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EXPERIMENTAL STUDY ON LOW-ENERGY $^2H(\alpha,\alpha)^2H$ ELASTIC SCATTERING

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1. - INTRODUCTION

Strong correlations between the constituent nucleons of ^6Li are responsible for some of its interesting features like stability, charge radius and structure of low excited states[1]. Due to the significance of these correlations, it is possible to study this nucleus as a two (α +d, p+^{5}He, n+^{5}Li, ^{3}He+^{3}H) or three (α +p+n) body problem, rather than as a problem with six independent nucleons: the advantage (with respect to a "true" three-nucleons system) is that ^6Li offers a rich structure from the point of view of resonances.

Investigations of deuteron-alpha elastic scattering have provided in the past experimental information concerning the low lying excited of ^6Li[2-8]. Glueber and collaborators[6-8] performed the most extensive experiments using gaseous ^{4}He targets, measuring both differential cross sections and vector and tensor analysing powers. The phase-shift analysis of their scattering data are in disagreement with the analysis by Senhouse and Tombrello[3], in particular for the $J=1^+$ mixing parameter of s and d waves; moreover their single-level R-matrix analysis did not confirm the existence of three p-levels as found in Ref. (3). Some difficulty in reproducing the data of the 1^+ resonance was attributed by the authors to the limits of the single-level approximation[8].

From a theoretical point of view, several works[9,10] have pointed out the importance of low energy α+d reaction in the framework of three body problems. The simplicity of the problem is related to the hypothesis

that at low energies the excitation of the α-particle may be safely neglected (the binding energy is 22 MeV). Whereas this hypothesis has been questioned[11,12] invoking three body forces with 2π exchange and α-excitation (to account for [illegible] induced deuteron breakup), a more recent analysis[13] of breakup [illegible] is no compelling need for three body forces with α-excitation. On the other [illegible] calculations[9,10], with two body potentials, show evident disagreement with the [illegible] experiments. In particular this is verified in the angular distributions for backward and [illegible]gies in the cm system and in the phase-shifts in the low energy region (e.g. the ones related to the 1^+ [illegible]nance).

Due to these interesting features of low energy α+d elastic scattering, we performed a D (α,α)D experiment concerning the ^{6}Li excitation spectrum between 3.45 and 6.13 MeV. The main purposes of this experiment were:

- to collect accurate measurements of the angular distributions of differential cross sections, in smaller energy steps than in previous works;
- to perform a phase-shift analysis of the scattering data (tested by calculating vector and tensor analysing powers and total reaction cross sections);
- to compare the results with the predictions of theoretical three-body calculations;
- to interpret the phase shifts in terms of R-matrix theory, with the aim of getting further information on the level parameters of the T=0 resonances of ^{6}Li, essentially 2^+ and 1^+: the former recently[7] quoted at 4.7 MeV (excitation energy) with respect to the previous 4.31 MeV value[1], the latter which gives an indication of the tensor force contributing to the interaction, through the mixing parameter of s and d waves.

Special attention was paid to the problem arising in the fitting procedure; in particular:

a) the dependence of the level parameters on the boundary condition;
b) the possibility of improving the quality of the fits by a multichannel multilevel R-matrix approach.

The experimental details are given in Sect. 2.; Sect. 3. deals with the phase shift analysis and the comparison wiht the theoretical predictions; the R-matrix analysis is presented and discussed in Sect. 4.

2. - EXPERIMENT

Measurements were made at 40 incident energies (E_α^L) between 5.962 MeV and 13.911 MeV, using the $^4He^{++}$ beam of the 7 MV Van de Graaff accelerator of the Laboratori Nazionali di Legnaro (Padova).

High counting rates, good angular and energy resolutions were achieved by using solid deuterated polystyrene targets of good stability and mechanical resistance. They were prepared by a method already described[14].

The thickness of the polystyrene layer was determined by measuring the energy loss of 5.477 MeV α-particles from a ^{241}Am source; targets of 20 to 50 μg/cm^2 were obtained and consequently the deuteron maximum energy losses were of about 40 keV.

Fig. 1 shows a simplified scheme of the experimental apparatus and of the associated electronis. A collimation system (2 mm in diameter) was used at the entrance of the scattering chamber. Both the elastically scattered α-particles and the recoiling deuterons were detected in coincidence, by means of surface barrier (300 μm thick) detectors. Four d-detectors (D1, D2, D3, D4), suitably collimated, were mounted on a rotating platform to allow measurements at different angles simultaneously; a large area α-detector was positioned, opposite with respect to the beam, on another rotating platform, in order to detect all the α-particles in coincidence with D1, D2, D3 and D4 at the same time.

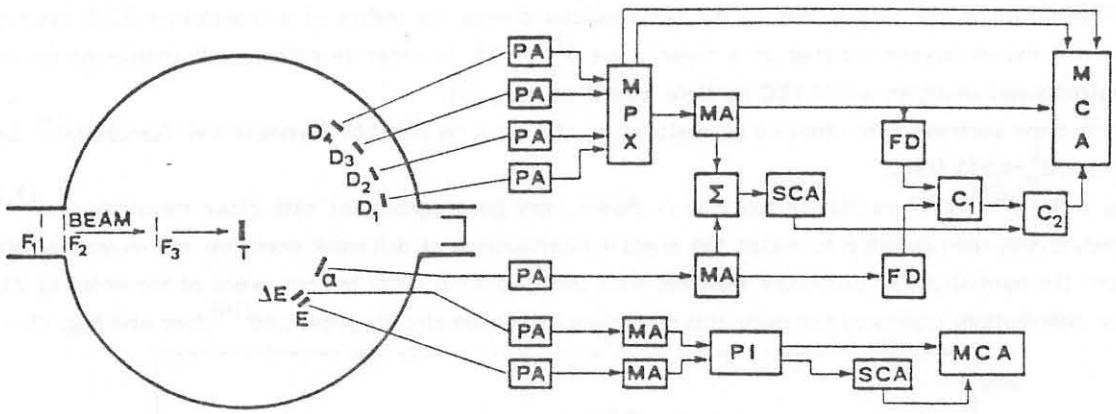

FIG. 1 - Simplified scheme of the experimental apparatus and of the associated electronics. F_1, F_2, F_3: collimators; T: target; D_1, D_2, D_3, D_4: deuteron detectors; α:α-particle detector; ΔE,E: monitor telescope; PA: preamplifier; MPX: multiplexer; MA: main amplifier; Σ: sum amplifier; SCA: single channel analyzer; FD: fast discriminator; C_1-C_2: coincidence; PI: particle identifier; MCA: multichannel analyzer.

