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CUT REGGEON FIELD THEORY

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Foreword

The theory of high energy small-momentum transfert interactions is far from being completely understood. It has become apparent that the essential requirement is the fulfilment of the unitarity in the s and in the t channel. Whereas the s-channel unitarity leads to rather intuitive requirements, like the optical theorem, the t-channel constraints act in a more subtle way- and they are basically responsible for the introduction of the moving singularities in the complex plane of the analytically continued angular momentum, i.e. the Regge poles.

It follows therefore that the major task of the theory is to put the Regge poles in agreement with the s-channel unitarity. At moderately high energies there are not many problems, since the large contribution is represented by the single Regge pole exchange, and the unitarity contraints, involving the exchange of more poles, can be satisfied perturbatively. And indeed the picture based on Regge poles is more or less satisfactory.

It is at very high energy, probably much higher than those experimentally reached today, that the dynamics of the leading Regge pole, the Pomeron, can no longer be treated in a perturbative way, if the total cross section maintains the present non-decreasing behaviour.

It appears that there is essentially one theoretical structure capable to handle the requirements of s and t channel unitarity, whereas providing an acceptable physical picture, and it is the Reggeon field theory.

In this contest the Reggeon or, more precisely, the Pomeron field represents the collective degrees of freedom that are relevant to the description of the high energy phenomena, more or less in the same way as a phonon represents a collective behaviour in a cristal; both descriptions suppose, and ultimately will be based on, a more fundamental and detailed theory.

Here we review some aspects of the theory which are relevant for the discussion of the expected distribution of the produced particles. Our aim will be to provide a general picture mostly of the present understanding of the asymptotic regime, where everything simplifies.

Of course, after the qualitative behaviour is well established, more phenomenological computation for the experimental energies will also be necessary. The theory has not be fully explored yet and some of the results we will describe are subject to be rediscussed.

We will try to relate the simplest version of the various theoretical phenomena we are concerned, still keeping the essential points. In general, except some explicitely stated or self-evident cases, the calculations not shown in detail can be easily reproduced as an exercise by the interested reader.

We have not attempted a review of all the work done in the field. Rather, we have chosen a number of representative developments, mainly with a pedagogical criterion, in order to present a definite comprehensive and self consistent picture. We apologise with those whose work has not been related here.

1. Multi Pomeron exchanges, without Pomeron interactions.

The basic ingredient of the theory of the high energy small transverse momentum interactions is the exchange of the leading Regge[1] pole, the Pomeron, which gives the contribution to the scattering amplitude

$$T = i \beta^2 s^\alpha \tag{1}$$

with the normalization $\sigma_T = \frac{1}{s} \mathrm{Im}\, T$. This exchange is represented by a graph (ordered in the rapidity $y = \ln s$)

The intercept α is around 1. We will consider also the possibility $\alpha > 1$. The multiparticle content of this contribution is known, according to the multiperipheral dynamics. In particular the leading term of the multiplicity of the produced particles is obtained by taking the derivative with respect to α:

$$\langle n \rangle \cdot \sigma_T = c \frac{d}{d\alpha} \sigma_T \qquad \text{where } c \text{ is a constant ,}$$

This is so because if $\sigma_T = \sum_n \sigma_n$, the generating function of the multiparticle distribution

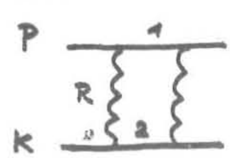
$$S(z, s) = \sum_n z^n \sigma_n(s) , \qquad S(1, s) = \sigma_T \tag{2}$$

can be interpreted in terms of a rescaling by a factor z of the underling coupling constant of the multiperipheral model, on which the intercept α depends.

More in general,

$$\langle n(n-1) \cdots (n-p+1) \rangle \cdot \sigma_T = c^p \frac{d^p}{d\alpha^p} \sigma_T . \tag{3}$$

A first candidate to a double Pomeron exchange is represented by the diagram



which defines the variables we need.

Actually, as it is well known, this diagram gives a non leading contribution at high energy. A quick way of obtaining this result, as well as of discussing other contributions, consists in factorizing out the phase space of the transverse momentum, which stays bounded whereas the longitudinal one expands, and in using the light cone variables in the CM frame. At high energy we can assume $P_- = K_+ = 0$ whereas $P_+ = K_- = \sqrt{s}$; due to the assumed "softness" of an underlying field theory, typically a φ^3 theory which in the ladder approximation gives the Regge poles, the invariant masses of the various lines of the diagram are effectively bounded. Therefore:

$(R_+ \cdot R_-)$ is small and therefore R_+ and/or R_- is small; suppose it is R_-.

Then the smallness of the mass of the line 2 gives

$$R_+ = O\left(\frac{1}{K_-}\right) = O\left(\frac{1}{\sqrt{s}}\right)$$

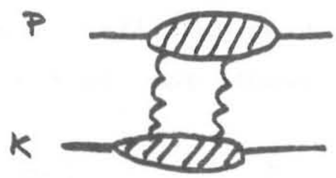
and the same for the line 1 gives $R_- = O\left(\frac{1}{P_+}\right) = O\left(\frac{1}{\sqrt{s}}\right)$.

Therefore we neglect the longitudinal part of the momentum transfert carried by the Pomeron with respect to the transverse one ($R_+ \cdot R_-$ as compared to a transverse squared mass $m_\perp^2$). This is what happens in other situations, too.

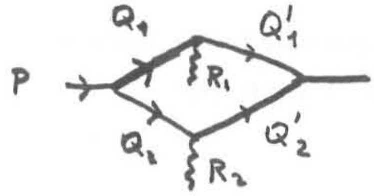
Then, in doing the required loop integration we can separately integrate over R_- and R_+. The singularities for R_- are at $(m_1^2 - i\varepsilon)/P_+$ (from the line 1), those for R_+ are at $-(m_2^2 - i\varepsilon)/K_-$ (from the line 2) and we separately close the contours to get: zero. The assumed quickly decreasing behaviour in the masses of the lines is actually realized, as usual, by mutually cancelling contributions of singularities at, say for the line 1, $m_1'^2, m_1''^2, \ldots$ Therefore, it can be said that the contribution of the elastic intermediate states is canceled by part of the contribution of the diffractively produced states.

In order to obtain a non vanishing contribution one has of course to look for situations in which the singularities for R_+ (and R_-) occur on both sides of the integration contour.

The simplest case in which this happens is the Mandelstam diagram



where the top looks like



and similarly the bottom.

The "softness" imply the smallness of

$$(R_{1+}\cdot R_{1-}),\ (R_{2+}\cdot R_{2-}),\ (Q_{1+}\cdot Q_{1-}),\ (Q_{2+}\cdot Q_{2-}),\ (Q'_{1+}\cdot Q'_{1-}),\ (Q'_{2+}\cdot Q'_{2-}).$$

Since $Q_{1+}+Q_{2+}=P_+$, Q_{1+} and/or Q_{2+} must be large, therefore Q_{1-} and/or Q_{2-} must be small; but $Q_{1-}+Q_{2-}=0$, therefore Q_{1-} and Q_{2-} must be small, $\leq O(1/P_+)$. Analogously, $Q'_{i-}\leq O(1/P_+)$. It follows that

$$R_{i-}\leq O(1/P_+).$$

An identical argument for the bottom gives $R_{i+}\leq O\left(\frac{1}{K_-}\right)$. Therefore we can neglect $(R_{i+}\cdot R_{i-})$ in the Pomeron exchange, R_{i+} on the top integrations, R_{i-} in the bottom integrations. This result is quite general; we can factorize the graph into two halves: the top, where only the R_{i-} are relevant, the bottom, where the R_{i+} are relevant. Now, the calculation for the top is completely independent from the calculation of the bottom. It gives

$$\int dQ_{1+}\cdot\int\frac{dQ_{1-}}{(Q_{1+}Q_{1-}-m^2+i\epsilon)}\,\frac{1}{((Q_{1+}-P_+)Q_{1-}-m^2+i\epsilon)}\cdot\int\frac{dQ'_{1-}}{(Q_{1+}\cdot Q'_{1-}-m^2-i\epsilon)}\,\frac{1}{(Q_{1+}-P_+)Q'_{1-}-m^2-i\epsilon)}\cdot$$
$$\cdot\, i\,(Q_{1+})^{\alpha}\cdot i\,(P_+-Q_{1+})^{\alpha}.$$

