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SUMMARY. -

In this paper we first "complete" a previous letter (wherein we derived, among the other things, a classical "quark confinement" from General Relativity plus dilatation-covariance), by showing our theory to be compatible also with quarks' "asymptotic freedom". Then - with in a "bi-scale" theory of gravitational and strong interactions - we propose a classical field theory for the (strong) interactions between different hadrons. Various, noticeable consequences are briefly analysed.

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1. - We have recently published a letter⁽¹⁾ proposing a classical "quark confinement" from General Relativity, on the basis of refs. (2,3). We want here to extend the content of ref. (1), thus suggesting a classical field theory of the strong interactions between hadrons (within a unified theory⁽³⁾ of gravitational and strong interactions). We shall also take advantage of the present occasion for improving some minor points^(x) of ref. (1).

Let us remember that, when physicists took due account of the electromagnetic phenomena (besides of the mechanical ones), it was necessary to leave Galilean Relativity⁽⁴⁾ in favour of Einstein's⁽⁵⁾. Since we have now to deal also with nuclear and subnuclear forces (in particular with strong interactions), it is perhaps advisable to look for a new Relativity⁽¹⁻³⁾. To such an aim, we can get inspiration from the "Erlangen program" of Physics put forth by Fantappiè⁽⁶⁾. Let us observe that the Galilei group $G_{1,3}^{10}$ (whose homogeneous part is isomorphic to $O(3)$) can be obtained - through a "contraction" - from the Poincaré one $L_{1,3}^{10}$ (whose homogeneous, "Lorentz" part is isomorphic from the complex viewpoint to $O(4)$, i. e. to the group of the "rotations" in a flat, four dimensional (1, 3)-space) as the "limit-case" (= boundary case) for $c \rightarrow \infty$. One can wonder whether the Poincaré group can be in its turn a "limit-case" of another, new group. Remaining in a four-dimensional space (considering only 10-parameters groups), in 1954 it was shown⁽⁶⁾ that a unique new group exists, depending with continuity on a parameter R , which is reduced to Poincaré's for $R \rightarrow \infty$ and which cannot be any more the "limit" of any other, different group. Such a new group $F_{1,3}^{10}$, happened to be that one of the motions into itself of the De Sitter space-time having constant curvature and metric

$$ds^2 = c^2 d\tau^2 - (d\xi_1^2 + d\xi_2^2 + d\xi_3^2) \exp(2c\tau/R),$$

with cosmological constant $\Lambda = 3/R^2$. Now, the De Sitter space-time is represented as a hypersurface, with equation

$$Z_0^2 - Z_1^2 - Z_2^2 - Z_3^2 - Z_4^2 = R^2 \tag{1}$$

embedded in a flat, five-dimensional space (here and in what follows we admit that some coordinates can be imaginary). From that viewpoint, then the De Sitter-Fantappiè group $F_{1,3}^{10}$ becomes the group $O(5)$ of the rotations in a flat, 5-dimensional (1, 4)-space; and this clearly shows that (and how) $F_{1,3}^{10}$ generalizes the Poincaré group⁽⁶⁾. Thus - roughly speaking - we have been considering the following "chain" of groups

$$G_{1,3}^{10}(c \rightarrow \infty; R \rightarrow \infty) \Leftarrow L_{1,3}^{10}(c; R \rightarrow \infty) \Leftarrow F_{1,3}^{10}(c, R), \tag{2}$$

(x) Let us immediately point out that the first addendum of the r. h. s. of eq. (5) in ref. (1) should read $\frac{1}{9}H^2 r^3$ instead of $\frac{1}{9}H^2 r$.

where the final, De Sitter-Fantappi  group "contains" two universal constants (a fundamental length, \underline{R} , and the light-speed in vacuum, \underline{c}). But we know⁽⁷⁾ that, in order to plan in a dimensionally correct way even only a mechanics (dynamics) theory, three universal constants are needed⁽⁷⁾. To lengthen the chain in eq. (2) one has however to abandon the 10-parameters groups (i. e., the fourdimensional Minkowski space). Following Arcidiacono⁽⁶⁾, it is then easy to "reach" the conformal group $C_{1,4}^{15}$, with 15 parameters, which is locally isomorphic to the rotations of a 6-dimensional space. It will now contain three independent universal constants: $C_{1,4}^{15}(\underline{c}; \underline{R}; \underline{h}^{-1})$, where the third constant, \underline{h} , must depend on a Mass (besides, possibly, on a Length and on a Time)⁽⁷⁾.