The signals coming from the preamplifiers of the four deuteron detectors were fed into a multiplexer[15]. This unit provides a linear multiplexed output and logical address outputs; the first, suitably amplified, together with the address signals, was fed into a multichannel analyzer.

In order to select only events coming from the elastic reaction, this analyzer was gated (C_2) by:

a) a coincidence (C_1) between signals coming from the deuteron and the alpha channels;

b) a window analyzer selecting events with total energy equal to the energy of the beam within 1 MeV.

Typical energy spectra are shown in Fig. 2, without and with α-d coincidence. The absence of spurious peaks in the lower part of the figure and the large suppression of continuous background are clearly displayed.

For each beam energy a measurement of the angular distribution was performed, detecting the recoiling deuterons at lab angles ϑ_d^L = 7° ÷ 69° in 2° steps: the statistical accuracy was better than 1%.

Four excitation curve were also measured (ϑ^{CM}=66°, 98°, 130°, 150°), at the same energies of the angular distributions. The yields were corrected for the change in the

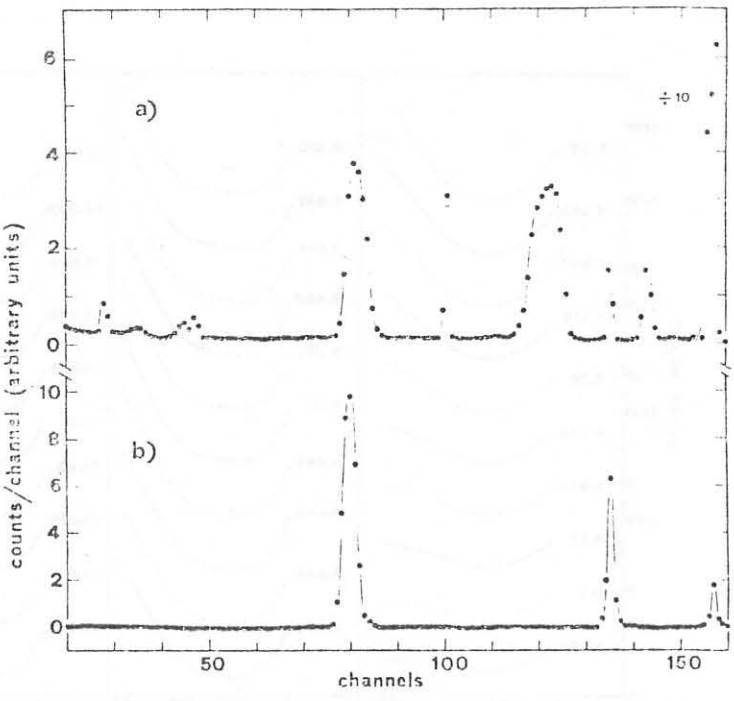

FIG. 2 - Typical energy spectra of recoiling deuterons (E_α^L=9.559 MeV: ϑ_d^L=13°): a) without α-d coincidence: b) with α-d coincidence.

number of deuterons in the target, due to the accumulated charge, by means of a monitoring ΔE-E system, which recorded the deuterons emitted at a fixed angle $\vartheta_d^L = 50°$. In order to collect only deuterons on the monitor multichannel analyzer, an ORTEC particle identifier was used.

Absolute cross sections were obtained normalizing to the value reported by Senhouse and Tombrello[3] for $\vartheta^{CM}=150°$ and $E_d^L=4.955$ MeV.

In Fig. 3 the $\vartheta^{CM}=150°$ excitation function is shown; very good agreement with other measurements[3,8] was obtained. It was then possible to match the angular distributions at different energies; the overall errors, coming from the normalization procedure together with the statistical contributions, were of the order of 2%. The angular distributions measured following this procedure have been already presented[16] (see also Fig. 4).

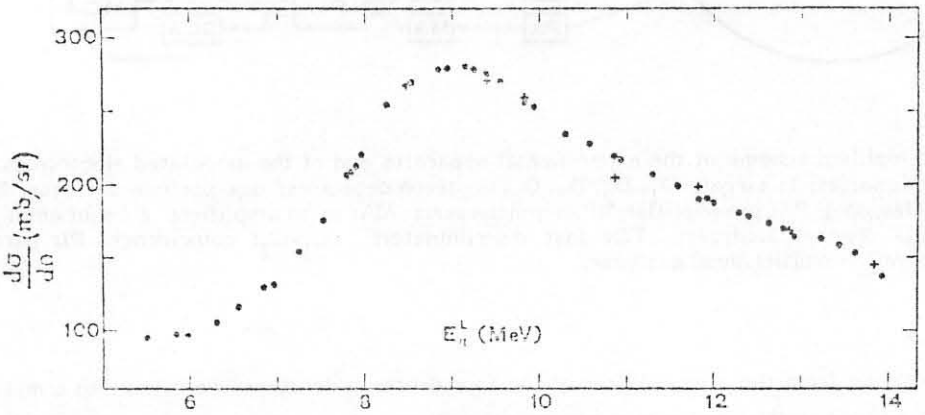

FIG. 3 - Excitation function of D (α, α) D scattering at $\vartheta^{CM}=150°$; (+) Ref. (3); (•) present work.

