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SMALL - SIGNAL GAIN OF THE TOK AMPLIFIER(x)

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ABSTRACT

The gain curve of the Transverse Optical Klystron in the small signal regime is found following the beam evolution through the device. Our results are in agreement with those deduced from the Madey's theorem.

In our previous works[1,2] the emission mechanism in a Tranverse Optical Klystron (TOK) has been studied.

This paper is devoted to examine more closely the gain curve and its characteristics with respect to that of the FEL for a beam having an initial gaussian energy distribution. To our knowledge this is the first paper where the TOK gain curve is calculated through the electron beam evolution and a free drift space is compared with a dispervive one. These calculations illuminate about the relation between the energy spread of the beam, the drift length and the gain width, reaching the result that the gain width depends only on the beam quality.

The TOK is described as a multi-particles process by means of the collisionless one-dimensional Vlasov equation. Then the properties of the device are derived following the evolution of the electron beam having an initial energy spread within the three sections of the device (buncher, drift-space and radiator, Fig. 1).

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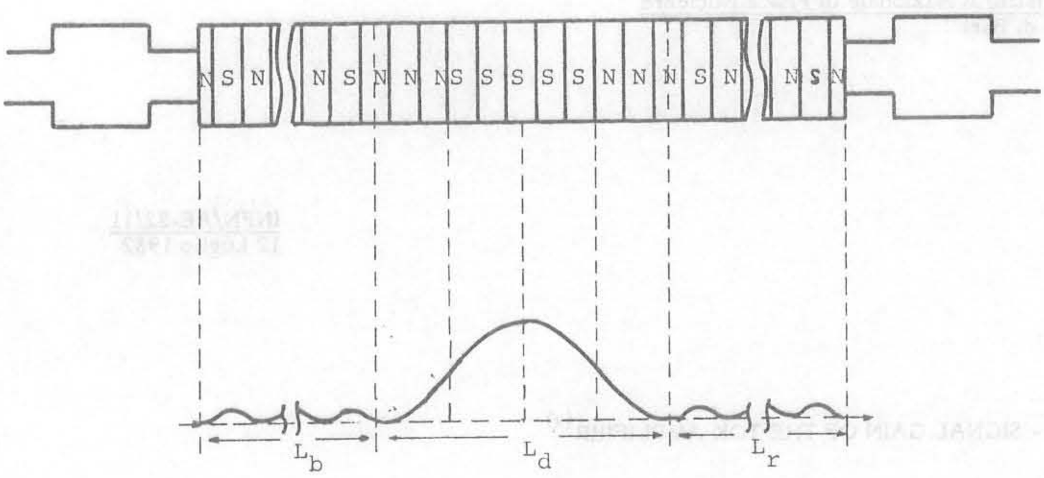

FIG. 1 - Sketch of a TOK configuration. The drift space length is $L_d = 5\lambda_w$.

We are considering the case of buncher and radiator enough short as the density evolution inside is negligible.

In order to reproduce the physical situation of a microwave klystron we choose to study the bunching in a system which moves with the average electron beam velocity in the buncher (EBS), where the interference of the wiggler and the laser field leads to a static potential pattern. In this frame the z-component of the electron velocity is non-relativistic. On the contrary the gain calculations are performed in the LAB frame where the physics is immediate. We repeat briefly the calculations performed in refs. (1) and (2) for easy reference.

In the buncher and radiator the interference of the laser (L) and wiggler (W) fields in the system (EBS) travelling with the electron average velocity $\beta_b c$, brings about a potential pattern

$$V_b(z) = V_{bo} \cos(k_{b+} z_b + \phi) \tag{1}$$

where $k_{b+} = k_b^{(L)} + k_b^{(w)}$, being $k_b^{(L)}$ and $k_b^{(w)}$ the laser and the wiggler field wavenumber respectively in the EBS.

In the buncher the evolution of the electron beam reads

$$\frac{\partial \varrho_b}{\partial t_b} + \dot{z}_b \frac{\partial \varrho_b}{\partial z_b} + \dot{p}_{bz} \frac{\partial \varrho_b}{\partial p_{bz}} = 0 \tag{2}$$

where $\dot{p}_{bz}$ is deduced from Eq. (1).

