P. Caldirola and E. Recami:
THE CONCEPT OF TIME IN PHYSICS. -

Istituto Nazionale di Fisica Nucleare
Sezione di Catania

P. Caldirola[*] and E. Recami :
THE CONCEPT OF TIME IN PHYSICS

1. - INTRODUCTION.

The perception of "time flowing" is one of the fundamental experiences of our reflective mental-life. And the sensation of phenomenon "duration" seems to be primeval and not liable to explanation in terms of simpler words.

However, the sensorial experience data are themselves fit for being organized into a succession that we call "temporal".

From the first viewpoint, we can then recall that cases are known of people (suffering from Korsakoff-syndrone) who are unable to perform that ordering, and therefore practically prevented from any organized activity. We can add that the perception itself of "time" as a one--dimensional quantity is perhaps bound to the fact that our mind is equal to a sole series of elementary attention-acts.

From the second wiewpoint, on the contrary, we can say that, if macro-objects (and our own bodies) were made e. g. of few molecules, we should objectively be unable to attain to such a temporal ordering, since the statistical laws would fail, which assign a univocal arrow to time (as we'll see). To be more exact, in the case of few molecules it would be no more possible to describe the behaviour of physical bodies by means of macrophysical quantities (macro-observables) for the definition of which one must necessarily have a statistical element, which is the true responsible of the irreversible dynamical evolution of such bodies (and that consequently assign a univocal arrow to time).

Let us see what the contribution can be of the physical science to understanding time and trying to get over the famous statement by St. Augustine ("If you don't ask me, I know what is time; but, if you ask me, I don't know it any more").

## 2. - TIME AS A MEASURABLE QUANTITY.

As is well-known, in physics we deal essentially with measurable entities ("physical quantities"). Let us start from afar and, as a first aim, investigate the properties of space and of time , as they appear to our immediate intuition (i. e. , as they appear in our limited, local space-time region):

A) One realizes that only one instrument (the first one) is enough to build up space-geometry, or in other words to measure "lengths", "widths" and "heigths": i. e. , a rod (or better "the rod plus our arms"). A posteriori, the quantity measured by the "rod" will be called length, L: it will be our first fundamental quantity.

(*) Istituto di Fisica, Università di Milano, Milano (Italy).

And, owing to what is seen above, surface and volumes will have the physical dimensions $[S] = [L^2]$; $[V] = [L^3]$, where Maxwell's symbols have been used.

It is interesting that our brain, when ordering the "world" of our sensations, distributes the sources of those sensations (e. g. of the tactile ones) in a three-dimensional space, that can be easily referred to three Cartesian axes (Fig. 1). In particular, the vision mechanism itself projects "outside us" the image of a 3-dimensional space.
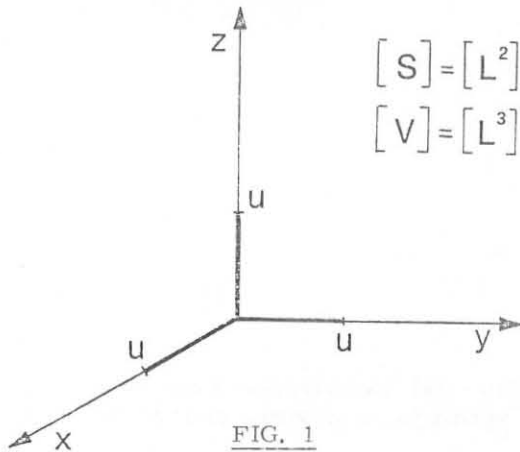
Incidentally, if at this point we ask that reality itself suggests to us a "natural measure-unit", which fixes the scale, then we should look for a (first) "universal constant" with the physical dimensions of a length. According to various Authors, this "fundamental length" has been searched for, either at cosmological[2] level, or at microphysical[3] level, or at ultra-microphysical[4] level. By the way, an acceptable view in such a respect (at least for theories at a certain stage of approximation) is that different "fundamental lengths" may exist, each one characterizing a certain "order" of phenomena.

$$[S] = [L^2]$$
$$[V] = [L^3]$$

FIG. 1

B) As a second step, let us come to what we are more interested in; i. e. , let us build up kinematics. We now need a second instrument, the clock, which measures a second fundamental quantity: time,T. In this respect, time is nothing but the physical quantity measured by the clock chosen.

Notice that time is tightly bound to space, since any clock requires a movement (in space).