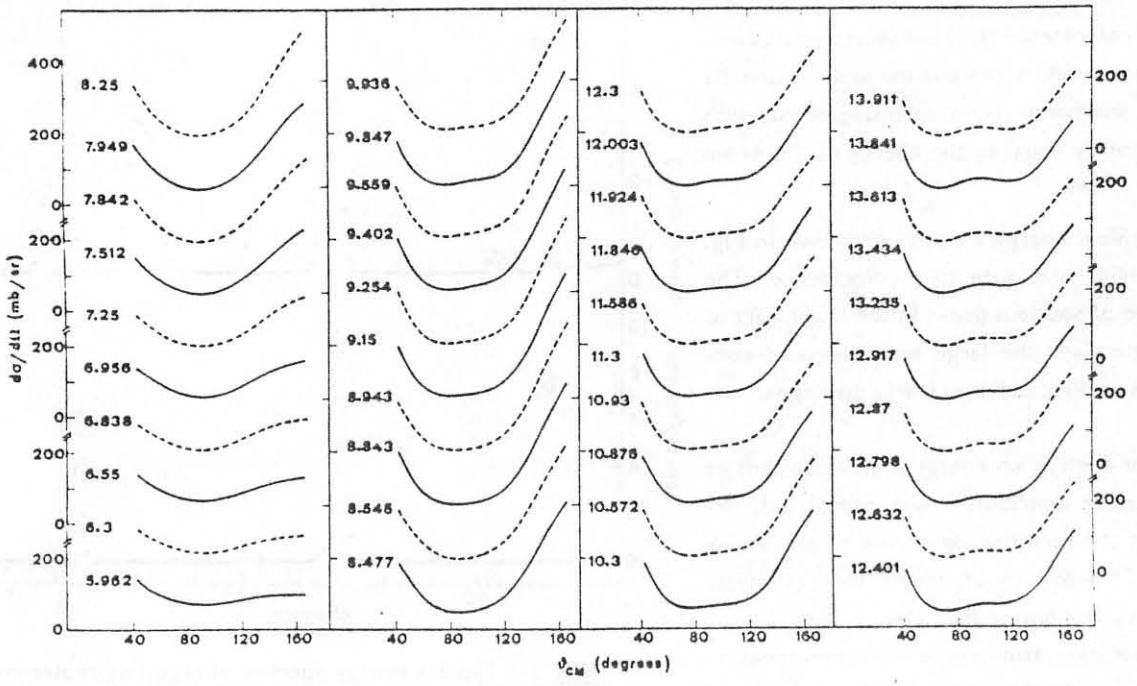

FIG. 4 - Angular distributions of the differential cross sections (left scales refer to continuous lines, right scales to dashed ones). The experimental points are not reproduced because they would be indistinguishable from the fitting curves.

3. - PHASE-SHIFT ANALYSIS

The centre of mass differential cross sections, for the α+d elastic scattering, may be written in terms of elements of the collision matrix U[2,3,17] (see Appendix).

If no mixing between partial waves is allowed, only the diagonal elements of the collision matrix are different from zero and can be expressed in terms of complex phase shift δ_ℓ^J

$$U^J_{\ell,\ell} = e^{2i\delta^J_\ell} \tag{1}$$

In our analysis only the values of orbital angular momentum up to $\ell_{max}=4$ were considered.

Moreover, since in the scattering of a spin-zero from a spin-one particle, tensor interactions do not conserve the orbital angular momentum ℓ, one has to introduce off diagonal U-matrix elements which describe scattering between states of the same total angular momentum J, but with orbital angular momenta ℓ differing by two units. We used the parametrization of Blatt and Biedenharn[18,19] (which retains the phase-shift description).

For example, in the case $J=1^+$ (s-d coupling):

$$U^1_{00} = e^{2i\delta^1_\alpha}\cos^2\varepsilon^1 + e^{2i\delta^1_\beta}\sin^2\varepsilon^1 \tag{2a}$$

$$U^1_{2,2} = e^{2i\delta^1_\alpha}\sin^2\varepsilon^1 + e^{2i\delta^1_\beta}\cos^2\varepsilon^1 \tag{2b}$$

$$U^1_{0,2} = U^1_{2,0} = \frac{1}{2}\sin(2\varepsilon^1)\left[e^{2i\delta^1_\alpha} - e^{2i\delta^1_\beta}\right] \tag{2c}$$

where δ^1_α, δ^1_β are the (complex) eigen phase-shifts and ε^1 the (complex) mixing parameter. Similar expressions were used for the $J=2^+$ (p-f coupling) case.

We note that:

- The mixing parameter ε^J is chosen by convention to approach zero in the limit of zero incident energy. It changes, as a rounded step function, from this value to about $-\pi/2$ at the resonance energy. For example (see Eq. (2a) and (2b)) if ε^1 is near zero δ^1_α is mostly s-wave and δ^1_β mostly d-wave; the opposite occurs for ε^1 near $-\pi/2$ (this "exchange of roles" prevents a crossing of the eigen phase-shifts)[4].
- The off diagonal element $U^1_{0,2}$ is a product of two terms (see Eq. (2c)); thus the magnitude of the mixing can still be small, even for $\varepsilon^1 \simeq -\pi/4$, if δ^1_α is very close to $\delta^{1(8)}_\beta$.

To determine the values of the free parameters (phase-shifts, eigen phase-shifts, mixing parameters) at each energy, a minimization procedure was carried out by means of a computer code which included the routine MINUIT[20]. It is welle know that, when the number of parameters involved is quite large, a dependence of the results on the starting values used in the minimization routine can occur. As usual, to overcome this difficulty, as a first step a continuity criterion was adopted, i.e. the best fit values at each energy were used as entry values for the next energy. The set of the resulting parameters was then tested with a high number of trials, coresponding to different starting values.

No problems arose below the inelastic (^{5}He,p) threshold, where the imaginary parts of the phase-shifts did not play a significant role (and were then setted at zero) and the minima were unambiguously determined. The errors were then computed by the routine MINOS of MINUIT, which takes into account the shape of the χ^2 surface without any approximation (these errors were generally larger than those obtained by a quadratic approximation of the minimum).

Above that inelastic threshold the increasing weight of the imaginary parts doubles the number of parameters and different results, depending on the starting values of the parameters, were obtained. When the resulting values differed by a larger amount than the MINOS errors, the best set on the basis of the continuity was chosen, and the errors were enlarged in order to include the other acceptable solutions.

The errors of the imaginary parts were particularly affected by such a procedure, and therefore the imaginary parts were not used in the successive determination of the level parameters (R-matrix analysis).

The angular distributions related to the best sets of phase-shifts are shown in Fig. 4; the experimental results are not displayed because they lie (with the errors) on the fitting curves.