I have assumed the outgoing longitudinal momentum equal to the incoming one, and written for the Pomeron exchange the factorized form

$$i\left((Q_{1+})_{\text{TOP}}\cdot(Q_{1-})_{\text{BOTTOM}}\right)^{\alpha}.$$

The singularities in Q_{1-} are at $(m^2-i\varepsilon)/Q_{1+}$ and $(m^2-i\varepsilon)/(Q_{1+}-P_+)$. Therefore in order to obtain a non vanishing contribution we must have $0 \leq Q_{1+} \leq P_+$. The result for the top is then:

$$-(2\pi)^2\int_0^{P_+} dQ_{1+}\ \frac{1}{(P_+-Q_{1+})m^2+Q_{1+}m^2}\cdot\frac{1}{(P_+-Q_{1+})m^2+Q_{1+}m^2}\cdot i\,(Q_{1+})^\alpha\cdot i\,(P_+-Q_{1+})^\alpha.$$

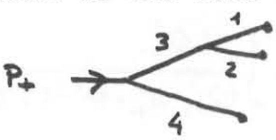
(To be more precise we should have distinguished the various transverse masses, which here are indicated with the same symbol m^2).

Let us note that:

1. The result obtained correspond to substituting one of the two propagators in Q and one of the two propagators in Q' (which one does not matter) with their discontinuities.

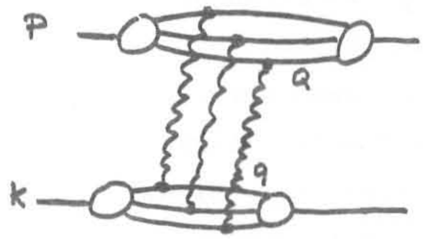
2. But for the Regge factors $(Q_{1+})^\alpha\cdot(P_+-Q_+)^\alpha$, the top is given as an integral over an integrand without singularities. In this sense "it is purely real".

3. Therefore taking the discontinuity of the whole graph does not alter numerically the top or the bottom. This is also explicitly seem from the point 1, since the top (and bottom) can already be expressed as a discontinuity . We can proceed analogously for the case of the exchange of more Pomerons. Since the (Pomeron) P-lines carry a flow of (-) component of momentum in the top part, that one can be in turn split into two halves as far as the integration over the $(Q_-)'s$ is concerned, separated by the points when the P_- lines are attached. As an example one can easily verify that in the case of the graph



the integration over $(Q_{1-}),(Q_{2-})$ gives a non vanishing result only if $Q_{1+}, Q_{2+}, Q_{3+}, Q_{4+} > 0$ (they are such that $P_+ = \sum Q_+$); it can be done by turning round the poles, giving the sum of two terms which can be expressed as the result of putting 1 and 2 on the mass shell and the result of putting 3 on the mass shell. The resulting function of the variables has no singularities in the integration range; a reason for that is that, for instance, 3 cannot be on the mass shell together with 1 and 2.

These results can be generalized for the case of the exchange of ℓ -Pomerons (of course, no more than one Pomeron can be attached to the same line):



Each Pomeron gives a factor $i(s_m)^\alpha = i\,(Q_{m+}\cdot q_{m-})^\alpha$.

The contribution to the scattering amplitude T is:

$$T_\ell = -i\cdot\frac{(-)^\ell}{\ell!}\cdot\left[\int_0^\infty\cdots\int_0^\infty\left(\prod_1^\ell dQ_{m+}\;(Q_{m+})^\alpha\right)\cdot\delta\left(P_+-\sum_1^\ell Q_+\right)\cdot F(Q_+)\right]\cdot$$

$$\cdot\left[\int_0^\infty\cdots\int_0^\infty\left(\prod_1^\ell dq_{m-}\;(q_{m-})^\alpha\right)\cdot\delta\left(K_- -\sum_1^\ell q_-\right)\cdot F(q_-)\right] \qquad (4)$$

We can compare with an eikonal model, where $T = -is\sum_\ell \frac{(i(is^{\alpha-1}))^\ell}{\ell!}$. T_ℓ must then be integrated over the transverse phase space. $F(Q_+)$ is a homogeneous function of the Q_{i+} of degree $2(1-\ell)$, which is purely real, that is, without singularities in the integration range: it can be represented as a sum over all the (meaningful) ways of taking the discontinuity of the top (or the bottom)(*).

Let us consider the discontinuity of the whole graph. To simplify the notation, let us call

$$-\eta = i\cdot i\cdot(Q_+\cdot q_-)^\alpha \qquad (5)$$

an uncut Pomeron (P_-) contribution. An uncut P_- exchange is then:

$$T_1 = -i\;(-\eta) = i\eta$$

The discontinuity of a Pomeron is twice the immaginary part,

(*) This can be verified explicitly for the case in which the blobs are a sum of tree diagrams. To get the general case one would probably have to specify more the "softness" of the theory.

therefore it is equal to

$$2\,\mathrm{Im}\,T_1 = +2\eta \tag{6}$$

which we call: a cut Pomeron contribution. It represents (with a factor 2s) the sum over all the production cross section of the multiperipheral model. The imaginary part of T_ℓ is given by $\frac{1}{2}$ * (the sum of all the contribution obtained cutting the graph). A cut divides the graph in a left and a right part, and a cut can pass through none, one or several Pomeron lines. A cut Pomeron line corresponds to a multiperipheral production, several cut Pomeron lines at once are often called to correspond to a polyperipheral production. Out of the P - lines, let us call ℓ_c the cut ones, ℓ_- the ones which belong to the left part, ℓ_+ the ones of the right part (an element of the left part is the complex conjugate of the corresponding element of the right part, but here everything is real). Of course, the P -lines cannot be all (-) or all (+). The top and the bottom are, for what said, already equal to their discontinuity for every configuration (+), (-) and (c) of the Pomerons. Therefore they are unaffected by the cutting operation (or, put in another way, they are real factors and therefore they can be taken out of the imaginary part).

If we indicate symbolically

$$T_\ell = -ic\,\frac{(-\eta)^\ell}{\ell!}$$

we get, by our rules,

$$\mathrm{Im}\,T_\ell = \frac{1}{2}\,\frac{c}{\ell!}\sum_{\ell_++\ell_-+\ell_c=\ell}{}' \frac{\ell!}{\ell_+!\,\ell_-!\,\ell_c!}\,(-\eta)^{\ell_+}\cdot(-\eta)^{\ell_-}\cdot(2\eta)^{\ell_c} \tag{7}$$

where $\sum'$ means that we exclude $\ell_+=\ell$ and $\ell_-=\ell$.
The fact that the sum gives back the correct result for $\mathrm{Im}\,T_\ell$ is a check of the consistency of the theory with the s -channel unitarity. The rules we have discussed so far are the famous AGK cutting rules.

It is not completely clear how much these rules are model dependent; in particular the exchange of two Pomerons has been discussed in detail, both in the Feynman graph approach and in the dual model[3], giving the above results.

We will see anyhow that a nice feature of those rules is the fact that they can be generalized to hold also in the more complex case in which the Pomerons interact each other. They are in fact almost unique in that respect(that is, within some general assumption we will discuss later on), and they guarantee the fulfilment of one of the s-channel requirements, i.e. the optical theorem.

