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Abstract

The reaction $\tau \rightarrow \eta \pi \nu_\tau$ is discussed, and indicative estimates are given, in the framework of current algebra-like models of second class currents.

Riassunto

Si discute la reazione $\tau \rightarrow \eta \pi \nu_\tau$, e si danno stime indicative, sulla base di modelli delle correnti di seconda classe di tipo algebra delle correnti.

Abnormal (or second class) vector and axial vector currents by definition have opposite G-parity to the standard, first class (V, A) currents[1]. Would they show up in some weak process, the theoretical description of weak interactions would become considerably more involved than the usual one, since second class currents do not fit in a natural way into renormalizable schemes with fermion fields[2]. Up to now there seems to be no evidence for such currents in nuclear β-decay and μ-capture. Second class nucleon matrix elements, however, are proportional to the momentum transfer, so that low energy nuclear processes, where momentum transfers are quite small, should not be expected to be very sensitive to second class currents effects. It has been pointed out recently that the heavy lepton decay $\tau \to \omega\pi\nu_\tau \to (4\pi)\nu_\tau$ might represent a clean test for abnormal axial vector currents[3]. In this note we would like to recall, as well as to present some qualitative estimates, that the reaction $\tau \to \eta\pi\nu_\tau \to (4\pi)\nu_\tau$ should be, in turn, a unique signature for abnormal vector currents, the conventional, electromagnetic branching ratio (via η-π mixing) being expectedly as small as 10^{-4}. Thus the (4π) channel in τ decays seems to be a really interesting one for weak interaction phenomenology.[4]

Assuming the standard $V-A$ lepton coupling (with $m_{\nu_\tau} = 0$), the amplitude for the decay $\tau(P) \to \eta(k)\pi(q)\nu_\tau(p)$ can be written as:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\, \varkappa\, \bar{u}_{\nu_\tau}(p)\gamma_\mu(1-\gamma_5)\, u_\tau(P) \langle \eta(k), \pi(q) | \bar{V}_\mu | 0 \rangle \tag{1}$$

with $\bar{V}_\mu$ the abnormal vector current and $\varkappa$ its coupling strength, relative to the conventional, first class currents. Defining

$$\langle \eta(k), \pi^a(q) | \bar{V}^b_\mu | 0 \rangle = \delta_{ab}\left[(k-q)_\mu f_+(t) + (k+q)_\mu f_-(t)\right] \tag{2}$$

with $t=(q+k)^2$, the decay width takes on the form:

$$\Gamma(\tau^+\to\eta\pi^+\nu_\tau)=\frac{G_F^2\varkappa^2}{128\pi^3}\frac{1}{m_\tau}\int_{(m_\eta+m_\pi)^2}^{m_\tau^2}\frac{dt}{t}\lambda^{1/2}(t,m_\eta^2,m_\pi^2)\cdot \tag{3}$$
$$\cdot\left(\frac{m_\tau^2-t}{t}\right)^2\cdot\left\{|f_+(t)|^2\frac{1}{3}\left(1+\frac{2t}{m_\tau^2}\right)\lambda(t,m_\eta^2,m_\pi^2)+|d(t)|^2\right\}$$

where

$$d(t)=(m_\eta^2-m_\pi^2)f_+(t)+t\,f_-(t) \tag{4}$$

is the current divergence form factor and $\lambda(t,m_\eta^2,m_\pi^2)=$ $=(t-m_\eta^2-m_\pi^2)^2-4m_\eta^2m_\pi^2$.

