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E. Recami. ELEMENTARY PARTICLES AS MICRO-UNIVERSES

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ELEMENTARY PARTICLES AS MICRO-UNIVERSES(x)

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ABSTRACT.

The First Part is just a large introduction, containing also various digressions - mainly from an anusual point of view - about projective and conformal Relativities.

We come to our main point in the Second Part. Namely, by postulating the covariance of physical laws under dilations, we describe gravitational and strong interactions in a unified way. In terms of the new (discrete) dilational degree of freedom, our cosmos and hadrons (= strong micro-cosmoses) can be regarded as finite, similar systems. Actually, a discrete hierarchy of finite "universes" can be defined, which are governed by fields with strength inversally proportional to their radii; and in each universe an Equivalence Principle holds, so that the relevant field can be geometrized there.

"Scaled down" Einstein equations (with cosmological term) are assumed to hold inside hadrons, and they yield in a natural way classical confinement - as well as "asymptotic freedom" - of the hadron constituents. In other words, applying the methods of General Relativity to subnuclear particle physics allows to avoid recourse to phenomenological models so as the "M. I. T. bag" model (which results to be advantageously substituted by the association of strong micro-universes of Friedmann type to the hadrons). Inside hadrons we have essentially to deal with a tensorial field (= "strong gravity"), and hadron constituents are supposed to exchange spin-2 "gluons".

Our approach allows also writing down a (tensorial, bi-scale) field theory of hadron-hadron interactions, by suggesting (modified) Einstein-type equations for strong interactions in our cosmos. We obtain in particular: (i) the correct Yukawian behaviour of the strong (scalar) potential for $r > 1$ fm and at the static limit; (ii) the value of the hadron radii in strong interactions.

As a by-product, we derive a whole "numerology" connecting our cosmos with the strong micro-cosmoses.

Finally, a structure of the "micro-universe" type seems to be characteristic also of leptons (P. Caldirola); a hope for the future is therefore including also weak interactions in our (classical) unification of the fundamental forces.

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FIRST PART : INTRODUCTION AND DIGRESSIONS.

1. - INTRODUCTION.

The thought that each microscopic "particle" of matter might be something like a whole "cosmos" - extremely reduced in size - has probably old origins. For instance, it appears in the papers (ca. 400 B. C.) by Democritus of Abdera. Namely, Democritus[1] - by reversing the analogy - spoke about giant atoms which could reach the size of our cosmos ; and - in order to be clear - he added : if one of such super-atoms (that consituted super-cosmoses) would leave its "giant cosmos" and fall on our world, our world would be destroyed.

That thought is connected with the meditations - very common as well[2] - on the effects of dilations and contractions on the physical objects, or even on the "world".

Within the scientific arena, let us recall[3] the old idea of a "hierarchy of cosmoses", corresponding to very different scale factors and possibly organized so as a series of "Russian dolls". We can say that in the microscopic analysis of matter one met - roughly speaking - a series of "Chinese boxes" ; and something analogous might happen also when investigating the universe, i. e. in the direction of the macro, besides of the micro. "Hierarchical" theories were formulated[3] e. g. by J. H. Lambert (1761) and later by Chalier (1909-1922) and Selety (1922), followed in more recent times by physicists so as O. Klein, H. Alfvén, J. E. Charon[4], K. P. Sinha and C. Sivaram[5], and M. A. Markov[6], D. D. Ivanenko[3] and some other Russian authors, till the papers by P. Caldirola, P. Castorina, G. D. Maccarrone, M. Pavšič and the present author[7], by A. Salam and coworkers[8], by P. Roman and collaborators[9], by H. J. Treder, etc.

In this article we shall essentially refer to the line followed by us (cf. Refs. (7)). Our approach starts from the wellknown empirical observation that the ratio $\bar{R}/\bar{r}$ between the Hubble radius $\bar{R} \simeq 10^{26}$ m of our cosmos ("gravitational cosmos") and the characteristic radius $\bar{r} \simeq 10^{-15}$ m of subnuclear particles equals grosso modo the ratio S/s of the strength S of the nuclear (or strong) field over the strength s of the gravitational field[(x)]. It will be tempting to think of a similitude between the macro-cosmos and hadrons (now conceived as "strong micro-universes"). We shall therefore assume cosmos and hadrons - both regarded as finite[(7)] objects[(+)] - to be similar systems, in a geometrico-physical sense : i. e., to be systems governed by similar laws, differing only for a scale-transformation which carries

(x) For the definitions of S, s see the following.

(+) For the moment, to fix our ideas, let us assume the naive model of a "Newtonian ball" - in 3-dimensional space - for both our cosmos and hadrons. More sensible models[(7)] (of Friedmann type) will be considered later.

$\bar{\bar{R}}$ into $\bar{r}$ and the gravitational field into the strong field (both fields being a priori tensorial, in our theory). Namely, mere dimensional evaluations will tell us that: by "contracting" the cosmos by the factor $\varrho^{-1} \equiv \bar{\bar{R}}/\bar{r} \simeq 10^{41}$ (i. e., by transforming it into a hadronic micro-cosmos), the associated field-strength increases[7] in the same ratio (passing from gravitational to strong).

Moreover, since the typical duration of a decay process is inversally proportional to the interaction strength, we shall analogously be able to explain why the typical life-time of our gravitational cosmos (e. g. equal to the duration of a complete expansion/recontraction cycle, in the case of the cyclic big-bang theory $\simeq 10^{18}$ s) is multiple of the typical life-time of hadrons ($\tau \simeq \simeq 10^{-23}$ s) by the same factor $\varrho^{-1} \equiv \bar{\bar{R}}/\bar{r} \equiv S/s \simeq 10^{41}$. At last, we shall explain, always in a simple way, the fact (itself already empirically known, too) that the cosmos-mass M equals ϱ^{-2} times the typical mass m of the reference hadron considered. And so on.

Before reaching the core of our arguments, however, let us start from the distance, spending some words about the mathematical "environment" useful to throw light on our initial motivations[x].

Moreover, since we are going to consider (besides usual transformations) also space-time dilations and contractions, let us at this point recall a passage from the last scientific writing of Einstein[10]: "... From the field equations one can immediately derive what follows: if $g_{ik}(x)$ is a solution of the field equations, then also $g_{ik}(x/\alpha)$ is a solution, where α is a positive constant ("similar solutions"). Let us for instance suppose system g_{ik} to represent a finte-size crystal embedded in a flat space. We could then have a second "universe" with another crystal, exactly similar to the previous one, but dilated by the factor α. As far as we confine ourselves to a universe containing nothing but a unique crystal, we do not meet any difficulties. We realize that the size of such a crystal ("standard of length") is not fixed by the field equations ... "[+]. This passage is part of the "Preface" to Ref. (10), written by Einstein at Princeton on April 4, 1955 (two weeks before dying).

2. - ON "PROJECTIVE RELATIVITY".

