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A STUDY OF A TRANSVERSE OPTICAL KLYSTRON EXPERIMENT IN ADONE (TOKA)

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ABSTRACT. -

The storage ring operation of a free electron laser can be improved by a prebunching of the electron beam. We study in this paper a layout working as a Transverse Optical Klystron. The enhancement of the single pass gain and the consequent reduction of the wiggler length in the TOK compared with the FEL suggest that the first device is more suitable for a storage ring as Adone, where the straight sections are about two meters. The figures of the TOKA are carried out using as much as possible the hardware of the FEL experiment which is in progress at Adone (LELA experiment).

1. - INTRODUCTION

Just after the first free electron laser (FEL) operation[1,2], it was pointed out that the storage rings (SR) are well suited for FEL operation in the short wavelength region. This for the good quality of the beam, the high peak current and the high efficiency due to the possibility to re-accelerate the electrons after their deceleration owing to the emission[3].

A great deal of papers have been produced so far about the installation of a FEL device in a SR[4].

At Frascati INFN Laboratories, Barbini and Vignola have proposed a FEL experiment for Adone (LELA)[5] and it will soon operate.

With the aim to enhance the oscillator gain we have studied a variation of the FEL generator. The idea is to prebunch the beam before the interaction with the wiggler and the optical wave[6,7,8]. Since in this way the principle of operation of the device is the same as the microwave klystron, it has been called transverse optical klystron (TOK)[7].

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In this paper we shortly review the theory presented in refs. (6, 7, 8) with more attention on the transverse beam motion.

The figures of the TOK for Adone are presented. These have been calculated using the parameters of the LELA experiment.

Here we want to say that our calculations, having considered the transverse motion of the electrons, give the same bunching coefficient of the beam as ref. (9).

2. - THE TOKA.

The schematic of the device is shown in Fig. 1.

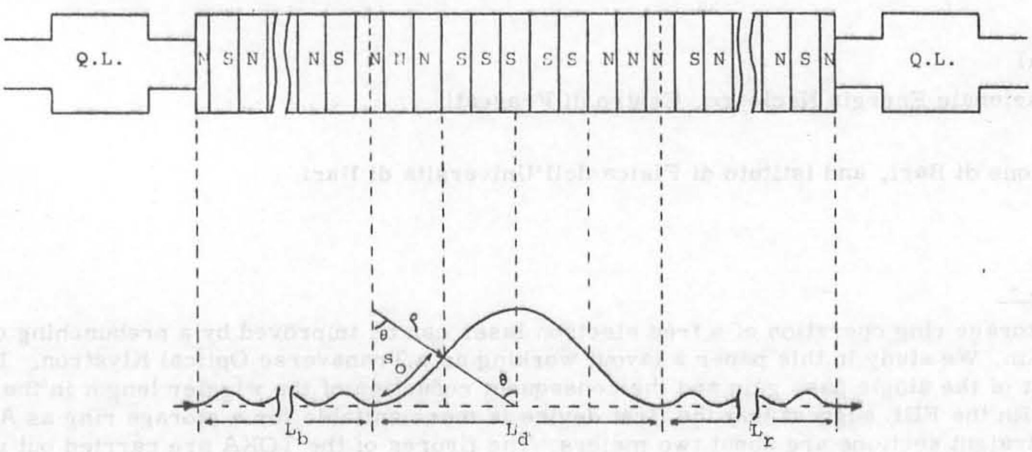

FIG. 1 - Schematic of the TOKA : the 20 periods of the wiggler are rearranged in a section $L_b = 8\lambda_w$ for the buncher, $L_d = 5\lambda_w$ for the dispersive drift region and a section $L_r = 7\lambda_w$ for the radiator.

The LELA wiggler is divided in three parts in order to have a section, the buncher, for the energy modulation of the beam, a section, the dispersive drift space, for the transformation of this modulation into a density modulation and ultimately the radiator for the coherent interaction of the bunched electron beam with the input wave for the emission.

3. - BUNCHING THEORY.

The evolution of the beam from the homogeneous distribution in space (along the z-axis) to a bunched one is studied with the one-dimensional collisionless Vlasov equation (avoiding the problem of the finite beam emittance at first stage) as done by Hopf et al. in ref. (10). The beam is taken with an initial gaussian energy distribution. We are used to treat the problem in the electron beam system (EBS) since in this reference frame in the buncher we have a static potential pattern which obviously brings out the energy modulation.

3. 1. - The buncher.

In the buncher the Vlasov equation reads

$$\frac{\partial \varrho_b}{\partial t_b} + \dot{z}_b \frac{\partial \varrho_b}{\partial z_b} + \dot{P}_{bz} \frac{\partial \varrho_b}{\partial P_z} = 0 \tag{1}$$

where the index b refers to quantities in the $(EBS)_b$ which travels with the mean velocity along the horizontal axis.

To find $\dot{z}_b$ and $\dot{P}_{bz}$ we start with the Hamiltonian of the system

$$H = c\left[(\vec{P}_b - e\vec{A}_b)^2 + m^2 c^2\right]^{1/2} \tag{2}$$

since them

$$\dot{z}_b = \frac{\partial H}{\partial P_{bz}} , \qquad \dot{P}_{bz} = -\frac{\partial H}{\partial z_b} . \tag{3}$$

We notice that in refs. (6, 7, 8) we have considered only the case of non-relativistic electrons in the $(EBS)_b$, that is the case with low static magnetic field B_o. In the present experiment B_o = = 4.5 KG, which means a $p_\perp \sim 1.2$ MeV thus the above approximation is no more good.

If we let the Lorentz factor of the $(EBS)_b$ relative to the LAB γ_b, considering the energy transformation between the two systems it is easy to find that

$$H = m_o \gamma_\perp c^2 \tag{4}$$

with

$$\gamma_\perp = \frac{\gamma}{\gamma_b} = (1 + h^2)^{1/2} \tag{5}$$

being $h = eB_o / 1.414\, m_o c^2 k_w$ (x).

The term $m_o \gamma_\perp$ can be thought as the effective electron mass in the buncher.

With the well known considerations on the Hamiltonian(8, 10) we get

$$P_{bz} = p_{bz} , \qquad \vec{p}_\perp = -e\vec{A}_\perp + \text{const} . \tag{6}$$

However $\langle \vec{p}_\perp \rangle = 0$ as $\langle \vec{A}_\perp \rangle = 0$ and besides $\vec{p}_\perp$ and $\vec{A}_\perp$ have always opposite directions, when $\vec{p}_\perp = 0$ (at the maximum of the magnetic field) $\vec{A}_\perp$ is zero too, thus the constant must be zero. In these hypothesis we have

$$\dot{z}_b = \frac{p_{bz}}{m_o \gamma_\perp} , \qquad \dot{p}_{bz} = \frac{1}{2} \frac{e^2}{m_o \gamma_\perp c} \frac{\partial A_b^2}{\partial z_b} \tag{7}$$

(x) - We notice that our parameter h corresponds in the FEL literature to parameter K ; the symbol change has been made to avoid misleading.

and eq. (1) reads

$$\frac{\partial \varrho_b}{\partial t_b} + \frac{p_{bz}}{m_o \gamma_\perp} \frac{\partial \varrho_b}{\partial z_b} + \left(\frac{e^2}{2 m_o \gamma_\perp c} \frac{\partial A_b^2}{\partial z_b} \right) \frac{\partial \varrho_b}{\partial p_{bz}} = 0 \qquad (8)$$

being $\vec{A}_b = \vec{A}_{wb} + \vec{A}_{sb}$ and $\vec{A}_{wb}$ and $\vec{A}_{sb}$ the vector potentials relative to the wiggler and laser fields respectively.