In order to test, a posteriori, the reliability of our procedure, from the same phase-shifts we calculate:

- the total reaction cross sections which are presented in Fig. 5. They are in reasonable agreement with previous ones[7,8];

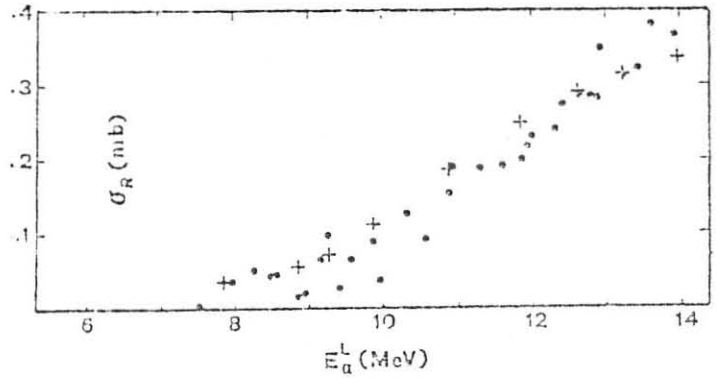

FIG. 5 - Total reaction cross sections vs α -incident energy; (•) present work; (+) prediction of Faddeev type calculations.

- the vector and tensor analyzing powers; there is a good agreement with the experimental values of Schmelzbach et al.[6] (see Fig. 6).

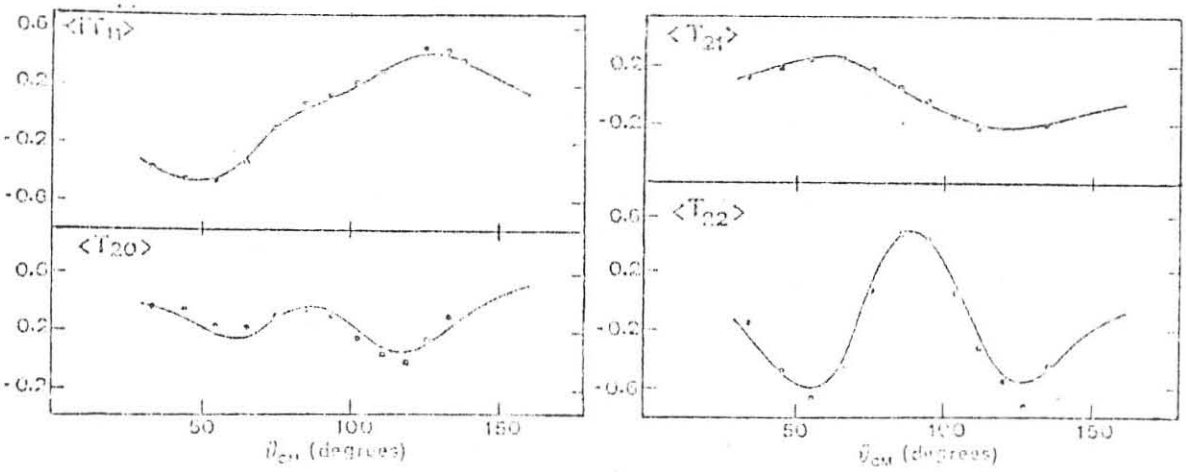

FIG. 6 - Vector and tensor analysing powers evaluated from the phase-shifts of this work (continuous line); the points are experimental values from Ref. (6). (E_d^L = 4.81 MeV).

With the exception of the four parameters related to the $2^+(\delta_2^2)$ and $1^+(\delta_\alpha^1, \delta_\beta^1, \varepsilon^1)$ resonances, our phase-shifts, presented in Fig. 7, in our energy range have a generally smooth behaviour, characteristic of hard-sphere scattering. In particular this holds for the previously mentioned p-levels.

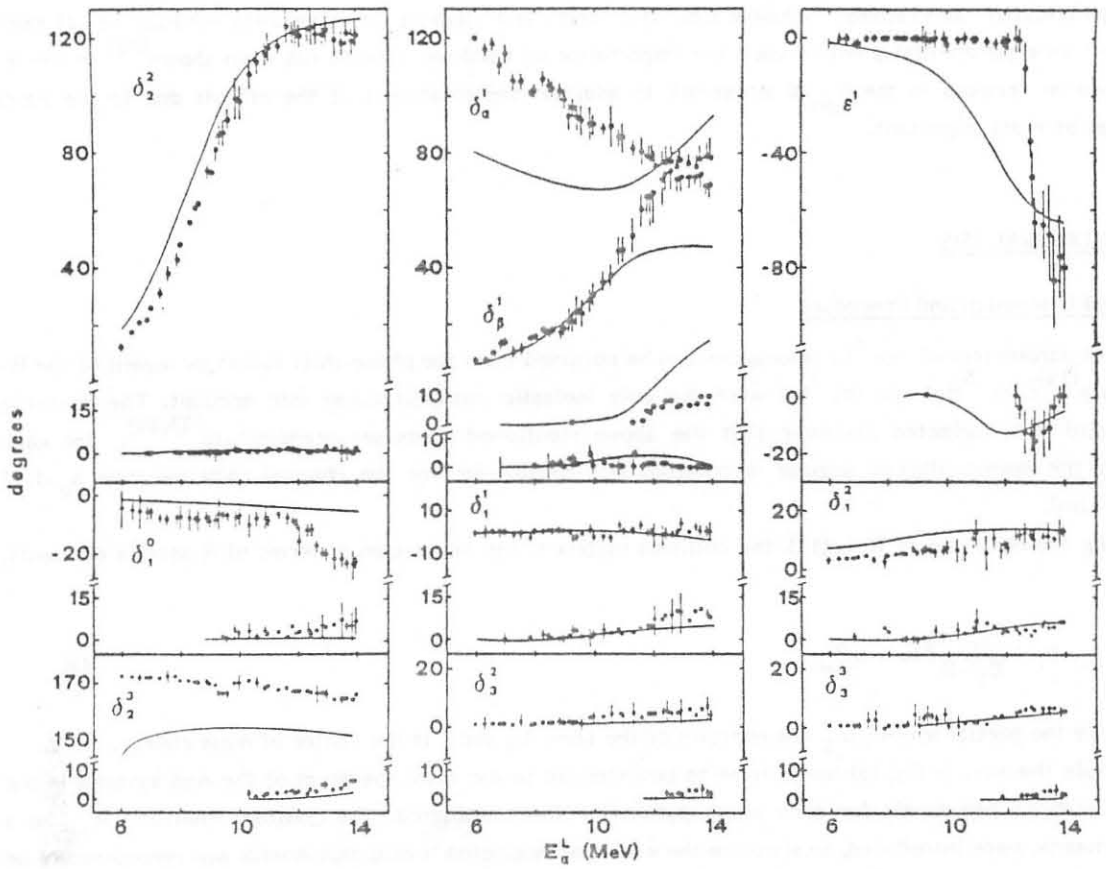

FIG. 7 - Real (up) and imaginary (down) parts of the phase-shifts and mixing parameter obtained in the present work as a function of the α_4-incident energy. The continuous lines are the previsions of Faddeev calculations (see text). The δ_3 is not presented in the Figure.