Special Relativity (in both its ordinary and "extended"[11] forms) refers to a pseudo-Euclidean chronotopous which is flat and infinite. One immediately realizes that such a 4-dimensional background constitutes a very risqué extrapolation of the local properties of our space and time, and does not adapt itself to the description of our cosmos. It is for instance difficult to believe that physical laws are covariant also under time-translations of thousand millions years

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(x) At the first reading, one might jump to Sect. 3.

(+) Our translation from German.

(in their ordinary form, at least) ; and so on.

An interesting step towards a space-time (s-t) that a priori is suitable to cosmological studies is the following one. Let us observe that the Galilei group G can be obtained (through a "contraction") from the Poincaré one P as the "limiting case" when $c \to \infty$. We can wonder whether the Poincaré group can be in its turn a "limiting case" of another, new group. Remaining in a four-dimensional space (only considering 10-parameter groups), in 1954 Fantappié showed that a unique new group exists, depending with continuity on a parameter R, which is reduced to Poincaré's for $R \to \infty$ and which cannot be any more the "limit" of any other group. Such a new group, F, happens to be that one of the motions into itself of a de Sitter space-time[12]. Now, the de Sitter s-t is representable as a hypersurface with equation

$$Z_0^2 + Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 = 0 \qquad (1)$$

embedded in a flat 5-dimensional space (here and in what follows we shall admit that some co-ordinates can be immaginary). From that point of view, then, the group F becomes the group of rotations in a flat, five-dimensional space ; and this clearly shows that F generalizes the Poincaré group (whose homogeneous - Lorentz - part, as wellknown, is isomorphic from the "complex" point of view to the group of rotations in a flat four-dimensional (1, 3) space).

A useful, important physical interpretation of the de Sitter group F has been put forth in 1959 by Arcidiacono[13], who distinguished the de Sitter s-t from the "relative" s-t of each observer living in it. He took into account the fact that every observer would perceive the events as though they happened in a flat s-t, any geodesic appearing to the observer as a straight line. In other words, each "relative" space time is a geodesic representation of de Sitter s-t on a tangent hyperplane. Thus, the de Sitter group becomes represented by the projections from the center of quadric (1) and sections with the tangent hyperplane. Or, rather, F becomes the group of projectivities which transform into itself the quadric

$$X_1^2 + X_2^2 + X_3^2 + X_4^2 + R^2 = 0 \qquad (2)$$

that is to say

$$R^2 = x^2 + y^2 + z^2 - c^2 t^2 \,. \qquad (2')$$

When introducing homogeneous coordinates, by setting $X_i \equiv R x_i / x_5$, eq. (2) writes $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 0$. In conclusion, from the projective point of view, the ordinary "physical" space-time is the region external to the Kayley-Klein "absolute" with equation (2). But the projective space defined as the region external to the quadric (2) is nothing but the Castelnuovo space-time[14], and only in this space the mathematical expressions receive a physical interpretation. A "(Special) Projective Relativity" follows[13], which reduces to the ordinary Relativity only when $R \to \infty$.

Here, let us add only the following. As wellknown, Einstein build up his General Relativity in such a way to include Special Relativity as a particular case. Subsequently, to build up "unitary theories", they tried to enlarge the Riemannian geometry of General Relativity (GR). That is to say, before enlarging the GR, they did not try to "bring to perfection" the Special Relativity. According to Ref. (13), the ordinary unitary theories result to be unsatisfactory also because all theories build up by enlarging the Riemannian geometry (or by passing to 5- or 6-dimensional manifolds) are still "based" on Special Relativity (SR) - set up in the Minkowski s-t - and on the Poincaré group. Such a group is not "simple"(12) and therefore splits in the chronotopical (6 parameters) rotations and (4 parameters) translations. This leads to the partition of ordinary SR in two independent parts (Mechanics - of continuous media -, and Electromagnetism), where a sharp distinction exists between "matter" properties and "electricity" properties. In Projective Relativity(12, 13), on the contrary, rotations and translations merge together into the rotations of (a 5-dimensional hypersphere) S^5 via the new fundamental lenght R ; as a consequence, a link is found between "matter" and "electricity", while remaining - nevertheless - inside the realm of the classical theories directly founded upon groups ("Erlangen Program" for physical theories).

If, afterwards, one wants to erect a "general relativity" starting from Projective Relativity (which is based on the de Sitter-Fantappié group), one expects that the new "projective general relativity" extends Einstein's gravitational theory on a cosmological scale, and therefore is particularly suited for astrophysical problems.

In order to blend the conceptions of those who prefer to rely directly upon the ds^2 and of those who on the contrary strictly appeal to the "path of groups" (step-by-step), we can take advantage of the unifying point of view by Cartan : who, by generalizing the idea of space, inserted the very Riemannian geometry in a group-context. In fact, following Cartan(15), a Riemannian variety V^4 can be regarded as constituted of infinite many - e. g. Euclidean - spaces tangent to it at each of its points, each one of those spaces having a geometry (in Klein's sense) grounded on the roto-translations group ; such a geometry was called holonomous by Cartan. Those infinite many, Euclidean space-elements are then linked together through a certain "connection" law (in this case called Euclidean by Cartan), which allows deducing both curvature and torsion (local properties) of V^4 by using infinitesimal closed cycles on the variety, and the "holonomy group" (global properties) of V^4 by using finite closed cycles on V^4. Vice versa, once the holonomy group is known, the connection law can be univocally determined(12, 13). Of course, what precedes can be at once extended to the cases when the tangent spaces possess a non-Euclidean geometry, based on a group G^r with r parameters (still in the sense of the "Erlangen Program"). Likewise, given any holonomous (= founded upon a group) geometry, anholonomous geometries can be constructed corresponding to it. For instance : in Minkowski s-t the holonomy group is obviously the identity, and such space-time is holonomous ; on the contrary, the Riemannian s-t of General Relativity is no more holonomous : however it admits (it being devoid of torsion) as holonomy group the Lorentz one, i.e. the group of rotations in S^4 space.

Let us summarize[13]: (i) For going beyond SR, Einstein jumped from a theory founded upon the rotations group R_4 (Lorentz group) to theories which rather utilize Riemannian manifolds V^4, V^5, V^6, ... and in such a way he skipped, in a sense, the direct (step-by-step) path of groups[16]; (ii) In the "theory of the universes" by Fantappié-Arcidiacono, on the contrary, the models of cosmoses (or of "universes") are build up on the basis of the rotations groups R_4, R_5, R_6, ..., thus establishing a link among physical laws[12], group, and geometrical model of the cosmos or universe. In fact, the chosen group acquires in that way the geometrical task of representing the motions into itself of the corresponding "universe-model", and - from the physical point of view - of expressing in mathematical form a "principle of relativity". Physics, for instance, can be build up by using the "topological group", i.e. n(n-1)/2 dimensional manifolds which possess both geometrical and group structures[12] (this is comparable[13] with what Lagrange did in his analytic mechanics, when he described a mechanical system in terms of its "Lagrangian parameters"); (iii) In order to reconcile the points of view of Einstein and of Fantappié-Arcidiacono, we can make use of the link established by Cartan between group theory and differential geometry. From this "third" point of view[13], we can set up a series of "special relativities", based on the rotation groups R_n, and than associate with each of them a "general relativity" by making recourse to an "anholonomous" geometry (that admits R_n as its holonomy group, and therefore is a Riemannian geometry).