As already claimed in ref. (1), out of all the elements of the conformal group we start considering only the space-time dilatations: $x'_\mu = \varrho x_\mu$, ($\mu = 0, 1, 2, 3$; ϱ discrete), besides the Lorentz transformations. Actually⁽³⁾, since a contraction (dilatation) of the chosen, chronotopical measure-units should not affect the form of physical laws, then (when fixing, conversely, those measure-units) physical laws ought to be covariant under space-time dilatations (contractions). And we postulate that physical laws are covariant also under (discrete) dilatations; and that in nature only discrete values of ϱ happen to have physical counterparts^(x)(1-3). Furthermore, let us here propose⁽³⁾ for a renewed consideration a passage from the last scientific writing^(o) by Einstein⁽⁹⁾:
"... From the field equations one can immediately derive what follows: if $g_{ik}(x)$ is a solution of the field eqs., then also $g_{ik}(x/a)$ is a solution, where a is a positive constant ("similar solutions"). Let us for instance suppose system g_{ik} to represent a finite-size crystal embedded in a flat space. We could then have a second "universe" with another crystal, exactly similar to the previous one, but having its linear sizes a times as big. As far as we confine ourselves to a universe containing nothing but a unique crystal, we do not meet any difficulties. We realize only that the size of such a crystal ("standard of length") is not fixed by the field equations. . . "⁽⁺⁾.

2. - Before going on, let us improve a couple of points contained in ref. (1):

(i) If we call $\underline{g}_0 = |\underline{g}'|_{av}$ the average magnitude of the quark strong-charges and \underline{g} the "hadron strong-charge" so as defined in ref. (1), then

(x) Such discrete values of ϱ can be a priori obtained either by quantum versions of the theory, or rather by imposing suitable boundary conditions e. g. in 5-dimensional spaces. See e. g. refs. (8).

(o) This passage was written by Einstein at Princeton on the April 4, 1955.

(+) Our translation. The original (complete) text, in German, can be found in ref. (9), and in refs. (3).

eq. (6) of ref. (1) writes⁽³⁾ ($n = 2, 3$):

$$g_0 = \frac{g}{n} \approx \frac{m}{n\sqrt{\rho}} \approx \sqrt{\frac{\hbar c}{G}} = \text{Planck mass},$$

where⁽¹⁾ $\rho \approx 10^{-41}$ and m is the mass (= gravitational charge) of the considered hadron. Therefore, we expect that the predicted "small black-holes" could merely be identified with quarks, which possess in our theory a strong-charge - 10^{-5} grams (in suitable units).

(ii) In ref. (1) we assumed: inside our cosmos (= gravitational "universe") the Einstein equations with cosmological term⁽³⁾ ($G = 1$):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\rho}_{\rho} - \Lambda g_{\mu\nu} = - \frac{8\pi}{c^4} T_{\mu\nu}; \quad 2\Lambda \equiv (m_g c/\hbar)^2;$$

and inside hadrons (= strong "universes") the "scaled" Einstein eqs. ($N = G = 1$):

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\rho}_{\rho} - H \tilde{g}_{\mu\nu} = - \frac{8\pi}{c^4} S_{\mu\nu}; \quad 2H \equiv (m_s c/\hbar)^2. \quad (3)$$

For details, see ref. (1). Here, let us observe that - because of what we are going to say in Sect. 4 - quantity m_s must be necessarily attributed the meaning of mass of the external strong-quanta (in fact, we got⁽¹⁾: $m_s \approx m_{\pi}$); and, analogously, quantity m_g must be necessarily attributed the meaning of mass of the external gravitation-quanta, for which by the way we got $m_g \approx 10^{-68}$ Kg. Namely: quantities m_s and m_g can bear the significance (also) of "internal" strong-quanta (spin-2 gluon) mass and of "internal" (spin-2) graviton mass, respectively, only if we assume the "external" quanta masses to be of the order of the "internal" ones⁽³⁾. On the contrary, in our theory we may a priori even assume the internal-quanta masses to be zero, both in the strong and in the gravitational case.