One of the immediate, and most celebrated, consequences of the cutting rule concerns the evaluation of the multiplicity moments

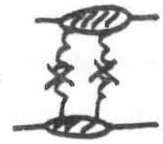
$$n_p \equiv \langle n(n-1)\cdots(n-p+1)\rangle .$$

Clearly, looking at the eq.s (2) and (3), we see that in order to get the moments we have to take the derivatives with respect to the intercept of the cut Pomeron, at fixed intercept of the uncut ones. This is so because only the cut Pomerons represent the sum over the cross sections for the production of different number of particles - the uncut Pomerons represent the absorption effects, necessary to restore the unitarity. We can graphically represent a derivative with a cross over a $\mathbb{P}$ -line. Therefore a contribution to n_p will be represented by a graph in which a number of cut Pomerons carry one or more cross, with any number m of cut and uncut Pomerons without crosses. For a fixed $m>0$ we have to sum over different possibilities and we get:

$$\sum_{m_+ + m_- + m_c = m} \frac{m!}{m_+!\, m_-!\, m_c!} (-\eta)^{m_+} \cdot (-\eta)^{m_-} \cdot (2\eta)^{m_c} = 0 \qquad (8)$$

because now the sum is unrestricted - it is not $\sum'$ like before - since there is at least one extra crossed cut Pomeron besides the m considered.

This is the AGK cancellation: only graphs in which all the exchanged Pomeron are cut and crossed contribute to the inclusive distribution. For instance

$\frac{d\sigma}{dy} =$  $\qquad \frac{d^2\sigma}{dy_1 dy_2} =$  $+$  (*)

Here we have put a cross at a given rapidity-we are considering Mueller diagrams; remember that $n_p \sigma_T = \int dy_1 \ldots dy_p \, d^p\sigma / dy_1 \ldots dy_p$. Clearly, what we have seen is relevant for high energies, where threshold effects can be neglected. For instance, in eq. (4) in order that we can speak of a Pomeron exchange the relative invariant energy $S_m = Q_{m+} \cdot q_{n-}$ must be sufficiently high (say, $S_m > 5\ GeV^2$). The energy momentum conservation factors, $\delta(P_+ - \Sigma Q_+) \cdot \delta(K_- - \Sigma q_-)$ prevent the ℓ P exchange to be meaningful if ℓ is too high at fixed total invariant energy S ; this threshold effect also distorts the energy dependence of the whole graph.[4]

We will assume in the following that for the values of we are interested in, graphs with too high ℓ give a negligible contribution.

Let us summarize the rules obtained so far:
since or graphs are in general non planar, it is convenient to think of them as three dimensional structures. In order to compute the discontinuity of a graph we immagine a cut plane through the graph. A Pomeron can appear as cut, when it lies in the cut plane; or it can be at the right of the plane and then it is called a (+) Pomeron or at the left and than it is a (-) one(**).

The cut Pomerons represent the sum over the cross section of the multiperipherally (without rapidity graphs) produced particles. More than one Pomeron can be cut at once, but the Pomerons cannot be all (+) or all (-).

(*)The second equation does not hold in general in the case of Pomeron interactions.

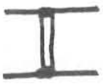
(**)A pomeron cannot be partially cut.

A (+) or a (-) Pomeron gives a contribution $(-\eta)$, a cut Pomeron gives a contribution 2η.

It is convenient to redefine the uncut contribution to be η : then we insert an i in the top and the bottom for each uncut $\mathbb{P}$ in our graph. It is also convenient to redefine the cut contribution to be η , too: then we insert a $\sqrt{2}$ in the top and the bottom.

The cut $\mathbb{P}$- lines will be represented by double lines : $\|$, the uncut ones by a wavy line: .

By defining an overall factor in such a way that

 $=$  $= - s^{\alpha - 1}$ (9)

(and therefore the sum of the cut diagrams is equal to $-\frac{1}{s}\,\mathrm{Im}\,T$) (*) we can formulate our cutting rules in terms of the top and bottom vertices as follows:

 $= \frac{i}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{i}\right)^{m_c} \cdot$  (10)

m_- m_c m_+

$m_- + m_c + m_+ = m$

m - uncut

The uncut top vertex of the r.h.s. of the above graphical equation is the one of the theory for the amplitude (as opposed to the theory for the discontinuity of the amplitude); it contains a factor $(i)^m$ which, combined with the bottom, gives the $(-1)^m$ of the alternating series of the $\mathbb{P}$ -exchanges.

(*) A ℓ-Pomeron exchange contributes a factor $(-1)^\ell \cdot s^{\ell(\alpha-1)}$ to $-1/s\ \mathrm{Im}\,T$. It can be understood in general by observing that the top is in the form of

$$\int \prod_1^\ell (dQ_+ Q_+^\alpha) \cdot \prod_1^\ell (dQ_-) \prod_1^\ell (dQ'_-) \cdot \delta(P_+ - \Sigma Q_+) \cdot \delta(\Sigma Q_-) \cdot \delta(\Sigma Q'_-) \cdot (\text{Denominators})$$

where the denominators are bounded functions of $Q_+ Q_-$ or $Q_+ \cdot Q'_-$. Therefore changing variables to $M_i^2 = Q_{i+} Q_{i-}$ and to $M_i'^2 = Q_{i+} Q'_{i-}$ (which are bounded) and rescaling $Q_{i+} = \sqrt{s}\, x_i$, with $P_+ = \sqrt{s}$, we get the result.

2. Pomeron interactions and Reggeon Field Theory.

Let us consider the usual one $\mathbb{P}$-exchange in more detail

$$= -\beta_1\beta_2\, e^{-(1-\alpha_0)Y - \alpha' k^2 Y}\,\theta(Y) =$$

$$= -\beta_1\beta_2 \int d^2\underline{B}\; e^{i\underline{k}\,\underline{B}}\; \frac{e^{-(1-\alpha_0)Y - B^2/4\alpha' Y}}{4\pi\alpha' Y}\,\theta(Y) \equiv -\beta_1\beta_2 \int d^2\underline{B}\, P(Y,\underline{B}) \tag{11}$$

We have written explicitly the dependence on the transverse momentum $\underline{k}_\perp$ and introduced the impact parameter $\underline{B}$, which is the conjugated variable, in the sense of the Fourier transform, of $\underline{k}_\perp$. In the phase space (rapidity-impact parameter) the one $\mathbb{P}$-exchange is represented as the propagation of a signal from the position of the first particle $\underline{b} = 0$, $y = 0$ to the position of the second particle $\underline{b} = \underline{B}$, $y = Y$. The propagator $P(y,\underline{b})$ is a Green function, it obeys the non homogeneous equation

$$\left(-\frac{\partial}{\partial y} - (1-\alpha_0) + \alpha' \nabla_{\underline{b}}^2\right) P(y,\underline{b}) = -\delta(y)\,\delta^{(2)}(\underline{b}). \tag{12}$$

This equation describes the propagation of a field ϕ, in a dynamical system corresponding to the action

$$S = \int dy\, d^2\underline{b}\, \left(-\phi^+ \partial_y \phi - \alpha' \underline{\nabla}_b \phi^+ \cdot \underline{\nabla}_b \phi - (1-\alpha_0)\phi^+\phi\right) \equiv$$

$$\equiv \int dy\, d^2\underline{b}\, \left(-\phi^+\partial_y\phi - \mathcal{H}_0\right). \tag{13}$$

ϕ^+ is a canonical conjugated variable with respect to ϕ. We may take $\phi^+(\underline{b})$ to be the creation operator of a Pomeron and $\phi(\underline{b})$ to be the annihilation at the point $\underline{b}$. Their commutator is therefore

$$\left[\phi(\underline{b}), \phi^+(\underline{b}')\right]_{\text{equal } y} = \delta^{(2)}(\underline{b} - \underline{b}'). \tag{14}$$

The one $\mathbb{P}$ -exchange is given by the vacuum expectation value

$$-\beta_1\beta_2 P(Y,\underline{B}) = -\beta_1\beta_2 \langle 0|\phi(\underline{B})\, e^{-H_0 Y}\, \phi^+(\underline{0})|0\rangle$$

where

$$H_0 = \int d^2\underline{b}\ \mathcal{H}_0 \, .$$

A multi $\mathbb{P}$ -exchange is similarly described by

$$\frac{(-)^n}{n!}\int d^2\underline{B}\ e^{i\underline{k}_\perp\cdot\underline{B}}\cdot\left(P(Y,\underline{B})\right)^n\cdot(\mathrm{Top})\cdot(\mathrm{Bottom}) . \tag{15}$$

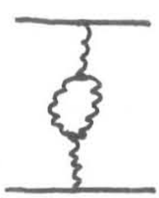
This term is generated through derivatives by

$$Z_0(\beta_1,\beta_2) = e^{-\beta_1\beta_2 P(Y,\underline{B})} = \langle 0|\, e^{-i\beta_1\phi(\underline{B})}\, e^{-H_0 Y}\, e^{-i\beta_2\phi^+(\underline{0})}|0\rangle . \tag{16}$$

As it is well known the Pomerons can interact among themselves.[1)] Technically, this amounts to the addition of an interaction term to the hamiltonian density . The simplest form is a $3\mathbb{P}$ interaction described by the hamiltonian

$$\mathcal{H} = \alpha'\nabla\phi^+\nabla\phi + (1-\alpha_0)\phi^+\phi + \frac{ig}{2}\left(\phi^{+2}\phi + \phi^+\phi^2\right) . \tag{17}$$

Let us remark the antihermitean nature of the interaction, related to the absorptive nature of the correction represented by the insertion of an extra $\mathbb{P}$ -line in a graph. For instance, the $O(g^2)$ correction to the Pomeron propagator, corresponding to the diagram



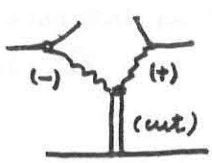
has the opposite sign as the unperturbed propagator  .