In Appendix A we show, for example, how to build up a "general relativity" when starting from Projective (Special) Relativity.

2.1. - An Alternative Approach.

Wanted we strictly to follow the "Erlangen Program" in physics, the following alternative approaches would be available[13, 17]. The investigation of de Sitter universe - projective relativity - and of the corresponding, generalized Maxwell equations[13] confirms the usefulness (besides of a straighforward group-theoretical foundation of physics) of resorting to the rotations groups R_n of n-dimensional spaces. We saw that in such a way a succession of "universe models" is obtained, represented by the hyperspheres S^{n-1} embedded in n-dimensional spaces E^n, $(n = 4, 5, ...)$; and the problem arose of developing a "Relativity" just based upon the group R_n of the motions into itself of the hyperspheres S^{n-1}. [Incidentally, in the groups R_n $(n > 3)$, with their projective coordinates x_i $(i = 1, ..., n)$, there appear n-3 universal constants, necessary for adding square lengths to the squares of the new coordinates (which come after the first three ones, x, y, z) without violating the requirement of dimensional homogeneity[18]].

If we set - so as in projective relativity - n-4 normalization conditions, we take back the "n-dimensional Relativity" to a 4-dimensional formulation (in terms of the space-time coordinates)[13]. At the limit for $R \to \infty$, besides, every hypersphere S^{n-1} is reduced to a flat space E^{n-1}, and its n projective coordinate become n-1 Cartesian coordinates ; consequently, the group R_n (with n(n-1)/2 parameters) decomposes into the product of rotations R_{n-1} and translations T_{n-1} (having (n-1)(n-2)/2 and (n-1) parameters, respectively), while the normalization conditions become n-5 independent equations with n-1 unknowns(13). For instance, for n=5 we get the projective transformations (projective geometry) ; for n=6 the conformal transformations (conformal geometry) ; for n >6 the Cremona-type transformations : In such a way, one succeeds in applying the algebraic geometry to physics.

At this point, the new alternative approach comes in. Let us notice, in fact, that within the aforesaid group-theoretical conception of physics a particular rôle is played by the generalized "Maxwell equations" of the various hyperspherical universes S^{n-1}, which are covariant under the group R_n. If we call $H_{ik} = -H_{ki}$ (i, k = 1, 2, ... , n) the generalized "electromagnetic field", possessing n(n-1)/2 distinct components, the generalized Maxwell equations then write(13):

$$\mathrm{Curl}\, H_{ik} = J_{ikl} \; ; \quad \mathrm{Div}\, H_{ik} = I_k \, , \qquad (i, k, l = 1, 2, \dots , n) \tag{3}$$

where J_{ikl} and I_k are the field "sources", and Curl, Div are understood to operate in n dimensions.

A relation has been uncovered between enlarging the basic group of physics and the possibility of unifying the various physical interaction-fields, such a synthesis being performed by the very algebraic structure of the various rotations groups.

Particularly interesting appears to be the extension from the group R_5 (projective relativity) to the group R_6 (conformal relativity), the latter comprehending also the uniform accelerations. In Refs. (13) it has been shown, in this connection, that the corresponding generalized Maxwell equations yield a unified theory of matter (gravitation plus "hydrodynamics" of continua) and of electromagnetism. In particular, for $R \to \infty$, one is taken back to a flat space E^5 and the abovementioned generalized Maxwell equations split, on one hand, in Corben's equations(17) (of the unified gravitational-electromagnetic field) and, on the other hand, in the mechanical equations of the generalized "hydrodynamical" field(13). By using such a "Conformal Relativity" (n=6), therefore, there is no need of passing - as done on the contrary in General Relativity - to "anholonomous" manifolds, but one succeeds in describing even gravitation without departing from a strict group-theoretical formulation of physics.

3. - ABOUT "CONFORMAL" RELATIVITY.

Historically, when they took due account of the electromagnetic phenomena, besides of the mechanical ones, it was necessary to abandon - as well known - Galilean relativity in favour of Einstein's. We could now wonder whether, once arrived at investigating also the nuclear and subnuclear forces, a further extension towards a new Relativity should be necessary. Actually, at the beginning of Sect. 2, we considered - roughly speaking - the following "chain" of groups:

$$G(c \rightarrow \infty ; R \rightarrow \infty) \Longleftarrow P(c ; R \rightarrow \infty) \Longleftarrow F(c ; R) \quad , \tag{4}$$

where the final, de Sitter group "contains"[13] two universal constants (a fundamental length, R, and the light-speed in vacuum, c). But, in order to plan in a dimensionally correct way even only a mechanical (dynamical) theory, three universal constants are needed[18]. To lengthen the chain (4) one has however to leave the 10-parameter groups (i. e., the 4-dimensional Minkowski space[13]). It is then easy to reach the conformal group C, with 15 parameters, which can be shown to be locally isomorphic to the rotations of a 6-dimensional space. That group will allow setting up[12] the new "Conformal (Special) Relativity", a generalization of the Projective one and a theory having as universe-model a 5-dimensional hypersphere (embedded in a flat 6-dimensional space). In such a Conformal Relativity, now, three independent universal constants c, R, h will enter, where the third constant, h, ought a priori to depend on a Mass (besides on a Length and Time)[13, 19]. Let us recall at this point also what we wrote in Sect. 2. 1.

SECOND PART: SUBNUCLEAR PARTICLES AS MICRO-UNIVERSES.

4. - A HIERARCHY OF "UNIVERSES".

We can also start from a different point of view (although within a more limited framework, in a sense) for generalizing the Special Relativity according to the spirit of the beginning of Sect. 3. In the following, we shall essentially refer to work done by the present author in collaboration with P. Caldirola and M. Pavsic, as it appears from the references. Let us, namely, observe that the symmetries of Maxwell equations have not been fully exploited by SR. In fact, Maxwell equations are known to be covariant - besides under Poincaré transformations - even under conformal transformations[20] and, in particular, under dilations[(x)]. Moreover, we have already recalled that Einstein gravitational equations too are covariant under dilations[(o)]. Then, let us fix our attention in particular on the (space-time) global dilations ($\mu = 0, 1, 2, 3$):

$$x'_\mu = \varrho x_\mu \tag{5}$$

and postulate[7] that physical laws are covariant also under the global dilations (5), where only

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(x) In the general case when charges are present, such covariance exists provided that also charges are suitably "scaled".