We note that:

a) below $E_\alpha^L \simeq 9$ MeV another solution for Re (ε^1) was found, with comparable confidence level. A similar trend was already observed by Senhouse and Tombrello[3] in the same energy range. This second solution has been disregarded since it was difficult to handle in the R-matrix analysis and anyway would not sizeably affect the parameters of the resonance (which corresponds to $E_\alpha^L \approx 13$ MeV, see Table II);

b) at these low energies the fitting procedure lead to very small values of the mixing parameter ε^2, and the simplest procedure of neglecting ε^2 was followed.

In Fig. 7, together with our "experimental" phase-shifts we plot the results of theoretical Faddev predictions[9].

The calculation have been made by a computer program[21] which allows to obtain the significant phase-shifts. As we pointed out in the introduction the most important feature of this type of calculation was the disagreement with experiment for the 1^+ complex at low energies[9]. This disagreement is clearly confirmed by our eigen phase-shifts and mixing parameter and it is not substantially changed by modifying the two body potentials as in Ref. (10).

We stress that this disagreement between theoretical previsions and experimental results indicates the need

of more sophisticated calculations, including Coulomb forces and allowing for a detailed scrutiny of off shell effects (from an experimental point of view the importance of Coulomb effects has been shown[22] in the α-induced deuteron breakup in the E_{pn}=0 situation). In addition the treatment of the effects due to the Pauli principle may be most important.

4. - R-MATRIX ANALYSIS

4.1. - General Discussion and Procedure

The level parameters of the ^{6}Li resonances can be obtained from the phase-shift values by means of the R-matrix theory[14]: (p, ^{5}He) and (n, ^{5}Li) were the only inelastic channels taken into account. The deuteron breakup (α,p,n) was neglected assuming that the above mentioned processes predominate[23,24]; for each channel only the lowest allowed angular momentum was considered (for the channel radii we take a_c=1.45 $(A_1^{1/3} + A_2^{1/3})$ fm).

Following the notations of Ref. (17), the collision matrix U can be written in terms of R-matrix elements, defined as:

$$R_{cc'} = \Sigma_\lambda \frac{\gamma_{\lambda c}\,\gamma_{\lambda c'}}{E_\lambda - E} + R^o_{cc'} \qquad (3)$$

where $\gamma_{\lambda c}$ are the partial widths, E_λ the energies of the level λ, and E is the centre of mass energy.

In principle the sum in Eq. (3) would have to be extended to the total spectrum of the A=6 system; in our case a maximum of two levels for each phase parameter was considered. The constant elements $R^o_{cc'}$ of a background matrix were introduced, to simulate the effect of neglected levels; this matrix was considered to be diagonal, assuming that the signs of the partial widths of the neglected levels are randomly distributed. The choice of the boundary condition B_c may be important. In a formal R-matrix theory this quantity is energy dependent. The choice B_c=$S_c(E_R)$, where S_c is the shift factor and E_R is the resonance energy, is very useful in the one-level approximation; in this case, in fact, the calculations are simplified and the energy shift vanishes at E=E_R. Other possible choices, among the most frequently used, are: B_c=0 and B_c=-ℓ (as in previous single-level analyses of d+α scattering). In the present work all the above mentioned boundary conditions have been used; their effect on the level parameters will be discussed in the following. A multichannel multilevel R-matrix computer code was developed using, also at this stage, the minimization routine MINUIT[20] to fit the phase-shifts; the partial widths $\gamma_{\lambda c}$ and the level energies E_λ were treated as free parameters. Neutron and proton emission partial widths were not necessarily assumed to be equal, contrary to previous analyses. A difference between γ_n and γ_p in the 1^+ and 2^+ states may be related to their possible interpretation in terms of threshold states[25,26]. Only the real part of the phase-shifts were analyzed in terms of R-matrix for the reason pointed out in Sect. 3.

The phase-shifts not related to the 2^+ and 1^+ resonances were fitted to check the absence of nearby levels. The expected hard-sphere behaviour was fully confirmed by the results of the R-matrix analysis, and in particular holds for the previously mentioned p-levels.

4.2. - Results and Discussion

4.2.1. - 2^+ Resonance

The real part of the δ_2^2 phase shift exhibits (see Fig. 8) a clear resonant behaviour, which is well fitted by a

strict one-level approximation and three channels with a weighted variance (χ^2 per degree of freedom) of about 1.1.

The addition of a background matrix R^o gives a more realistic shape of the fitting curve, in particular in the higher energy range (although the weighted variance is not improved); this trend matches the behaviour at even higher energies[7]. No further improvement was achieved by introducing a second level, thus we present in Table I the results in the one-level + R^o hypothesis, for the three considered boundary conditions.

The values of Table I confirm the expected dependence of the level energy and the reduced widths on the boundary conditions (in particular E_λ); whereas the resonance energy E_R (*) and total level width Γ turn out to be more stable.

Since, however, only in the case $B_c=S_c(E_R)$, E_λ lies within the width of the observed resonance, this boundary condition seems to be the most adequate[17] to fit the data in a single-level approximation.

The values of the reduced widths, independently on the boundary conditions, are larger than those previously obtained. This can be mainly due to a difference in the anlysis, since our phase-shifts do not differ appreciably from previous ones.

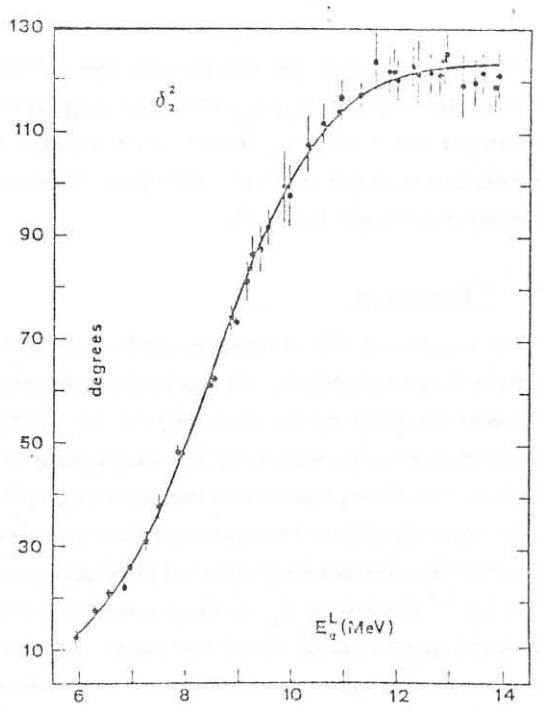

FIG. 8 - Real part of the δ_2^2 phase-shift. The continuous line is the fitting curve corresponding to the boundary condition $B_c=S_c(E_R)$. Very similar curves were obtained in the other conditions (see Table I).

