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E. Recami: A NEW INTRODUCTORY VIEW ABOUT SUPERLUMINAL FRAMES AND TACHYONS.

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E. Recami: A NEW INTRODUCTORY VIEW ABOUT SUPERLUMINAL FRAMES AND TACHYONS(o).

1. - RIVISTING THE POSTULATES OF SPECIAL RELATIVITY (SR). -

A suitable choice of postulates for the theory of Special Relativity (SR) is the following one: (see ref. (1) and (2))

1) First Postulate: Principle of Relativity (PR): "Physical laws of Mechanics and Electromagnetism are covariant (=invariant in form) when going from an inertial observer to another inertial observer". Notice that this postulate does not impose any constraint on the relative speed u of the two inertial observers; and it is inspired to the consideration that all frames should be equivalent [for a careful definition of 'equivalence' see refs. (3) and (1).]
2) Second Postulate: "Space-time is homogeneous and space is isotropic". As wellknown, this postulate is justified by the fact that from it the conservation laws (of energy, momentum, angular momentum) follow.

   Since 1910 it was shown that the postulate of ligh-speed invariance is not strictly necessary, since it can be derived[4] from the above postulate 1) and 2). Let us moreover observe that the particular role of light-speed in SR is due to its invariance and not to the fact that it is (or is not) the maximal one.

   If we want - as we do - to avoid information transmission into the past, a third postulate is necessary[1]:
3) Third Postulate: "Negative-energy objects or particles, travelling forward in time, do not exist (and physical signals are carried only by objects that appear to carry positive energy)". This postulate will be shown to be equivelent to the Principle of (Retarded) Causality: "For every observer, "causes" chronologically precede their own 'effects' [for a definition of "causes" and "effects" see e. g. ref. (1)]". Moreover, from Postulate 3) the existence of anti-matter will be inferred.

From postulate 1) and 2) it follows[2] that one, and only one, quantity w^2 (having the physical dimensions of a speed square) must exist, which has the same value according to all the inertial frames: w^2=invariant. If we assume $w=\infty$, as in Galilean relativity, then we'ld get classical (Galilei-Newton's) physics. In such a case the invariant speed would be the infinite one, and we could write: $\infty \oplus v = \infty$; we indicate by $\oplus$ the operation of "speed composition". But experience has shown to us that the invariant speed is finite (and real); it is the light speed, c, in vacuum. In this case

$$c \oplus v = c\,, \tag{1}$$

and we immediately get Einstein's Relativity and physics. Let us emphasize that, in this second case, the infinite speed is no more invariant: when eq. (1) holds, then $\infty \oplus v \neq \infty$. Moreover, postulates

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1) and 2) require the existence of one invariant speed, and not of a maximal speed; the ligh speed will result to be in SR a limiting speed, but any limit is wellknown to have a priori two sides.

2. - CAUSALITY IN SR. -

Let us consider Fig. 1, where for simplicity a two-dimensional space-time is depicted. When we are in the position x=0 at time t=0, we usually incline to consider as 'existing' all the x=axis events. However, if another inertial observer, 0', moving along the positive x-axis, overtakes us at the origin-event, then at the same time t=t'=0 he will tend to consider as existing all the x'-axis events. Therefore, if we want to be able to start discussing and exchanging information with him, we must first be prepared to consider that all chronotopical events 'exist' (at least the ones outside the past-future zone of the ligh-cone). Then, nothing a priori prevents event A from influencing event B (see Fig. 1).

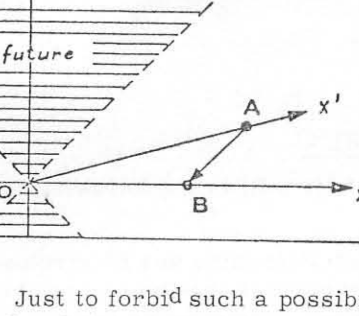

FIG. 1

Just to forbid such a possibility we introduce the Third Postulate. Our point is that, since we 'explore' the Minkowski space-time going forwrd in time (along the direction determined by thermodynamics and by cosmological evolution[5]) any observer will see the event B of Fig. 1 as the first one and the event A as the last one.

It can be, moreover, shown that[6] an object going backwards in time (Fig. 1) corresponds in the dual space, i. e. in the four-momentum space (see Fig. 2a), to an object carrying negative energy; and vice-versa. Let us start from the safe consideration of a positive-energy object, going forward in time; if we want to turn its motion backwards in time, then postulates 1) and 2) oblige us to apply to it a non-orthocronous Lorentz transformation. But any Lorentz transformations changing the sign of time will change also the signs of the fourth components of any other 4-vectors associated to the observed object (and in particular of the energy). This is true also in relatistic quantum mechanics (QFT); for example; if $f(\vec{p}, E)=1/(2\pi)^2 \int \tilde{f}(\vec{x}, t)\cdot \exp[ip\cdot x - iEt]\cdot d^4x$, then[6]

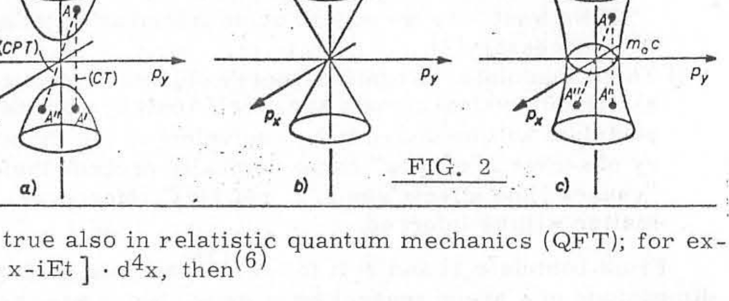

FIG. 2

$$f(\vec{p}, -E)=\frac{1}{(2\pi)^2}\int \tilde{f}(\vec{x}, -t)\cdot \exp\left[ip\cdot \vec{x} - iEt\right]\cdot d^4x . \qquad (2)$$

Let us now apply our Third Postulate (or "RIP", see the following): The two paradoxical occurrences "motion backwards in time and negative energy" can be reinterpreted in an orthodox way by any observer when they are - as they actually are - simultaneous[6]. In fact, let us suppose (Fig. 3) that a Particle P, with negative energy and e. g. charge[7] - e, travelling backwards in time, is emitted by A at time t_1 and observed by B at time $t_2 < t_1$.

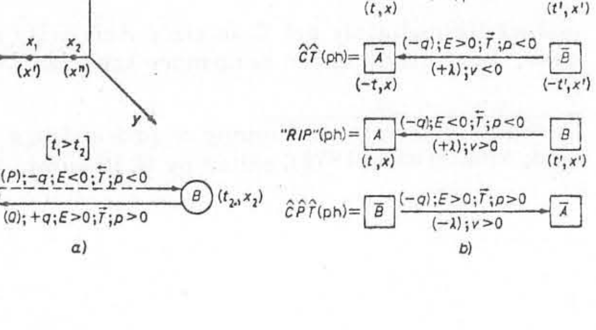

FIG. 3

Therefore at time t_1 object A 'loses' negative energy and charge - e, i. e. gains positive energy and charge +e. And, at time $t_2 < t_1$, object B 'gains' negative energy and charge -e, i. e. loses positive energy and charge +e. The physical phenomenon here depicted will of course appear to be nothing but the exchange from B to A of an (ordinary) particle Q with positive energy, charge +e, and travelling forward in time.

We have seen, however, that Q has the charge opposite to P; this means that our 'reinterpretation procedure' operates-among other things-a 'charge conjugation'[7], C. A closer inspection (see refs. (6) and (8)) of the 'RIP' tells us that Q will indeed appear as the antiparticle of P:

$$Q = \overline{P} \; ; \tag{3}$$

(actually, the mere 'RIP' in this case yields the particle except for the helicity sign: but the full result, eq. (3), is immediately got when considering the action of the complete Lorentz transformation-together with the 'RIP').