(o) When in presence of matter and of a cosmological term, such covariance is preserved - as we shall see - provided that also the gravitational charge (= mass) and the cosmological constant are suitably scaled.

discrete values of ϱ are supposed to have in nature a physical counterpart. (Such discrete values might be derived e.g. by imposing suitable boundary conditions in five-dimensional spaces[21]: but we leave this problem open here).

At this point, let us recall that natural objects seem essentially to interact via (at least) four fundamental forces: the gravitational, the "weak", the electromagnetic, and the "strong" one, here listed according to decreasing strength. For instance, the strength of the electromagnetic force is measured by the dimensionless coefficient $(4\pi\varepsilon_0)^{-1} e^2/\hbar c$. In general, the strength of an interaction can be measured by the dimensionless square of the corresponding "coupling constant". Here we are interested in particular in the strength of the gravitational interaction:

$$\frac{Gm^2}{\hbar c} \simeq 1.3 \times 10^{-40} \tag{6}$$

and of the strong one:

$$\frac{Ng^2}{\hbar c} \begin{cases} \simeq 3 \\ \simeq 15 \end{cases} \tag{6'}$$

where: (i) G and N are the gravitational and strong universal constants in vacuum, respectively; (ii) quantities m and g represent the gravitational-charge (= mass) and the strong-charge (see the following) of one and the same hadron[7], e.g. of a nucleon or a pion-meson. The value in eq. (6) is calculated for the pion mass, $m=m_\pi$; in eq. (6') the first number typically corresponds to the value of the $\pi\pi\varrho$ coupling-constant square[22] (whilst the second number represents, more generally, the value of the $pp\pi$ coupling-constant square).

With regard to the above expression "strong-charge of a hadron", let us regard the quarks[(x)] as the actual sources of the strong field, i.e. the real carriers of strong-charge, and let us call "color" the sign s of quark strong-charges[7]; more precisely, the hadrons can be considered as endowed with zero total strong-charge, each quark possessing a strong-charge $g_i = s_i|g'|$ where $\Sigma_i s_i = 0$. The ordinary strong-interactions among hadrons should, in a sense, originate from Van-der-Waals-type forces[7]. In correspondence to quantity m of eq. (6), in eq. (6') the quantity $g \equiv ng_0$ will enter, quantity g_0 being the average magnitude of the constituent-quark charges and n being the quark number.

Let us put

$$\varrho \equiv \frac{Gm^2}{Ng^2} \begin{cases} \to \simeq 4 \times 10^{-41} \to \\ \to \simeq 0.9 \times 10^{-41} \nearrow \end{cases} \simeq 10^{-41} \tag{7}$$

(x) Let us recall, incidentally, that the hadron constituents (2 for mesons and 3 for baryons) have been named "quarks" by M. Gell-Mann. The Anglo-Saxon word quark is usually ennobled by literary citations (e.g., Gell-Mann got inspiration - as wellknown - from J. Joyce's "Finnegans Wake" (1939)). Let us here quote that Goethe properly used such a word in "Faust", verse 292, where Mephistopheles referring to mankind exclaims: "In jeden Quark begräbt er seine Nase".

and notice that, if we conventionally choose $m \equiv g$, then the "strong universal constant" N becomes :

$$N = \varrho^{-1} G \approx 10^{41} G \simeq \frac{hc}{m^2} \simeq 7 \times 10^{30} \ m^3 Kg^{-1} s^2 \ ; \tag{8}$$

conversely, if we choose units such that $[N] = [G]$ and moreover $N = G = 1$, then we get :

$$g = \frac{m}{\sqrt{\varrho}} \simeq 2 \times 10^{-33} \ cm \simeq 3 \times 10^{-5} \ gr \simeq \sqrt{\frac{\hbar c}{G}} \equiv \text{Planck-mass} \ , \tag{9}$$

where we eventually chose in eqs. (6') and (7) the upper value. Eq. (9) tells us, by the way, that the "Planck-mass" $\sqrt{\hbar c/G} \equiv m\sqrt{\varrho^{-1}}$ is nothing but the typical hadron (or, perhaps, quark) "strong charge", in suitable units. We do not expect, therefore, existence of further, new small black-holes - as predicted by other Authors - with a mass of the order of the Planck-mass, since we already met hadrons (or, perhaps, quarks) with strong charges of the order of Planck-mass (in suitable units).

The most important observation is, however, the following one. Let us regard both hadrons ("typically" the pions, or the nucleons), and our cosmos as finite objects. Then, relation (7) and the fact that, when calling $R(U) \equiv R$ the Hubble radius of our cosmos[7] and $r(h) \equiv r$ the hadron (pion) radius in strong interactions, one gets

$$\frac{r(h)}{R(U)} = \frac{10^{-15} \ m}{10^{26} \ m} \simeq 10^{-41} = \varrho \ , \tag{10}$$

suggest that our cosmos and hadrons can be considered as (finite) similar systems[(x)] governed by similar laws that differ only in the scale-factor ϱ (which carries R into r and the gravitational field into the strong one). Roughly speaking, we can imagine that - by shrinking the cosmos by the factor $\varrho \simeq 10^{-41}$ - we can get the hadrons (see the following, and Refs. (7)), that is to say that, by dilating a hadron by the factor $\varrho^{-1} \simeq 10^{41}$, we can get a cosmos. In Refs. (7), indeed, after having called "universe" any almost-isolated system, governed by one of the fundamental forces, we have analogously introduced a "hierarchy of universes"[(3)], which can be obtained through a series of suitable discrete dilations (or contractions).

Drawing our inspiration, as said above, from the hypothesis that physical laws are covariant under (discrete) dilations, we are led to assume, briefly, that[(7)] :

(x) Just to fix our ideas, let us think here in terms of "Newtonian balls" both for hadrons and our cosmos. Later on, we shall adopt less naive models (namely, Friedmann models) for both our cosmos and hadrons, consistently with General Relativity.

A) inside our cosmos (gravitational case) the Einstein equations, with attractive cosmological constant[5] Λ, hold (G = 1) :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - \Lambda g_{\mu\nu} = - \frac{8\pi}{c^4} T_{\mu\nu} \; ; \qquad (11)$$

B) inside hadrons ("strong" case) the "scaled down" Einstein equations hold (N = G = 1) :

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\varrho}_{\varrho} - H\tilde{g}_{\mu\nu} = - \frac{8\pi}{c^4} S_{\mu\nu} \qquad (12)$$