The $3\mathbb{P}$ interaction we have considered is known to be different from zero (see discussion below), buth other multi-$\mathbb{P}$ interactions may also play a rôle. In the following we will essentially concerned with the pure $3\mathbb{P}$ case, which is the one most explored.

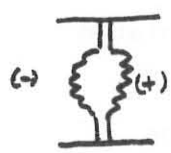
Similar things can be repeated for the more complex world of the uncut Pomerons. We have three kind of fields: the (+) fields, the (-) fields and the cut fields. Their free propagator, and therefore their respective free hamiltonians, are the same. As far as interactions are concerned, the (+) Pomerons can interact among themselves and the (-) Pomerons also among themselves with an interaction which must be equal to the one of the uncut theory. This is clear since the (-) Pomerons, for instance, are just the ordinary uncut Pomerons to the left of the cut plane(*) .

Beside that, there are the interactions of the (+),(-) with the cut ones, and of the cut ones among themselves.

There is an interaction (cut) (+)(-) which represents the diffractive production of a large mass and is the one experimentally observed in the inclusive distribution corresponding to the well known diagram:



This interaction must be real, and indeed instance the graph



adds with a + sign to

since they both represents cross sections of physical processes.

Finally, there is a real (cut)→(cut) (cut) interaction, which represents an inelastic rescattering of the particles represented by the cut Pomeron with production of new multi peripheral states, and the imaginary (cut)→(cut) (+) and (cut)→(cut) (-) interactions, representing absorption corrections

(*) Since the (+)-P's represent a complex conjugated dynamics with respect to the (-)-P's , the 3(+)P interaction should be the negative of the 3(-)P one. We can however redefine the sign of the (-) field to have a symmetrical picture.

to the inelastic processes. Therefore the interaction hamiltonian will be of the form:

$$\mathcal{H}_{int} = \frac{i}{2} \sum g_{ijk} \left(\phi_i^+ \phi_j \phi_k + \phi_j^+ \phi_k^+ \phi_i \right) \qquad (18)$$

where the non-zero elements of g_{ijk} are:

$$g_{+++} = g_{---} = g \qquad\qquad g_{cc\pm} = g_{c\pm c} = g_A$$

$$g_{c-+} = g_{c+-} = \frac{i}{\sqrt{2}}\, g_D \qquad\qquad g_{ccc} = \frac{\sqrt{2}}{i}\, g_P$$

Let us notice that the vertex (cut) $\rightarrow \phi_i \phi_j \equiv V_{cij}$ is such that $V_{cij} = \frac{i}{\sqrt{2}} \left(\frac{\sqrt{2}}{i}\right)^{m_c} i g$, m_c being the number of cut fields among i and j , provided $g_A = g_D = g_P = g$.
This would give for V_{cij} the same rule which we have seen to hold for the coupling of the Pomerons with the external particles (see fig-eq.10) Indeed we can formally bend upward the external particle lines in fig.10 to recostruct the symbol for the cut propagator, remembering that the cut plane stays in the middle.
In general, we ask the validity of the optical theorem in the cut theory. At the level of one P- exchange, this requires the cut and uncut complete propagators to be the same. The one loop correction gives (besides a factor which is the same for both)

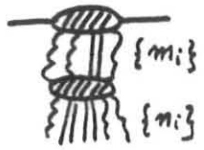
$$g^2 = 4 g_A^2 - 2 g_P^2 - g_D^2 \ .$$

A more involved relation comes,[5] in the next step, from the comparison of the vertex corrections in the cut and uncut theory, which should also be proportional. This set of equations give two solutions

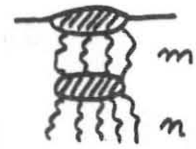
$$a)\ g_A = g_D = g_P = g \qquad\qquad b)\ g_A = g \quad g_P = \frac{1}{\sqrt{2}}\, g \quad g_D = \sqrt{2}\, g .$$

Of course, we consider the solution a), which is the one consistent with our rules for the particle-Pomerons coupling.
In order to insure the fulfilment of the optical theorem in general, we have then to be sure that the rules we have established for the cutting of a n-Pomeron exchange diagram in the case of no interaction still hold when the interactions

are present. This amounts to require that the top of the n - Pomeron exchange diagram still satisfies the equality of fig.(10) even when the blob contains Pomeron interactions. Both sides of the equality of fig.(10) are modified in the way respectively shown here :



$$= \frac{i}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{i}\right)^{n_c} \cdot \tag{19}$$



where we indicate with $\{m_i\}$, $\{n_i\}$ a set of m_+ P-lines of the kind (+) etc. with $m_+ + m_- + m_c = m$, $n_+ + n_- + n_c = n$. Calling $G_c^{(m_i, n_i)}$ the Green function of the cut theory and $G_u^{(m,n)}$ the Green function of the uncut one, the equality of fig.(10) clearly remains satisfied, as in fig.(19), if

$$\sum_{m_+ + m_- + m_c = m} \left(\frac{\sqrt{2}}{i}\right)^{m_c} G_c^{(m_i, n_i)} = \left(\frac{\sqrt{2}}{i}\right)^{n_c} G_u^{(m,n)} \tag{20}$$

with an extra obvious constraint, that if the m are all (+) or all (-) also the n must be all (+) or all (-) respectively. It can be shown,[5] with an ingenious trick, that indeed eq.(20) is satisfied (for the solution a)), together with the constraint. Those relations can be put in compact form by introducing the generating functional of the Green functions:

$$Z_c(f_+, f_-, f_c; g_+, g_-, g_c) =$$

$$= \sum_{\{n_i\}} \sum_{\{m_i\}} \frac{(-if_+)^{m_+}}{m_+!} \frac{(-if_-)^{m_-}}{m_-!} \frac{(-f_c)^{m_c}}{m_c!} G_c^{(m_i, n_i)} \frac{(-ig_+)^{n_+}}{n_+!} \frac{(-ig_-)^{n_-}}{n_-!} \frac{(-g_c)^{n_c}}{n_c!}$$

$$Z_u(h;k) = \sum_n \sum_m \frac{(-ih)^m}{m!} G_u^{(m,n)} \frac{(-ik)^n}{n!} . \tag{21}$$

The relations are:

$$Z_c(f,0,0;g_+,g_-,g_c) = Z_u(f;g_+)$$

$$Z_c(0,f,0;g_+,g_-,g_c) = Z_u(f;g_-)$$

$$Z_c(f,f,\sqrt{2}f;g_+,g_-,\tfrac{1}{\sqrt{2}}(g_++g_--g_3)) = Z_u(f,g_3). \qquad (22)$$

