oxford, 1982), p. 401 ; P. Caldirola, M. Pavšič and E. Recami, Nuovo Cimento B48, 205 (1978) ; Phys. Letters A66, 9 (1978) ; Lett. Nuovo Cimento 24, 565 (1979) ; P. Caldirola, Riv. Nuovo Cimento 2, no. 13 (1979).

(15) - E. Recami and G. D. Maccarrone, Lett. Nuovo Cimento 28, 151 (1980).

(16) - E. Recami, K. T. Shah and W. Yourgrau, "On a possible experiment concerning tachyons", unpublished preprint (1978).