Simple dimensional evaluations immediately tell us that (within our "dilation covariant" relativity)[7] :

$$H = \varrho^{-2}\Lambda \quad ; \qquad m_G = \varrho\, m_S \; , \qquad (13)$$

where $m_G \equiv \hbar\sqrt{2\Lambda/c}$ and $m_S \equiv \hbar\sqrt{2H/c}$ are the average mass - small, but finite - of the "external" gravitons and the average mass of the "external" strong-quanta, respectively; cfr. Refs. (7) and Sect. 6. Moreover, the strong-charge tensor $S_{\mu\nu}$ is essentially $S_{\mu\nu} = \varrho^{-1} T_{\mu\nu}$, where $T_{\mu\nu}$ is a priori the ordinary matter-tensor (containing e. g. the Dirac spinorial functions, etc.). For example, if we require (also) the "external" gravitational interactions (see Refs. (7)) to have a range of the order of $R = R(U) \simeq 10^{26}$ m, then we obtain at once[7] :

$$m_G \simeq 10^{-68}\ \text{Kg} \; ; \qquad \Lambda \simeq 10^{-56}\ \text{cm}^{-2} \; ; \qquad m_S \simeq m_\pi \; , \qquad (13')$$

as well as :

$$H^{-1} \equiv \varrho^2 \Lambda^{-1} \simeq 10^{-25}\ \text{cm}^2 \simeq 0.1\ \text{barn} \; . \qquad (14)$$

The present, elementary theory[7] allows proving in a systematic way all the empiric relations (which connect macro - with micro- cosmoses) heuristically discovered by Weyl, Eddington, Dirac, etc.[23] ; although our own numerology[7] connects the gravitational interactions with the strong ones [that are -like the former - always attractive, non-linear, and eventually associable with non-Abelian gauge theories : these Sections propose, indeed, an ante litteram geometrical interpretation of those theories], and not with the electromagnetic ones (as suggested, on the contrary, by Dirac). For instance, it is straightforward to prove from our "dilation-covariant Relativity" that the mass M of our cosmos and the mass m of the pion are linked together as follows :

$$M = \varrho^{-2}\, m \simeq 10^{54}\ \text{Kg} \; ; \qquad m = \varrho^2 M \simeq 10^{-28}\ \text{Kg} \; . \qquad (15)$$

The numerology derived by our theory, moreover, deos not predict any dependence of G on time : see Ref. (22).

Before going on, let us specify that we shall confine the models to be adopted (for both our cosmos and hadrons) within the realm of Friedmann models. In particular, we shall take advantage of the fact that the Friedmann models are compatible with Mach Principle[24], and are embeddable in five dimensions[25].

Moreover, always consistently with eqs. (11), for the spatial part of our cosmos we can choose the simple model of the 3-dimensional hypersurface of a hypersphere. Analogously we can proceed for hadrons (strong universes), so as to be able to extend e. g. the Mach Principle for them : in the sense that the inertia of every hadron-constituent (parton) will coincide with its strong-charge (and not with its gravitational charge!). In such a way, we shall be able to consider an "Equivalence Principle" as locally valid even inside hadrons, so as to justify from the point of view of General Relativity the present geometrization of the strong field (first of all inside hadrons, and then - as we shall see - even in their surroundings). It is apparent that also the other fundamental fields could a priori be geometrized in the same way.

5. - INSIDE A "UNIVERSE". QUARK CONFINEMENT.

Let us now find out an exact solution of eqs. (12) - inside a hadron - for a spherically-symmetric distribution g' of strong-charge. The geodesic equation for a (small) test-constituent with strong-charge g'' in the vacuum ($i, j = 1, 2, 3$; $N = 1$):

$$\frac{d^2 x^i}{dt^2} = \frac{c^2}{2} g^{ij} g_{oo,j}$$

yields in the radial case :

$$\frac{d^2 r}{dt^2} = -\frac{c}{2}\left(1 - \frac{2g'}{c^2 r} + \frac{Hr}{3}\right)\left(\frac{2g'}{cr} + \frac{2Hr}{3}\right). \qquad (N = 1) \tag{16}$$

The spherically-symmetric distribution g' can be identified e. g. with a quark ; on the contrary, g'' must be a (small) test-particle. When g'' is another quark, eq. (16) holds only approximately and merely furnishes an idea about the radial behaviour of g'' in the field of g'. Nevertheless, eq. (16) yields the so-called "asymptotic freedom" of quarks (or, rather, "small constituents") for small distances r, as well as the quark (constituent) "confinement" for large values of r.

Let us examine the case of small values of r, when the attractive term $\propto -1/r^2$ dominates (so as in the gravitational case). (Notice that the repulsive term $\propto 1/r^3$ effectively works only at extremely small values of r, so that the radial acceleration in the gravitational case would vanish only for $r \simeq 2Gm/c^2$!). However, if we attribute an angular momentum J with respect to g' to the considered constituent g'', i. e. if we add the "kinetic-energy term" to the radial potential corresponding to eq. (16), then - with the choice (8) for the measure units - we can write for small r ($r \ll r(h)$; $r \ll 1$ fm) :

$$V \simeq \frac{(J/g'')^2}{r^2} - \left(\frac{Ng'}{r} - \frac{Ng'^2}{2c^2 r^2} - \frac{c^2 H}{3} + \cdots\right) \approx -\frac{Ng'}{3} + \frac{(J/g'')^2}{r^2}. \tag{16'}$$

We see that g'' will tend to stabilize dynamically itself at a distance r_o from g' where the (total) interaction-potential vanishes. This result seems to render simply reason of the asymptotic freedom of hadron constituents.

In the case of two quarks, some approximate considerations do even allow to write down a Regge-like relation(7) between J and the hadron mass.

For large distances, when $r \simeq r(h) \simeq 1$ fm, one gets(7) from eq. (16) a radial attractive force (confining the constituents inside hadrons) which is proportional to $-r$ ($N = 1$; $N = G$):

$$F \simeq -\frac{g''}{3}(c^2 Hr - 2H) \approx -g''c^2 Hr/3 \propto -r, \quad (r \simeq \sqrt[3]{\frac{6Ng}{cH}} \simeq 1 \text{ fm}) . \tag{17}$$

In other words, by applying the methods of General Relativity to hadron structure, we get in a very natural way also a confining potential $V \propto r^2$ for large values of r. For very large values of r, attainable e. g. when the considered hadron starts to get deformed under a high-energy collision, we would get(7) an even stronger confining force, proportional to $-r^3$:

$$F \simeq -g''c^2 (\frac{H^2}{g} r^3 + \frac{H}{3} r + \cdots\cdots), \qquad (r > r(h)) . \tag{18}$$

In conclusion, through eq. (16), our unified (classical) approach to strong and gravitational interactions yields a completely defined (radial) potential for constituent-constituent interaction inside hadrons. Such interesting potential appears to deserve further attention. Another conclusion is that the introduction of our micro-universes (for which, essentially, the theory of general relativity can be used) can advantageously substitute models so as the "M. I. T. bag" model.

Let us notice that all our previous results must (and can) hold also inside our cosmos, mutatis mutandis.

6. - IN THE SURROUNDINGS OF A "COSMOS".