We are meaning that the concept of anti-matter is a purely relativistic one; and that, on the basis of the double sign (Fig. 2a)

$$E = \pm\sqrt{\vec{p}^{\,2} + m_o^2} \tag{4}$$

existence of antiparticles could been predicted since 1905,-exactly with the properties they actually showed to posses when later discovered, - provided that recourse had been made to the above 'reinterpretation'. We therefore mean that the points of the lower hyperboloid-sheet in Fig. 2a represent the kinematical states of the anti-particle $\overline{P}$ of the particle P represented by the upper hyperboloid-sheet.

Our Third Postulate, together with the above reinterpretation procedure, can assume the following form, that - after STÜCKELBERG[9] and FEYNMAN[9] - we shall call 'Reinterpretation Principle' (RIP): "Negative-energy objects travelling forward in time do not exist; and any negative-energy object P travelling backwards in time can, and must, be reinterpreted as its anti-object $\overline{P}$ going the opposite way (but endowed with positive energy and travelling forward in time)" (see refs. (1) and (8)). Notice that our three postulates imply also that: positive-energy objects travelling backwards in time do not exist; moreover, not only we can apply the 'RIP', but we must apply it (since we must 'explore' space-time in the positive t-direction).

It is now clear that the 'RIP', by eliminating any information transmission into the past, implements the validity of the law of (retarded) causality ("causes happen before their own effects"). Our 'RIP' finds a more elegant formulation in a five-dimensional space, where the fifth axis is related to rest-mass (see the following).

3. - SOME CONSEQUENCES. -

Inspection of Fig. 3b) shows e. g. that the 'RIP' does change -among other things - the 3-momentum sign but doesn't affect the 3-velocity sign: i. e. it changes the rest-mass sign. The 'RIP' can be recognized[1, 10] from Fig. 3b) to be formally equivalent to change the sign of all additive charges and of the rest-mass m_o (besides changing emission into absorption and vice-versa); we shall call 'strong conjugation' $\overline{C}$ the discrete operation

$$\overline{C} \equiv CC_{m_o} \tag{5}$$

where C is the conjugation of all additive charges and C_{m_o} is the rest-mass sign inversion. (Notice that, in quantum mechanics, our operator $\overline{C}$ will be a unitary operator when acting on the state-space[10]). Neglecting the operation X that effects the charge emission $\rightleftarrows$ absorption, we can write

$$\text{'RIP'} \equiv \overline{C}. \tag{6}$$

We can conclude that antiparticles must be formally attributed negative rest-masses (but positive total energies, of course). For clarity's sake, let us remember that in covariant form for any free particle:

$$E \equiv p_o \doteq m_o u_o c^2 \tag{7}$$

where u_o is the time-component of four-velocity. Now, let us consider a non-orthochronous Lorentz- transformation, L, changing (for simplicity) only the sign of all time-components:

$$E' = -E = m_o(-u_o)c^2 = m_o u_o c^2.$$

Afterwards, when applying the 'RIP' so as to get the corresponding antiparticle we finally have (for the antiparticle):

$$E'' = -E' = (-m_o)(-u_o)c^2$$

so that the antiparticle (endowed of course with 4-velocity component $-u_o$) remains with a negative rest-mass. We shall therefore write

$$\begin{cases} E = + m_o c^2 & \text{for free particles} \quad (m_o > 0) \\ E = - m_o c^2 & \text{for free antiparticles} \quad (m_o < 0) \end{cases} \tag{8}$$

so that always $E = + |m_o| c^2$. Eqs. (8) do not violate covariance since they both descend from the covariant eq. (7).

However it should be clear that nothing prevents us from introducing a new formalism, where e. g. a new 'proper mass' (as distinct from the 'rest-mass') is invariant when going from particles to antiparticles[11]; what we wanted to notice is that, in the usual formalism, ordinary rest-mass possesses the abovementioned property. For instance, let us shift to QFT: if we correcly insist in associating positive energies to both electrons e^- and positions e^+, then we get that the free Dirac equation yields[10] opposite intrinsic parities for e^- and e^+ - as required - only under condition m_o (fermion) = m_o(antifermion). Still within the realm of QFT, it is easy to observe also that (when we deal , as usual, with states of definite parity[10]):

$$\overline{C} \equiv P_5 \tag{5 bis}$$

quantity P_5 being the chirality operation $\left[P_5^{-1} \psi P_5 \equiv \gamma^5 \psi = \overline{C}^{-1} \psi C \right]$, so that[10]

$$'RIP' \equiv P_5. \tag{6 bis}$$

We shall come back to similar considerations in the following.

4. - EXTENDED RELATIVITY : HISTORICAL REMARKS.

All the previous considerations assume a more compact form if we allow room also for Super-luminal (=faster-than-light) frames of references and for tachyons[6], so to consider all space-time 'rotations' (in Fig. 4, e. g. , relative to $0 < \alpha < 2\pi$) as generalized Lorentz transformations. In particular, the same set of three postulate previously introduced, are enough for deriving a causal theory even in presence of faster-than-light objects. Such objects have been given the name 'Tachyons' (T) in ref. (12), from the Green word ταχύς = =fast.

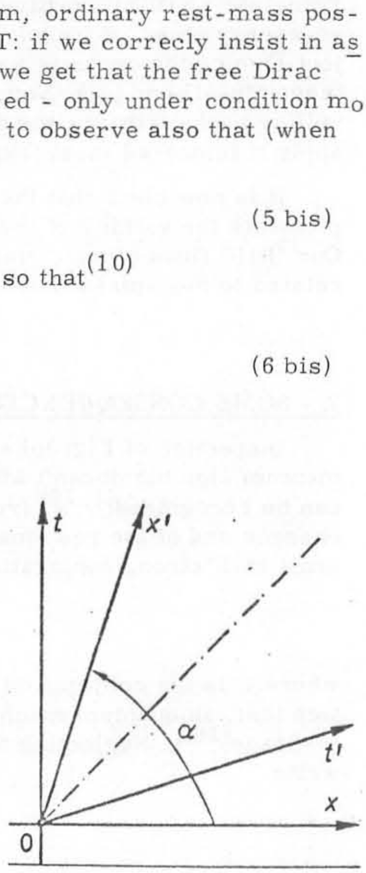

FIG. 4

"Une particule qui a un nom possed dejà un début d'existence", will later be commented[13]. We shall call 'Bradyons' (B) the usual, slower-than light objects[14] from the Green word βραδύς = slow. At last, we shall call 'luxons' (ℓ) the objects - like photons - travelling exactly at the speed of light[15].

As regards tachyons, as far as we know the first author mentioned them was LUCRETIUS, as outlined by CORBEN during this Meeting as well as in ref. (16). Let us here explicity quote another passage from 'De Rerum Natura'[17]:

> "Quone vides citus debere et longius ire
> multiplexque loci spatium transcurrere eodem
> tempore quo Solis pervolgant lumina coelum? ",

which means: "Don't you see how thay must go faster and farther/And travel a larger interval of space in the same amount of/Time than the Sun's light as it spreads across the sky? "

After Lucretius we don't know about any other progress untill THOMSON's[18], Heaviside's, Des Coudres', and particularly SOMMERFELD's works[18]. In 1905, however, together with Relativity, the convinction that ligh-speed in vacuum was the upper limit of any speed spread over, the early -century physisicist being led (and possibly misled) by the evidence that ordinary particles cannot overtake that speed. They behaved like SUDARSHAN's imaginary demographer studying the population patterns of the Indian subcontinent: "Suppose a demographer calmly asserts that there are no people North of the Himalayas since none could climb over the mountain ranges! That would be an absurd conclusion. People of central Asia are born there and live there: They did not have to be born in India and cross the mountain range. So with faster-than-light particles" (cfs. Fig. 5a). Moreover TOLMAN[19] believed to have shown, in his old 'paradox' of the anti-telephone, that the existence

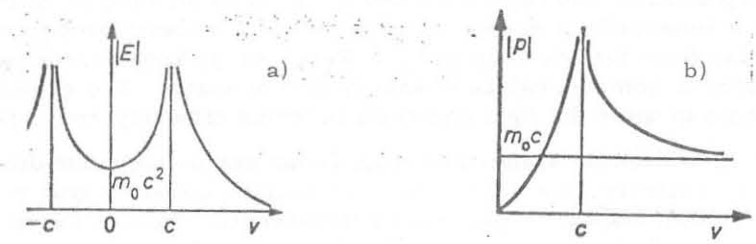

FIG. 5

of Superluminal particles allowed information transmission into the past.