(iii) We already mentioned⁽¹⁾ that our theory⁽¹⁻³⁾ allows deriving (proving) in a systematic way all the empiric relations - which connect macro- with micro-cosmoses - heuristically discovered by Weyl, Eddington, Dirac, etc. Let us here clarify that our "numerology", however, connects the gravitational cosmos with the strong ones (= hadrons); actually, gravitational and strong interactions are both: always attractive, non-linear, and eventually associable in a natural way with non-Abelian gauge-theories of which we are indeed proposing an "ante litteram" geometrical interpretation. On the contrary, Dirac's suggestions regarded gravitational and electromagnetic interactions.

3. - We have shown⁽¹⁾ that our classical theory naturally yields - in side hadrons - the so-called "infrared divergency" (or confinement) for the constituent partons (and quarks): cf. eqs. (12-16) in ref. (1). Let us here add the derivation also of their "asymptotic freedom". I. e., let us consider also the behaviour for small values of r of the geodesic equation (eq. (12) in ref. (1)) holding for a (small) test-"parton" with strong-charge g' in vacuum⁽³⁾ (N = 1):

$$\frac{d^2 \vec{r}}{dt^2} = \frac{c^2}{2} \left(1 - \frac{2g}{c^2 r} + \frac{Hr^2}{3} \right) \left(\frac{2g}{c^2 r^2} + \frac{2Hr}{3} \right) \frac{\vec{r}}{r}, \quad (4)$$

which will approximately hold also for a "test-quark". In the case of small values of r, the attractive term $\propto -r^{-2}$ dominates (so as in the gravitational case), while the repulsive term $\propto +r^{-3}$ effectively works only at extremely small values of r, so that the radial acceleration vanishes only for $r \approx 10^{-33}$ cm (and, in the gravitational case, only for $r \approx \approx Gm/c^2$!). However, if we add the "kinetic-energy term" to the (radial) potential corresponding to eq. (4), i. e. if we attribute an angular momentum J to the considered quark (with respect to the set of the other constituents), then we can write for small r ($r \ll r(h)$; $\bar{N} = g^{-1}G$):

$$V \approx \frac{(J/g')^2}{r^2} - \left(\frac{Ng}{r} - \frac{N^2 g^2}{2c^2 r^2} - \frac{c^2 H}{3} + \dots \right) \approx -\frac{Ng}{r} + \frac{(J/g')^2}{r^2}; \quad (5)$$

so that in the quark case ($g' \approx (g-g')/n'$, with $n' = 1, 2$) we get:

$$V \approx 0 \quad \text{for} \quad r \approx 10 \times J^2 / (Ng)^3.$$

If we attribute a speed $v \approx c$ to the considered, moving quark, then^(x) $J \approx \hbar$ and $V \approx 0$ for $r \approx 0.1$ Fermi. Conversely, if we assume, - for instance in the baryons' case⁽³⁾, when $\bar{N} \approx 10^{40} G$, - the "stability radius" to be of the order of 1/100 of the "strong Schwarzschild radius" $r_0 \approx 2Nm/c^2$ of our hadron (considered as a "strong black-hole": cf. refs. (2, 3), then we get the Regge-like⁽¹⁰⁾ relation $J \approx (N/100 c) m^2$, where m is the baryon-mass in Kg; this relation also reads⁽¹⁰⁾ (with m now measured in GeV/c²):

$$J/\hbar \approx m^2. \quad (6)$$

(x) We can also borrow from quantum mechanics the suggestion that $J \approx \hbar$.