We shall come back to the problem of choosing a clock.

Of course, no other instuments[3], besides rod and clock, are required to measuring the derived quantities, as the velocity v, where $[v] = [LT^{-1}]$ and so on.

Following the example passed on to us by our natural "representation mechanism" itself, - which developed during our past biological evolution, - we now will need a four-dimensional space in order to represent the space-time points (or kinematical events); the fourth axis being that of time, of course.

Since daily-experience brings us in touch with small speeds only, our brain found it more economical keeping the "geometrical", 3-dimensional space separate from the one-dimensional, temporal one. As we know, only Special Relativity - when analyzing a broader experience field- makes us realize the link between all the chronotopical coordinates, and adopt a pseudo- Euclidean space-time as the "background" for events (cf. Fig. 2). But it is beyond doubt that, should mankind some day come in daily contact with relativistic speeds, it will develop - during its future evolution - an intuitive, immediate "representation" of the Minkowski chronotopous (and a "future Kant" will assign to men the pseudo-Euclidean space-time as a "mental category").

In any case, in order to be able to sum length and times in space-time, as required e. g. by the generalized "theorem of Phythagoras", we need (see refs. 1, 4) in our theory a second universal constant c, the quantity c being for example a velocity, so that $[cT] = [L]$

$$[v] = [LT^{-1}]$$
$$ds^2 = dx^2 + dy^2 + dz^2 + c^2 d(it)^2$$

FIG. 2
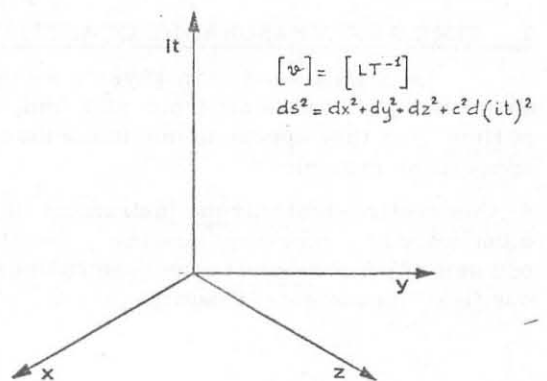
C) If we eventually wanted to build up dynamics, as our third step, then we should need a third instru
ment, able to measuring e. g. either forces or masses (a "dynamometr", for instance). As the
third fundamental quantity we could choose e. g. the mass, M. The most natural framework for
mechanical phenomena would then be a five-dimensional "space"[5], having for instance the fifth
axis related to rest-mass . And so on.

3. - CHOOSING THE CLOCK.

Let us go back to the abovementioned problem of choosing the clock. The concept of time is
tightly connected with the variability of our (external and internal) perceptions, and then with the be-
coming of reality. And time, as well as space, appears to be linked to the existing bodies, i. e. to
the properties of matter. For the physicsts, today, it is obvious that even time would not exist any
more if physical beings - and our own bodies, - subject to becoming, did not exist; because in such a
case it should not be possible to distinguish the "before" and the "after". According to C. D. Broad:
"Time consists in the "before/after" relation among the events".

We see, and we shall see, that today we are pretty far - as well known - from Newton's
view, who believed in the existence of an "absolute, true and mathematical time, which flows equal-
ly without regard to anything external"[6], as opposite to "relative, apparent and common time, whi -
ch is some sensible and external measure of the former, by the means of motion"[6, 7].

We mean that today physicists are still far from Newton's ideas, even if the belief in exi-
stence of privileged, "absolute" references-frames is growing up again in our days. The privileged
frames would be those frames (at rest with respect to the "heaven of fixed stars") in which the va-
rious radiations, coming from far, cosmic sources, result to be isotropic[*].

In fact, already in the second half of the XIX century, E. Mach objected that "absolute time"
does not possess any value, neither practical nor scientific, and consequently maintained that phy-
sics cannot refer to any other time but the "relative" one (measured by movements). All that, of
course, holds as well for space. And ought to be valid also at the metaphysical level, since even
philosophycally there is no reason for extrapolating, beyond sensible reality, a strictly phenomenal
experience as that one of "duration" and forawarding to time an independent existence.

The aptitude to temporally ordering the sense-data requires inside ourselves the capabili-
ty, - besides to feel the phenomena duration (as one of the most immediate aspect of physical reali-
ty),- also to compare durations one with the other, by means of a "biological, internal clock",
and eventually to fix sensations and ratioes in our memory. (For instance, even our heart is a
clock). It is better to speak, thus, not of time, but rather of "time intervals", or durations. Let
us remember that measuring a time interval means to find out its ratio to a "standard time inter-
val".