Therefore, one had to wait almost untill the sixties before seeing the tachyon problem re-examined, apart from the mathematical considerations by WIGNER[20] and by SCHMIDT[20]. The pioneering works are the ones by ARZELIES[21], by TANAKA[21], by TERLETSKY[21], and by SUDARSHAN and Coworkers[15]. After ref. (15), a number of people started studyind the subject, among whom, e. g. , in USA Feinberg[12] and in Europe the present author and Coworkers[22]. One of the main reason of interest in 'Extended Relativity' (which includes Superluminal frames and objects[23]) is bound to the fact that it yields also a better understanding of ordinary SR, even if tachyons would not exist as 'asymptotical objects'. However, no essential reason against the existence of free,'asymptotical' tachyons will be apparently met, so that we might get inspired - following Murray Gell-Mann - from the 'principle'[24] asserting that "anything not forbidden is compulsory". Let us remember that most experimental search looking for tachyons has been till now lacking of a good theoretical background; in particular, most experiments looked for Cherenkov radiation supposedly emitted by tachyons in vacuum, whilst our theory of SR extended to Superluminal frames and tachyons does not predict any such radiation in vacuum.

In 'Extended Relativity'[6] (ER) both sub-luminal (=slower-than-light) and Superluminal frames are considered; the problem of finding out the 'Superluminal Lorentz transformation' (SLT) connecting a frame s of the former class to a frame S of the latter class has been first considered in the pioneering (independent) work by PARKER[25] (who studied the 2-dimensional case) and by OLKHOVSKY et al.[26] and then by MIGNANI et al.[27]. The four-dimensional extension has been first attempted in refs.(26), and then - by complex transformations-in ref. (27) .

5. - PRELIMINARIES (AND CAVEATS) ON TACHYONS.

In a bidimensional space-time, or in the case of purely collinear motions, it is possible to define rapidity the quantity $R \equiv c \cdot tgh^{-1}\beta$, so that $R=0^{\pm}$ for $\beta \to 0^{\pm}$ and $R \to \pm\infty$ for $\beta \to \pm c$ (and one gets an additive 'rapidity composition law'). But this cannot be meaningfully done in more dimensions, so that - even from this point of view - space-like objects cannot be 'squeezed away' from space-time.

The theory of ER can be based on our postulates 1), 2), 3) of Sect. 1; to make our arguments simpler, let us however substitute now postulate 2) with the more conventional one about light-speed invariance in vacuum. Therefore we shall base ER on the assumtions: (1) Principle of Relativity; (2) light-speed invariance in vacuum; (3) Third Postulate: principle of retarded causality (or equivalent ones: see above). We are releasing[29] the additional postulate the $|\vec{v}| \leq c$ for all velocitites $\vec{v}$. Let us choose throught these lectures the signature (+---); natural units (c=1) will be adopted when convenient. We shall make recourse to Einsten's notation and to the 'Euclidean metric' $g_{\mu\nu} = \delta_{\mu\nu}$, by writing the chronotopical vectors as $x \equiv (x_0, x_1, x_2, x_3) \equiv (ct, ix, iy, iz)$. We shall not have to distinguish between covariant and contravariant components, and the light-cone will write $\sum_{\mu=1}^{4} x_\mu^2 = 0$. Formally we shall have: time = i · space, $[c = 1]$. As noticed by MINKOWSKI himself, we can formally write: 1 second ≡ i (3×10^8) meters.

Extension of SR to Superluminal frames and objects is straightforward when we have a symmetry between the numbers of space and time dimensions, like in the two-dimensional M^2 case[25] or when we introduce a $M(3,3) \equiv M^6$ space or a C^3 space[30]. If we stick - as in the following - to the usual Minkowki space-time, in order to get equivalence between s and S frames we shall have to introduce sometimes some imaginary units[23]. Some authors (as CORBEN[16, 23] and SHAH[23]) are satisfied by the situation and the present interpretation possibilities[23]; others looked for a wider interpretation on the basis of complex (or real multi-dimensional) space times. From a 'conservative' viewpoint, one can regard the use of SLT's as an analytic-extrapolation procedure (leading to deal, in the intermediate steps, with complex - or at least purely imaginary - space-time coordinates), not far from the one adopted by T. Regge in his known theory where scattering amplitudes are extrapolated to complex values of energy or momentum. The essential point is that in ER we shall always be able to write the final equations in terms of (purely) real quantities.

In Minkowski space-time (Fig. 4) our world-line coincides - in our frame - with the time-axis t; on the contrary, the world-line of an infinite-speed tachyon moving along x coincides with the x-axis itself (with respect to us). Such a 'trascendent' tachyon ($V=\infty$) will then consider as his time-axis t' the one called x-axis by us, and analogously will consider as space-axes (x', y', z') the ones called t, y, z by us. On the contrary, such a'trascendent' observer will appear to us - owing to the structure of ER - as possessing one space-axis and three time-axes; and the same will happen for rods and clocks. Let us repeat that (free) bradyons always admit a class of s-frames (the rest-frames) according to which they are space-'points' extended in time along a line; whilst (free) tachyons always admit a class of s-frames[31] (the 'critical' ones[6]) whereform they appear with divergent speed, i.e. as 'points' in time extended in space along a line. This is important for understanding the tachyon 'localization' with respect to us and corresponds to the fact that the 'little groups' of time-like and space-like representations of the Poincaré group are SO(3) and SO(2,1) respectively.

6. - THE GENERALIZED LORENTZ TRANSFORMATIONS (GLT). -

From the postulates (1), (2), (3) of Sect. 5 we get[6] immediately as a corollary the 'Duality Principle': "the terms B, T, s, S are not absolute but only relative, and

$$B(S) = T(s); \; B(s) = T(S); \; \ell(s) = \ell(S); \tag{9}$$

moreover the relative speed between two frames S_1, S_2 (or s_1, s_2) is always smaller than c, and the one between two frames s, S is always larger than c". Therefore, the transformations L between two inertial frames f_1, f_2 must be such that[6]

$$x'_\mu \; x'^\mu = \pm \; x_\mu \; x^\mu \tag{10}$$

for every fourvector x_μ, where the sign plus refers to the the ordinary case ($u^2 < c^2$) and the minus to the Superluminal one ($U^2 > c^2$). Of course, according to postulate (1), frames S are supposed to have at their disposal exactly the same physical objects as frames s have, and vice-versa. When two frames s, S observe the same event, time-like vectors transform into space-like vectors, and vice-versa, in going from s to S or from S to s; the "Superluminal Lorentz transformations" (SLT) are expected to be such that $\left[\beta \equiv u/c\right]$:

$$c^2 t'^2 + (i\vec{x}')^2 = -\left[c^2 t^2 + (i\vec{x})^2\right]. \qquad \left[\beta^2 > 1\right] \tag{11}$$

Of course also tachyons will posses real rest-masses. If we apply eq. (11) to 4-momentum vectors, we derive for tachyons

$$E^2 - \vec{p}^{\,2} = -m_o^2 < 0. \qquad \left[m_o \text{ real}\right] \tag{12}$$

In Figs. 3 the three cases (for B's, ℓ's, T's, respectively) are depicted. Any SLT maps the 'interior' of the light-cone $p_\mu p^\mu = 0$ into its 'exterior', and vice-versa (as it can be shown e. g. within the mathematical 'theory of catastrophes': see SHAH ref.(23)), even if such a mapping is one-to-one quasi everywhere only. Tachyons will slow down when energy increases (cf. Fig. 5 b). In particular, divergent energies are needed to slow down tachyon speed to its lower limit c; and, on the contrary, when a tachyon tends to have divergent speed, its energy tends to zero (see Fig. 3c and Fig. 5a). Incidentally, since trascendent tachyons transport zero energy but finite (minimal) momentum (with magnitude $m_o c$), they allow getting the rigid body behaviour even in SR; as a consequence, in elementary particle physics tachyons can be useful for describing diffraction scatterings, pomeron-exchange reactions, and elastic scatterings (see the following) on a classical basis.