As an example, let us check the optical theorem taking the coupling of n_-,n_+,n_c Pomerons to the top particle line to be $\frac{i}{\sqrt{2}}\cdot\frac{(-if)^{n_-}}{}\cdot\frac{(-if)^{n_+}}{}\cdot\frac{(\sqrt{2}f)^{n_c}}{}$, in agreement with the eq.(10) $(f\to g$ for the bottom line$)$. Then

$$-\frac{1}{s}\,\mathrm{Im}\,T = -\frac{1}{2}\sum{}'\frac{(-if)^{n_+}}{n_+!}\frac{(-if)^{n_-}}{n_-!}\frac{(\sqrt{2}f)^{n_c}}{n_c!}G_c^{(n_i,m_i)}\frac{(-ig)^{m_+}}{m_+!}\frac{(-ig)^{m_-}}{m_-!}\frac{(\sqrt{2}g)^{m_c}}{m_c!}$$

where $\sum'$ means not all n or m equal to zero nor all of the kind (+) or all of the kind (-). With the previous formulae we get

$$-\frac{1}{s}\,\mathrm{Im}\,T = -\frac{1}{2}\left(Z_u(f;0)-1-2\left(Z_u(f;g)-1\right)\right) = \left(Z_u(f,g)-1\right)$$

which is the optical theorem. We have used the fact that $Z_u(f;0)=1$, i.e.

$$\langle 0|(\phi^+)^\ell|0\rangle = \langle 0|(\phi)^n|0\rangle = 0, \qquad \text{for} \quad \ell,n\geq 1.$$

3. Applications.

a) As a first illustration we will consider the multiplicity distribution[6,7] in the "critical" theory,[1] which is by definition the case in which the renormalized intercept α is equal to 1. In this case it is convenient to work in the phase space $(E, \underset{\sim}{k}_\perp)$ where E is the Laplace transform variable with respect to the rapidity Y $\left(\tilde{A}(E)=\int_0^\infty dY e^{EY} A(Y)\right)$ and $\underset{\sim}{k}_\perp$ is the transverse momentum. The inverse free propagator is then

$$\Gamma_0^{(1,1)} = E-(1-\alpha)-\alpha' k_\perp^2$$

therefore $E=1-J$, in terms of the analytically continued t-channel angular momentum J. In the critical theory, for the complete (inverse) propagator we have

$$\Gamma^{(1,1)}(E=0,\ \underset{\sim}{k}_\perp=0)=0.$$

The dominant contribution to the cross sections (total or inclusive) comes from the most singular diagram (for $E\to 0, \underset{\sim}{k}_\perp\to 0$), which is the 1 - $\mathbb{P}$ exchange, or $(\Gamma^{(1,1)})^{-1}$. As explained before, in order to obtain the inclusive distribution we take the derivatives with respect to α_c at $\alpha_c=1$ of the complete propagator for a 1- cut $\mathbb{P}$ -exchange:

$$L(\sigma_T\cdot m_p) = z^p \frac{\partial^p}{\partial\alpha_c^p}\left(\Gamma_c^{(1,1)}\right)^{-1} \tag{23}$$

where z is a constant and L means Laplace transform. A(re-) normalization is necessary in order to eliminate a divergence :

$$\left.\frac{\partial}{\partial\alpha_c}\left(\Gamma_c^{(1,1)}\right)^{-1}\right|_{\substack{\alpha_c=1\\ E=\bar{E}}} = -1/\bar{E}^2 \quad .$$

Then an equation follows from the rescaling of E and $\bar{E}$ (for $\alpha_c-1=\alpha-1=0$ we do not have "mass" in the propagator); this is a well known renormalization group equation

$$\left(E\frac{\partial}{\partial E}+(p+1)+p\gamma+\eta\right)\left(\frac{\partial^p}{\partial\alpha_c^p}\Gamma_c^{-1}\right)=0 \,. \tag{24}$$

We have considered that the dimensionality in E is $\left[\frac{\partial^p}{\partial\alpha_c^p}\Gamma_c^{-1}\right]=\left[E^{-p-1}\right]$. η and γ are called anomalous dimensions.

η determines the behaviour of the total x-section

$$\Gamma_c^{-1} \sim E^{-(1+\eta)} \Rightarrow \sigma_T \sim Y^{\eta}.$$

The behaviour of the multiplicities can be read from the equation (24) :

$$\left(\frac{\partial^p}{\partial \alpha_c^{\,p}} \Gamma_c^{-1}\right) \sim E^{-(1+\eta)-p(1+\gamma)} \Rightarrow n_p \cdot \sigma_T \sim Y^{\eta+p(1+\gamma)}.$$

γ is easily computed by taking $E = \bar{E}$ in the equation (24) for $p=1$:

$$\bar{E}\frac{\partial}{\partial E}\left(\frac{\partial}{\partial \alpha_c}\Gamma_c^{-1}\right)_{\bar{E}} = (2+\gamma+\eta)\frac{1}{\bar{E}^2}.$$

In terms of radiative corrections

$$\Gamma_c^{-1} = \frac{1}{E-(1-\alpha_c)+\Sigma_c};$$

since the normalization is $\Sigma_c(\bar{E}) = 0$ and the definition of η is $\eta = \frac{1}{\bar{E}^2}\frac{\partial \Sigma_c}{\partial E}\Big|_{\bar{E}}$, we get

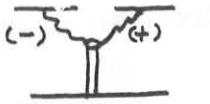
$$\bar{E}\frac{\partial}{\partial E}\frac{\partial}{\partial \alpha_c}\Sigma_c\Big|_{\bar{E}} = \eta - \gamma.$$

Now Σ is in general a bubble: $\Sigma_c =$  .

For any number of Pomeron exchanged the derivative with respect to α_c gives zero, as we have seen in sect. 1. Therefore $\eta = \gamma$ (the ε- expansion at order ε gives $\eta = 1/6$; more refined estimates give $\eta \sim .24$).

Of course, the interesting thing is that η does not depend on any measured quantity, it is a pure number determined by the structure of the theory (even if it may be difficult to compute). An analysis of the same kind[7] shows that also the correlation coefficients, defined as $c_p = n_p/(n_1)^p$ only depend on the structure of the theory. So far only the first order ε- expansion has been computed for c_2 and c_3 (giving 1 + 1/4 and 1 + 7/4 respectively). It is necessary to extend the application of alternative techniques also to the evaluation of those fundamental outputs of the critical theory.

b) Triple Regge region for the inclusive distribution in a theory in which the intercept is $\alpha > 1$. The basic diagram for this region is

 . (25)

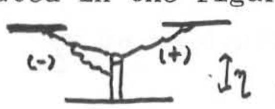
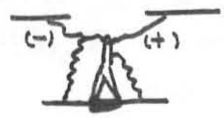
This defines the coupling constant of the theory. Indeed we have

$$\frac{d\sigma}{d\eta\, dt} = \frac{1}{16\pi} \cdot g \cdot \sqrt{2} \cdot \beta^3 \cdot e^{(2\mu - 2\alpha' t)\cdot(Y-\eta)} \cdot e^{\mu\eta}$$

with $\mu = \alpha - 1$, and $\eta = \ln M^2$. Roughly this gives for $\mu = 0.06$ and $\beta \approx 8$ GeV^{-1}, ($\beta^2 e^{6\mu}$ = 100 GeV^{-2}),

$$g \simeq 0{,}5 \text{ GeV}^{-1}!$$

Of course the task is to compute corrections like the ones illustrated in the figure:

$i)$  $ii)$  (26)

Let us define the 3P region in the inclusive distribution as the configuration in which there is production of a cluster of particles more or less uniformely distributed covering the rapidity range from zero to η .

In our theory this corresponds to having at least one cut Pomeron between $y = 0$ and $y = \eta$ and no cut Pomerons between $y = \eta$ and $y = Y$. This calculation has been attempted[8] so far in a drastically approximated scheme:

1) neglecting the internal Pomeron loops (e.g. neglecting the graph of fig. 26 i) but retaining the one of fig. 26 ii).
2) in the zero slope limit $\alpha' = 0$.

The two approximations are not so bad after all since, calling β a typical coupling of a Pomeron to an external line, the statement 1) corresponds to retain powers of βg compared to powers of g^2 (at very high energy however the loops will anyhow dominate) and the statement 2) corresponds to neglect the slope of the Pomeron trajectory as compared to the slope of the Regge residue (also this approximation will not be valid at very high energy). Moreover this calculation will give an idea of the general trend of the theory.