The initial condition (in the LAB) is

$$\varrho(z,\varepsilon,0) = \frac{\varrho_o}{\sqrt{2\pi}\,\sigma_\varepsilon\,\varepsilon} \exp\left\{ -\frac{(\varepsilon-\varepsilon_o)^2}{2\sigma_\varepsilon^2 \varepsilon_o^2} \right\} \tag{3}$$

By choosing the more convenient parameters

$$\zeta = k_{b+} z_b; \quad \tau = (V_{bo}/m)^{\frac{1}{2}} k_{b+} t_b; \quad q = (m V_{bo})^{-\frac{1}{2}} p_{bz}$$
$$\sigma = (V_{bo}/mc^2)^{-\frac{1}{2}} \sigma_\varepsilon \tag{4}$$

Eqs. (2) and (3) become in the EBS

$$\frac{\partial \varrho b}{\partial \tau} + q \frac{\partial \varrho b}{\partial \zeta} + \sin\zeta \frac{\partial \varrho b}{\partial q} = 0 \tag{5}$$

$$\varrho_b(\zeta, q, 0) = \frac{\varrho_{bo}}{\sqrt{2\pi}.\sigma} \exp\left\{ -\frac{(q-q_o)^2}{2\sigma^2} \right\} \tag{6}$$

Eq. (5) can be exactly solved with the standard method of characteristics in terms of Jacobi elliptic functions[3]. However, in many cases, the dimensionless interaction time is very short ($\tau \ll 1$), therefore a solution of Eq. (5) can be found in terms of a Fourier-Taylor series. We restrict our analysis to this case.

At the end of the buncher section the electron beam distribution function is given by

$$\varrho_b(\zeta, q, \tau_b) = \varrho_b(\zeta, q, 0) \sum_{r_o}^{\infty} D_r(\zeta, q)\, \tau_b^r \tag{7}$$

where the coefficients D_r are periodic function of ζ. After the integration over all the momenta it can be shown that the first Fourier coefficient is negligible, instead with the integration over ζ the beam results energy modulated.

The spatial bunching occurs then essentially in the drift region. If the drift is a free space the distribution function is found solving the Vlasov equation

$$\frac{\partial \varrho_d}{\partial \tau} + q \frac{\partial \varrho_d}{\partial \zeta} = 0 \tag{8}$$

with the initial condition $\varrho_d(\zeta, q, 0)$ given by the distribution (7) at the exit of the buncher section. The solution of Eq. (8) is trivial. After the integration over all the momenta, the final spatial electron density can be expressed as

$$\varrho_d(\zeta, \tau) = \varrho_{do} \sum_m A_m(\tau_b, \tau_d) \exp\left\{ j\, m \zeta \right\} \tag{9}$$

The bunching coefficients A_m depend on the interaction times in the buncher and in the drift space τ_b, τ_d. The first, A_1, reads

$$A_1(\tau_b, \tau_d) = \frac{1}{2}\left[\tau_b\, \tau_d + \frac{\tau_b^2}{2}(1 - \sigma^2 \tau_d^2) \right] \exp\left\{ -\frac{\sigma^2}{2} \tau_d^2 \right\} \tag{10}$$

The maximum of the bunching, reached for $\sigma\tau_d = 1$, is

$$A_{1max} = 0.15 \frac{e^2}{m_o^2 c^3 \gamma^2} \frac{E_o^{(L)} B_o^{(w)}}{\sigma_\varepsilon k_w} L_b \tag{11}$$