From our postulates (1), (2), (3) it follows that GLT's must be linear transformations satisfying eq. (10); they costitute[1, 32] a new group G which is the extension of the usual (proper, orthochronons) Lorentz group L_4^+ by the two operations CPT and $\mathcal{S}$:

$$G = \varepsilon\,(L_4^+, \text{CPT}, \mathcal{S}), \tag{13}$$

where $\mathcal{S}$ is the product of the two operators $A \equiv x^\mu \rightarrow i x^\mu$ and $B \equiv \beta \rightarrow 1/\beta$. Notice that: $\det L = +1$, $\forall\, L \in G$; and that, if $L \in G$, then $-L \in G$, $\forall\, L \in G$. Briefly speaking, if we call LT the usual (proper, orthochronous) Lorentz transformations, then

$$\text{SLT}(1/\beta) = \pm \mathcal{S}\left[\text{LT}(\beta)\right]. \qquad \left[\beta^2 < 1;\ \frac{1}{\beta^2} > 1\right] \tag{14}$$

At this point, let us observe that $\sqrt{\beta^2 - 1} = \pm i\sqrt{1-\beta^2}$ since $(\pm i)^2 = -1$; let us choose the sign plus. We shall also understand that, for $\beta^2 > 1$, quantity $\sqrt{1-\beta^2}$ represents the upper half-plane solution. Then, for a Superluminal boost along the(positive) x-direction with speed U, eq. (14) yields[26, 27, 33]:

$$\begin{cases} x' = \pm \dfrac{ct - ux/c}{\sqrt{1-(u/c)^2}} = \mp \dfrac{x - Ut}{\sqrt{\beta^2 - 1}}\,; \\[2ex] t' = \pm \dfrac{x - ut}{c\sqrt{1-(u/c)^2}} = \mp \dfrac{t - Ux/c^2}{\sqrt{\beta^2 - 1}}\,; \qquad \left[\beta \equiv \dfrac{U}{c} > 1\right] \\[2ex] y' = \pm B\, i y; \qquad z' = \pm B\, i z, \end{cases} \tag{15}$$

where we put $u \equiv c^2/U < 1$. In eqs. (15) the relative signs depend on our conventions above. In the two dimensional case, the GLT's simply read (in G-covariant form):

$$x' = \pm \frac{x - ut}{\sqrt{|1-\beta^2|}}; \quad t' = \pm \frac{t - ux/c^2}{\sqrt{|1-\beta^2|}}; \qquad \left[-\infty < \beta \equiv \frac{u}{c} < +\infty\right] \tag{16}$$

in such a (or similar) form eqs. (16) first appeared in refs. (26), (27), and then in a number of subsequent papers[34]; in refs. (27), (35) the eqs. (16) have been shown to be - in the Superluminal case - essentially an improved version of the pioneering Parker's equations[25].

Let us consider an application: if a tachyon has rest-mass m_o (relative to its rest-frames) and moves with speed U relative to us, we shall then observe the total mass

$$m = \frac{m_o}{\sqrt{|1-\beta^2|}} = \frac{-im_o}{\sqrt{1-\beta^2}} = \frac{m_o}{\sqrt{\beta^2-1}} \qquad \left[\beta^2 > 1;\ m_o \text{ real}\right] \tag{17}$$

where tachyons are evidently attributed real rest-masses.

A 'rule of ER' easily follows: the relativistic laws for tachyons can be obtained from the corresponding laws for bradyons by applying a SLT, for instance the 'trascendent' one $K_+ \equiv + \lim_{\beta \to \infty} \left[SLT(\beta)\right]$; more precisely, $K_+ \equiv \begin{pmatrix} \sigma_2 & 0 \\ 0 & i\sigma_o \end{pmatrix}$, where $\sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_o \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ are Pauli matrices. Actually, at this stage we have still some freedom; for instance, we may have a rotation of axes y, z around the motion line (axis x) when 'crossing' the light-cone, so that a priori the above quantity $i\sigma_o$ can be e.g. chosen with the sign minus.

7. - TACHYONS AND CAUSALITY. -

In the case of tachyons it is even clearer (cf. Sect. 2) that our Third Postulate does easily eliminate any motion backwards in time. In fact (see Fig. 2c), to get transition from an (ordinary) tachyon A to a negative-energy tachyon A', it is enough an ordinary LT. The fact that such a LT will change sign not only to energy but also to time is easily seen by comparing Figs. 2c and 6. Let us first look at Fig. 2c, and consider a frame s_o and then a continuous succession of frames, with increasing positive speeds $u < c$ along the x-direction, that observe the same free tachyon T. When varying observer, the point K representing the kinematical state of the observed tachyons moves from its initial position $A \equiv K(s_o)$, which represents e. g. tachyon T travelling with speed $V > c$ along the positive x-direction. In order to go from the upper $(E > 0)$ region to the lower $(E < 0)$ one, point K must cross the hyperplane E=0 where it refers to a tachyon T endowed with infinite speed (since $\vec{V} = \vec{p}/E$) and minimal momentum $m_o c$. It is easy to calculate that the critical frame s_∞ wherefrom T appears to be trascendent is the one with speed $u = c^2/V < c$

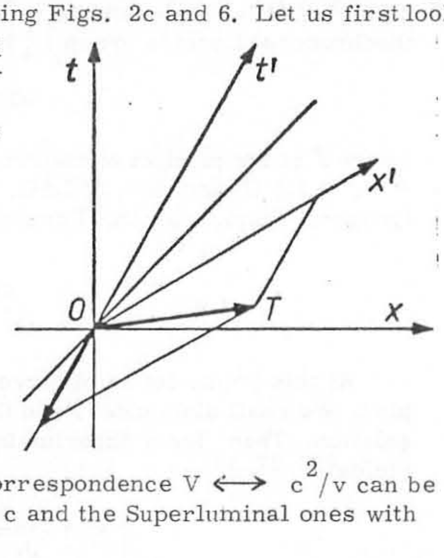

FIG. - 6

(relative to s_o). Incidentally, in two dimensions the one-to-one correspondence $V \longleftrightarrow c^2/v$ can be easily set between subluminal frames (or objects) with speed $v < c$ and the Superluminal ones with $V \equiv c^2/v > c$.

Any observer coming after s_∞ in the above succession of frames should therefore see T andowed with a negative energy E (see point A'). Let us pass to Fig. 6; the frame s_∞ will be represented by axes x_∞, t_∞ rotated with respect to $s_o(x, t)$ by an angle α_∞ such that x_∞ is superimposed to the world-line OT of the considered free tachyon. The frames, in the above succession, attributing $E > 0$ to T correspond to $0 < \alpha < \alpha_\infty$, and the ones that should attribute $E < 0$ to T are rotated by $\alpha > \alpha_\infty$; but inspection of Fig. 6 confirms that the latter ones should also see T moving backwards in time. It is straightforward to conclude that, owing to the 'RIP', point A' actually represents nothing but an anti tachyon $\bar{T}$, travelling the opposite way (with positive energy, and forward in time). We are left with no motion backwards in time.

This success of eliminating any causality violation is obtained at the price of abandoning the conviction that the judgement about what is 'source' (or 'cause') and what is 'detector (or 'effect') is independent of the observer[6, 37]. Actually, in Relativity only lwas, and not the description 'details', must be covariant[6, 8]. In fact, the initial observer s_o in the case above examined judges the event at A as causing the event at B (see Fig. 3,and 2c), whilst any observer s' (which interprets the same phenomenon as exchange of an antitachyon $\overline{T}$ from B to A) judges the event B as cause of the event at A[38]. Nevertheless, all observers will always see the cause chronologically to precede its own effect; the law of 'retarded causality' is relativistically covariant and holds for all inertial observers, both s and S[6, 8, 3].

However, the relativity of judgement about cause and effect (and even more of existence of a 'causal correlation'[3, 6, 8]) led to a series of apparent 'causal paradoxes' that - even if solvable[38]- gave rise to some perplexities. Let us here recall only the 'paradox' proposed by PIRANI[39] in 1970 and essentially solved by PARMENTOLA and YEE[38] in 1971 on the basis of refs. (38). For such a point, refs. (3),(6),(8), (38). For instance, in refs. (3),(6) the paradox by Pirani is formulated also in a 'strong version' and then solved (following ROOT and TREFIL[38]). See the "Note Added" at the end.