As we have shown in our previous work, when the synchronism condition is met (in LAB)

$$\lambda_s = \left(\frac{1}{\beta_b} - 1 \right) \lambda_w \simeq \frac{\lambda_w}{2\gamma_b^2} \qquad (9)$$

a static potential pattern is built up

$$V_b = V_{bo} \cos (k_{b+} \; z_b + \varphi) \qquad (10)$$

which causes the energy modulation of the beam.

With the force derived from that potential the electron motion is periodic with a frequency

$$\Omega_b \simeq \left(\frac{V_{bo}}{m_o \gamma_\perp c} \right)^{1/2} k_{b+} \qquad (11)$$

and the maximum bunching with a buncher long enough would occur approximately at

$$t_b \approx \frac{1}{4} \frac{2\pi}{\Omega_b} . \qquad (12)$$

Recalling that

$$V_{bo} = \frac{e^2}{2 m_o \gamma_\perp c} \frac{E_o B_o}{k_s k_w} , \quad k_{b+} = k_{bw} + k_{bs} \simeq \gamma_b k_w \qquad (13)$$

it is easy to calculate the bunching time in LAB frame

$$t \approx \pi m_o \gamma c / (e^2 c E_o B_o)^{1/2} \qquad (12')$$

E_o is the laser field.

3.2. - Drift region.

When the beam is energy modulated with the wavelength λ_s (since the potential pattern period is $\lambda_+ \simeq \lambda_s$) the two beam sections, ($\lambda_s/2$) long, belonging to each wavelength tend to superimpose owing to their energy gap. In the $(EBS)_d$ the two sections travel one against the other (see Fig. 2).

The maximum bunching length can be assessed with the following elementary argument. From Fig. 2 we see that the crest particles must drift for a time

$$t = \frac{\lambda_s}{4 \Delta v} \qquad (14)$$

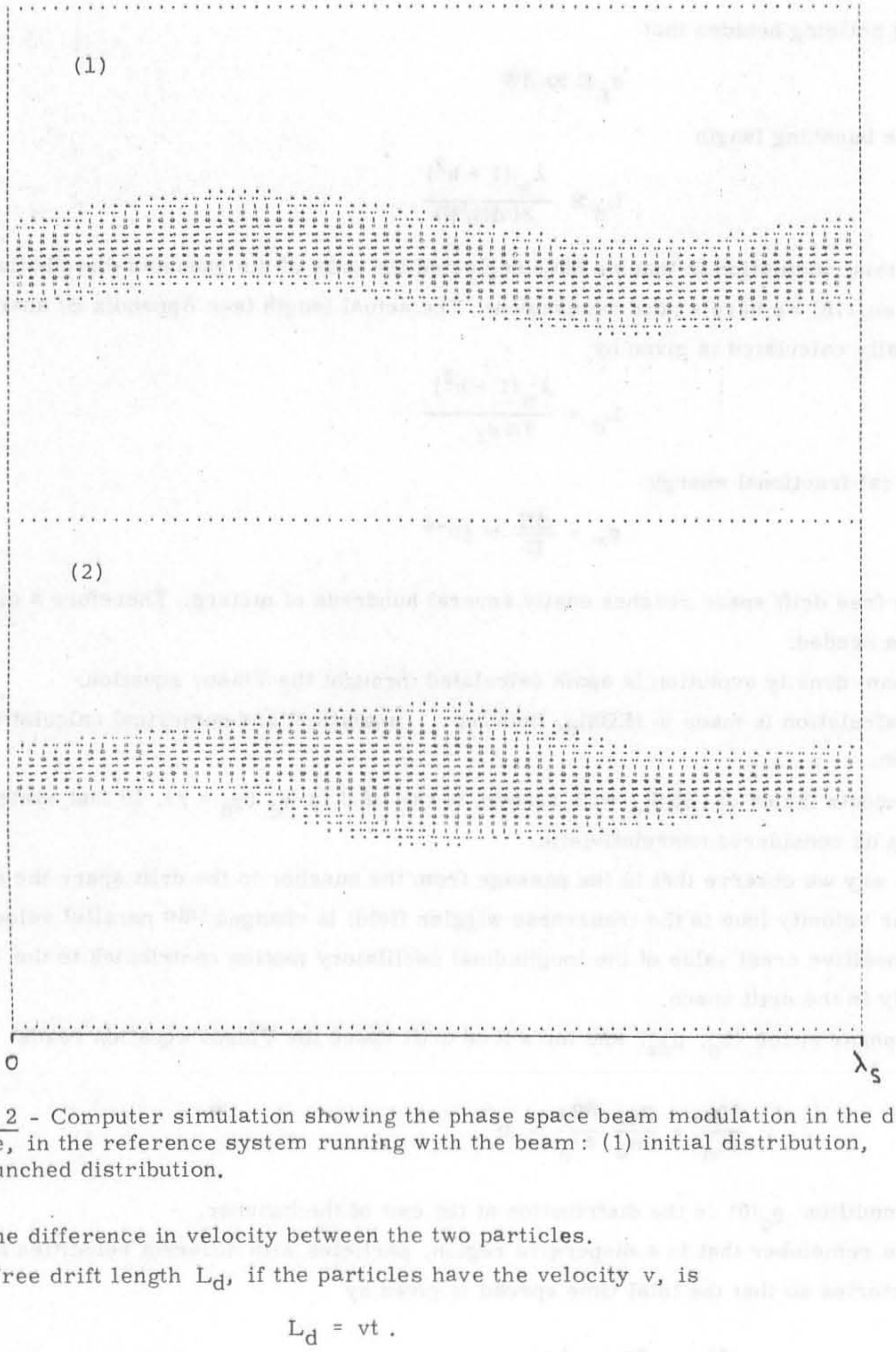

FIG. 2 - Computer simulation showing the phase space beam modulation in the drift space, in the reference system running with the beam : (1) initial distribution, (2) bunched distribution.

with Δv the difference in velocity between the two particles.

The free drift length L_d, if the particles have the velocity v, is

$$L_d = vt . \tag{15}$$

With the (14) and observing that

$$\frac{\Delta v}{v} = \frac{1}{\gamma^2} \frac{\Delta E}{E} , \quad \lambda_s = \frac{\lambda_w}{2\gamma^2}(1 + h^2) , \quad \Delta E \simeq 2(\sigma_\varepsilon E + \Delta W) \tag{16}$$

σ_ε being the initial energy spread, ΔW the modulation induced by the interaction within the buncher, and noticing besides that

$$\sigma_{\varepsilon} E \gg \Delta W \tag{17}$$

we get for the bunching length

$$L_d \approx \frac{\lambda_w (1 + h^2)}{8(\Delta E/E)} \quad . \tag{18}$$

The limit of this calculation is that we have neglected the tails of the gaussian distribution. Nevertheless with eq. (18) we have a good assessment. The actual length (see Appendix B) analitically and numerically calculated is given by

$$L_d = \frac{\lambda_w (1 + h^2)}{4\pi \sigma_{\varepsilon}} \quad . \tag{18'}$$

With the typical fractional energy

$$\sigma_{\varepsilon} = \frac{\Delta E}{E} \sim 10^{-4} \tag{19}$$

of the SR the free drift space reaches easily several hundreds of meters. Therefore a dispersive drift space is needed.

The beam density evolution is again calculated throught the Vlasov equation.

This calculation is made in $(EBS)_d$, both for analytical and numerical calculations simplification.

The Lorentz factor of $(EBS)_d$ with respect to the LAB is γ_d ($\gamma_d = \gamma$). In that system the particles can be considered nonrelativistic.

By the way we observe that in the passage from the buncher to the drift space the electron perpendicular velocity (due to the transverse wiggler field) is changed into parallel velocity and besides the positive crest value of the longitudinal oscillatory motion contributes to the z-axis beam velocity in the drift space.