At this point, the problem for physics (which is inspired by inter-subjectivity) is measu-
ring durations in the way as little subjective and particular as possible.

From time immemorial, aim of science has been measuring time by adopting suitable
"standard durations", yielded by clocks as much indifferent to external, contingent influences as
possible.

It should be clear, however, that such a choice is intrinsically conventional: in fact, a prio-
ri it may be implemented by choosing any movement whatever (in general, periodic) as a clock,
provided that it lasts indefinitely. And of course, it has then no meaning at all to ask ourselves if
each periodic cycle of the chosen clock takes always the same amount of time! The main choice-
-criterium is that the clock allow to come to natural laws in a particularly simple form[7].
This criterium recalls that particular aspect of Science that Mach named "intelligent economy
of thought". Moreover, the clock must be easily reproducible, or easily available to everybody.

A priori, we might choose as clock the pulse-beating of a person P; except that such a clock

---

(*) In other words, the privileged frames would be the ones approximately at rest with respect to
the universe itself "as a whole".

would have the practical drawback of beeing not reproducible, and before all it would cause numberless physical processes to depend on the health condition of P, against the "principle of sufficient reason".

Just to avoid unnecessary complications in formulating nature laws, physicists have been continuously modifying their choice of the standard clock, passing e. g. from the Sun and planet motion to that one of the electromagnetic waves emitted by a suitably perturbed Cesium-133 atom[*] (see ref. 7).

At this point, let us observe that, if e. g. all movements in the universe did abruptly slow down one thousand times, then we could not notice any change (since also times shown by clock would have increased one thousand times). So that even speaking of such a scale change would not have any meaning, since the ratioes between different durations would be the only meaningful quantities.

Time, as well as space, does not have an existence independent of matter. We shall indeed see that identical clocks beat different "times" when subjected to different gravitational fields (whose source is just the matter).

4. - THE ARROW OF TIME. THE IRREVERSIBILITY IN PHYSICS[8].

The concept of irreversibility has appeared in modern science only in relatively recent times (XIX century). In fact, it was substantially absent in the work of the founding fathers of mechanics. This is easily understandable if one thinks that the main object of interest of mechanics until XIX century was the motion of heavenly bodies, in which the ideal conditions (system isolation, absence of friction, etc.) for a purely mechanical analysis are realized. In fact planetary motion exibit a periodic character without any element of irreversibility, at least on the time scale of human observation.

Only the development of thermodynamics of continuous systems, by the study of macroscopic bodies with the size of the objects of our daily life, introduced the concept of irreversibility[8] in scientifically precise terms. The second principle of thermodynamics led to the concept of entropy, introduced by Clausius as a "measure of transformability" of a system it, "entropy", increases in irreversible transformations (roughly speaking, the entropy is a measure of the disorder).

The evolution of physics and chemistry from the end of XIX century onwards has led successively to a deep investigation of the phenomena which occur at microscopic level, both atomic and subatomic. In such phenomena we find some elements of irreversibility, as the decay of the radiactive nuclei or the violation of the time reversal invariance in "weak" interactions. Such small irreversibility can perhaps suggest[9], - as we shall see - that in "weak interactions" even subatomic particles behave as composite objects and not as elementary bodies. Even if, however, those phenomena may not forward a clear tendency (which is a characteristics of macro-system) to well determined equilibrium state.

In any case, irreversibility remains the main aspect of the macroscopic phenomenology.

Hence, irreversibility may be considered as a typical and unavoidable characteristic of macroscopic phenomena. It may be analysed in a precise way as it follows[10]:

Ia) An isolated macroscopic body tends to a condition of equilibrium, in which it remains as long as it is undisturbed by external influences. In this approach to the equilibrium state, the system forgets almost completely its initial state. In fact, the final condition of equilibrium depends on the initial state only via very few thermodynamically-relevant parameters (the mean energy per unit volume, the mean density, etc.);

(*) For instance, in 1964 the XX General Conference on Weights and Measures defined as one second the time interval during which 9, 192, 631, 770 waves are emitted of the electromagnetic radiation produced in the transition between two certain energy levels of the iperfine structure of Cesium-133 fundamental state.