Let us add some considerations about antimatter. We have seen that, given a tachyon T, an ordinary LT can transform it into an object $\overline{T}$ expected to have exactly all the properties that antiparticles actually showed to have in the experiments. Therefore, in the case when we confine ourselves to ordinary LT's, the character matter/antimatter is invariant for B's but it is relative to the observer for T's. However, in ER that character is relative to the observer also for B's. Moreover, let us confine ourselves for simplicity to boosts along x; then, when overtaking the trascendent (relative to s_o) frame $f(U = \infty)$, we pass from frames f^R (e. g. with a right-handed spatial frame) to totally-inverted frames $f^L = (PT) f^R$ with a left-handed spatial frame, a reversed time-axis, and so on. See Fig. 7. In other words, we pass from frames f^R to frames f^L with space-parity and with particles transformed into antiparticles. This could have been expected, since the total inversion PT is a 'rotation' of space-time, so that $PT \in G$.

A close inspection of Fig. 3 reveals that the Third postulate cannot be applied if we do not take account of the proper sources and detectors (for each B or T), or - more generally - of the proper 'interaction regions'. This leads to completing the 'RIP' by saying that "Under a trans-critical GLT, when the rôles of emitter and absorber happen to be interchanged, any negative-energy object in the initial 'state' physically corresponds to its positive-energy antiobject in the final 'state', and vice-versa".

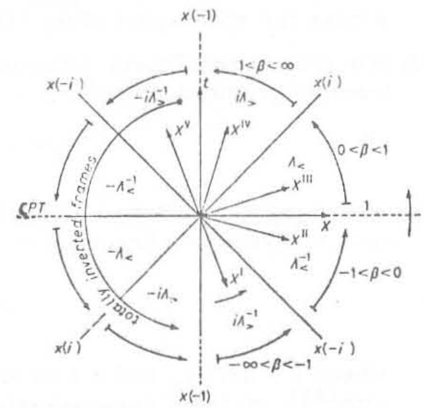

FIG. 7

It is worthwhile to repeat that, if a GLT acts on a fourvector associated with a body, then it analogously act on the other fourvectors associated with that body. In particular, the 'total inversion' operation $\overline{P}\overline{T} = \mathbb{1}$, which changes sign to $\vec{x}$ and t, will change sign also to $\vec{p}$ and E, ect. We used the now symbols $\overline{P}$ (strong parity) and $\overline{T}$ (strong time-reversal) for meaning the sign-inversion of the first three components and of the fourth component of all fourvectors, respectively.

Besides, if we call $\Lambda_< \equiv \Lambda\ (\beta^2 < 1)$ the ordinary, proper, orthochronous (homogeneous) LT's in 4x4 matrix-form, then $SLT = \pm i\Lambda_>$, where $\Lambda_> \equiv \Lambda(\beta^2 > 1)$ are (complex) matrices formally identical to the $\Lambda_<$'s but corresponding to values $|\beta| > 1$. One can verify that $[i\Lambda_>(\beta)] \cdot [-i\Lambda_>^{-1}(\beta)] = \mathbb{1}$, but $[i\Lambda_>(\beta)] \cdot [i\Lambda_>^{-1}(\beta)] = -\mathbb{1}$. In general, the product of two SLT's (which is always a LT's) can yield a LT both orthochronous and non-orthocronous. In Particular $GLT(\alpha = 180^o) = \overline{P}\overline{T} \equiv -\mathbb{1}$. Since, in order to reach $\alpha = 180^o$ (starting from 0^o) we have to bypass the case $\alpha = 90^o$, then we have to apply the 'RIP' (cf. Figs. 7 and 3), so that actually

$$GLT\ (\alpha = 180^o) = CPT; \qquad \overline{P}\overline{T} \overset{(RIP)}{=} CPT, \tag{18}$$

and CPT-covariance is directly required by ER as a particular case of G-covariance[6, 3]. At a classical (purely relativistic) level, it is moreover possible to derive the so-called 'Crossing Relations' of high energy (elementary particle) physics, also for ordinary bodies (bradyons[6,8]). At last let us mention that within ER it seems easy to explain why relativistic equations are expected to admit both retarded and advanced solutions[40].

8. - CLASSICAL PHYSICS FOR TACHYONS. -

By applying the 'rule of ER' (Sect. 6) one can predict the classical laws obeyed by tachyons, - such laws being got in terms of purely real quantities. -

For instance:

(1) The fundamental law of dynamics for bradions reads $F^{\mu} = c\frac{d}{ds}(m_o c\frac{dx^{\mu}}{ds})$ and for tachyons according to ref. (6) will read:

$$F^{\mu} = -c\frac{d}{ds}(m_o c\frac{dx^{\mu}}{ds}), \qquad \left[\beta^2 > 1\right] \tag{19}$$

so that in G-covariant form we should have

$$F^{\mu} = \frac{d}{d\tau_o}(m_o \frac{dx^{\mu}}{d\tau_o}), \qquad \left[\beta^2 \lessgtr 1\right]$$

where dx/ds is a four-vector only with respect to the group L_4^+, whilst $dx/d\tau_o$ is a four-vector with respect to the whole group G. Notice that by suitably choosing the Lagrangian, the 3--momentum of tachyons can result to be opposite to their velocity: $\vec{p} = -m\vec{v}/\sqrt{\beta^2 - 1}$; in such a case the space-part of eq. (19), e. g., would be written $\vec{F} = +(d/dt)\,\vec{p}$ even for tachyons.

(2) In a gravitational field (associated e. g. to subluminal sources), where a bradyon feels an attractive gravitational force, a tachyon will experience the repulsive 4-force[6]

$$F^{\mu} = +m_o\, \Gamma^{\mu}_{\varrho\sigma} \frac{dx^{\varrho}}{ds} \frac{dx^{\sigma}}{ds}, \qquad \left[\beta^2 > 1\right] \tag{20}$$

where m_o is the tachyons (real) rest-mass. However, due to eq. (19), the eqs. of motion for both tachyons and bradyons in a gravitational field will still read (in G-covariant form)[6]:

$$a^{\mu} + \Gamma^{\mu}_{\varrho\sigma}\, u^{\varrho} u^{\sigma} = 0, \qquad \left[\beta^2 \lessgtr 1\right]$$

where $u \equiv dx/d\tau_o$ and $a \equiv du/d\tau_o$ are 4-velocity and 4-acceleration, respectively. In conclusion[41]: (a) from the energetical and dynamical point of view, tachyons appear to be gravitationally repulsed by ordinary matter, i. e. to be the 'anti-gravitational' particles; (b) from the kinematical viewpoint, however, tachyons appear as bending (or 'falling down') towards the gravitational source.

(3) As a constant-speed bradyon in vacuum does not emit radiations, so a constant-speed tachyon in vacuum will emit no radiations: in particular, no Cherenkov's[6,42].

(4) As regards Doppler-effect for Superluminal sources, in the case of relative motion parallel to the x-axis, we shall have in both the sub- and Superluminal cases

$$\nu = \nu_o \frac{\sqrt{|1-\beta^2|}}{1+\beta\cos\alpha}, \qquad \left[u^2 \lessgtr c^2\right] \tag{21}$$

where $u \equiv u_x \equiv \beta c$ is the relative speed and $\alpha \equiv \widehat{\vec{u}\vec{\ell}}$, the vector $\vec{\ell}$ going from the observer to the source. The same shift will be observed for both $u=v<c$ and for $U = c^2/v > c$. For Superluminal approach, the radioemission will be received in reversed chronological order, and this fact corresponds to the negative sign appearing in eq. (21) in such a case.