In the phase space (z_d, p_{dz}) and for a free drift space the Vlasov equation reads

$$\frac{\partial \varrho_d}{\partial t_d} + \frac{p_{dz}}{m_o} \frac{\partial \varrho_d}{\partial z_d} = 0 \quad . \tag{20}$$

The initial condition $\varrho_d(0)$ is the distribution at the exit of the buncher.

Now we remember that in a dispersive region, particles with different velocities follow different trajectories so that the total time spread is given by

$$\frac{\Delta t}{t} = \frac{\Delta s}{s} - \frac{\Delta v}{v} \tag{21}$$

due to the additional effect of both the trajectory and velocity spread. The right hand side of eq. (21) can be expressed in terms of the momentum spread[11]

$$\frac{\Delta s}{s} = \alpha_c \frac{\Delta p}{p} \quad , \qquad \frac{\Delta v}{v} = \frac{1}{\gamma^2} \frac{\Delta p}{p} \tag{22}$$

therefore

$$\frac{\Delta t}{t} = (\alpha_c - \frac{1}{\gamma^2})\frac{\Delta p}{p} \tag{23}$$

where the drift region "momentum compaction" α_c is vanishing for a free space. In this last case we recall that

$$\frac{\Delta t}{t} = -\frac{1}{\gamma^2}\frac{\Delta p}{p} \; . \tag{24}$$

For this we can define an effective Lorentz factor

$$\frac{1}{\gamma_{eff}} = \frac{1}{\gamma^2} - \alpha_c \tag{25}$$

so that eq. (23) can be written

$$\frac{\Delta t}{t} = -\frac{1}{\gamma_{eff}^2}\frac{\Delta p}{p} \tag{26}$$

and our dispersive drift space can be considered as a free space for particles having a Lorentz factor γ_{eff}.

The momentum compaction α_c for the particular case of TOKA is calculated in Appendix A.

About the bunching length z we have to observe that in free space the particle trajectory lengthening Δz and the momentum spread are related by

$$\frac{\Delta z}{z} = \frac{1}{\gamma^2}\frac{\Delta p}{p} \; . \tag{27}$$

In dispersive space, that is for particles having an effective Lorentz factor γ_{eff}, the dispersive drift length z_{eff} corresponding to the same momentum modulation and bunching amplitude is

$$\frac{\Delta z}{z_{eff}} = \frac{1}{\gamma_{eff}^2}\frac{\Delta p}{p} \; . \tag{28}$$

From eqs. (27), (28) and (29) for the length in dispersive space we obtain

$$z_{eff} = \frac{z}{1 - \alpha_c \gamma^2} \; . \tag{29}$$

Since $\alpha_c \gamma^2 \gg 1$

$$z_{eff} - \frac{z}{\alpha_c \gamma^2} \qquad \text{(with regard to sign)} \; . \tag{29'}$$

For very short interaction time the evolution eq. (8) can be solved analytically with the method of Fourier analysis[(6)]. The obtained solution is then used as initial condition for eq. (20). The electron density at the output of the bunching section is found after the integration over all the momenta of the distribution function $\varrho_d(z_d, p_{dz}, t_d)$. Here t_d is the transit time in the drift space.

At the end the electron beam density has the form (in LAB)

$$\varrho(z,t) = \sum_{0}^{\infty}{}_m \; A_m(t_1,t_2)\cos\left[m(k_+z-\omega_+t)\right] . \tag{30}$$

The coefficients A_m depend on the time interaction within the buncher t_1, and the time interaction within the drift space t_2.

The analytical expression of the first two-three harmonics can be found (in Appendix B the first harmonic calculation is made). For the maximum value of the first real harmonic coefficient the result is

$$\left| A_1 \right| \simeq \frac{0.61}{2} \; \frac{e^2}{m_o^2 \gamma^2 c^3} \; \frac{B_o^{(b)} E_o^{(b)}}{\sigma_\varepsilon k_w^{(b)}} \; L_b \tag{31}$$

the superscript index b remembers the buncher parameters.

For the corresponding dispersive drift length it is found

$$L_d \approx \frac{\lambda_w^{(b)}(1+h^2)}{4\pi\alpha_c \gamma^2 \sigma_\varepsilon} . \tag{32}$$

To obtain the first harmonic amplitude at any time the formula (B.12) of Appendix B must be used.

The evolution eqs. (8) and (20) have been solved numerically with the method of finite differences[12] in order to find all the harmonic coefficients A_m at any time of interaction.

To find these coefficients we can alternatively use the exact solution of the evolution equations given in ref. (13).

4. - EMITTED POWER FROM A BUNCHED BEAM.

During the interaction within the radiator for the coherent emission, the beam will be considered "frozen". This assumption is shurely true because of the ultrarelativistic electron energy[14]. The high velocity allows us to neglect, in first approximation, the coulomb interaction in spite of the high electron density usual in storage rings.

Two cases will be considered : the first one refers to the spontaneous coherent emission from a bunched electron beam within a radiator, the second one refers to the stimulated emission, that is to the emission occurring when an input electromagnetic wave runs with the electron bunches in the radiator.

In the optical klystron or converter this is simply realized by adding two mirrors to the radiator.

4.1. - Spontaneous coherent radiation.

In this case the emitted power can be calculated with the classical electrodynamics method of retarded potentials[15].

The electron wave current is given by

$$\vec{J} = e\,c\,\vec{\beta}(z)\,\varrho_o \sum_0^{\infty} m\, A_m(t_1, t_2) \cos\left[m(k_+ z - \omega_+ t)\right] \tag{33}$$

where

$$\vec{\beta}(z) = \beta b \cos k_r z\, \hat{y} + \beta_r\, \hat{z}\,. \tag{34}$$

With some complicated mathematics, the emitted power per unit solid angle in the forward direction and on the m-th harmonic can be expressed as[6]

$$\frac{dP_m}{d\Omega} = \left(\frac{dP_s}{d\Omega_s}\right) \frac{\beta^2}{\beta_r}\, \varrho_o N_r \frac{1}{4} \left(\frac{\lambda_s}{m}\right) A_m^2 \tag{35}$$

with

$$\frac{dP_s}{d\Omega_s} = \frac{c\,e^2}{4}\, b^2 \left(\frac{\beta_r}{1-\beta_r}\right)^3 N_r k_r \beta_r \varrho_o\,. \tag{36}$$

This last is the spontaneous emitted power per unit solid angle in the forward direction from an unbunched beam.

The parameter $b = eB_o^{(r)}/k_r p$ gives the sine of the maximum deflexion angle of the electron trajectory with respect to the magnet axis and k_r, N_r are respectively the wave-number and the number of periods of the radiator. Finally ϱ_o is the initial (unmodulated) electron beam density.

The power (35) is due to the coherent emission of the particles packed in a quarter wavelength

$$\varrho_o N_r \frac{\lambda_s}{m} \approx \varrho_o N_r \frac{\lambda_r}{2\gamma_r^2} = \varrho_o \frac{L_r}{2\gamma_r^2}\,. \tag{37}$$

Here L_r is the radiator length and γ_r the effective Lorentz factor associated with the average velocity of the particles along the z-direction within the radiator $\beta_r c$.

This number represent a very small fraction of the total electrons contained in the radiator. This can be understood with an heuristic argument.

In order to assess the number of the particles which are coherently emitting, we look at this problem in a system $(EBS)_r$ which moves with the average velocity $\beta_r c$ along the radiator z-axis. In that frame the electrons are at rest or slowly moving. However the radiator moves in the negative z-direction, then the particles which at the time t_r cooperate to the coherent emission are those ones contained in the Lorentz contracted radiator length L_r/γ_r, and their number is $\varrho_r L_r/\gamma_r$. Furthermore for the Lorentz expansion of the beam length the particles density is lowered by a factor γ_r: $\varrho_r = \varrho_o/\gamma_r$. At the end the number of electrons which are coherently emitting will be : $N_e \approx \varrho_o L_r/\gamma_r^2$. This number is of course the same in the laboratory frame.