I<u>b</u>) A macroscopic body, not isolated but subjected to external influences (thermal bath, external forces, etc.) tends to a stationary state dependent on external actions (e. g. on the temperature of the bath);

II) The approach to the equilibrium state or to the stationary state <u>cannot be reversed in time</u>. Ruoghly speaking, a time evolution <u>cannot</u> be realized in which the state of the system at a time $t_1 > t_0$ is <u>more far</u> from equilibrium that the state of the system at time t_0.

Of course, fluctuations of the system around equilibrium are always possible. However, such fluctuations are in general extremely small on a macroscopic scale.

With the enunciation of the principles of thermodynamics, the problem naturally arose of connecting them with the laws of mechanics. In fact, it was generally eccepted the opinion that material bodies are formed by a <u>very large number</u> of elementary constituents ("atoms"). Then one should be able to deduce the whole phenomenology of the "heat" from the atomic motions, whi ch are quite naturally assumed to obey the same laws of dynamics as macroscopic bodies. This is the programme of the so-called kinetic theory, which Maxwell and Boltzmann (above all) began and developed.

From such a point of view, the mathematical interpretation of the "<u>Second Principle</u>" of thermodynamics was rather harder and originated a discussion which has been lasting till our days.

Boltzmann studied the dynamics of dilute gases and from the analysis of the binary colli-sions of particles deduced his <u>equation</u> describing the time-evolution of the joint distribution of position and velocity of the particles.

The most interesting consequence of the Boltzmann equation is the possibility of defin-ing a quantity which <u>always increases</u> during the time evolution of the system, for any initial condition (<u>H-theorem</u>) ; such a quantity can be immediately associated to the <u>entropy</u> of the sys-tem. From the H-theorem it follows that the distribution of position and velocity of the particles tends to an equilibrium configuration.

However the Boltzmann equation is not a consequence only of the laws of mechanics, but also of the hypothesis of "molecular chaos"("Stosszahlansatz"), which amounts to <u>assume</u> the most comple te uncorrelation between the positions and the velocities of any two particles before their scattering. The "molecular chaos" is purely statistical hypotesis, quite independent of the laws of dynamics.

Some objections were raised to Boltzmann's analysis, when the necessity of the presence of the <u>statistical</u> element in his deduction had not yet been recognized. The first objection consists in the so-called "Loschmidt's paradox": Loschmidt said that if the "entropy associated to Boltzmann equation increases in the direct motion, it must necessarily decrease in the inverse motion, which is in contrast with Boltzmann's H-theorem".

Another paradox of Boltzmann equation was indicated by Zermelo. In fact, a theorem due to Poincaré says that for a <u>spatially limited</u> system the motion of its <u>representative point</u> in phase--space or in suitable spaces is "almost periodical". Roughly speaking, this means that the system goes repeatedly back as near as we want to its initial state. The "period" characterizing this phenomena is astronomically long for a macroscopic system. However, this theorem contra-dicts the H-theorem according to which the system reaches an equilibrium state, in which it re-mains for ever.

Loschmidt's and Zermelo's paradoxes point out in the clearest way the "problem of irreversi bility", i. e. the problem of reconciling the <u>irreversible</u> character of <u>macroscopic</u> physics with the fundamental <u>reversibility</u> of dinamical laws ruling the motion of the <u>atomic</u> constituents of the system.

A convincing explanation of the mentioned paradoxes has been given only in recent times by developing a theory of macro-systems, starting from their atomic composition and on the ba-sis of the so-called generalized "<u>Master-equation</u> formalism". In such a formalism, the macro-copic description of a large system is performed through suitable macroscopic variables (named "coarse-grained" variables), obtained by averaging - over an infinite number of values- the quan-tities relative to the system single atoms <u>taken at different times</u>; so to introduce a time-interval Δt which is <u>long</u> at the <u>microphysical</u> level.

In other words, in the macroscopical description, "value of a physical quantity at time t" means actually a time-average over a great number of states so that - even if in its own evolution the system strictly approaches the initial microscopical state several times (as required by Poincaré theorem) - nevertheless the system itself will display an irreversible macroscopical behaviour: in the sense that the time-averages over the abovementioned interval Δt will always be independent of the initial conditions.

From the proceding considerations it is clear that the notion of the "time", at which the physical quantities of a system are given, is different according to whether we deal with a microscopical description of the system (i. e. by means of physical-quantities associated with single atoms) or we deal with a macroscopical description (i. e. by means of global-quantities associated with the whole system, so as for instance the "thermodynamical" quantities).