We may regard the spatial parts of our cosmos and of hadrons (time aside) as embedded in a 4-dimensional flat space E^4. The problem of strong interactions between two hadrons (e. g. two nucleons) requires considering what heuristically we can call the "intersections" of hadrons with our cosmos: such intersections being 2-dimensional spherical surfaces, that we just call "hadrons" tout court. Since (in our cosmos) two "hadrons" interact strongly - e. g. via Van-der-Waals-like forces - we need therefore to describe the (strong) interactions between the aforesaid "intersections". To this end, when considering the motion of a hadronic test-particle - possessing both strong and gravitational charges -, a "bi-scale" theory is required (inside our cosmos) in the surroundings of hadrons and in presence of subnuclear interactions. In other words, we need to modify the gravitational Einstein equations by introducing, in the micro-neighbourhood of the abovementioned intersections (hadrons), a strong metric-deformation $s_{\mu\nu}$ affecting (only) the objects with strong charge (i. e. with scale-factor $k = \rho \simeq 10^{-41}$), and not affecting the ones with gravitational-charge only (i. e. with scale-factor $k = 1$). Around a hadron we can assume the gravitational metric-tensor to be $f_{\mu\nu} \cong \eta_{\mu\nu}$ (in suitable coordinates); and set

$$g_{\mu\nu} \equiv f_{\mu\nu} + s_{\mu\nu} \cong \eta_{\mu\nu} + s_{\mu\nu} \tag{19}$$

where the components of the strong métric-tensor $s_{\mu\nu}$ have to vanish for $r \gg 1$ fm. In Refs. (7), we proposed the following field-equations (for test-objects having both gravitational and strong charges, in the surroundings of a hadron, inside our cosmos):

$$R_{\mu\nu} + Hs_{\mu\nu} = -\frac{8\pi}{c^4}(S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho}) \, , \tag{20}$$

with $S_{\mu\nu} = NT_{\mu\nu}$; $N = G\varrho^{-1}$; and where the "cosmological (strong) term" with the hadronic constant H takes care of the geometric properties of the strong field around the "source hadron".

Let us here verify that, at the static limit and for "weak" field ($r \gtrsim 1$ fm), we do correctly reduce ourselves to deal with a scalar field s_{oo} having the required Yukawian behaviour (of course, with $|s_{oo}| \ll 1$ for $r \gg 1$ fm). Eq. (20) in suitable coordinates writes

$$R_{\mu\nu} + H(g_{\mu\nu} - \eta_{\mu\nu}) \simeq -\frac{8\pi}{c^4}(S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho}) \, , \tag{20'}$$

which can also read

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - Hg_{\mu\nu} = -\frac{8\pi}{c^4} S'_{\mu\nu} \\ S'_{\mu\nu} \cong S_{\mu\nu} - \frac{c^4}{8\pi} H(\eta_{\mu\nu} - \frac{1}{2} g^{\alpha\beta}\eta_{\alpha\beta} g_{\mu\nu}) \, , \end{cases} \tag{21}$$

where the last term has the meaning of <u>interference</u> between the two tensorial fields. Notice that our strong-field tensor is precisely

$$\Phi_{\mu\nu}/g' \equiv \frac{1}{2} s_{\mu\nu} \cong \frac{1}{2}(g_{\mu\nu} - \eta_{\mu\nu}) \, . \tag{22}$$

By linearizing with respect to the flat metric, from eq. (20') we get the linearized equations with "hadronic (cosmological) term" ($r > 1$ fm) :

$$\begin{cases} \partial^{\mu}\partial_{\mu} s_{\alpha\beta} + 2Hs_{\alpha\beta} \simeq \frac{16\pi}{c^4}(S_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} S^{\varrho}_{\varrho}) \\ \partial_{\mu} s^{\mu}_{\ \nu} = \frac{1}{2} \partial_{\nu} s^{\mu}_{\mu} \, ; \qquad |s_{\mu\nu}| \ll 1 \quad \text{for} \quad r \gg 1 \text{ fm} \, . \end{cases} \tag{23}$$

Eq. (23) is a (relativistically covariant) equation for a massive tensorial field[26].

In the static limit ($\partial g_{\alpha\beta}/\partial t \simeq 0$), when

$$S^{\varrho}_{\varrho} = S_{oo} = Nc^2\gamma \, , \tag{24}$$

quantity $N\gamma$ being the density of strong-charge magnitude (and γ the ordinary mass-density), one gets for the scalar field $V_{ext} \cong \frac{1}{2} c^2 s_{oo}$ the equation

$$\nabla s_{oo} - 2Hs_{oo} \simeq \frac{8\pi}{c^4} N\gamma \, . \tag{25}$$

Finally, for a point-particle with strong charge g at rest in the origin, a spherically symmetric solution of eq. (25) is ($g_{oo} = 1 + s_{oo}$) :

$$s_{oo} \equiv \frac{2V_{ext}}{c^2} = - \frac{2g}{cr} \exp(-r\sqrt{2H}) . \tag{26}$$

In the case of a nucleon, eq. (6') holds with the second value, $Ng^2/\hbar c \simeq 15$. It is enough to identify

$$\sqrt{2H} \equiv m_S c/\hbar , \tag{27}$$

in order to get for the (external) field-mass(26) the value

$$m_S = \hbar\sqrt{2H}/c \simeq m_\pi . \tag{27'}$$

In conclusion (for test-particle low speeds,and "weak" field) we actually obtained a scalar field with the correct Yukawian behaviour :

$$V_{ext} \simeq - \frac{g}{r} \exp(-rm_\pi c/\hbar) . \tag{28}$$

7. - FURTHER REMARKS, AND SOME SPECULATIONS.

If, in our space (inside our cosmos), we want to associate with ordinary hadrons - i. e. with the aforesaid "intersections" (cf. Sect. 6) - a spherically symmetric source of the strong field $\Phi_{\mu\nu}/g' = \frac{1}{2} s_{\mu\nu}$, we can try to solve the "Schwarzschild-type problem" for the Einstein-type equations (20). Remember that eqs. (20), differently from eqs. (12), are no more Einstein equations (but they are modified Einstein equations). In particular, we shall eventually be able to calculate the "strong Schwarzschild-type radii" corresponding (in our cosmos) to hadrons. The results $r_S^{(s)}$ that we shall obtain appear to yield actually the "effective radii" of hadrons in strong interactions.

In order to perform our task, we shall write eqs. (20) as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\varrho_\varrho - H(g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g^{\alpha\beta}\eta_{\alpha\beta} g_{\mu\nu}) = - \frac{8\pi}{c^4} S_{\mu\nu} , \tag{20"}$$

and then use the trick of transforming them into

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\varrho_\varrho \simeq \frac{8\pi}{c^4} (S_{\mu\nu} + t_{\mu\nu}^{(strong)}) , \\ t_{\mu\nu}^{(strong)} \equiv - \frac{c^4 H}{8\pi} (g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g^{\alpha\beta}\eta_{\alpha\beta} g_{\mu\nu}) , \end{cases} \tag{29}$$

with $S_{\mu\nu} = NT_{\mu\nu}$; $N = \varrho^{-1} G$.