In the literature N_e is the well known "cooperation number".

The general relation joining the radiator and buncher periods for small deflexion angles ($\beta_b \simeq \beta_r \simeq 1$) is given by

$$\lambda_r = \lambda_w/m \tag{38}$$

and if $m \neq 1$ the device works as a frequency up-converter.

It can be shown that the most part of the radiation is concentrated in a solid angle[16]

$$\Delta\Omega = \frac{1+h^2}{2N_r\gamma^2} \tag{39}$$

where the power has approximately the peak value of the first harmonic. In eq. (39)

$$h = eB_o^{(r)}\lambda_r/2.83\pi m_o c\ . \tag{40}$$

For relativistic electrons the incoherent power on the peak of the first harmonic is obtained from the formula (36) (in MKS units)

$$P_s \simeq \left(\frac{dP_s}{d\Omega}\right)\Delta\Omega \simeq \frac{\pi e}{\varepsilon_o}\frac{h^2}{(1+h^2)^2}\frac{\gamma^2 I}{\lambda_r} \tag{41}$$

where I is the beam current.

Therefore the coherent spontaneous power on the m-th harmonic emitted in the same angle $\Delta\Omega_m \approx \Delta\Omega/m$, from (35) and (41) is given by

$$P_m \simeq \frac{\pi}{2c}\frac{h^2}{1+h^2}N_r I^2 \frac{A_m^2}{m} \tag{42}$$

and

$$P_m(\text{watts}) \simeq \frac{\pi}{8e\varepsilon_o}\frac{h^2}{(1+h^2)}N_r I^2\frac{A_m^2}{m} = \left[\frac{\pi}{8}\frac{h^2}{1+h^2}N_r\frac{A_m^2}{m}\right]\eta_o I^2\ . \tag{43}$$

In eq. (43) η_o is the vacuum impedence. From the last equation we deduce that the radiator "impedence" on the m-th harmonic is dependent on the harmonic amplitude and on the periods number N_r; the parameter h gives the dependence on the wiggler period and magnetic field.

It is worthwile to remark that from the formula of the spontaneous coherent power, for instance emitted on the first harmonic, we can get out the relative requirements on the beam quality and buncher parameters. In fact from eq. (42) we have

$$\frac{dP_1}{d\Omega} \simeq \left(\frac{dP_s}{d\Omega}\right)\varrho_o\frac{\lambda_s}{4}N_r A_1^2\ . \tag{44}$$

This coherent emission has a significant value compared with the spontaneous one only if

$$\varrho_o\frac{\lambda_s}{4}N_r A_1^2 > 1 \tag{45}$$

or in other terms

$$\frac{E_o^{(b)2}B_o^{(b)2}L_b^2 L_r\gamma_\perp^2\lambda_w^2\varrho_o}{\sigma_\varepsilon^2\gamma^6} > 3.4\times10^3\left(\frac{m_o^2c^3}{e^2}\right)\ . \tag{46}$$

The coherence properties of the emitted radiation impose an upper bound for the radiator periods number, as matter of fact the electrons in a bunch have an energy spread. This is substantially the initial energy spread (see (17)). Because of this emitted wavelength will have the fractional width

$$\frac{\Delta\lambda}{\lambda} \simeq 2\,\frac{\Delta\gamma}{\gamma} = 2\,\sigma_\varepsilon\ . \tag{47}$$

From considerations on the coherence length[17] we deduce the upper limit

$$N_r < \frac{1}{2\sigma_\varepsilon}\ . \tag{48}$$

It corresponds to an impressive long wiggler.

4.2. - Stimulated emission.

If an input wave is introduced into the radiator and the frequency of the wave is resonant with the radiator periodic magnetic field (synchronism condition), we will have in the radiator a potential pattern running with about the same velocity of the electron bunches. The potential depth is

$$V_o^{(r)} = \frac{e^2}{2m_o\gamma c}\,\frac{E_o^{(r)}B_o^{(r)}}{k_r k_s} \tag{49}$$

where r refers to the radiator.

If the initial condition is choosen so that the bunches are on the negative peak of the ponderomotive force, the electrons are decelarated and so they are stimulated to give the maximum amount of energy to the radiation field.

To evaluate the stimulated emitted power we can assume, for short radiator, that the decrease of the electron average velocity and consenquently the slippage between the potential and the electron bunches is negligible. Furthermore we do not take into account the electron oscillatory motion along the z-axis.

We want to stress that the beam does not evolve appreciably within the radiator, since all the bunches are on the crests of the sinusoidal force and, in first approximation, each electron is acted upon by the same force, beyond the fact that the electrons are ultrarelativistic (see Fig. 3).

In the radiator matched on the m-th harmonic of the electronic wave ($\lambda_{rm} = \lambda_r/m$), the average energy lost by each electron during the trip is

$$\Delta W_e = \int_0^{L_r/\beta_r c} \vec{F}\cdot\vec{v}\,dt\ . \tag{50}$$

Remembering that

$$\begin{aligned} \vec{F} &= k_{r_+} V_o^{(r)} \cos(k_{r_+} z - \omega_+ t + \varphi)\,\hat{z}\ , \\ \vec{v} &= \beta_r c\,\hat{z} + c\,\frac{\sqrt{2}h}{\gamma}\cos k_r z\,\hat{y}\ , \\ z &\approx z_o + \beta_r c t \end{aligned} \tag{51}$$