3. - ARROW OF TIME AND COSMOLOGY.

It is interesting to notice that the Poincaré's "recurrence theorem" - which yielded one of the well known paradoxes of statistical mechanics for ordinary bodies - might on the contrary be considered as showing a way for allowing our universe (which is probably a confined system[11]) to undergo successive phases of expansion and contraction, according to the well known "big-bang" theory.

We are particularly interested to this point, since the cosmic evolution forwards to us a second way for assigning an arrow to time[12].

Before going on, let us recall the following informations. The presently known forces in nature are four[11], here listed together with their relative strength and together with their characteristic time durations (the latters behaving as inverse power of the formers):

	Force Fields	Strengths	Durations (sec)
(1)	Strong interactions (short range)	1	10^{-23}
	electromagnetic " (long ")	10^{-4}	10^{-19}
	weak " (short ")	10^{-13}	10^{-10}
	gravitational " (long ")	10^{-40}	10^{+17}

Let us underline that the characteristic time of an object decaying through gravitational interactions is 10^{17} seconds, i. e. about 3×10^9 years.

Now, let us build up a very simple cosmological model[13], which apparently accords with the big-bang theory, and which will provide us with the second way to give time an arrow.

Let us consider the spatial part of our universe, and suppose it to be finite. Then, the simplest hypothesis is imaging it with constant, positive curvature, apart from possible local deformations. We are thus led to a 3-dimensional spherical hyper-surface, embedded in a four-dimensional (Euclidean), outer, "abstract" space, whose fourth Cartesian-axis we shall call the "abstract-coordinate" axis.

Our universe in then the "surface" of a hyper-ballon, which started with a radius $R_0 \simeq 0$, is expanding untill a maximal radius $\overline{R}$, and then will contract again to $R_0 \simeq 0$. If the galaxies are like dots on the ballon hyper-surface, then during universe expansion they will recede far away from each other. All the points of the universe are equivalent; the "center" of the universe belongs to the abstract space, and not to the universe itself.

Moreover, the fact that the older the detected galaxy-image is, the faster the galaxy appears to move, suggests that the speed $\dot{R}(t)$ is decreasing with time.

In Fig. 3 we sketch the possible trajectory of the light carrying on old image of galaxy A to observer P. Of course P will deem the light to come from A', since every observer will see everything "projected" onto his tangent space (extrapolation of his local, flat space). Cf. also ref. (2).

Since in its expansion the universe is slowed dawn by its own gravitation, roughly speaking we can assume its radius R to change with time in this way[*]

$$(2) \qquad R \simeq v_o t - \frac{1}{2} a t^2 ,$$

where the initial speed v_o will remain the maximal speed (in the abstract space), and can be assumed to be the light-speed, c_o, at that time. We shall assume moreover c_o to be not far from the present-time light-speed, c.

FIG. 3

galaxy apparent position from P

galaxy actual position

tangent space of P

$R(t_2)$

$R(t_1)$

espanding universe

The maximal radius $\bar{R} = R(\bar{t})$ will satisfy (if $c_o \sim c$: see eq. (9b) in the following) the relation:

$$(3) \qquad \dot{R} \simeq c_o - \bar{a} t \simeq c - \bar{a} t = 0 \implies \boxed{\bar{a} \approx \frac{c}{\bar{t}}} \quad ,$$

whence:

$$(4) \qquad \bar{R} \simeq c \bar{t} - \frac{1}{2} c \bar{t} \simeq \frac{1}{2} c \bar{t} \implies \bar{t} \simeq \frac{2\bar{R}}{c} ;$$

i. e.

$$\boxed{\bar{R} \simeq \frac{c \bar{t}}{2}} .$$

If the negative acceleration, - a, of R=R(t) in the abstract space is due - as previously said - to the gravitational effect of the universe-mass M on itself, then (see Ref. (13)):

$$(5) \qquad \bar{a} \simeq \frac{GM}{\bar{R}^2} ,$$

where G is the gravitation universal constant. But, since

$$\bar{a} \simeq \frac{c}{\bar{t}} \simeq \frac{1}{2} \frac{c^2}{\bar{R}} ,$$

then - in accord with Mach's Principle - it is[+]:

$$(6a) \qquad G \simeq \frac{1}{4} \frac{c^3 \bar{t}}{M} \simeq \frac{1}{2} \frac{c^2 \bar{R}}{M} ;$$

$$(6b) \qquad M \simeq \frac{1}{4} \frac{c^3 \bar{t}}{G} \simeq \frac{1}{2} \frac{c^2 \bar{R}}{G} ;$$