From the condition of maximum, the free drift length L_f in the LAB frame (L_b is the length of the buncher) results

$$L_f = \frac{(1+K^2)}{4\pi(\sigma_\varepsilon + \Delta\varepsilon/\varepsilon)} \;;\; (K = \frac{e B_o^{(w)}}{\sqrt{2}\, m_o c k_w} \text{ for plane wiggler}) \tag{12}$$

where $\Delta\varepsilon/\varepsilon$ is the beam energy modulation within the buncher. The length (12) in practical cases reaches easily several hundreds of meters, so the drift space must be dispersive. It can be realized connecting suitably some wiggler poles so that a compensated system of three larger poles ($\int B_y(y, z) \,.\, dz = 0$) is obtained (see Fig. 1). This dispersive system can be thought as a free drift space if we assign to particles the new Lorentz factor

$$\gamma_{eff} = \gamma / (1 - \alpha_c \gamma^2)^{\frac{1}{2}} \tag{13}$$

Here the dispersive properties of the system are determined by the "momentum compaction" α_c defined as the ratio of the relative particle path lengthening $\Delta s/s$ to the relative momentum spread $\Delta p/p$, i.e. $\alpha_c = (\Delta s/s/(\Delta p/p)$. In this way the effective bunching length is strongly reduced and is

$$L_d = L_f/(1 - \alpha_c \gamma^2) \tag{14}$$

In the last section of the device (radiator) the buncher condition are reproduced and in the hypothesis $\tau_r \ll 1$ a negligible density evolution of the electron beam occurs.

<u>Small signal gain:</u> We perform the gain calculations in the LAB frame.

In the radiator we have the travelling potential wave obtained by the Lorentz transformation of Eq. (1)

$$V(z, t) = V_o \exp\left\{ j(k_+ z - \omega_+ t + \Delta\phi)\right\} \tag{15}$$

And the electronic wave

$$\varrho(z, t) = \varrho_o \Sigma_m A_m (t_b, t_d) \exp\left\{ j m (k_+ z - \omega_+ t)\right\} \tag{16}$$

where t_b and t_d are the interaction times within the buncher and drift space respectively. The phase shift $\Delta\phi$ is essentially due to the different velocities of the two waves in the drift since the phase lag in the radiator and buncher can be considered negligible (short interaction time).

Between the two waves occurs an energy exchange whose rate is calculated from the relation

$$\frac{dW}{dt} = -\frac{1}{e}\int_{-L_r/2}^{+L_r/2} \overline{F} \cdot \overline{J}\, d^3x \tag{17}$$

where

$$\begin{cases} \overline{F} = -\dfrac{\partial V}{\partial z}\ \hat{z} \\ \overline{J} = e\, c\, \varrho(z, t)\, \overline{\beta}(z) \end{cases} \tag{18}$$

The energy lost by the electron beam within a radiator matched on the first harmonic is

$$\Delta W = \frac{e^2}{2 m_o c \gamma} \frac{L_r}{k_w} E_o^{(L)} B_o^{(w)} \varrho_o A_1 \mathrm{V} \sin\Delta\phi \tag{19}$$

V being the interaction volume.

From this equation we see that for $\Delta\phi = \pi/2$ the electron beam is stimulated to give up the maximum amount of energy ΔW_{max} to the radiation field. In this condition together with a drift leading to the maximum bunching, the gain defined as the fractional increase of the radiation energy, is

$$G_{max} = \frac{2\,\Delta W_{max}}{\varepsilon_o E_o^2 \mathrm{V}} = 0.30\ \frac{e^3}{\varepsilon_o m_o^3 c^5 \gamma^3} \frac{B_o^2}{\sigma_\varepsilon \lambda_L k_w^2} \frac{L_b L_r}{L_w} I \tag{20}$$

having assumed for the electron and radiation beams the same transverse section $S = \lambda_L \cdot L_w/2$.

If we consider the gain of a FEL having a length L_w [4], the enhancement with the TOK configuration results

$$\Delta = \frac{G_{TOK}}{G_{FEL}} \simeq \frac{0.18}{\sigma_\varepsilon} \frac{N_b N_r}{N_w^3} \tag{21}$$

Here N_b, N_r and N_w are the period number of the buncher, radiator and FEL respectively.

From this last relation we deduce that the TOK gain exceeds the FEL one only for short wiggler and good beam quality. Eq. (21) can be obtained from the Madey's theorem [5].