By setting:

$$\overline{S}_{\mu\nu} \equiv S_{\mu\nu} + t_{\mu\nu}^{(strong)} , \tag{30}$$

we are left with the equations:

$$R_{\mu\nu} \simeq - \frac{8\pi}{c^4}(\overline{S}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\overline{S}_{\varrho}^{\varrho}) ,$$

$$\left| t_{\mu\nu}^{(strong)} \right| \ll 1 \quad \text{for} \quad r \gg 1 \text{ fm} . \tag{30'}$$

For a spherically-symmetric distribution of hadronic charge

$$t_{oo}^{(strong)} = \frac{\Phi_{oo}}{g'} u(r) \simeq \frac{1}{2}(g_{oo} - 1) u(r) , \tag{31}$$

the structure of eqs. (29) suggests[27, 7] to write - in analogy with what one does when in presen ce of an electromagnetic field -

$$u(r) = \frac{1}{8\pi g'^2} \left[\left| \vec{\nabla}\Phi \right|^2 + \tilde{\mu}^2 \left| \Phi \right|^2 \right] \simeq \frac{1}{32} \left[\left| \vec{\nabla}(g_{oo} - 1) \right|^2 + \tilde{\mu}^2 \left| g_{oo} - 1 \right|^2 \right], \tag{31'}$$

where we put

$$\Phi \equiv \Phi_{oo} \; ; \quad \tilde{\mu} \equiv m_S c/\hbar \simeq m_\pi c/\hbar . \tag{31''}$$

Eqs. (31), and the following ones, are better dealt with by means of the choice (9), with N = 1.

To solve our problem, we can thus adopt an iterative procedure. For the first iteration, in the static limit, we can take for its zeroth-order approximation ($r \gtrsim 1$ fm):

$$\frac{1}{2}(g_{oo} - 1)\Big|_I \simeq \frac{\Phi^I}{g'} \simeq - \frac{g}{r} \exp(- r\tilde{\mu}) , \tag{26'}$$

and get

$$u(r) \approx \frac{1}{8\pi} \frac{g^2 \exp(-2\tilde{\mu}r)}{r^2} \left(\frac{1}{r^2} + \frac{2\tilde{\mu}}{r} + 2\tilde{\mu}^2\right) . \tag{32}$$

We shall put as usual $\left[\Phi_{oo}/g' \equiv \frac{1}{2} s_{oo} \cong \frac{1}{2}(g_{oo} - 1) \equiv \exp\left[\nu(r)\right] \right]$:

$$s_{\mu\nu} \cong g_{\mu\nu} - \eta_{\mu\nu} = 2 \begin{pmatrix} \exp\left[\nu(r)\right] & & & \\ & \exp\left[\lambda(r)\right] & & \\ & & -r^2 & \\ & & & -r^2\sin^2\theta \end{pmatrix} , \tag{33}$$

where ν, λ are functions still to be determined. Notice that we are looking for "strong black-hole"-type solutions, inside our cosmos and in vacuum (and considering the spece outside the horizon).

By inserting eqs. (31) and (33) into eqs. (29) one gets, among the others, the equation[7, 27]:

$$-\frac{8\pi g^2}{m'^2 c^2} u(r) \approx \exp\left[-\lambda(r)\right]\left(\frac{1}{r^2} - \frac{1}{2}\frac{d\lambda(r)}{dr}\right) - \frac{1}{r^2}\ , \tag{34}$$

where $Ng^2/\hbar c \approx 15$, and m' is the mass of the hadronic test-particle (we can choose e. g. $m' \approx$ $\approx m_q$ = average quark-mass, the "test-quark" being considered a priori as situated initially outside the horizon). The exact solution of eq. (34) is

$$\exp\left[-\lambda(r)\right] = 1 - \frac{2\ell}{r} + \left(\frac{\tilde{\mu}k}{r} + \frac{k}{r^2}\right)\exp\left[-2\tilde{\mu}r\right]\ , \tag{35}$$

where $k \equiv g^4/c^4 m'^2$, and where $\ell = g^2 m/c^2 m'^2$ is an integration constant with the dimensions of a length, quantity m being the hadron mass (e. g., the nucleon mass).

Obviously, in our Schwarzschild-type geometry, the strong "Schwarzschild-type radii" $r_S^{(s)}$ will be calculated in correspondence with:

$$\exp\left[-\lambda(r)\right] = 0\ ; \qquad \exp\left[\nu(r)\right] = 0\ . \tag{36}$$

The first one of eqs. (36) yields values of $r_S^{(s)}$ depending only slightly on $\tilde{\mu}$. Almost the same results are got, e. g., for $\tilde{\mu} \approx m_\pi c/\hbar$ or $\tilde{\mu} \approx 0$. In the simpler case $\tilde{\mu} \approx 0$, the first one of eqs. (36) becomes

$$r_S^{(s)2} - 2\ell r_S^{(s)} + k = 0\ , \tag{37}$$

that is to say

$$r_S^{(s)} = \ell \pm \sqrt{\ell^2 - k}\ , \tag{37'}$$

For the nucleon, e. g., we have the two approximate solutions:

$$r_S^{(s)}(N) \begin{cases} \nearrow r_1 \approx 10^{-15}\ \mathrm{cm}\ ; \\ \searrow r_2 \approx 0.8\ \mathrm{fm}\ . \end{cases} \tag{38}$$

While the larger value is in good agreement with the ordinary nucleon-radius in strong interactions, many alternative interpretations might be suggested for the smaller value.

We have now to verify that on the Schwarzschild-type horizon also the second one of eqs. (36) holds. But the calculation of $\nu(r)$ can be performed only at the price of further approximations. We limit therefore ourselves only to verify that, in the present case, it is actually

$$\exp\left[\nu(r)\right] \approx \left\{\exp\left[\lambda(r)\right]\right\}^{-1} \approx 1 - \frac{2g^2 m}{rc^2 m'^2}\ . \tag{39}$$

It is finally worthwhile to notice that our "strong" metric

$$ds^2 = \exp\left[\nu(r)\right] c^2dt^2 - \exp\left[\lambda(r)\right] dr^2 - r^2(d\theta^2 + \sin^2\theta \, d\varphi^2) , \tag{33'}$$

together with eqs. (35) and (39), has been shown by Mignani to be identifiable with 'tHooft monopole-metric in curved space-times[28].

In conclusion, ordinary hadrons can be considered in our cosmos as "strong black-hole"-type objects, in the sense seen above. This is a very peculiar sense, since e. g. our "strong black-holes" at the static limit are surrounded (for $r > 1$ fm) by a strong (scalar) field with Yukawian behaviour, which has nothing to do of course with ordinary "strong black-holes". However, if the usual physics, valid for ordinary (strong) black-holes, can be extended - partially at least - to our peculiar "strong black-holes", then many stimulating questions would arise, which have been mentioned elsawhere[7]. In particular, our "strong horizon" might play - at a classical level - a rôle similar to the one of the already mentioned "MIT bag". [Let us recall that strong black-holes can be characterized (besides by mass, spin, electric charge and pseudoscalar charge) also by further quantum numbers or charges, since the ordinary "short range" fields can be regarded as "long range" fields at their scale]. Moreover, let us add that the "classical confinement" here obtained for hadron constituents can be violated by quantum effects so as Hawking's. The "Hawking temperature" for a strong black-hole, e. g., results[7] to be of the order of $T \simeq 2 \times 10^{11}$ K, and corresponds a priori to an "evaporation time" of the order of $\Delta t \simeq \simeq 10^{-23}$ s, unless we do impose some stability-conditions of the kind of Bohr's[7, 5]. [In any quantum theory, however, quarks may be again "totally confined" - if you want - by associating with their classical Schwarzschild (strong) horizon a suitable barrier of strict "super-selection rules" and of " conservation super-laws"].