TABLE I - Level parameters of 2^+ (T=0) resonance in ^{6}Li.

E^* (MeV)	E_R (MeV)	E_λ (MeV	$\lvert\gamma_\alpha\rvert$ ($MeV^{1/2}$)	$\lvert\gamma_p\rvert$ ($MeV^{1/2}$)	$\lvert\gamma_n\rvert$ ($MeV^{1/2}$)	Γ (MeV)	a_c (fm)	Ref.	Boundary cond.	Analys.
4.87	3.39	7.923	2.4	-	-	-	3.5	(3)	$B_c=-\ell$	a
4.6±0.1	-	6.2	1.3	1.1	1.1	-	4.3	(4)	$B_c=-\ell$	b
4.7	3.2	6.0	1.3	1.1	1.1	-	4.2	(6)	$B_c=-\ell$	b
4.31±0.03	-	-	-	-	-	1.5±0.2	-	(1)	-	
4.55±0.01	3.08±0.01	3.08±0.01	3.41±0.04	3.36±0.04	5.10^{-4}±5.	1.61	4.13	Present work	$B_c=S_c(E_R)$	c
4.54±0.01	3.07±0.01	-4.89±0.13	1.78±0.01	3.45±0.01	0.14±2.	1.61	4.13	"	$B_c=0$	c
4.54±0.02	3.07±0.02	8.24±0.08	2.75±0.06	2.55±0.12	0.34±0.18	1.58	4.13	"	$B_c=-\ell$	c
4.55±0.01	3.08±0.01	3.08±0.01	3.39±0.58	3.34±0.70	3.34±0.70	1.61	4.13	"	$B_c=S_c(E_R)$	d
4.54±0.02	3.07±0.02	-3.02±0.29	1.86±0.04	1.28±0.08	1.28±0.08	1.61	4.13	"	$B_c=0$	d
4.54±0.02	3.07±0.02	14.9±1.7	2.79±0.17	2.60±0.21	2.60±0.21	1.58	4.13	"	$B_c=-\ell$	d

a) One level, one channel
b) One level
c) One level + R^o
d) One level + R^o; $\lvert\gamma_n\rvert=\lvert\gamma_p\rvert$.

(*) We define E_R as the energy where the resonant phase-shift is equal to $\pi/2$; Γ the energy interval in which the resonant phase increases from $\pi/4$ to $3\pi/4$.

On the other hand, the results with the constraint $\gamma_n = \gamma_p$ (as used in previous works) have comparable confidence level as for $\gamma_n \neq \gamma_p$. This fact suggest that γ_n plays a minor role in the fitting procedure. For the case without constraint γ_n results much smaller than γ_p. It is therefore not possible to give any definite interpretation in terms of threshold states. Moreover the elastic reduced width is not drastically affected by this constraint, except for $B_c=0$.

4.2.2. - 1^+ Resonance

The analysis of this resonance, performed in Ref. (8), within a strict single-level approximation, showed that there is some inadequacy in reproducing the data, mainly in the energy region far from the resonance. For this reason we have taken into account the contributions of a possible additional level and of a diagonal background matrix. Moreover, as previously done in the 2^+ case, we allowed for a flexibile parametrization of the inelastic channels, leaving the corresponding reduced widths as independent parameters.

The most significant results of our fitting procedure, for various boundary conditions, are shown in Table II, where also the corresponding values of previous references are given.

As for 2^+ resonance E_λ is very sensitive to the choice of the boundary condition and $B_c=S_c(E_R)$ in the single-level approximation seems the most consistent. The results with and without the constraint $\gamma_n=\gamma_p$ have a comparable confidence level. When the two reduced widths are independent there is a clear tendency to a solution with $\gamma_p \gg \gamma_n$ (the same effect observed for the 2^+ level), whereas the constraint $\gamma_n=\gamma_p$ leads to results in a good agreement with previous ones.

We stress that the confidence level of the fit greatly improves in respect to Ref. (8) using a background matrix R^o (see Fig. 9); no similar effect arises when introducing additional levels. These good values for the weighted variance (~ 1.5) are not only due to a more realistic evaluation of the phase-shifts errors but also to a better agreement between fitting curves and our data. The imaginary part of ε^1, predicted by R-matrix fit of the real parts of the phase-shifts and mixing parameter, shows a qualitative agreement with the "experimental" imaginary part, in particular as far as the sign is concerned.

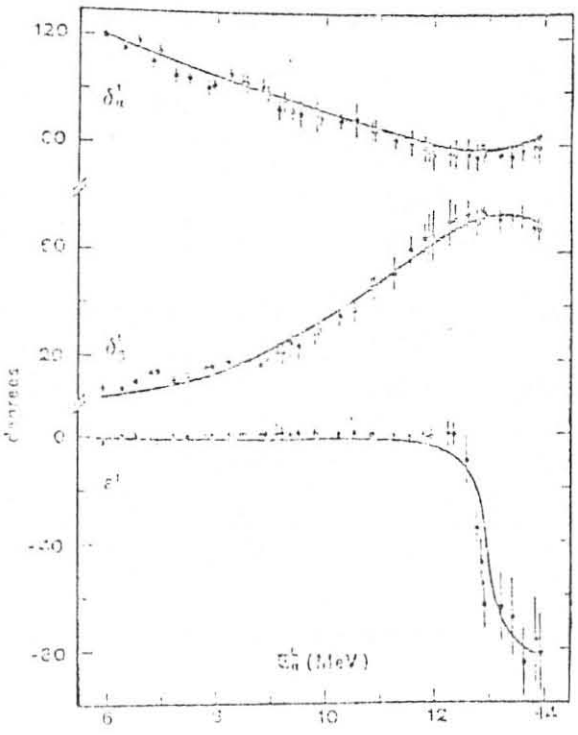

FIG. 9 - Real part of the δ_α^1 and δ_β^1 eigen phase-shifts and ε^1 mixing parameter. The continuous lines are the fitting curves corresponding to the boundary condition $B_c=S_c(E_R)$. Very similar curves were obtained in the other conditions (see Table II).