(5) With regard to Maxwell eqs., if one assumes the usual quantity $F_{\mu\nu}$ to be still a tensor under the new group G of GLT's, then he gets[42a] that Maxwell equations are G-covariant. However, if - more consistently - one firstly generalizes[1] the transformation-laws for electric and magnetic fields, $\vec{E}$ and $\vec{H}$, then new generalized Maxwell eqs. are got. Namely, in presence of both subluminal, $j_\mu(s)$, and Superluminal, $j_\mu(S)$, four-currents, we should have[43] in Lorentz-covariant form:

$$\begin{cases} \vec{\nabla}\cdot\vec{D} = \varrho(s), \\ \vec{\nabla}\,\vec{B} = -\varrho(S), \\ \vec{\nabla}\wedge\vec{B} = \vec{j}(S) - \dfrac{\partial\vec{B}}{\partial t}, \\ \vec{\nabla}\wedge\vec{H} = \vec{j}(s) + \dfrac{\partial\vec{D}}{\partial t}. \end{cases} \qquad \left[v^2 \gtrless c^2\right] \tag{22}$$

In other words, if we define the usual dual tensor $\left[\mu\nu\varrho\sigma = 0,1,2,3\right]$:

$$F^{*}_{\mu\nu} \equiv \frac{i}{2}\,\varepsilon_{\mu\nu\varrho\sigma}\,F_{\varrho\sigma}\;;\;(F^{*}_{\mu\nu})^{*} = -F_{\mu\nu} \tag{23}$$

and introduce the quantities[43]

$$\vec{F} \equiv i(\vec{E} + i\vec{H})\;;\;T_{\mu\nu} \equiv F_{\mu\nu} - iF^{*}_{\mu\nu}\;;\;J_\mu \equiv j_\mu(s) - i\,j_\mu(S),$$

then eqs. (22) write

$$\partial_\nu T_{\mu\nu} = J_\mu\;;\;T^{*}_{\mu\nu} = +i\,T_{\mu\nu} \qquad \left[v^2 \gtrless c^2\right] \tag{22'}$$

and a connection is met between the electromagnetic duality in eq. (23) and the 'dual correspondence' (see ref.(6)) bradyons $\longleftrightarrow$ tachyons (Sect. 6). Moreover, if we introduce also the complex four-potential[43]

$$C_\mu \equiv A_\mu - iB_\mu$$

where, following CABIBBO and FERRARI[44],

$$F_{\mu\nu} \equiv A_{\nu/\mu} - A_{\mu/\nu} - i\varepsilon_{\mu\nu\varrho\sigma}B_{\sigma/\varrho}\;;\;F^{*}_{\mu\nu} \equiv B_{\nu/\mu} - B_{\mu/\nu} - i\varepsilon_{\mu\nu\varrho\sigma}A_{\sigma/\varrho},$$

then the genaralized Maxwell equations (22') will read[43]

$$\Box C_\mu = J_\mu\;;\;\partial_\mu C_\mu = 0. \qquad \left[v^2 \gtrless c^2\right] \tag{22''}$$

Of course, also eqs. (22'), (22'') can split into purely real equations. Notice that in our theory A_μ is only a Lorentz-vector and not a G-vector, since under GLT's it behaves do as dx/ds; for instance, under a SLT=L:

$$A_\mu \rightarrow A'_\mu = -iL_{\mu\varrho}A_\varrho\;;\;T_{\mu\nu} \longrightarrow T'_{\mu\nu} = -iL_{\mu\varrho}L_{\nu\sigma}T_{\varrho\sigma},$$

and analogously for $F_{\mu\nu}$ and $A_{\mu/\nu}$. Finally, the structure of this theory reveals[43], however, that $B_\mu = -L^{1}_{\mu\nu}A'_\nu$. For further details or comments, see refs. (43). Here, let us only add that our version of 'extended electromagnetism' predicts existence of both sub- and Super-luminal 'electric' charges, rather than of (subluminal) magnetic monopoles. In other words, following our formulation of ER, we can expect existence of only one (electromagnetic) charge, both sub- and Super-luminal; the latter ones bring into the field equations a contribution analogous to that one expected to come from magnetic poles (if you want, you may call electric the subluminal charges and magnetic the Superluminal charges: but our 'magnetic charges' are faster-than-light 'electromagnetic' charges) (see ref. (43)). See Fig. 8. On the contrary, for Corben's version of 'extended electromagnetism' see CORBEN: these Proceedings. See also the contribution of TERLETSKY (these Proceedings) on the same topic.

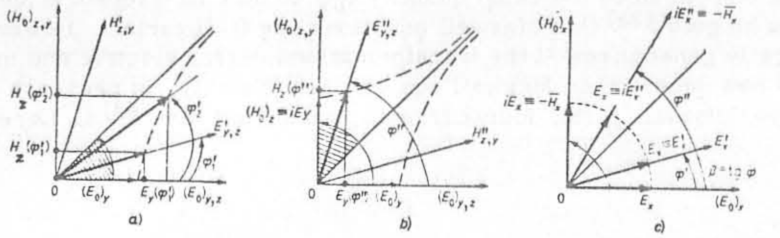

FIG. 8

9. - TACHYONS AND BLACK-HOLES. -

Let us pass to consider for a moment General Relativity (GR). Instead of accepting completely general coordinates, however, let us limit in the following way the adoptable coordinates. Given a set of general coordinates (a, b, c, d) and a space point P, we shall associate to them the (local) observer 0 which is at rest at P with respect to those coordinates. Then, let us require that we can go from (a, b, c, d) to other general coordinates (a', b', c', d') only if the (local) observer 0', associated to (a', b', c', d') at the same point P, (locally) moves with slower-than-light speed with respect to 0. Let us remember that, in GR, continuity and double-derivability of our space-time manifold are usually assumed, so that the geodesics never change their type and bradyons (tachyons) always remain bradyons (tachyons). Our previous requirement, however, leads us to abandon the manifold-derivability requirement, at least on some 'special' (not necessarily singular) varieties: so that the geodesic type can change there.

For instance, let us consider the Szekeres -Kruskal (SK) coordinates[45] as a priori constituting two sets of general coordinates for describing the (Schwarzschild) solution of Einstein equations in the case of spherically-symmetric mass-distribution. Namely - if we write down for simplicity only the radial and the time coordinates, - let us consider the set of SK coordinates $(u_>, v_>)$ defined outside the event horizon (i. e. for $r > 2M$; $c=G=1$), and the set of SK coordinates $(u_<, v_<)$ defined for $r < 2M$. However, we assume the definition of each set to be so extended to cover the whole space-time manifold (both outside and inside the event-horizon). It is then immediate to realize that [now in $c=2MG=1$ units]:

$$v_<(1/r) = \mathcal{J}\left[u_>(r)\right] \; ; \; u_<(1/r) = \mathcal{J}\left[v_>(r)\right] \qquad \left[r \geq 1;\ 1/r \leq 1\right] \tag{24}$$

where $\mathcal{J}$ is here the operator changing $r \rightarrow 1/r$ and multipying the whole function by the imaginary unit. The operator $\mathcal{J}$ is formally identical to the operator entering eq. (14) and effecting the transition from sub- to Super-luminal frames. Incidentally, that operator $\mathcal{J}$ of eq. (14) coincides with the 'trascendent boost' K_+ (cf. Sect. 6), which from eqs. (15) results to effectuate in two dimensions the transition[1]: $x \rightarrow x' = t;\ t \rightarrow t' = x$. Moreover, let us notice that, if we define[45]

$$u_{\parallel} \equiv \sqrt{|r/2M - 1|} \cdot \exp(r/4M) \cdot \cosh(t/4M);$$

$$v_{\parallel} \equiv \qquad\qquad \cdot \sinh(t/4M),$$

so that, for $r > 2M$, one has $u_> \equiv u_{\parallel}$; $v_> \equiv v_{\parallel}$, then for $r < 2M$ we get : $u_< \equiv i v_{\parallel}$; $v_< \equiv i u_{\parallel}$. Therefore, going from $r > 2M$ to $r < 2M$ (with t fixed) means exchanging the rôle of u, v: namely, $u_{\parallel} \rightarrow u' = v_{\parallel}$; $v_{\parallel} \rightarrow v' = u_{\parallel}$. This shows again the formal identity between going from s to S frames and going from the 'exterior' to the black-hole 'interior'. In ref. (45) it is concluded that the internal SK-coordinates $(u_<, v_<)$ are associated to observers that move faster-than-light relative to the observers associated (at the same point) with the external SK-coordinates $(u_>, u_>)$. In such a case, we would have a violation of our initial postulate and we should confine ourselves - for instance - to choose everywhere either the 'external' SK-coordinates or the 'internal' ones. The same could be said for other coordinates, as FINKESTEIN's.