It is worth noting that technically it is not always possible to adjust the magnetic field in the drift space in order to reach the maximum bunching [6], thus in general Eq. (21) must be modified taking into consideration the effective bunching amplitude, i.e.

$$\Delta = \frac{0.18}{\sigma_\varepsilon} \frac{N_b N_r}{N_w^3} \frac{A_{1\ \mathrm{effective}}}{A_{1\ \mathrm{maximum}}} \tag{22}$$

To have a concrete idea we report in Fig. 2 the comparison between the maximum gain versus σ_ε of the TOK and FEL experiment in progress at the Frascati Storage Ring [6]. The main differences are the length of the flatness and the rate of the falling off.

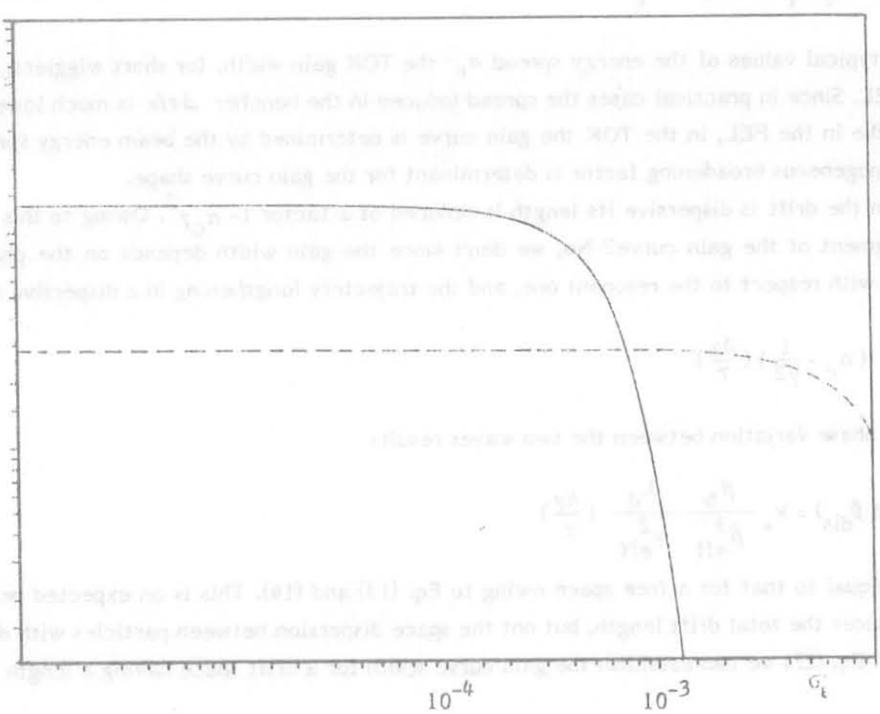

FIG. 2 - Maximum gain versus electron beam energy spread for a TOX (full line) and a FEL (dashed line).

We recall that Eq. (22) has been stated in the case $\Delta\phi = \pi/2$. The effective phase difference between the electronic and potential wave can be calculated hypothizing (owing to the continuity relation between buncher and radiator) that the potential pattern covers also the drift region. Thus, we have a potential wave which runs with a velocity $\beta_b c = \omega_+/k_+$ throughout the device. On the contrary,the electronic wave sees a different "refractive index" in the three sections of the TOK and so has different velocities in the buncher, drift and radiator. The phase lag between the two waves is due to the drift space, since they run with different speed in the drift section, while they have practically the same velocity within the buncher and radiator, and moreover, these last are very short.

If the electronic wave velocity in the drift space is $\beta_d c = \omega_e/k_e$, the refractive index with respect to the buncher is $n = \beta_b/\beta_d$ and so the phase lag is

$$\Delta\phi = k_+ \left(1 - \frac{\beta_b}{\beta_d}\right) L \tag{23}$$

For a free drift space, β_d and L coincide respectively with β and L_f, so an energy spread $\delta\gamma/\gamma$ leads to a phase variation

$$\delta(\Delta\phi) = k_+ \frac{\beta_b}{\beta^3} \frac{L_f}{\gamma^2} \left(\frac{\delta\gamma}{\gamma}\right) \tag{24}$$