4. - Let us finally "complete" our ref. (1) by discussing the important problem of strong interactions^(x) between different hadrons (in our cosmos). As already said, for the spatial part of both our cosmos and hadrons, we choose the finite model constituted by the 3-dimensional hyper-surface of (4-dimensional) hyperspheres. Let us remember⁽¹⁾ that, from this point of view, the intersection of (the space-part of) a "hadron" with (the space-part of) our cosmos is a 2-dimensional spherical-surface (which we usually call hadron "tout-court"). We need modifying the gravitational Einstein eqs. in our cosmos by introducing - in the micro-neighbourhood of the abovementioned intersections (hadrons) - a strong metric-deformation affecting (only) objects with strong-charge (i. e. with scale-factor $\kappa = \varrho \simeq 10^{-41}$), and not affecting the ones with gravitational-charge only (i. e. with "scale-factor" $\kappa = 1$). Around a hadron, we can assume (in suitable coordinates) the gravitational metric-tensor $g^{(grav)}$ to be $g^{(grav)} \cong \eta_{\mu\nu}$; and set⁽³⁾:

$$g_{\mu\nu} = g_{\mu\nu}^{(grav)} + s_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu} , \quad (7)$$

where the components of the strong metric-tensor $s_{\mu\nu}$ have to vanish for $r \gg 1$ Fermi. We can geometrize the strong-field acting, in the surroundings of a hadron, on a hadronic test-particle by attributing to the latter an inertia coinciding with its "strong" (and not "gravitational") charge^(3, 10): i. e., by extending from the hadron-interior to the micro-neighbourhood of the hadron the strong-case "Equivalence Principle"^(1, 3).

We wish to propose the following field equations⁽³⁾ for test-objects having both gravitational and strong charges, in the surroundings of a hadron (in our cosmos):

$$R_{\mu\nu} + Hs_{\mu\nu} = - \frac{8\pi}{c^4} (S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S_{\varrho}^{\varrho}) , \quad (8)$$

where: $N = \varrho^{-1}G$; $S_{\mu\nu} \equiv NT_{\mu\nu}$; $s_{\mu\nu} \cong g_{\mu\nu} - \eta_{\mu\nu}$ (in suitable coordinates); and where the "cosmological (strong) term", with the hadronic constant $H \equiv \varrho^{-2}\Lambda$, takes care of the geometric properties of the strong-field around the "source-hadron" (and is effective in a region with linear sizes of the order of 1 Fermi: see the following). In suitable coordinates, eqs. (8) write:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R_{\varrho}^{\varrho} - H(g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \eta_{\alpha\beta}) \simeq - \frac{8\pi}{c^4} S_{\mu\nu} , \quad (8')$$

which can be regarded as a particular case^(o) of the general "bi-scale"

(x) Originating e. g. from Van-der-Waals-like mechanism. Cf. e. g. refs. (1, 3, 11).

(o) I. e., after elimination of the terms here negligible (in our case).

equation describing the simultaneous presence in the source-hadron neighbourhood of two fields with different "hierarchical order". The last addendum in its l. h. s. plays the rôle of an interference (mixing) term. Eqs. (8) or (8') can provide us with a classical field theory of strong interactions, where the strong field is the second-rank tensor $\phi_{\mu\nu}/g' \equiv \frac{1}{2} s_{\mu\nu} \approx \frac{1}{2} (g_{\mu\nu} - \eta_{\mu\nu})$. More details will be given in the future. Now, let us observe that :