With our postulate (and therefore with our limitations), it is easy to realize that a free-falling B outside the event-horizon will become a T inside the horizon, and vice-versa[45]. And black-holes will be classical sources of tachyons.

What we want here to stress is that the same mathematical problems, met in SR for extending LT's to Superluminal frames, seem to be present in GR when going from the exterior to the interior of a 'horizon'. In particular, for non-spherically symmetric mass distributions (when, besides r and t, further space-coordinates enter in an essential way), the same difficulties with imaginary units should be met. Again, a useful tool on this respect seems to be the 'catastrophe theory'[46]. Actually, when analysing perturbed Schwarzschild problems, some authors[47] had to suggest the existence of coordinate-independent 'singular' surfaces.

10. - VIRTUAL PARTICLES AND TACHYONS. -

From the four-momentum conservation law, it is immediate to deduce that a body at rest cannot emit any tachyon, unless it lowers its rest-mass by a discrete 'jump'. For instance, a trascendent tachyon (bearing E=0, but $|\vec{p}| = m_o c$) cannot be emitted - nor absorbed - by any body in its rest-frame (unless the body rest-mass performes a classical jump to lower values). Let us remember that infinite-speed tachyon emission is completely equivalent to infinite-speed antitachyon absorption.

Moreover, when two moving bodies A, B exchange a trascendent tachyon (i. e., either a trascendent tachyon T is emitted by A and absorbed by B, or equivalently a trascendent antitachyon T is emitted by B and obserbed by A)[13], then we shall observe an elastic interaction of A, B due to an infinite-speed transmission of momentum. Since the infinite speed is not Lorentz-invariant, then other observers will deem the same process to be due to exchange of finite-speed tachyons (or antitachyons).

In other words, let us consider two interacting bodies A, B which do not change their rest-mass; then, in the c. m. s., the two bodies appear as exchanging momentum but no energy. Therefore, in the c. m. s., they can naturally be considered as connected through a trascendent-tachyon exchange[48]. By applying the LT's, this fact means that the elestic scatterings can be in general described by means of suitable, finite-speed tachyon exchanges[49,3]. Even more generally, the tachyon-exchanges can be useful to interpret (at a classical level) also the inelastic interactions between elementary particles.

Let us perform some calculations. A body (or particle) A can emit in its test-frame a tachyon T with rest-mass m only if the rest-mass M_A of A jumps to a lower value M'_A such that $\Delta(M_A^2) \equiv M'^2_A - M_A^2 = -m^2 - 2M_A E_T \leq -\vec{p}^2 \leq -m^2$, where $\vec{p}$ is the tachyon 3-impulse and $E_T = \sqrt{\vec{p}^2 - m^2}$; in fact, it must be

$$M_A = \sqrt{\vec{p}^2 - m^2} + \sqrt{\vec{p}^2 + M'^2_A}$$

In the infinite-speed case ($E_T = 0$), we have

$$\Delta(M_A^2) - m^2 . \tag{25}$$

Let's now consider a second body (or particle) B moving with (subluminal) speed w along the x-axis, and call M_B and $\vec{P}$ its rest-mass and 3-momentum, respectively. Owing to 4-momentum conservation, B can absorb a tachyon T (having rest-mass m and 3-momentum $\vec{p} // \vec{P}$) only if

$$|\vec{p}| = \frac{m}{2M_B^2}\left[m|\vec{P}| + \sqrt{(\vec{P}^2 + M_B^2)(m^2 + 4M_B^2) + M_B^2 \Delta}\right], \tag{26}$$

where now $\Delta \equiv M'^2_B - M'^2_B = 2p_\mu P^\mu - m^2 \gtrless 0$. In the rest-frame of B, i. e. when $\vec{P}=0$, we get $|\vec{p}| = (m/2M_B)\sqrt{m^2 + 4M_B^2 + \Delta}$; it means that a particle B at rest absorb only tachyons T endowed with the speed V such that (c=1):

$$V = 1+\sqrt{4M_B^2/(m^2+\Delta)} \quad ,$$

which tells us again that trascendent tachyons (having any direction whatsoever can be absorbed by B only if eq. (25) is satisfied. Notice that, if two bodies have infinite relative speed, then the product $p_\mu\ P^\mu$ of their 4-momenta is zero.

Considerations of this kind for elemental particles - althought accomplished within the realm of QFT - recently led CORBEN(50) to explain many hadrons as compound states of (other) bradyonic and tachyonic hadrons, thus proposing a Lorentz-covariant 'bootstrap' theory. With regard to refs. (50), let us notice that, if hadron B with rest-mass m_1 absorbs a tachyon T with rest-mass m_2 and $m_1 > m_2$, then the compound particle is always a bradyon(49). Moreover, if a tachyon is bound by a repulsive central force (so as in the gravitational field generated by a subluminal source, and - by extension(51) - in the strong field of a hadron),it reaches minimal (potential) energy when its speed diverges(51), i. e. the fundamental state of the system corresponds to a 'trascendent', periodic motion of the tachyon. Actually, we should remember that a tachyon experiencing a central force can easily perform a harmonic motion (by inverting its direction in the points where it reaches $|V| = \infty$)(6, 49) or move along closed paths(49). Now, within ER(6) (applied to QFT), CORBEN(50) derived masses and quantum numbers of a host of baryon and meson 'resonances' by considering them as composed of one bradyonic and 1 to 3 tachyonic hadrons. If hadrons, incidentally, can moreover be considered as 'strong black-holes'(51), the tachyonic constituens can then be emitted in bradyonic form, - when crossing the 'event-horizon' owing e. g. to quantum effects as Hawking's evaporation(51). Besides, CORBEN found for example the mass-differences among the members of various isospin-multiplets by binding Superluminal leptons to suitable (subluminal) hadrons(50, 49). By generalizing to the quark level such an approach, the quarks themselves could once more be assumed to be 'strings' or 'loops' made of Superluminal leptons(49) (in such a philosophy, quarks would of course be structures made of 'partons', where partons would be nothing but tachyonic leptons).

If we insist to invade the field of strong interactions (usually reserved for Q. M.), we easily meet the fact that 'virtual particles' bear in general a negative square-momentum: $t \equiv p^2 \equiv E^2 - \vec{p}^{\,2} < 0$. This suggests too that subnuclear particles can interact by exchanging objects classically interpretable as tachyons(49). About ten years ago it was verified(49) - within 'one-particle-exchange' models like the peripheral models 'with absorption' - that the hadron 'virtual-clouds'(6, 49, 52) ough to be associated to Superluminal speeds. Besides, if we want to adopt the ordinary terminology (where everthing is related to subluminal frames, so that eq. (17) is naïvely interpreted by attaching imaginary rest-masses to tachyons), then it is intuitive to consider the hadronic 'resonances' as consisting of bradyons and tachyons(49). In connection with this, it is interesting to study how a non-free bradyon appears to a Superluminal observer, or, in particular, how a tachyon harmonically oscillating (in a frame S) will appear to us.

One should not forget also: (i) that the existence of space-like components always appeared as a natural, and perhaps unavoidable(53), feature of interacting fields: e. g. , it has been shown(53) that - if the Fourier transform of a local field vanishes on a domain of space-like vectors in four -momentum space - the field is a generalized free field; (ii) the rôle of 'dual theories', string models, Higgs mechanisms, 'instantons', etc. , in the theory of elementary particle physics; (iii) that we have seen (Sect. 8) the essential analogy between the bradyon/tachyon duality and the electric/magnetic charge duality, but that new work, in progress, is revealing the connections (e.g.) between suitably modified Dirac strings and the dual strings; (iv) the proposed identifications between quark and magnetic monopole; interesting results have been for example obtained by assuming quarks simply to be quantized (closed) fluxes of magnetic field(54).