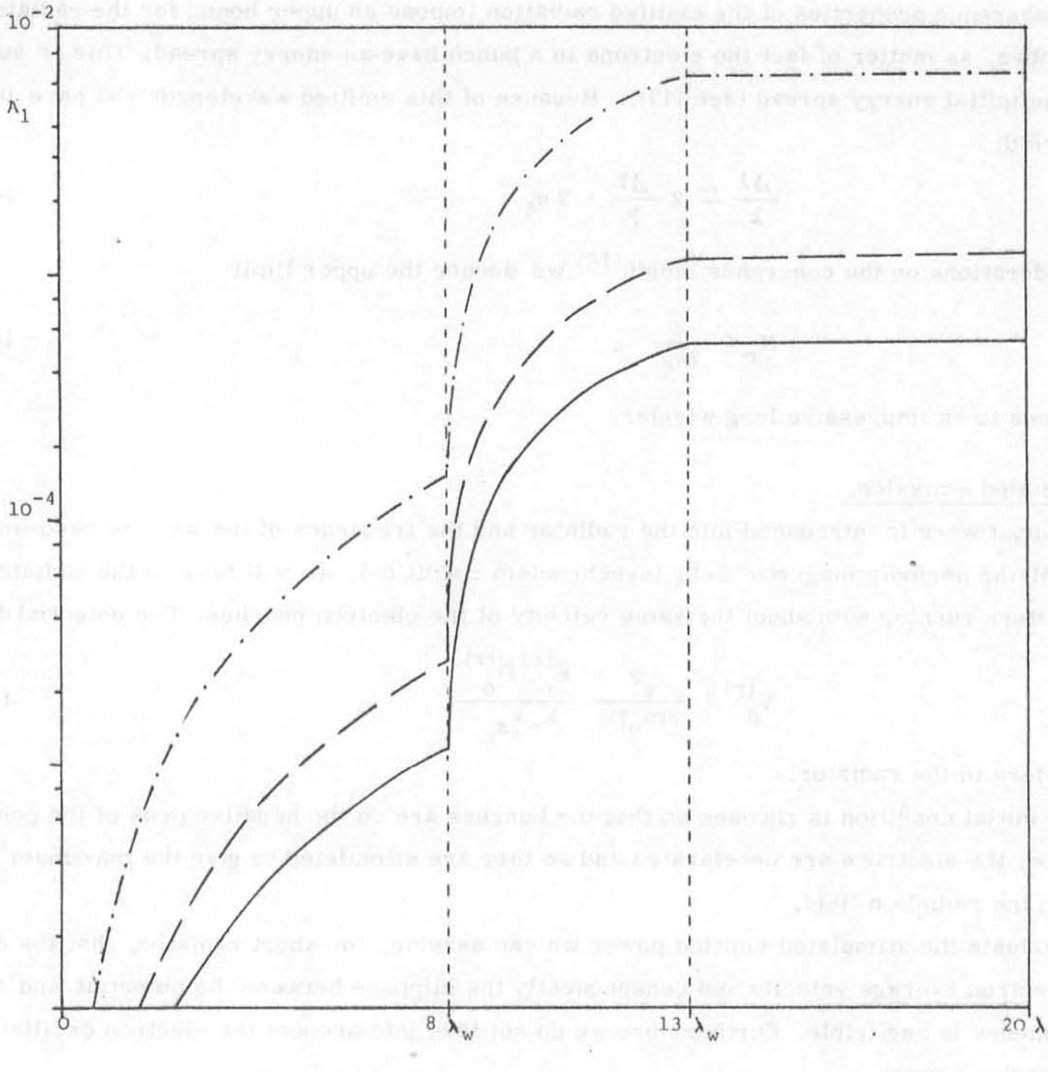

FIG. 3 - First harmonic evolution for a) $E_o = 0.08$ MV/m continuous line ; b) $E_o = 0.18$ MV/m dashed line ; c) $E_o = 1$ MV/m dotted line.

and the synchronism conditions

$$k_s = m\, k_{r_+} - k_r \, , \qquad \omega = m \omega_+ \tag{52}$$

the integral (50) reads

$$\Delta W_e = k_{r_+} V_o^{(r)} L_r \cos (k_{r_+} z_o + \varphi) \, . \tag{53}$$

If now we consider the density modulated electron beam given by (30), the energy exchange between the electrons and the ponderomotive force, if they are correctly phased, is

$$\Delta W = \frac{e^2}{4\, m_o c \gamma} \, \frac{L_r}{k_r} \, E_o^{(r)} \, B_o^{(r)} \, \varrho_o A_m V \tag{54}$$

where V is the volume of the beam interacting with the electronic wave and $k_r = k_{rm}/m$. In fact it is reasonable to extend the result of a line beam to a beam with a very small section S.

The small signal gain (G) on the first harmonic defined as the fractional increase of the radiation energy will be

$$G = \frac{2\Delta W}{\varepsilon_o E_o^2 V} \quad . \tag{55}$$

If we now assume that the electron beam section and the radiation waist coincide, we can write

$$\varrho_o = \frac{I}{ecS} \tag{56}$$

and

$$S \approx \lambda \frac{L_r}{2} \quad . \tag{57}$$

In (57) I is the electron beam current.

The ss-gain reads

$$G = \frac{e}{\varepsilon_o m_o c^2 \gamma} \frac{B_o^{(r)}}{E_o^{(r)} K_r \lambda_s} A_m I \quad . \tag{58}$$

Recalling the maximum amplitude of the first harmonic coefficient (31) the ss-gain for the first harmonic, that is the single pass gain of the TOK, is

$$G_{TOK} = 0.3 \frac{e^3}{\varepsilon_o m_o^3 c^5 \gamma^3} \frac{L_b B_o^2}{k_w^2 \sigma_\varepsilon \lambda_s} I \tag{59}$$

having assumed the same fields for the buncher and the radiator.

In order to compare this gain with the FEL gain we remember that this latter at the maximum is (see ref. (18), the gain has been divided by a factor two since our wiggler is plane[(19)])

$$G_{FEL} = \frac{1.08}{4\pi\varepsilon_o} \frac{e^3}{m_o^3 \gamma c^5} \frac{B_o^2}{(1+h^2)} L_{FEL}^2 I \quad . \tag{60}$$

The ratio between the two maximum gains, observing that in a SR we can reasonably write $2L_b \approx L_{FEL}$, reads

$$\Delta = \frac{G_{TOK}}{G_{FEL}} \approx \frac{10^{-1}}{\sigma_\varepsilon N_{FEL}} \quad . \tag{61}$$

We want to emphasise that this result is obtained with the maximum gains, dividing the FEL length in two equal parts. In ref. (9) the authors have taken into consideration the case of equal length for the radiator and FEL. Eq. (61) says that the TOK gain exceeds the FEL gain of 1-2 orders of magnitude with the usual parameters of SR accelerators. Maybe the ratio is a bit higher than the value of (61) since G_{FEL} is calculated for a monochromatic beam, contrary to G_{TOK}. The energy spread typical of a SR lowers the gain of FEL[20].

This enhancement of the gain per pass allows us to conclude that the threshold current for the generation is lowered. In fact if α is the mirror loss, the condition for generation

$$G_m \gtrsim \alpha \tag{62}$$

states that the threshold current I_{min} in TOK goes down of the factor Δ.

In addition, this current is further depressed by the consistent spontaneous coherent radiation given by eq. (44)[8, 16].

For TOKA we obtain

$$\Delta \simeq 10 .$$

5. - POSSIBLE EXPERIMENTAL LAYOUTS FOR ADONE.

In a circular machine as Adone (Fig. 4) two configurations for a Converter or a TOK experiment seem feasible: in the first the buncher and the radiator are set in two next straight sections of the machine and as dispersive drift region the bending magnet in between is used, in the second both wigglers are set in the same straight section and a very short drift space is interposed (Fig. 1).

However the first possibility cannot be taken into consideration because from eqs. (29) and (32) the physical space containing the bending magnets and the quadrupoles is too long as dispersive drift space.

In the second case the LELA wiggler used for the experiment of ref. (5) can accomplish the double function of buncher and radiator if it is assembled as two wigglers separated by a dispersive drift region.

For a correct realization of the drift space the condition

$$\int B_y(y, z)\, dz = 0 \tag{63}$$

must be fulfilled separately in the three regions of the device[24]. Consequently the minimum drift length can be L_d = 34.8 cm corresponding to three full wiggler periods.

To evaluate quantitatively the possibility of the experiment in Adone we start selecting the external wavelength of the LELA (Argon laser λ = 5145 Å) for the buncher operation.

The electron beam energy and the wiggler characteristics (period and magnetic field) are related to the input wavelength by the synchronisn conditions (9) (our calculations refer to a plane polarized input wave and transverse undulator). The choice of the drift space is determined by its dispersive properties through the momentum compaction α_c, eq. (32). This is in turn established by the number of magnetic poles constituing the magnetic drift space and the magnetic field amplitude (Appendix A).