TABLE II - Level parameters of $1^+(T=0)$ resonance in ^6Li.

E^x (MeV)	E_R (MeV)	E (MeV)	$\lvert\gamma_\alpha\rvert$ $\ell=0$ ($MeV^{1/2}$)	$\lvert\gamma_\alpha\rvert$ $\ell=2$ ($MeV^{1/2}$)	γ_p ($MeV^{1/2}$)	γ_n ($MeV^{1/2}$)	Γ (MeV)	a_c (fm)	Ref.	Boundary cond.	Analys.
6.24	7.16	10.925	-	2.4	-	-	-	3.5	(3)	$B_c=-\ell$	a
8.4	5.7 ± 0.1	1.4	0.1	1.4	0.7	0.7	-	4.3	(4)	$B=-\ell$	b
5.7	4.26	9.8	0.05	1.8	0.9	0.9	-	4.1	(6)	$B_c=-\ell$	b
5.65 ± 0.01	4.16	8.75 ± 0.01	0.07 ± 0.01	1.6	0.9	0.9	-	4.12	(8)	$B_c=-\ell$	b
5.65 ± 0.05	-	-	-	-	-	-	$1.0^{+0.6}_{-0.4}$	-	(1)	-	-
5.77 ± 0.02	4.30 ± 0.02	4.30 ± 0.02	0.02 ± 0.01	1.48 ± 0.02	2.43 ± 0.78	$2.10^{-5}\pm0.6$	1.71	4.13	Present work	$B_c=S_c(E_R)$	c
5.88 ± 0.02	4.41 ± 0.02	1.82 ± 0.015	0.03 ± 0.01	1.49 ± 0.03	2.66 ± 1.00	$6.10^{-3}\pm0.2$	1.81	4.13	"	$B_c=0$	c
5.88 ± 0.02	4.41 ± 0.02	13.57 ± 0.14	0.03 ± 0.01	1.50 ± 0.02	2.71 ± 0.02	$3.10^{-4}\pm0.2$	1.81	4.13	"	$B_c=-\ell$	c
5.80 ± 0.01	4.33 ± 0.01	4.33 ± 0.01	0.03 ± 0.01	1.50 ± 0.02	0.89 ± 0.77	0.89 ± 0.77	1.74	4.13	"	$B_c=S_c(E_R)$	d
5.88 ± 0.02	4.41 ± 0.02	2.20 ± 0.10	0.03 ± 0.01	1.50 ± 0.02	0.89 ± 0.20	0.89 ± 0.20	1.81	4.13	"	$B_c=0$	d
5.88 ± 0.02	4.41 ± 0.02	17.31 ± 0.01	0.03 ± 0.01	1.51 ± 0.01	2.39 ± 0.01	2.39 ± 0.01	1.81	4.13	"	$B_c=-\ell$	d

a) One level, one channel
b) One level
c) One level + R^o
d) One level + R^o; $\lvert\gamma_n\rvert=\lvert\gamma_p\rvert$.

Finally we note that the ratio of s-wave to d-wave squared reduced widths is less than 4×10^{-4} (to be compared to 2×10^{-3} found in Ref. (8)) suggesting that the effects of the tensor interaction are rather small in this resonance. Indeed our data are consistent with smaller values for $|U^1_{0,2}|$ than those found in Ref. (8) which were already lower than resonating group estimates[27] and theoretical Faddeev calculations[22] (see Fig. 10).

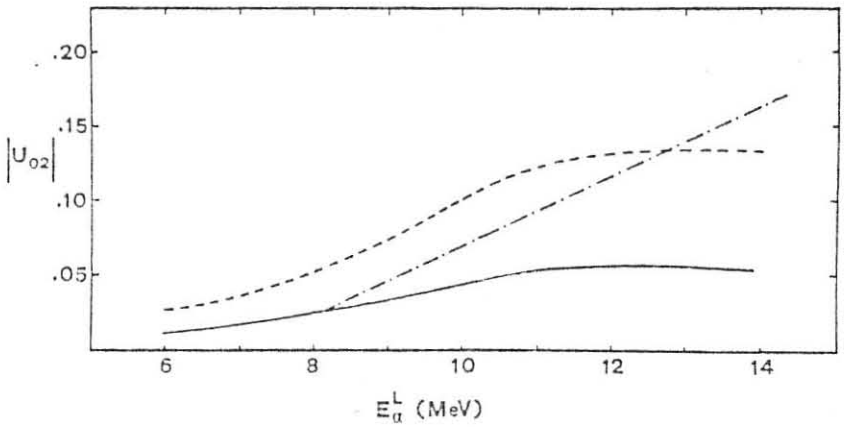

FIG. 10 - Absolute value of the off-diagonal matrix element for the $J=1^+$ complex. Continuous line: evaluated from phase-shifts of the present work; Dashed line: predictions of Faddeev calculations[22]; Dashed dotted line: results from a resonating group calculation[27]; Results from Ref. (8) (not represented here) lie between continuous and dashed-dotted lines.

5. - CONCLUSIONS

We shortly summarize our main results:

- We performed a phase-shift analysis of a large amount of angular distributions at different energies. The good confidence level of the phase-shifts was tested with the predictions for total reaction cross sections and vector and tensor analyzing powers.
- Our phase-shifts agree with previous results[6,8] (e.g. no evidence for p-wave levels near our energy range). As a general trend the shapes of our phase-shifts are better determined since we have performed measurements in much smaller energy steps.
- The comparison of the phase-shifts with three-body calculations, based on Faddeev equations, suggest that the 1^+ resonance has to be theoretically investigated with better microscopic inputs for the two body potentials, e.g. for the strenght of the tensor force.
- A multichannel multilevel R-matrix analysis shows that the 2^+ resonance can be described by a strict single-level approximation; in the parametrization of the 1^+ resonance, on the contrary, a background diagonal matrix greatly improves the quality of the fits. The reduced widths, however, show a tendency to larger values than those previously obtained.
- The R-matrix analysis exibits a dependence of E_λ and $\gamma_{\lambda c}$ on the boundary conditions, whereas E_R and Γ are rather stable. In any case the values obtained for Γ support the use of $B_c = S_c(E_R)$ in the single-level approximation.