Let us go back to eqs. (25), (26). With regard to eq. (26), let us notice that - if $\Delta = 0$, or Δ assumes 'discrete' values - body B can absorb (for every m) only tachyons with a definite (discrete) $\vec{p}$; and vice-versa. With regard to the former equation, if Δ (M_A^2) assumes only discrete values, then eq. (25) yields a constraint-expression for m (in term of M_A and $\vec{p}^2$), like in the case e. g. of the possible process Δ_{33} (1232) $\rightarrow$ p+t in the Δ_{33}-resonance rest-frame; by the way, if we compare this process with the electromagnetic decay of an axcited atom $A^* \rightarrow A + \gamma$, we meet again the hypothesis that the strong-field quanta can be (meson) tachyons.

11. - ASTROPHYSICS AND SUPERLUMINAL OBJECTS. -

We already considered (Sect. 8) the Doppler effect for Superluminal cosmological objects. Let us here consider a macro-object C emitting spherical electromagnetic waves. When we see it travelling with Superluminal, constant speed $\vec{V}$, because of the 'distorsion' due to the large relative speed $|\vec{V}| > c$ we shall observe the electromagnetic waves to be internally tangent to an 'enveloping' cone Γ having as axis the motion-line of body B (this cone has nothing to do with Cherenkov: cf. Sect. 8). As we hear a sonic 'boom' when we have the first sound-contact with a (constant speed) supersonic airplane(55), so we shall see an 'optic boom' when we first enter in radio-contact with body C, i. e. when we meet the surface of cone Γ. In fact, when C is seen under the angle α such that (see Fig. 9a):

$$V \cos \alpha = c, \tag{27}$$

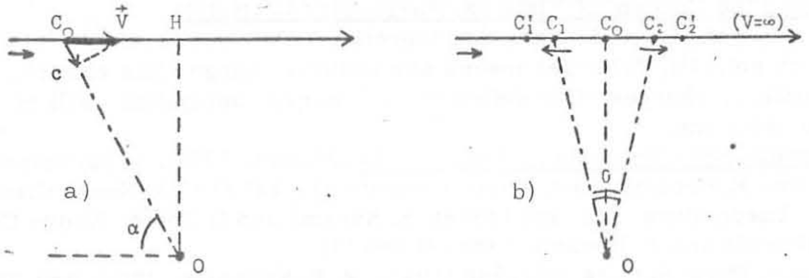

FIG. - 9

all the radiations emitted by C in a certain interval around its position C_o reach us simultaneously. If C_o is at cosmological distances, we can expect the 'optic boom' conditions to hold (when they hold) for a long time. Soon after the first optic (or radio) contact, we shall simultaneously receive the light emitted from suitable couples of points, one on the left and one on the right of C_o, respectively: We shall thus 'see' the initial body at C_o to split in two luminous objects receding along a line from each other with Superluminal (relative) speed U. In the simple case when C moves with almost infinite speed along r (see Fig. 9b), the apparent relative speed of C_1 and C_2 in the intial stage is

$$U \simeq \sqrt{\frac{dc}{t}} \propto t^{-\frac{1}{2}} \tag{28}$$

where $d \equiv \overline{OH} \equiv \overline{OC_o}$ and $t=0$ is the instant when 0 sees $C_1 \equiv C_2 \equiv C_o$.

Such considerations may be interesting e. g. in connection with the 'experimental' fact that about 50% of certain strong radio-sources reveal a structure apparently interpretable in terms of Superluminal expansion(56). Typically, they just appear as consituted of two sources collinearly receding from each other with (apparently) Superluminal relative speed, whilst 'covergent' Superluminal motions have not been observed. It is clear that phenomena of this kind can catch the observer's attention only when the angular separation θ between C_1 and C_2 is small, i. e. when C_1 and C_2 still appear near the position C_o. Then Fig. 9a clarifies that - according to the present working-hypothesis - both the bodies C_1 and C_2 should present a Doppler 'blue-shift', since thay are the images of a (unique) approaching body C. However if the Superluminal bodies C exist only at cosmological distances (like in the above-mentioned observations)(56), then one has to take account also of the cosmological red-shift, which can mask the initial 'kinematical' blue-shift.

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NOTE ADDED. -

In this Post Scriptum let us append an 'addendum' to Sect. 7, by concretely showing how the 'RIP' allows one to solve the causality paradoxes (also) for tachyons. Namely, we shall refer to Pirani's 'paradox'[39]. Let us consider four observers A, B, C, D having some given velocities in the plane (x,y) with respect to a fifth observer s_0. Let us suppose that the four observers are given in advance the instruction to emit a tachyon as soon as they receive a tachyon from another observer, so that the following chain of events takes place (see Fig. 10). Observer A initiates the experiment by sending tachyon 1 to B; observer B immediately emits tachyon 2 towards C; observer C sends tachyon 3 to D, and observer D sends tachyon 4 back to A, with the result that A apparently receives tachyon 4 (event A_1) before having initiated the experiment[39] by emitting tachyon 1 (event A_2). The sketch of this 'gedanken experiment' is in Fig. 10, where oblique vectors represent the observer velocities relative to s_0, and lines parallel to the Cartesian axes represent the tachyons paths. It is important to notice that in Fig. 10 the arrow of each tachyonic line simply denotes its motion direction with respect to the observer that emitted that particular tachyon (but we cannot, of course, mix together observations by different observers). Therefore, the figure does not represent the actual description of the process by any observer (on the contrary, tachyons' and observers' velocities can be chosen in such a way that all tachyons effectively appear to s_0 to move in directions opposite to the ones indicated in the figure 10). Thus, it is necessary to investigate how each observer describes the event-chain.

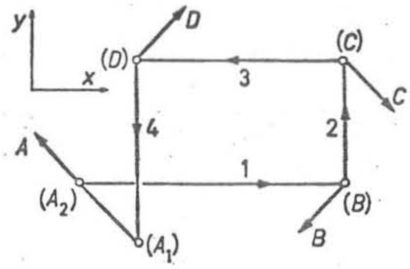

FIG. 10

Following ref. (38), by Parmentola and Yee, let us pass - for this end - to Minkowski space and study the space-time description given e. g. by observer A. From a dynamical point of view, the other observers may be replaced by external force-fields that scatter the tachyons (or by atoms, able to absorb and emit tachyons).

In Fig. 11 it is clearly shown that the absorption of 4 happens before the emission of 1. It might seem that one can send signals into the past of A. However, observer A will effectively see an orthodox sequence of events, as follows: Event D consists in the creation of pair 3 and 4 by the external field; tachyon 4 is then absorbed at A_1, while 3 is scattered at C (transforming into tachyon 2); event A_2 is the emission, by A itself, of tachyon 1 that annihilates at B with tachyon 2. Therefore, according to A, one has essentially an initial pair-creation at D, and a final pair-annihilation at B; and tachyons 1, 4 do not appear causally correlated at all. In other words, according to A the emission of 1 does not initiate any chain of events that leads to the absorption of 4, and we are not in the presence of any effect preceding his own cause.

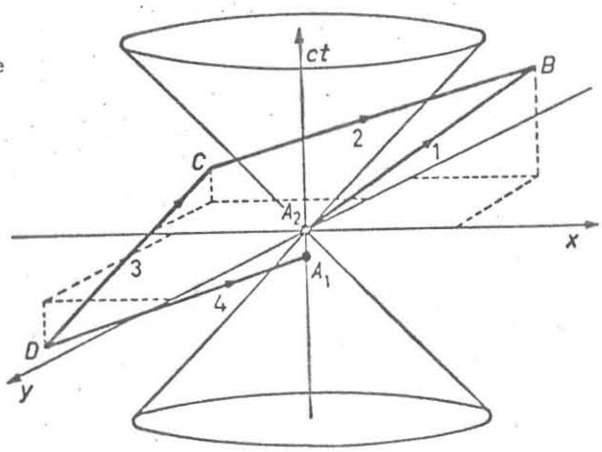

FIG. 11

Analogous, orthodox descriptions (i. e. the descriptions put forth by the remaining observers) may be obtained by Lorentz-trasforming the above description supplied by A.[3,6]

As we already mentioned in Sect. 7, the same 'paradox' can be formulated in a strong version (see refs. (3) and (6)) and then solved by following Root and Trefil[38].