In order to try a numerical estimate of eq. (3), explicit theoretical parametrizations of the form factors $f_+(t)$ and $d(t)$ are necessary. A possible procedure, to that purpose, is to assume that abnormal currents obey equal-time commutation relations with the standard (V,A) current algebra charges, and make use of soft-pion techniques. In this way:

$$\lim_{q_\mu\to 0}\langle\eta(k),\pi^a(q)|\bar{V}_\mu^b|0\rangle=-\frac{i}{f_\pi}\langle\eta(k)|[Q_5^a,\bar{V}_\mu^b]|0\rangle\equiv\frac{E}{f_\pi}\delta_{ab}k_\mu \tag{5}$$

where Q_5^a is the axial charge and $\sqrt{2}f_\pi\approx .93\,m_\pi$ is the $\pi\to\mu\nu$ coupling constant. Inserting eqs. (2) and (3) eq. (5) one then obtains:

$$d(m_\eta^2)=\frac{m_\eta^2}{f_\pi}E \tag{6}$$

apart from corrections $O(m_\pi^2/m_\eta^2)$. The RHS of eq. (6) obviously depends on the theoretical model we adopt to describe currents. As it is not possible to construct abnormal currents from the usual quark fields, one may resort to σ-like models. Also, although in principle a $SU(3)$ treatment, taking mixing angles into account, would be appropriate, we limit ourselves, for an order of magnitude evaluation, to the simpler scheme offered by the $SU(2) \times SU(2)$ σ-model. In that framework different possible realizations are available from the existing literature, and we choose to discuss just two outstanding examples.

In the current algebra treatment proposed in Ref. (5), where

$$-i\bar{V}_\mu^b \equiv K_\mu^b = \pi^b \partial_\mu \eta - \eta \partial_\mu \pi^b + \delta^b \partial_\mu \sigma - \sigma \partial_\mu \delta^b \tag{7}$$

$$\partial_\mu K_\mu^b = f_\delta m_\delta^2 \delta^b + (m_\delta^2 - m_\pi^2)(\delta^b \sigma - \pi^b \eta) \tag{8}$$

with K_μ hermitian, and with $\pi, \eta, \sigma, \delta$ respectively the pion, eta, sigma and the $\delta(980)$ $J^P = 0^+$ $I^G = 1^-$ meson fields, one has

$$f_\delta \approx f_\pi \tag{9}$$

and $[Q_5^a, \bar{V}_\mu^b] = i\varepsilon_{abc}\bar{A}_\mu^c$ leading to $E = 0$ on the RHS of eqs. (5) and (6), so that

$$d(m_\eta^2) = 0 \tag{10}$$

In addition, as a consequence of generalized sum rules, analogous to the Adler-Weisberger relation, which follow from the extended algebra obeyed by second class currents, one expects in the scheme of Ref. (5):

$$|f_+(0)| \approx 1 \tag{11}$$

Simple, subtracted parametrizations of form factors are possible, which are consistent with eqs. (9), (10) and (11). For example:

$$d(t) \approx \frac{m_\delta^2 (t - m_\eta^2) f_\delta g_{\delta\eta\pi}}{(m_\delta^2 - m_\eta^2)(m_\delta^2 - t)} \tag{12}$$

$$|f_+(t)| \approx |f_+(0)| \simeq \frac{f_\delta g_{\delta\eta\pi}}{m_\delta^2 - m_\eta^2} \tag{12'}$$

or, with M a typical hadronic mass $(M \simeq 1.5\ \mathrm{GeV})$

$$d(t) \simeq \frac{m_\delta^2 f_\delta g_{\delta\eta\pi}}{m_\delta^2 - t} - \frac{m_\delta^2 (M^2 + m_\eta^2) f_\delta g_{\delta\eta\pi} t}{m_\eta^2 (m_\delta^2 - m_\eta^2)(M^2 + t)} \tag{13}$$

$$|f_+(t)| \approx |f_+(0)| \simeq \frac{f_\delta g_{\delta\eta\pi}}{m_\eta^2 - m_\pi^2} \tag{13'}$$

or finally , to consider a case where there is t-variation of $f_+(t)$ (although vector mesons with $J^P = 1^-, I^G = 1^-$ do not seem to exist), we can take $d(t)$ as it is given in eq. (13), but

$$f_+(t) \simeq \frac{f_\delta g_{\delta\eta\pi}}{m_\eta^2 - m_\pi^2} \left(1 - \frac{m_\delta^2 (M^2 + m_\eta^2) t}{m_\eta^2 (m_\delta^2 - m_\eta^2)(M^2 + t)} \right) \tag{13''}$$