- The strength of the tensor interaction in the 1^+ complex is much weaker than theoretically estimated by resonating group method[27] and by a Faddeev three-body approach[21].

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APPENDIX

The centre of mass differential cross sections, for elastic scattering of spin-one by spin-zero particles, may be written as in Ref. (2):

$$\frac{d\sigma(\vartheta^{CM})}{d\Omega} = \frac{1}{3k^2}\left\{2|A|^2+|B|^2+\left[|C|^2+|D|^2\right]\sin^2\vartheta^{CM}+|E|^2\sin^4\vartheta^{CM}\right\}$$

where k is the wave number associated with the relative motion and ϑ^{CM} is the centre of mass scattering angle. We report here, for completeness, the formal dependence of the quantities A, B, C, D, E on the elements of the collision matrix; we remark the sign changes in the terms containing off diagonal elements of the collision matrix, as assumed first in Ref. (3).

$$A = R_\sigma + \sum_\ell (e^{\frac{1}{2}i\alpha_\ell}/4i)\left\{e^{\frac{1}{2}i\alpha_\ell}P_\ell\left[(\ell+2)U_{\ell,\ell}^{\ell+1}+(2\ell+1)U_{\ell,\ell}^{\ell}+(\ell-1)U_{\ell,\ell}^{\ell-1}-2(2\ell+1)\right]\right.$$
$$\left.+e^{\frac{1}{2}i\alpha_{\ell+2}}P_{\ell+2}\left[(\ell+1)(\ell+2)\right]^{\frac{1}{2}}U_{\ell,\ell+2}^{\ell+1}+e^{\frac{1}{2}i\alpha_{\ell-2}}P_{\ell-2}\left[\ell(\ell-1)\right]^{\frac{1}{2}}U_{\ell,\ell-2}^{\ell+1}\right\}$$

$$B = R_\sigma + \sum_\ell (e^{\frac{1}{2}i\alpha_\ell}/2i)\left\{e^{\frac{1}{2}i\alpha_\ell}P_\ell\left[(\ell+1)U_{\ell,\ell}^{\ell+1}+\ell U_{\ell,\ell}^{\ell-1}-(2\ell+1)\right]-\right.$$
$$\left.-e^{\frac{1}{2}i\alpha_{\ell+2}}P_{\ell+2}\left[(\ell+1)(\ell+2)\right]^{\frac{1}{2}}U_{\ell,\ell+2}^{\ell+1}-e^{\frac{1}{2}i\alpha_{\ell-2}}P_{\ell-2}\left[\ell(\ell-1)\right]^{\frac{1}{2}}U_{\ell,\ell-2}^{\ell-1}\right\}$$

$$C = \sum_\ell (e^{\frac{1}{2}i\alpha_\ell}/2i)\left\{\left[e^{\frac{1}{2}i\alpha_\ell}P'_\ell/\ell(\ell+1)\right]\left[\ell(\ell+2)U_{\ell,\ell}^{\ell+1}-(2\ell+1)U_{\ell,\ell}^{\ell}-(\ell^2-1)U_{\ell,\ell}^{\ell-1}\right]-\right.$$
$$\left.-e^{\frac{1}{2}i\alpha_{\ell+2}}P'_{\ell+2}\left[(\ell+1)/(\ell+2)\right]^{\frac{1}{2}}U_{\ell,\ell+2}^{\ell+1}+e^{\frac{1}{2}i\alpha_{\ell-2}}P'_{\ell-2}\left[\ell/\ell-1\right]^{\frac{1}{2}}U_{\ell,\ell-2}^{\ell-2}\right\}$$

$$D = \sum_\ell (e^{\frac{1}{2}i\alpha_\ell}/2i)\left\{e^{\frac{1}{2}i\alpha_\ell}P'_\ell\left[U_{\ell,\ell}^{\ell+1}-U_{\ell,\ell}^{\ell-1}\right]+e^{\frac{1}{2}i\alpha_{\ell+2}}P'_{\ell+2}\times\left[(\ell+1)/(\ell+2)\right]^{\frac{1}{2}}U_{\ell+2}^{\ell+1}-\right.$$
$$\left.-e^{\frac{1}{2}i\alpha_{\ell-2}}P'_{\ell-2}\left[\ell/(\ell-1)\right]^{\frac{1}{2}}U_{\ell,\ell-2}^{\ell-1}\right\}$$

$$E = \sum_\ell (e^{\frac{1}{2}i\alpha_\ell}/2\sqrt{2}\,i)\left\{\left[e^{\frac{1}{2}i\alpha_\ell}P''_\ell/\ell(\ell+1)\right]\left[\ell(U_{\ell,\ell}^{\ell+1}-(2\ell+1)U_{\ell,\ell}^{\ell}+(\ell+1)U_{\ell,\ell}^{\ell-1}\right]+\right.$$
$$\left.+(e^{\frac{1}{2}i\alpha_{\ell+2}}/\left[(\ell+1)(\ell+2)\right]^{\frac{1}{2}})P''_{\ell+2}U_{\ell,\ell+2}^{\ell+1}+(e^{\frac{1}{2}i\alpha_{\ell-2}}/\left[\ell(\ell-1)\right]^{\frac{1}{2}})P''_{\ell-2}U_{\ell,\ell-2}^{\ell-1}\right\}$$

where:

- $U_{\ell,\ell}^{J}$ is the element of the scattering matrix.
- R_σ is the Rutherford amplitude, expressed by:

$$R_\sigma = -\frac{1}{2}\eta\,\csc^2\frac{\vartheta^{CM}}{2}\exp\left[i\eta\ln\csc^2\frac{\vartheta^{CM}}{2}\right]$$

with $\eta = \frac{z_1 z_2 e^2}{\hbar v}$; $\alpha_o = 0$; $\frac{\alpha_\ell}{2} = \arg\Gamma(1+\ell+i\eta)$

- $P_\ell(\cos\vartheta^{CM})$, $P'_\ell(\cos\vartheta^{CM})$, $P''_\ell(\cos\vartheta^{CM})$ are the Legendre polynomyals and their derivatives.

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