The approximation 1) technically means to compute the Pomeron Green functions, before attaching them to the external particle lines, in the tree graph approximation. As it is well known, the tree graphs are generated by the functional

$$Z(\text{tree}) = e^{S_{cl} - \bar{J}\phi_{cl} - J\phi^{+}_{cl}}$$

where S_{cl} is the action, given in eq.(13) and (18), evaluated over the classical solutions, i.e. the solutions of the field differential equations

$$\frac{\delta S}{\delta \phi^{+}} = J \qquad\qquad \frac{\delta S}{\delta \phi} = \bar{J}.$$

In our case the sources term $\bar{J}\phi$ and $J\phi^{+}$ have to be interpreted as

$$\bar{J}\cdot\phi = i\beta\left(\phi_{(-)} + \phi_{(+)}\right)\Big|_{y=Y}$$

$$J\cdot\phi^{+} = \beta\left(i\phi^{+}_{(+)} + i\phi^{+}_{(-)} + \sqrt{2}\,\phi^{+}_{c}\right)_{y=0}$$

since at $y=Y$ there is a sink for the (+) and (-) fields and at $y=0$ there is a source for the fields (+), (-) and (cut).

As for the hamiltonian, it is the one of the cut theory, see eq.(18), with the extra prescription that for $y>\eta$ the interactions involving the cut field are zero, since there cannot be sink or sources of the cut field for $y>\eta$. The inclusive distribution (at fixed impact parameter) is then proportional to

$$\frac{d\sigma}{d\eta} \sim \phi_c(\eta)\,\phi^{+}_{(+)}(\eta)\,\phi^{+}_{(-)}(\eta)\cdot e^{S_{cl} - \bar{J}\phi - J\phi^{+}}. \qquad (27)$$

The factor in front of the esponential represents the basic graph of fig.(25), while the development of the exponential generates the other corrections in the tree approximation. At the end one should attach the Green functions to the particle line. As it stands eq. (27) represents the case in which the coupling of ℓ-Pomerons follow a Poisson distribution law. To cover other cases one has to perform proper convolutions with respect to the sources.

The key point[9] of the theory is that for large rapidities the fields must have values near the fixed points of the Hamiltonian.

In the problem[9] of the uncut fields it is convenient to redefine $\phi = -ip$, $\phi^{+} = -iq$; then , only real terms appear in the "classical "equations of motion. p and q play the role of the usual canonically conjugated variables and the hamintonian is

$$H = \mu pq - \frac{g}{2} pq(p+q).$$

The fixed points are at

$$(q=0, p=0), \left(q=\frac{2\mu}{3g}, p=\frac{2\mu}{3g}\right), \left(q=\frac{2\mu}{g}, p=0\right), \left(q=0, p=\frac{2\mu}{g}\right).$$

Actually only the last two are attractive in the limit of large rapidities. Which one of the two is reached depends on the value for the sources; in any case one of the two, p or q , is zero. The classical action, calculated for the solutions attracted by the fixed points, takes a finite value for $Y \to \infty$.

A similar situation also occurs for the more involved case of the cut theory[8]. The relevant attractive fixed points are such that or $\phi_i = 0$ or $\phi_i^{+} = 0$. Actually this means that in the expression of eq.(27) , or $\phi_c(\eta) \to 0$

or $\phi^{+}_{(+)}(\eta), \phi^{+}_{(-)}(\eta) \to 0$, like $e^{-(Y-\eta)\mu}$ or $e^{-\eta\mu}$ for $(Y-\eta), \eta \to \infty$. The classical action in the same limit remains finite. The conclusion is that the triple Regge configuration gives a negligible contribution to the total X-section, $d\sigma/d\eta$ actually going to zero in this region.

This is due of course to the absorptive effects, which will become more and more important with the growth of the rapidity graps. In view of the smallness of the triple Pomeron coupling constant the asymptotic result is expected to appear at rapidity values far behond the reach of the present research.

We will see that this kind of picture is common to the various more or less approximate schemes which investigate the Pomeron theory for $\alpha > 1$. Everything at the end turns out to be determined by the fixed points, and this is the mechanism which tames the apparent too violent behaviour of the theory.

Reassuring as it is, this ultimate pattern will likely set in at astronomycal energies. Therefore, the necessary strategy will be to turn to numerical evaluations relevant for experimental energies, once completely clarified the general qualitative behaviour.

c) Inclusive distribution - and diffractive production of many particles - in a theory with intercept $\alpha > 1$.

In order to discuss this topic is necessary to review[10,11] very briefly the main points of the uncut theory for $\alpha > 1$. The multi $\mathbb{P}$ exchange Green functions are generated by

$$Z_u(\beta_1;\beta_2) = \langle 0| e^{-i\beta_1\phi} e^{-HY} e^{-i\beta_2\phi^+}|0\rangle \qquad (28)$$

β_1 is centered at impact parameter $\underset{\sim}{B}$, β_2 is centered at impact parameter $\underset{\sim}{0}$. We have already written the hamiltonian; let us now add to it a specially chosen quartic term.[11] The reason for this is that, first of all, nobody knows the structure of the interaction besides the trilinear term, so we have some freedom, and, second, this extraterm semplifies enormously the following discussion. It actually plays the role of regularizing the theory; as matter of fact the theory can also be treated without it. Since however at the present stage we only can deal with models which more or less approximately reproduce the theory, let us consider the simplest of them. Our hamiltonian is

$$H = \int d^2b \left(\alpha' \underset{\sim}{\nabla}\phi^+ \cdot \underset{\sim}{\nabla}\phi - \mu\phi^+\phi + \frac{ig}{2}\left(\phi^{+2}\phi + \phi^+\phi^2\right) + \frac{\lambda}{4}\phi^{+2}\phi^2\right).$$

If $\mu \equiv \alpha - 1 = g^2/\lambda$ (and we will stick to this case), H can be transformed by a similarity transformation into a honest hermitian semipositive definite hamiltonian:

$$e^{\frac{2\mu}{ig}\int d^2b\,\phi} H e^{-\frac{2\mu}{ig}\int d^2b\,\phi} = \int d^2b \left(\alpha' \underset{\sim}{\nabla}\phi^+ \cdot \underset{\sim}{\nabla}\phi + \frac{\lambda}{4}\phi^+\left(\phi^+ - \frac{2ig}{\lambda}\right)\left(\phi + \frac{2ig}{\lambda}\right)\phi\right).$$

The spectrum of H is therefore real positive. It is easy to find out the lowest eigenstates, for which the eigenvalue is zero. They are:

right eigenstates

$$|0\rangle, \qquad |\Psi_0\rangle = |0\rangle - e^{\frac{2\mu}{ig}\int d^2b\,\phi^+}|0\rangle\,;$$

left eigenstates

$$\langle 0|, \qquad \langle\bar{\Psi}_0| = \langle 0| - \langle 0|e^{\frac{2\mu}{ig}\int d^2b\,\phi}\,.$$

Notice that $\langle\bar{\Psi}_0|$ is not the hermitean conjugate of $|\Psi_0\rangle$.

The normalization is

$$\langle 0|0\rangle = 1 \qquad \langle\bar{\Psi}_0|\Psi_0\rangle = -1 \qquad \langle\bar{\Psi}_0|0\rangle = \langle 0|\Psi_0\rangle = 0.$$

Let us now consider the continuous $\underline{b}$ space replaced by a lattice whose intersite distance is d. Then $\int d^2\underline{b} \Rightarrow \sum_{\underline{\ell}} d^2$, and redefining ϕ to be ϕ/d we find the one site hamiltonian H_s to be

$$H_s = -\mu\,\phi^+\phi + \frac{ig}{2d}\left(\phi^{+2}\phi + \phi^+\phi^2\right) + \frac{\lambda}{4d^2}\,\phi^{+2}\phi^2.$$

Again, the lowest state are :

right: $\chi = |0_s\rangle, \quad |1_s\rangle = \chi - \varphi \qquad$ where $\varphi = e^{\frac{2\mu d}{ig}\phi^+}|0_s\rangle.$

left: $\bar{\chi} = \langle 0_s|, \quad \langle\bar{1}_s| = \bar{\chi} - \bar{\varphi} \qquad$ where $\bar{\varphi} = \langle 0_s|e^{\frac{2\mu d}{ig}\phi}.$

Now

$$\langle 0_s|0_s\rangle = 1\,, \qquad \langle\bar{1}_s|1_s\rangle = -1 + e^{-\frac{4\mu d^2}{g^2}}.$$

For simplicity we will consider g so small to neglect $e^{-\frac{4\mu d^2}{g^2}}$. The intersite distance d plays the role of a cut-off, which somehow the theory must provide.