The coupling constant $g_{\delta\eta\pi}$ can be taken to be, from the δ-width, of the order of 2 GeV(*). The numerical results which are obtained by inserting eqs. (12)-(13") into eq. (3) (with, as usual, the pole

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(*) Actually eq. (12') would then result in a value of $|f_+|$ much smaller than unity. Nevertheless we still take the simple forms eqs. (12), (12') into account, since it is anyway understood that relations derived in the framework of Ref. (5) may be affected by non negligible corrections.

at $t = m_\delta^2$ replaced by a Breit-Wigner form), are displayed, for $\varkappa = 1$, in Tab. 1, where $\Gamma(\tau \to \eta\pi\nu_\tau)$ is compared to $\Gamma(\tau \to \pi\nu_\tau)$

$$\Gamma(\tau \to \pi \nu_\tau) = \frac{G_F^2 f_\pi^2 m_\tau^3}{8\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \tag{14}$$

Since, from eq. (14), the $\tau \to \pi\nu_\tau$ branching ratio is expected to be 10 %, the $\tau \to \eta\pi\nu_\tau$ branching ratio predicted in the scheme of Ref. (5) range, according to Tab. 1, from 5 % to 12%, depending on the parametrization.

Another simple $SU(2) \times SU(2)$ σ-model realization of second class currents has been proposed in Ref. (6). According to that model one may take

$$K_\mu^b \equiv -i\bar{V}_\mu^b = \frac{1}{2\langle\sigma\rangle}\left(\pi^b A_\mu^{(0)} + A_\mu^{(0)} \pi^b\right) \tag{15}$$

with π^b the pion field, $\langle\sigma\rangle$ the vacuum expectation value of the sigma field and $A_\mu^{(0)}$ the isoscalar axial current. Standard $SU(2) \times SU(2)$ commutation rules give $[Q_5^a, \bar{V}_\mu^b] = \delta_{ab} A_\mu^{(0)}$ and one obtains in this case, from eqs. (5), (6)

$$d(m_\eta^2) = m_\eta^2 \frac{f_\eta}{f_\pi}, \tag{16}$$

where we have defined $\langle 0 | A_\mu^{(0)} | \eta(k) \rangle = i f_\eta k_\mu$. The simplest parametrization of the form factors $d(t)$, $f_+(t)$ consistent with eq. (16) is:

$$d(t) = \frac{m_\delta^2 f_\delta g_{\delta\eta\pi}}{m_\delta^2 - t} \tag{17}$$

$$f_+(t) \simeq f_+(0) \simeq \frac{f_\delta g_{\delta\eta\pi}}{m_\eta^2 - m_\pi^2} \tag{18}$$

with

$$f_\delta g_{\delta\eta\pi} = \frac{m_\eta^2}{m_\delta^2}\left(m_\delta^2 - m_\eta^2\right)\frac{f_\eta}{f_\pi} \tag{19}$$

The value of the $\tau\to\eta\pi\nu_\tau$ width which is obtained (for $\kappa = 1$), by inserting eqs. (17), (18) and (19) into eq. (3) (and using, as an indication, $f_\eta \approx f_\pi$), is reported in Tab. 1: the resulting $\tau\to\eta\pi\nu_\tau$ branching ratio, is, accordingly, of the order of 6 %.

Summarizing our results, we may say that according to current algebra models of second class currents, if these couple to weak interactions in a "universal" way, the branching ratio for $\tau\to\eta\pi\nu_\tau$, where the vector cúrrent is involved, may be appreciable, of the order of several percents. Thus, taking into account the discussion of Ref. (3) concerning the axial abnormal currents, the (4π) channel in τ decay really seems to be encouraging, in order to test whether such currents exist[(7)].

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Table captions

Tab. 1: The ratio $R = \frac{\Gamma(\tau \to \eta \pi \nu_\tau)}{\Gamma(\tau \to \pi \nu_\tau)}$ according to the parametrizations adopted in the text.

Tab. 1

	Eqs. (12),(12')	Eqs. (13),(13')	Eqs. (13),(13")	Eqs. (17),(18)
R	0.5	0.6	1.2	0.6