The gain curve width σ_g can be deduced from Eq. (19) for a variation $\delta(\Delta\phi) = \pi$ and with the optimized drift length

$$\sigma_g = \left(\frac{\delta\gamma}{\gamma}\right)_f \simeq \pi\left(\sigma_\varepsilon + \frac{\Delta\varepsilon}{\varepsilon}\right) \tag{25}$$

For typical values of the energy spread σ_ε the TOK gain width, for short wigglers, is narrower with respect to the FEL. Since in practical cases the spread induced in the buncher $\Delta\varepsilon/\varepsilon$ is much lower than σ_ε, we conclude that, unlike in the FEL, in the TOK the gain curve is determined by the beam energy spread. That is, in the TOK the inhomogeneous broadening factor is determinant for the gain curve shape.

When the drift is dispersive its length is reduced of a factor $1-\alpha_c\gamma^2$. Owing to this reduction do you expect an enlargment of the gain curve? No, we don't since the gain width depends on the phase of the non-resonant particles with respect to the resonant one, and the trajectory lengthening in a dispersive system of length L_d is

$$\frac{\Delta L}{L_d} = \left(\alpha_c - \frac{1}{\gamma^2}\right)\left(\frac{\delta\gamma}{\gamma}\right) \tag{26}$$

Then the phase variation between the two waves results

$$(\Delta\phi_{dis}) = k_+ \frac{\beta_b}{\beta^3_{eff}} \frac{L_d}{\gamma^2_{eff}} \left(\frac{\delta\gamma}{\gamma}\right) \tag{27}$$

which is equal to that for a free space owing to Eq. (13) and (14). This is an expected result since the dispersive space reduces the total drift length, but not the space dispersion between particles with different velocities.

From Eq. (27) we can establish the gain curve width for a drift space having a length $L_d = N_d\lambda_w$.

$$\left(\frac{\delta\gamma}{\gamma}\right)_d \simeq \frac{1+K^2}{4\,|\alpha_c|\gamma^2 N_d} \tag{28}$$

being $|\alpha_c|\gamma^2 \gg 1$ and $k_+ \simeq k_L$. This formula reduces to (24) when $L_d(1-\alpha_c\gamma^2)$ is optimized to obtain the maximum bunching.

We recall that the momentum compaction α_c depends on the field intensity and length of the drift space by

$$\alpha_c \approx -\frac{1}{3}\left(\frac{c\,e\,B_o L_d}{4\,\varepsilon}\right)^2$$

where ε is the electron energy. Thus we may say that in general the gain curve width becomes narrower with longer drift space. This is well illustrated in Fig. 3b where the gain curves of the TOK experiment of Frascati Storage Ring are reported. Therefore in practical cases when the field intensity in the drift is fixed, it could be a better choice to reduce the drift length to ensure an enlargement of the gain curve. About Fig. 3b we have to add that the decrease of the maximum depends on the beginning of the debunching of the beam.