The other, more crucial, approximation is to truncate[10] the Hilbert space to be the space spanned by $|0_s\rangle$ and $|1_s\rangle$. We remember that we are indeed interested in the large Y behaviour of the theory, where the relevant states will be those of minimal energy.

The completeness is then $|0_s\rangle\langle 0_s| - |1_s\rangle\langle\bar{1}_s| = \mathbb{1}$. Accordingly, we represent the ϕ and ϕ^+ operators in this basis as

$$\phi = \frac{2\mu d}{ig}\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \equiv -i\frac{2\mu d}{g} P \qquad \phi^{+} = \frac{2\mu d}{ig}\begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix} \equiv -i\frac{2\mu d}{g} Q$$

and the total hamiltonian reduces to its intersite part

$$H = \frac{4\mu^2}{g^2}\alpha' \sum_i \sum_{\underset{\sim}{r}} \left(Q_{\underline{r}+\underline{\delta}_i} \cdot P_{\underline{r}} + Q_{\underline{r}} \cdot P_{\underline{r}+\underline{\delta}_i} \right)$$

where the sites nearest to $\underline{r}$ are $\underline{r}+\underline{\delta}_i$.

The lowest eigenstates of H, with $E=0$, are, as seen, the collective states

$$|0\rangle = \prod_{\underset{\sim}{r}} \chi_{\underline{r}} \qquad \text{and} \qquad |\psi_0\rangle = \prod_{\underset{\sim}{r}} \chi_{\underline{r}} - \prod_{\underset{\sim}{r}} \varphi_{\underline{r}} .$$

Since

$$P\chi = 0 \qquad (P)^n \varphi = \varphi$$

and

$$(Q)^n \chi = \chi - \varphi \qquad Q\varphi = 0$$

the expression of Z_u in the limit of large Y (at fixed $\underset{\sim}{B}$), when the only contributing intermediate states are $|0\rangle$ and $|\psi_0\rangle$, becomes

$$Z_u \underset{Y\to\infty}{\longrightarrow} 1 - \left(1 - e^{-\beta_1 \frac{2\mu d}{g}}\right) \cdot \left(1 - e^{-\beta_2 \frac{2\mu d}{g}}\right) .$$

Therefore at high energy in this model the matter appears as grey (remember $T = is(1-Z)$).

If we look at the model in more detail,[10] reading the expression for Z_u given in eq.(28) from right to left, we find that, first, the operator $Q_{\underline{r}=0}$ acts on the state $|0\rangle = \prod_{\underset{\sim}{r}} \chi_{\underline{r}}$.
This gives

$$(Q_{\underline{r}=0})^n \prod_{\underset{\sim}{r}} \chi_{\underline{r}} = \prod_{\underset{\sim}{r}} \chi_{\underline{r}} - \varphi_{\underline{r}=0} \prod_{\underline{r}\neq 0} \chi_{\underline{r}} .$$

We may say that the second state at the r.h.s. has an impurity φ at the site $\underline{r}=0$. The point is now that the evolution operator e^{-HY} expands the impurity, whose edges travel with a velocity $v = 4\mu^2\alpha' d/g^2$. Since there is at the end the action of the operator $P_{\underline{r}=\underline{B}}$, which annihilates $\chi_{\underline{r}=\underline{B}}$, we obtain a result different from zero only if the impurity has succeeded in traveling up to $\underset{\sim}{B}$. Therefore the maximal $|\underline{B}| \sim vY$

and the cross section turns out to be

$$\sigma_T \sim Y^2 .$$

It is now easy to consider in this model the diffractive production of a number of particles.[12] Since by definition the diffractive production is mediated by the Pomeron exchange, each produced particle is represented by the insertion of the operator, in our basis,

$$c\,\phi^+\phi = c' \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \equiv c'\,\Omega$$

(actually $c' \sim e^{-4\mu^2 d^2/g^2} c$, therefore we would have obtained zero with the previous expressions for ϕ and ϕ^+. We have to suppose that c compensates the small factor).

The action of Ω is

$$\Omega_{\underset{\sim}{\gamma}_1} \prod_{\gamma} \chi_{\underset{\sim}{\gamma}} = 0 \qquad \Omega_{\underset{\sim}{\gamma}_1} \prod_{\gamma} \varphi_{\underset{\sim}{\gamma}} = \prod_{\gamma} \varphi_{\underset{\sim}{\gamma}} - \chi_{\underset{\sim}{\gamma}_1} \prod_{\gamma \neq \gamma_1} \varphi_{\underset{\sim}{\gamma}} .$$

Therefore $\Omega_{\underset{\sim}{\gamma}_1}$ on an impurity configuration gives back the impurity minus the impurity with a hole at the position $\underset{\sim}{\gamma}_1$. Since under the action of the evolution operator the impurity expands over χ , the hole will be filled, unless another produced particle $\Omega_{\underset{\sim}{\gamma}_2}$ or the final $P_{\underset{\sim}{B}}$ operator is acting at the same site before too much rapidity is elapsed. If that were not the case, we would get zero. As a result, if the particles are produced at $(\underset{\sim}{b}_1, y_1), (\underset{\sim}{b}_2, y_2) \ldots, (\underset{\sim}{b}_n, y_n)$ we get a non zero cross section only if $(\underset{\sim}{b}_\ell - \underset{\sim}{b}_{\ell-1})$ and the rapidity intervals $(y_\ell - y_{\ell-1})$ are bounded (it turns out, they are exponentially bounded). Therefore strictly speaking there is asymptotically no diffractive production,(*) in the model. This process gives a negligible contribution to the total X-section.

The inclusive distribution in the cut theory with $\alpha > 1$ is a little bit more complex to discuss,[13] but the relevant phenomena are always the same.

(*) The diffractive production of a bunch of particles at the ends of the total rapidity interval is of course allowed.

It appears useful to redefine

$$\Phi_{(+)} = -i\,\Phi_1 \qquad \Phi_{(-)} = -i\,\Phi_2 \qquad \Phi_c = -\,\Phi_3$$

$$\Phi^\dagger_{(+)} = -i\,\bar{\Phi}_1 \qquad \Phi^\dagger_{(-)} = -i\,\bar{\Phi}_2 \qquad \Phi^\dagger_c = -\,\bar{\Phi}_3$$

and a scalar product $\underset{\sim}{f}\cdot\underset{\sim}{\Phi} = f_1\Phi_1 + f_2\Phi_2 - f_3\Phi_3$, where f_i is a c-number source or it is the field $\bar{\Phi}_i$.

In order to simplify the description of the essential points, we will take a somewhat axiomatic point of view, defining as a model for our theory the one in which the right-eigenstates with zero energy eigenvalue are, at one site,

$$\chi = |0_s\rangle \qquad \varphi_1 = e^{-\frac{2\mu d}{g}\bar{\Phi}_1}|0_s\rangle \qquad \varphi_2 = e^{-\frac{2\mu d}{g}\bar{\Phi}_2}|0_s\rangle$$

$$\varphi_3 = e^{-\frac{2\mu d}{g}(\bar{\Phi}_1+\bar{\Phi}_2-\sqrt{2}\bar{\Phi}_3)}|0_s\rangle \qquad \varphi_4 = e^{-\frac{2\mu d}{g}(\bar{\Phi}_1+\bar{\Phi}_2-\frac{1}{\sqrt{2}}\bar{\Phi}_3)}|0_s\rangle.$$

The left-eigenstates are $\bar{\chi} = \langle 0_s|$, $\bar{\varphi}_1 = \langle 0|e^{-\frac{2\mu d}{g}\Phi_1}$ etc.
The point is that, by using the truncated Hilbert space spanned by χ, φ_i , we get, at one site,

$$Z_c(\underset{\sim}{f};\underset{\sim}{g}) = \langle 0|e^{-\underset{\sim}{f}\cdot\underset{\sim}{\Phi}}\,e^{-HY}\,e^{-\underset{\sim}{g}\cdot\underset{\sim}{\bar{\Phi}}}|0\rangle = \langle 0|e^{-\underset{\sim}{f}\cdot\underset{\sim}{\Phi}}\,e^{-\underset{\sim}{g}\cdot\underset{\sim}{\bar{\Phi}}}|0\rangle$$

and we can easily verify, by using a matrix representation for Φ_i and $\bar{\Phi}_i$ analogous to the one introduced before, that

<u>indeed satisfies the basic relations</u> of eq. (22)

(The form of the states has actually been obtained by some variational technique from the cut Pomeron hamiltonian).