In the LELA wiggler it is not possible to enhance furtherly the magnetic field (as for instance in the experiment of Novosibirsk where this can be done lowering the gap of the permanent

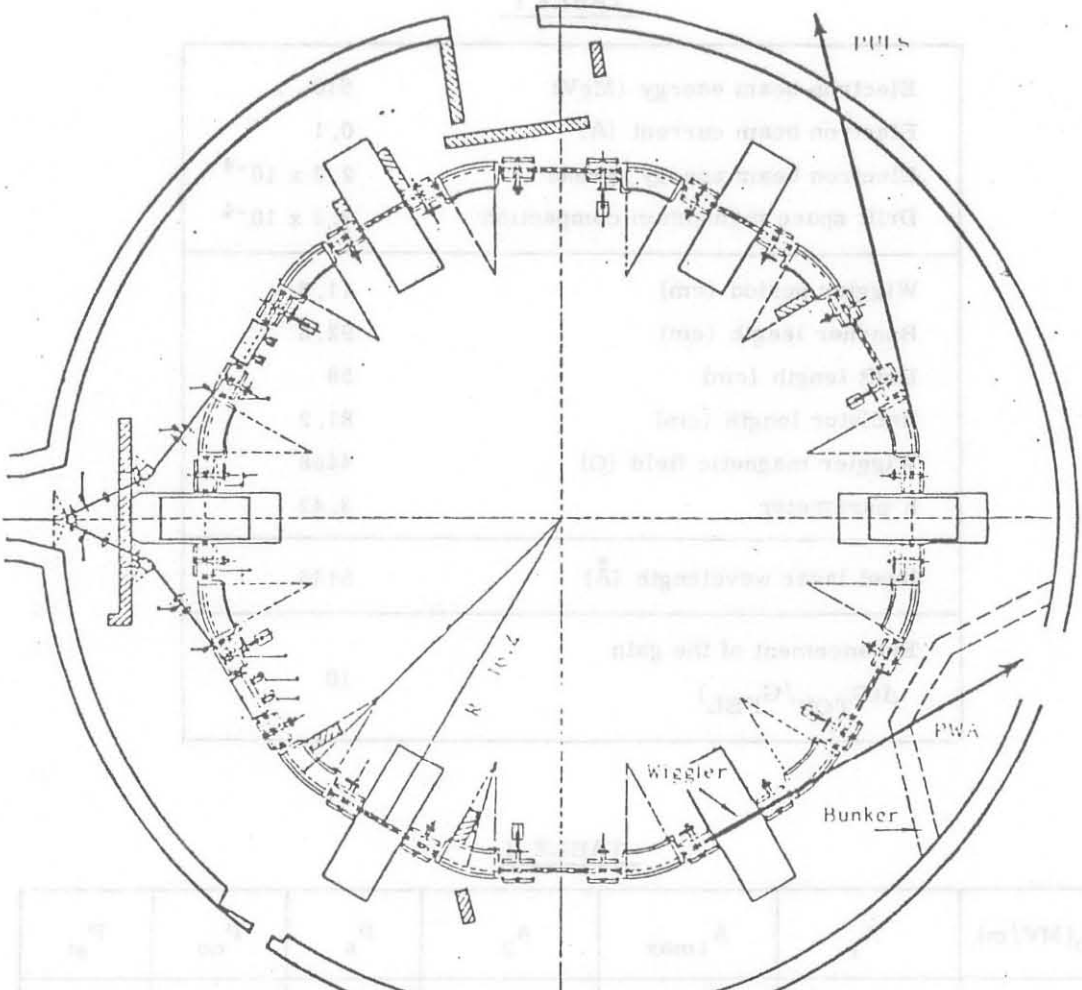

FIG. 4 - Comprehensive view of Adone storage ring with the straight section holding the LELA wiggler.

magnet[9]) thus the best configuration for TOKA is that having a drift length of 5 periods which corresponds approximatelly to the maximum bunching length. Of the remaining 15 periods, 7 can be devoted to the buncher and 8 to the radiator or viceversa, since with a longer buncher we have a more bunching and with a shorter buncher we have a longer interaction time in the radiator and the two arrangements give clearly the same result.

Once choosen the magnetic set-up (see Table I) the last choice is the amplitude of the input wave E_0 and therefore the power of laser. The first obvious choice is the use of the available 2 watts Argon laser of LELA experiment. With this laser and the beam section less than 1 mm^2 the input electric field is 0.08 MV/m.

We have also examined the figures of the device when the 10 watts Argon laser (commercially available) is used, when these lasers are used in the mode-locked (see Table II).

TABLE I

Electron beam energy (MeV)	610
Electron beam current (A)	0.1
Electron beam energy spread	2.3×10^{-4}
Drift space momentum compaction	3.3×10^{-4}
Wiggler period (cm)	11.6
Buncher length (cm)	92.8
Drift length (cm)	58
Radiator length (cm)	81.2
Wiggler magnetic field (G)	4458
h parameter	3.42
Input laser wavelength (Å)	5145
Enhancement of the gain $\Delta(G_{TOK}/G_{FEL})$	10

TABLE II

E_o(MV/m)	A_1	A_{1max}	A_2	P_s	P_{co}	P_{st}
0.08	5.1×10^{-4}	6.5×10^{-4}	2×10^{-7}	5 mW	2.5 μW	3.4 mW
0.18	1.2×10^{-3}	1.5×10^{-3}	10^{-6}	5 mW	15 μW	20 mW
1	6.4×10^{-3}	8.1×10^{-3}	3×10^{-5}	5 mW	4 mW	5.3 W
5	3.2×10^{-2}	4.1×10^{-2}	7.3×10^{-4}	5 mW	10 mW	13.3 W
15	0.10	0.12	6.5×10^{-3}	5 mW	90 mW	0.12 kW

E_o peak electric field ;

A_m m-th beam harmonic coefficient ;

P_s spontaneous incoherent power on the first harmonic ;

P_{co} spontaneous coherent power on the first harmonic ;

P_{st} stimulated power on the first harmonic.

In Figs. 3 and 5 are shown the evolution of the beam harmonics with the four different electric fields. The steep slope of the diagrams in the drift space, shows that this latter is decisive for the longitudinal modulation of the beam. The calculated flatness of the curve in the radiator confirms that the beam structure does not change during the interaction within the radiator.