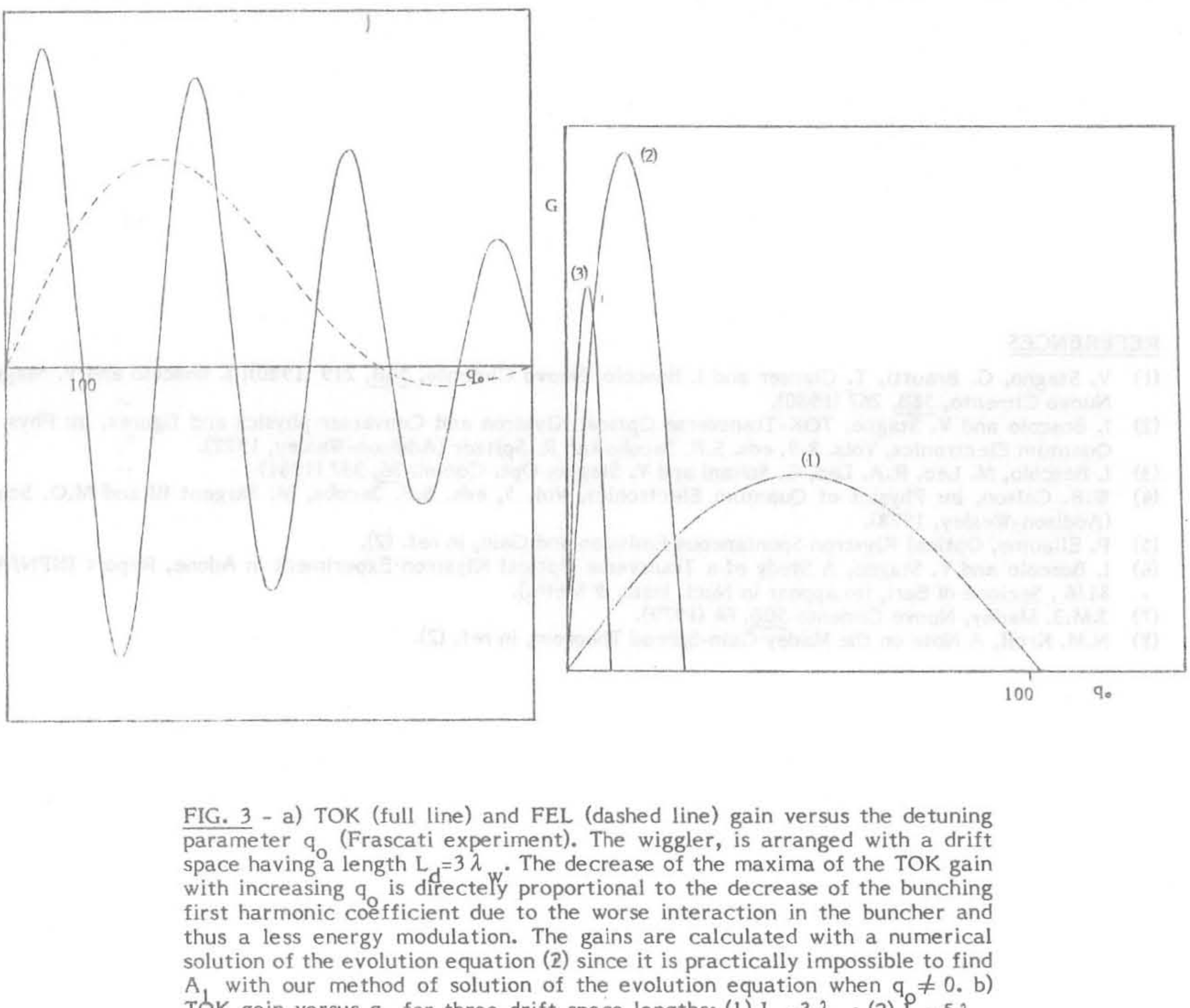

FIG. 3 - a) TOK (full line) and FEL (dashed line) gain versus the detuning parameter q_o (Frascati experiment). The wiggler, is arranged with a drift space having a length $L_d=3\,\lambda_w$. The decrease of the maxima of the TOK gain with increasing q_o is directely proportional to the decrease of the bunching first harmonic coefficient due to the worse interaction in the buncher and thus a less energy modulation. The gains are calculated with a numerical solution of the evolution equation (2) since it is practically impossible to find A_1 with our method of solution of the evolution equation when $q_o \neq 0$. b) TOK gain versus q_o for three drift space lengths: (1) $L_d=3\,\lambda_w$; (2) $L_d=5\,\lambda_w$ (optimized drift length for the gain amplitude); $L_d=7\,\lambda_w$. The gain curve width is $\Delta q_o=(m_o c^2/V_{bo}).\,(\delta\gamma/\gamma)_d$.

Conclusions: The TOK theory shows that an enhacement of the gain with respect to the FEL configuration, for short wiggler and appropriate electron beams, can be achieved.

Our theory based on the beam evolution yields for the gain the same shape derived from the Madey's theorem[7] (and generalized by Kroll[8]), which says that the gain is the derivative of the spontaneous spectrum.

The most important section of the TOK is the drift space which determines all the gain characteristics. The dispersive drift, necessary to reach the electron bunching in a reasonable space length, behaves as the free space.

The width of the TOK gain curve is much narrower than that of the FEL and reduces with longer drift space. It is proportional to the energy spread when the drift length is that requested for the maximum bunching.

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