(i) By linearizing eq. (8') with respect to the flat metric⁽³⁾, one gets a (relativistically-covariant) massive equation for a tensorial field with field-mass $m_s = \frac{\hbar}{c} \sqrt{2H} \approx m_\pi$ (which in this case is the "external strong-quanta" mass). In the static limit we have $S_{00}^0 = S_{00} = N\gamma c^2$, where $N\gamma$ is the density of the strong-charge magnitude. One would get the equation⁽³⁾ ($\gamma \equiv$ mass-density) :

$$\nabla_{s_{00}} - 2Hs_{00} \approx \frac{8\pi N}{c^4} \gamma \quad (9)$$

Finally, in the case of a point-particle with strong-charge g at rest in the origin, a spherically-symmetric solution of eq. (9) is^(1,3) :

$$s_{00} \equiv \frac{2V_{\text{ext}}}{c^2} = - \frac{2g}{c^2 r} \exp(-rm_s c/\hbar), \quad (9')$$

so that we have obtained the correct Yukawian potential V_{ext} (for test-particle low-speeds and for weak fields). Incidentally, one verifies that: $|s_{00}| \ll 1$ for $r \gg 1$ Fermi.

(ii) If our cosmos is similar to a hadron, it might then be regarded - for instance - as a "Super-pion", and therefore as constituted by one matter "half-cosmos" (or "Meta-galaxy"⁽¹²⁾) and by one antimatter "Meta-galaxy" (so as each pion consists of one quark and one antiquark).

(iii) If hadrons are systems similar to our cosmos, then (within a cyclic "big-bang" theory) they too could perform successive cycles of expansion and contraction, with a period - however - of about $\Delta\tau \approx \approx 10^{18}/10^{41} \text{ s} \approx 10^{-23} \text{ s}$. We could thus get that hadrons can be regarded as point-like only at certain successive, discrete positions along their trajectory (associable with a fundamental chronon); this would be important e. g. with regard to their behaviour when "quantum-mechanically" interacting. It is very interesting that Caldirola⁽¹³⁾ (starting from a finite-difference equation for the electron) reached the conclusion that even leptons can be regarded as a point-object moving on a De Sitter space which performs cycles of expansions and reconstructions. Such a structure (with 1, 2 or 3 point-objects) seems therefore to be possibly characteristic of all elementary particles, and points towards a new (geome-

trical) approach to the unified theory of gravitational, strong and electromagnetic interactions.

Let us take the liberty, at this point, of calling attention⁽¹⁴⁾ to an early, interesting use of the Anglo-Saxon word "Quark" made by Goethe in his verse: "In jeden Quark begräbt er seine Nase"⁽¹⁵⁾.

5. - At last, we can also evaluate the "strong Schwarzschild radii" r_S^S for spherically-symmetric strong-charge distributions, both starting from the "inside" viewpoint (i. e. from eq. (3), and then using a limiting-procedure⁽³⁾) and from the "outside" viewpoint (i. e. from eqs. (8)(8'), and then following ref. (2)). For strong-charges g , these radii appear to be clearly related to the ones experimentally revealed by hadrons in strong interactions: for instance $r_S^S(N) = 0.8 \text{ Fermi}^{(2, 3)}$. This therefore seems to support, for the "strong universes" (hadrons), the particular model of "strong black-holes" (1-3, 16). If such a model is taken seriously, we can imagine the "second law of black-hole thermodynamics" (17) to hold even for "strong black-holes", and in particular for neutrons e. g. when they melt together during the final stage of the cosmos-contraction. Thence the "super-neutron", arising from the "fusion" of the $\sim 10^{80}$ neutrons constituting our cosmos, must eventually possess a Schwarzschild horizon with area

$$S > 10^{80} \times 4\pi r_0^2 ; \quad (r_0 \equiv r(N) \simeq 10^{-13} \text{ cm}) ;$$

in other words, at the end of the cosmos-recontraction we must have a process that builds up a new cosmos with radius

$$R > 10^{25} \text{ m} , \quad (10)$$

and such a consideration may be a hint to investigating the big-bang "explosions".

Our last remark is the following. One of our starting point was assuming an attractive cosmological term, both in the gravitational and in the strong cases. However, if the "moving" (satellite) quarks can be considered as tachyonic⁽¹⁸⁾, then - due to tachyon mechanics and tachyon gravitational-interactions⁽¹⁹⁾ - the "cosmological" terms could on the contrary be chosen as repulsive.

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