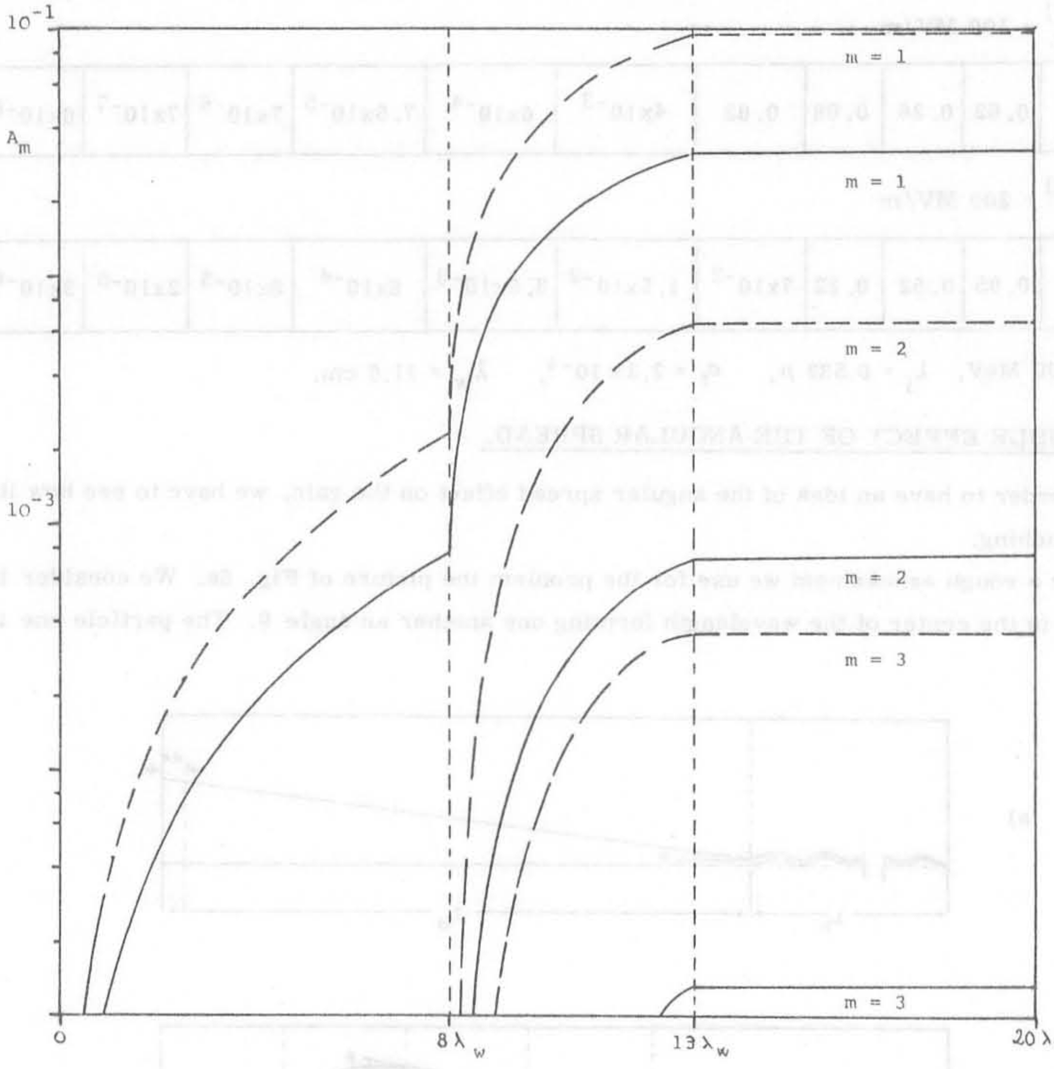

FIG. 5 - Some harmonic evolution with E_o = 5 MV/m continuous line, and E_o = 15 MV/m dashed line.

In Table III the harmonics amplitudes of the case with ε = 600 MeV, λw = 11.6 cm, $\lambda^{(i)}$ = = 0.532 μ and the electric field E_o = 100 e 200 MV/m are reported. The input wavelength has been changed because with the Argon laser it is not possible to achieve so high fields as instead it is with N_e-YAG laser.

TABLE III

m	1	2	3	4	5	6	7	8	9	10
a) $E_o^{(i)}$ = 100 MV/m										
A_m	0.62	0.26	0.08	0.02	4×10^{-3}	6×10^{-4}	7.5×10^{-5}	7×10^{-6}	7×10^{-7}	6×10^{-8}
b) $E_o^{(i)}$ = 200 MV/m										
A_m	0.95	0.52	0.22	7×10^{-2}	1.5×10^{-2}	3.5×10^{-3}	6×10^{-4}	8×10^{-5}	2×10^{-5}	3×10^{-6}

ε = 600 MeV, λ_i = 0.532 μ, $\sigma_\varepsilon = 2.3\times10^{-4}$, λ_w = 11.6 cm.

6. - POSSIBLE EFFECT OF THE ANGULAR SPREAD.

In order to have an idea of the angular spread effect on the gain, we have to see how it acts on the bunching.

For a rough assessment we use for the problem the picture of Fig. 6a. We consider two particles in the center of the wavelength forming one another an angle θ. The particle one is a

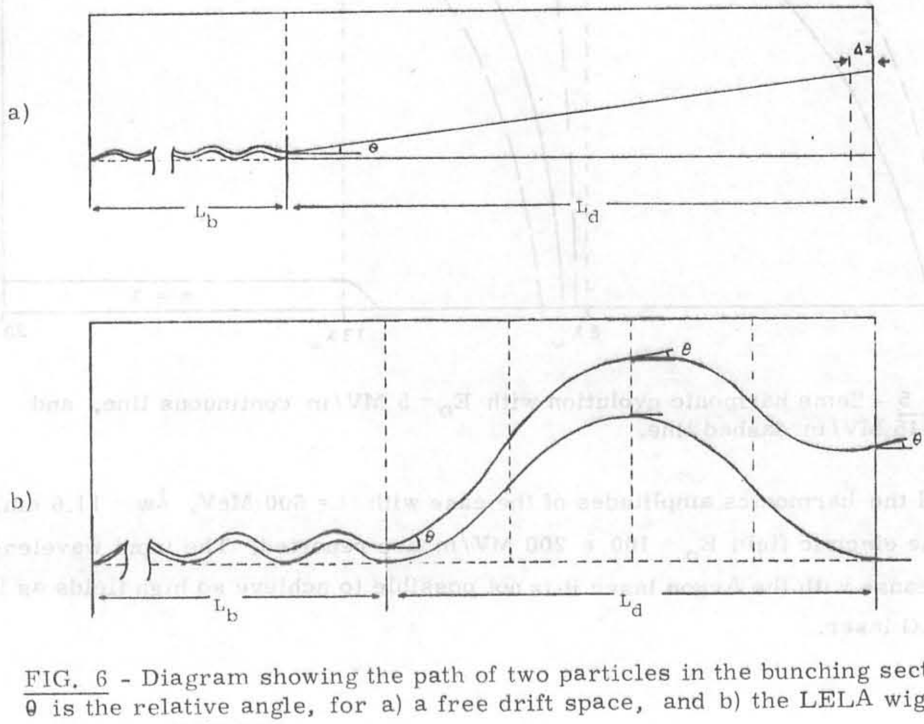

FIG. 6 - Diagram showing the path of two particles in the bunching section, θ is the relative angle, for a) a free drift space, and b) the LELA wiggler dispersive drift space.

synchronous particle. They will be yet bunched at the output of the drift space if they will be within $\lambda_s/8$, that is

$$(L_b + L_d) - (L_b + L_d)\cos\theta \lesssim \frac{\lambda_s}{8} \quad . \tag{64}$$

We can write in first approximation with a free drift space

$$L_b + L_d \simeq L_d \ , \qquad \cos\theta \simeq 1 - \frac{\theta^2}{2} \tag{65}$$

and thus

$$\theta^2 \lesssim \frac{\lambda_s}{4L_d} \quad . \tag{66}$$

Because the exact free drift length is given by (18') eq. (66) becomes

$$\theta \lesssim 1.7\,\sigma^{1/2}/\gamma \ . \tag{67}$$

This strong influence of the angular spread on the bunching with a free space is confirmed in ref. (22).

When the space is dispersive as in Fig. 1, since the trajectory of the particle is (see Fig. 6b), for symmetry considerations

$$s = 2R\left[\arcsin\left(\frac{L_{d_{eff}}}{4R} + \sin\theta\right) + \arcsin\left(\frac{L_{d_{eff}}}{4R} - \sin\theta\right)\right] \tag{68}$$

with the approximations

$$\arcsin x \simeq x + \frac{x^3}{6} \ , \qquad \sin\theta \approx \theta$$

the path difference between the two particles is

$$\Delta z \simeq s - s(\theta = 0) \simeq \frac{L_{d_{eff}}\,\theta^2}{2} \quad . \tag{69}$$

This means that with a dispersive drift space we have

$$\theta^2_{max} \lesssim \frac{\lambda_s}{4\,(L_b + L_{d_{eff}})} \tag{70}$$

which for Adone parameters give a θ_{max} about two orders of magnitude higher than the corresponding one with a free drift space (eq. (67)). Thus with a dispersive drift space the angular divergence has not a strong effect.

May be it is worth noticing that we have extended the result of the two central particles to all the particles inside a wavelength. This extension come out from the consideration that the particles with an angle θ can be treated as slower particles than the synchronous one. Thus we can guess that the right and left section of the beam with respect to the center particle counterbalance their different behaviour.

7. - CONCLUSIONS.

A Transverse Optical Klystron seems easily feasible in Adone making some modifications on the LELA experimental set-up.

Its gain should be a factor 10 greather than the FEL gain, having assessed the angular spread effect.

The three components of the emitted radiation, the spontaneous incoherent, the spontaneous coherent and the stimulated can be easily measured only when the electric field E_0 is more than 1 MV/m because in this case the spontaneous coherent radiation is high enough.

The measure of the coherent radiation would be important in order to chek the possibility of the Converter.

The measure of the stimulated power can be programmed in order to verify the enhancement of the TOK small signal gain compared with the FEL gain.