The total hamiltonian, intersite interactions included, now reads (with $\Phi_j = \frac{2\mu d}{g}A_j$)

$$H = \frac{4\mu^2}{g^2}\alpha'\sum_i\sum_{\underset{\sim}{r}}\left(\underset{\sim}{\bar{A}}_{\underset{\sim}{r}+\underset{\sim}{\delta}_i}\cdot\underset{\sim}{A}_{\underset{\sim}{r}} + \underset{\sim}{\bar{A}}_{\underset{\sim}{r}}\cdot\underset{\sim}{A}_{\underset{\sim}{r}+\underset{\sim}{\delta}_i}\right).$$

The eigenstates of zero energy are the five collective states $\prod_{r}\chi_r,\ \prod_{r}\varphi_{i\,\underset{\sim}{r}}$. In order to get the moments of the inclusive distribution one has to insert the operator

$$c\,\bar{\Phi}_c\Phi_c = c\,\frac{4\mu^2}{g^2}d^2\,\bar{A}_3A_3 \equiv c\,\frac{4\mu^2}{g^2}d^2\,R.$$

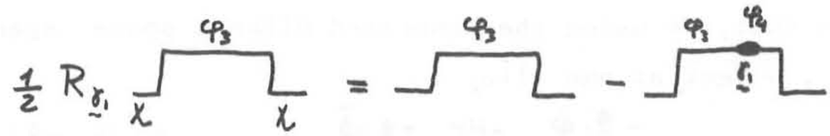
The basic property of R is that

$$R \cdot \chi, \varphi_1, \varphi_2, \varphi_4 = 0 \qquad R\,\varphi_3 = 2(\varphi_3 - \varphi_4).$$

Let us now follow, like before, the evolution of our system using the rapidity as a time variable. At the beginning the action of $\bar{A}_{\underline{\gamma}=0}$ on $\prod_{\gamma} \chi_{\underline{\gamma}}$ creates at the site $\underline{\gamma}=0$ impurities of the kind φ_1, φ_2 and φ_3. Every one of those impurities expands over χ with the same velocity, equal to the one of the uncut theory. Consider a first insertion of the operator $R_{\underline{\gamma}_1}$: it gives a non zero result only if the impurities have expanded up to $\underline{\gamma}_1$. $R_{\underline{\gamma}_1}$ annihilates everything but φ_3 . The result is (2 times) the same impurity state φ_3 - minus a state which contains a site φ_4 inside the impurity φ_3 .

It can be seen that φ_4 expands over φ_3 , at the same rate at which φ_3 expands over χ .

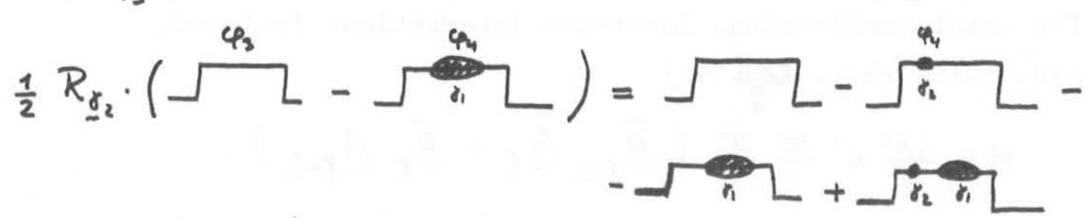
We will then see an expanding φ_3 impurity with a growing φ_4 hole inside.



The next insertion $R_{\underline{\gamma}_2}$ gives zero if it hits the outside χ region, otherwise it can :

a) hit the position of the φ_4 hole. In this case the state with the hole is annihilated and the state without hole is trasformed into a φ_3 -minus a state with a φ_4 hole at $\underline{\gamma}_2$.

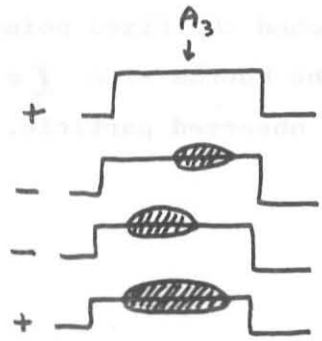
b) hit a φ_3 position. It results the situation of the figure



Suppose that next it comes the end, that is the sandwich with $\prod_{\gamma} \chi_\gamma e^{-\underline{\beta} \cdot A_{\gamma = B}}$. It is only A_3 which is relevant, so that let us consider $\prod_{\gamma} \chi_\gamma \, A_{3\,\underline{\gamma}=\underline{B}}$:

$$\chi A_3 \chi = 0 \qquad \chi A_3 \varphi_3 = \sqrt{2} \qquad \chi A_3 \varphi_4 = 1/\sqrt{2} .$$

In the case a) of before it is clear that A_3 must hit a position within the expanded hole φ_4 , giving in the matrix element $1/\sqrt{2}$. In the case b) of before we have the situation represented in the figure



(the blobs representing the expanded φ_4 holes). It is clear that the result is $\neq 0$ only if A_3 hits in a position in which two expanded holes overlap.

Cases a) and b) can be summarized requiring that: first, the insertions of $R_{\gamma_1}, R_{\gamma_2}$ must occur at positions within the expanded φ_3 , it does not matter if within or not possible φ_4 holes; second, the final operator $A_{3_{\underset{\sim}{\gamma}=\underset{\sim}{B}}}$ must occur at a position within the overlapping of the expanded φ_4 holes, previously created by the operators R . This result can be easily generalized and we get for the ℓ- particles inclusive distribution $T^{(\ell)}(\underset{\sim}{B}\, Y; \underset{\sim}{b}_i\, y_i)$ at fixed impact parameter $\underset{\sim}{B}$ of the colliding particles and $\underset{\sim}{b}_i$ of the observed ones

$$T^{(\ell)}(\underset{\sim}{B}\, Y; \underset{\sim}{b}_i\, y_i) = (u \cdot w) . c^{\ell} \prod_{j=1}^{\ell} \theta(v^2 y_j^2 - \underset{\sim}{b}_j^2)\,\theta(v^2 (Y-y_j)^2 - (\underset{\sim}{B}-\underset{\sim}{b}_j)^2) .$$

when u and w are constants — the coupling to the incoming particle lines.

The multiplicity moments m_ℓ are then obtained:

$$\sigma_T \cdot m_\ell = \int d^2\underset{\sim}{B} \prod_{j=1}^{\ell} d^2\underset{\sim}{b}_j\, dy_j\, T^{(\ell)} .$$

By rescaling $\underset{\sim}{b}_i = Y\underset{\sim}{x}_i$, $\underset{\sim}{B} = Y\underset{\sim}{x}$, $y_i = Y t_i$, we get $m_\ell \sim Y^{3\ell}$.

For the multiplicity m_1 this result is easily understood. Due to the AGK mechanism the only graph is

where the blobs contain P-interactions. As a consequence of those interactions the field evolves both at the top and the bottom towards the fixed point. The result is essentially non zero if the two regions in the impact parameter space where the field has reached the fixed point, coming from the top site $\gamma = B$ or the bottom site $\gamma = 0$, overlap with the impact parameter of the observed particle.

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