All what previously said, of course, can be "translated" so to hold for the case of "gravitational universes" as our cosmos.

Let us add three speculations.

First, if our cosmos is similar to a hadron, it might e. g. be conceived - following the calculations in this Second Part - as a Super-pion, and therefore as constituted by one matter half-cosmos (or "Meta-galaxy"[3]) and by one antimatter half-cosmos (so as each pion consists of a quark and an antiquark).

Second, let us assume that for ordinary neutrons (from the point of view of our peculiar "strong black-holes") it can be extended the validity of the Second Law of black-hole thermodynamics, saying that when two black-holes melt together the Schwarzschild area of the final black-hole must be larger than the sum of the two initial Schwarzschild areas. Then, when neutrons would melt together (within the cyclic big-bang theory) at the end of an expansion/recontraction cycle, the "Super-neutron" born out from the fusion of those 10^{80} neutrons ought eventually to possess - by <u>extrapolation</u> - a Schwarzschild horizon with area

$$S \gg 10^{80}\pi r_n^2 , \qquad (r_n \simeq 10^{-13} \text{ cm}) \tag{40}$$

so that the melting process (= big-bang explosion!) should rebuild up a new object with radius

$$R \gg 10^{27} \text{ cm} . \qquad (40')$$

Third, if hadrons are similar to our cosmos, they too could perform successive cycles of expansion and recontraction, with a period - however - of about $\Delta\tau \simeq 10^{18}/10^{41}$ s $\simeq 10^{-23}$ s. We should thus get that subnuclear particles can be regarded as pointlike only at certain successive, discrete positions along their trajectory (associable with a fundamental chronon). It is interesting that Caldirola[29], with regard to this argument, started from a "finite difference" equation for the electron motion and ended with the conclusion that leptons too can be considered as pointlike objects moving on a 4-dimensional de Sitter micro-universe (to which, in our ordinary space, a spherical object can be associated - by suitably projecting onto a tangent hyperplane, - such a sphere performing successive cycles of expansions and recontractions with a period of about 10^{-23} s).

A structure of the type of the "micro-universes" could therefore be characteristic of all subnuclear particles[7], and show a classical path to unification also of weak interactions (together with the strong and gravitational ones).

8. - CONCLUSIONS.

We have shown that, when applying the methods of General Relativity to subnuclear particle physics, one can advantageously substitute models so as the "MIT-bag" model with our classical theory which considers hadrons to be (finite, "strong") micro-universes. According to our unified (classical) approach to gravitational and strong interactions, inside hadrons there is essentially a tensorial field (= "strong gravity"), and the constituents are supposed to exchange spin-2 "gluons". In other words, "scaled down" Einstein equations - with "strong" cosmological term - hold for the hadron interior, and they yield in a natural way the confinement and "asymptotic freedom" of the hadron constituents.

Our approach allows also writing down a (bi-scale, tensorial) field-theory of hadron-hadron interactions by suggesting (modified) Einstein-type equations for strong interactions. In particular, we obtain in this way: the correct Yukawian behaviour of the strong potential (for $r > 1$ fm) at the static limit; and the value of hadron radii in strong interactions. In a sense, we carry out the old idea by Riemann and later Clifford that the matter-particles were merely the manifestation of a local strong curvature of space.

Our hope for the future is that the internal symmetries of the micro-cosmoses associated with hadrons (for instance via a projection onto a tangent hyperplane) can lead to some basic elements of the theory of quantum-chromodynamics so as the "color SU(3)".

As a by-product, we derived a whole "numerology" connecting our gravitational cosmos to the strong micro-cosmoses (hadrons). For further details, cf. Refs. (7).

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APPENDIX A

In this Appendix, let us - for example - hint at the construction of the "general relativity" by starting from Projective Relativity. We have then to introduce(13) an anholonomous X^4 space (in general, a variable-curvature Riemannian manifold) that admits the de Sitter group as its holonomy group. Since that group is isomorphic to the one of S^5 rotations (where S^n indicates - let us repeat - the n-dimensional hyperspherical space), we have to resort to the geometry of a Riemannian variety V^5 (which just admits as holonomy group the one of the rotations of S^5). Such a geometry of V^5 will have then to be interpreted in terms of projective differential geometry of a four-dimensional manifold X^4. It is known that the projective differential geometry of a X^n allows in fact a (n+1)-dimensional interpretation in terms of the Riemannian geometry of V^{n+1}.

Following again Cartan, a space X^4 with projective connection is a space having the characters of a projective space in the (infinitesimal) neighbourhood of each point P of its, and endowed with a projective (homographic) connection-law between the neighbourhoods of two infinitely-close points of its. To such a purpose, it is necessary to provide a suitable field of quadrics Q, placed in the spaces tangent to the single points P of X^4 (cf. Ref. (13)). Once fixed the point P, the corresponding quadric Q(P) constitutes the "absolute" of the local, non-Euclidean metric. The parallel transport in a Riemannian V^4 preserves the isotropous cones; analogously, the projective connection must yield a projective transport law that preserves the aforesaid field of quadrics Q. After having thus determined the projective connection(13), one can build up in the usual way the "curvature projective tensor" $P_{\alpha\beta\gamma\delta}$ $(\alpha, \beta, \gamma, \delta = 1, \ldots, 5)$, whose vanishing is the necessary and sufficient condition for the given space to be "projectively flat" (i.e., with constant curvature). In fact, the constant-curvature varieties are locally representable onto the Euclidean space with preservation of the geodesics.

The vanishing of the curvature Riemannian tensor leads, in GR, to get again the Minkowski s-t; on the contrary, the vanishing of the curvature projective tensor $P_{\alpha\beta\gamma\delta}$ leads back - in projective general relativity - to the de Sitter s-t with constant curvature. Finally, the tensor $P_{\alpha\beta\gamma\delta}$ has the important property of including the torsion tensor (so that Cartan called it the "curvature-and-torsion-tensor"). Actually, at variance with what happens in the ordinary spaces endowed with affine connection, now the curvature of a projective-curvature-space implies a torsion: this is due to the fact that the de Sitter group (holonomy group of X^4) decouples - at the "relativistic" limit - in the rotations and translations of S^4, to which the "rotation curvature" and the "translation curvature" (= torsion) correspond, respectively.

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