Another stimulating measure is that of the bunching throught the synchrotron radiation emitted in a bending (non dispersive) magnet. For this measure the bending magnet successive to the LELA wiggler, can be used. Because of its high momentum compaction, it needs to detect the radiation just after the first part of it. The FEL and TOK electron bunching comparison could be decisive to explain the possible difference in the gain.

We want to stress that the most important results have been obtained without taking into account the angular spread of the elactron beam. This assumption means that the beam can be considered one-dimensional and the involved Vlasov equation is in this case more simple both in the analytical and numerical approach. Really the emittance of the electron seems to be unimportant for the evaluation of the bunching and the emitted radiation.

An exact formulation of the problem imposes the solution of the complete Vlasov equation in the six dimensional phase space (x_i, p_i) with an evident both analytic and numerical complication.

We must furthermore observe that our results refer to a single passage through the system; the recirculated beam TOK is a necessary next step.

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APPENDIX A - Calculation of the drift space momentum compaction.

The trajectory length within the drift space is (see Fig. 3)

$$s = 4\, s_o \qquad (A.1)$$

where s is the electron path within the drift region before the entrance into the reversed polarity magnet.

From the figure we can see that

$$s_o = \varrho \theta \qquad (A.2)$$

i.e.

$$s = \frac{4p}{eB}\,\theta\ . \qquad (A.3)$$

Observing that

$$\theta = \arcsin\left(L_d \frac{eB}{4p}\right)$$

we have

$$s = \frac{4p}{eB} \arcsin\left(L_d \frac{eB}{4p}\right)\ . \qquad (A.4)$$

Since the deflection angles are very small we can assume

$$\arcsin x \approx x\left(1 + \frac{x^2}{6}\right)\ .$$

Then eq. (A. 4) finally become

$$s = L_d\left(1 + L_d^2 \frac{e^2 B^2}{96\, p^2}\right)\ . \qquad (A.5)$$

However the drift space momentum compaction has been defined as

$$\alpha_c = \frac{p}{s}\frac{ds}{dp}$$

(see eq.(22), and with a very good approximation in our case is $s \simeq L_d$, thus

$$\alpha_c = -\frac{1}{48} e^2 B^2 \frac{L_d^2}{p^2} = -\frac{c^2}{48} e^2 B^2 \frac{L_d^2}{\varepsilon^2}\ . \qquad (A.6)$$

With the parameters of tables we deduce finally

$$\alpha_c = 3.3 \times 10^{-4}\ . \qquad (A.7)$$

APPENDIX B - Calculations of the bunching first harmonic coeffient.

We define in the $(EBS)_b$ and $(EBS)_d$ the dimensionless parameters

$$\zeta = k_{b+} z \;, \qquad \tau = \Omega_{b+} t \;, \qquad q = (m_o \gamma_\perp V_{b_o})^{-1/2} p_z \;, \qquad \sigma = \frac{(m_o \gamma_\perp c^2)}{V_{b_o}} \sigma_\varepsilon^2 \tag{B.1}$$

so that the Vlasov eq. (8) in the buncher reads

$$\frac{\partial \varrho_b}{\partial \tau} + q \frac{\partial \varrho_b}{\partial \zeta} + \sin \zeta \frac{\partial \varrho_b}{\partial q} = 0 \;. \tag{B.2}$$

With LELA parameters and $E_o = 0.08$ MV/m the dimensionless interaction time within the buncher of length $L_w = 92.8$ cm is $\tau_b \simeq 5 \times 10^{-3}$.

This means that the solution of the eq. (B.2) can be expressed with a very good approximation as[6]

$$\varrho_b(\zeta, q, \tau) = \varrho_b(\zeta, q, 0) \left[1 + B_1(\zeta, q)\tau + B_2(\zeta, q)\tau^2 \right] \tag{B.3}$$

where

$$\varrho_b(\zeta, q, 0) = \frac{\varrho_o}{(2\pi\sigma)^{1/2}} \exp\left\{ - \frac{q^2}{2\sigma} \right\} \tag{B.4}$$

is the initial distribution function of the beam ($\sigma \simeq 1.7$).

Substituting the solution (B.3) in (B.2) we obtain

$$B_1(\zeta, q) = \frac{q}{\sigma} \sin \zeta \quad , \quad B_2(\zeta, q) = \left(\frac{q^2}{4\sigma^2} - \frac{1}{4\sigma} \right) - \frac{q^2}{2} \cos \zeta - \left(\frac{q^2}{4\sigma^2} - \frac{1}{4\sigma} \right) \cos 2\zeta \;. \tag{B.5}$$

At the exit of the buncher region ($\tau = \tau_b$) the beam distribution function will be given by

$$\varrho_b(\zeta, q, \tau_b) = \varrho_b(\zeta, q, 0) \left[1 + \tau_b B_1(\zeta, q) + \tau_b^2 B_2(\zeta, q) \right] . \tag{B.6}$$

Within the drift space Vlasov equation simplifies

$$\frac{\partial \varrho_d}{\partial \tau} + q \frac{\partial \varrho_d}{\partial \zeta} = 0 \tag{B.7}$$

having a solution in terms of the initial condition

$$\varrho_d(\zeta, q, \tau) = \varrho_d(\zeta - q\tau, q, 0) \;. \tag{B.8}$$

However for $\tau = 0$ the initial condition is the distribution function at the exit of the buncher, i.e.

$$\varrho_d(\zeta, q, 0) = \varrho_b(\zeta, q, \tau_b) \tag{B.9}$$

so that finally

$$\varrho_d(\zeta, q, \tau) = \varrho_b(\zeta, q, 0)\left[1 + B_1(\zeta - q\tau)\tau_b + B_2(\zeta - q\tau, q)\tau_b^2\right] \qquad (B.10)$$

where $B_1(\zeta - q\tau, q)$ and $B_2(\zeta - q\tau, q)$ can be directly obtained by (B.5).

The density distribution function is carried out by integration over all the momenta eq. (B.10)

$$\varrho_d(\zeta, \tau) = \varrho_o(1 + A_1(\tau)\cos\zeta) \qquad (B.11)$$

where the first harmonic coefficient is given by

$$A_1(\tau) = -\left[\tau_b\tau + \frac{\tau_b^2}{2}(1 - \sigma\tau^2)\right]\exp\left(-\frac{\sigma}{2}\tau^2\right). \qquad (B.12)$$

The maximum of the bunching is reached for $\sigma\tau_d^2 = 1$ when

$$A_{1max} = \tau_b\,\sigma^{-1/2}\exp\left\{-1/2\right\}. \qquad (B.13)$$

For LELA parameters this means $A_{1max} = 6.5 \times 10^{-4}$.

The free drift space length to reach the maximum of the bunching can be calculated having in mind that

$$\frac{\tau_d}{\tau_b} = \frac{1}{\gamma_\perp}\,\frac{t_d}{t_b} \simeq \left(\frac{\gamma_b}{\gamma}\right)^2 \frac{L_f}{L_w}$$

where t_d and t_b are the interaction time respectively in the $(EBS)_d$ and in the $(EBS)_b$ and L_w is the buncher length.

With the relations (B.1) finally we get ($k_{b_r} \simeq k_s/\gamma_b$)

$$L_f \simeq \gamma^2\lambda_s/2\pi\sigma_\varepsilon. \qquad (B.14)$$

In our case $L_f \simeq 500$ m.

If the drift is dispersive with a momentum compaction α_c the length is reduced by a factor $g = (1 - \alpha_c\gamma^2)$. If $\alpha_c = 3.3 \times 10^{-4}$ then $L_d = L_f/g \simeq 1$ m.

However the LELA effective drift length is $L_{eff} = 58$ cm so the effective first harmonic amplitude at the end of the drift space is

$$A_{1_{eff}} \simeq 5.1 \times 10^{-4} \qquad (B.15)$$

This is exactly the same result obtained solving the Vlasov equation by computer analysis.

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