(*) Strictly speaking, the negative ecceleration, - a, is a function of time. Since the universe presently-evaluated age is just 10^{10} years, we can assume our universe to be not far from its maximal expansion ($R \lesssim \bar{R}$); and eq. (2) to hold at least in a certain range of values $R' \equiv R(t) = \bar{R} - \Delta R \leq R \leq \bar{R}$, where now $v_o = v_o(t') \approx c$. In this last case, we can now assume that $v_o(t')$ is not far from the present-time light-speed. See ref. (13).

(+) Relation (6a) does not mean that G grows with time, since $\bar{t}$ and $\bar{R}$ are (maximal-expansion) constants.
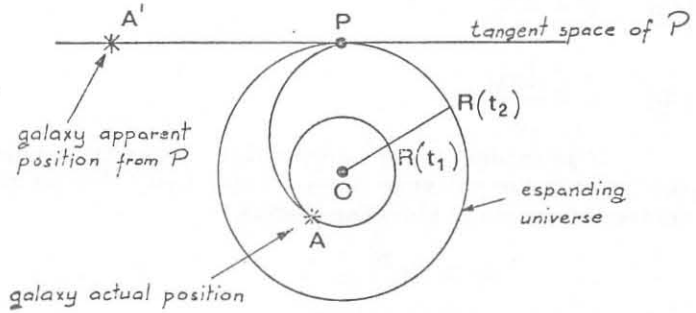
or, vice-versa,

(7a) $\bar{R} \simeq \dfrac{2GM}{c^2}$;

(7b) $\bar{t} \simeq \dfrac{4GM}{c^3}$.

Our model, though very rough, forwards acceptable results. For instance, using as input--datum only the universe mass , estimated[13] to be about $(10^{40})^2$ times the proton-mass, we can derive the maximal universe-radius

$$R \simeq 10^{26} \text{ m}$$

and the universe expansion time:

(7b) $t \simeq 10^{10}$ years,

in full accord - as we can see - with the estimates of modern astophysics for our **universe** and age[*]. Or, viceversa, by using as imput only universe expansion-time, $\bar{t}$, we can immediately derive not only the universe expansion-radius, $\bar{R}$, but also the correct universe mass, M. More-over, let us underline that eq. (7a) yields for the universal maximal radius, $\bar{R}$, exactly the universe "Schwartschild radius" $2G\bar{M}/c^2$, as though our universe were a black-hole (or better a confined "white-hole"[13]) in 3-dimensions.

Our previous cosmological model is interesting for us not only since it allows deriving e. g. the universe age (from universe mass and the value of gravitation constant) , but also for the following reason. Owing to the fact that, during expansion, R = R(t) is an increasing function of t, we could choose the axis R(t) as the axis of a certain "cosmological time" $\tau = R/c_o$;

We can thus interpret why we can stop our movement in space, but not our "movement" in time (i. e. along the "abstract" radial axis). Moreover, the cosmic expansion gives time an arrow. We have therefore met a second (cosmological) way - besides the statistical one - for assigning an arrow to time.

Those two differently defined "arrows" have been shown to coincide, and have been traced back to one and the same origin. But, here, we confine ourselves merely to quote refs. (12) about this question.

Our suggestion to consider the "abstract", fourth dimension of our model as a time-coordinate (except for a multiplicative constant with the physical dimensions of a speed) is supported by the following considerations:

a) Eq. (7a), as we noticed, reveals that in our four-dimensional model the universe maximal radius is equal to its "Schwarzschild radius" (as calculated, however, in a three-dimensional space): Therefore, the universe expansion contraction theory (which indeed requires even photons to go back to the initial singularity, after that expansion is finished) suggests for the universe a particular motion "inside a black-hole", where the expansion ("white-hole" phase) turns into collapse ("black-hole" phase) as soon as the maximal radius $\bar{R} = R_S$ is reached[13].

Even if some problems are of course left unsolved on this respect, nevertheless we can now recall that, inside a black-hole, tha radial coordinate does actually play the role of a time. In other words, during the expansion phase the universe might behave as the interior of a "white-hole", and during contraction as a black-hole's interior[13].

Therefore, our impossibility to stop our motion along the time axis would simply become the well-known impossibility of stopping the motion along the radial coordinate inside a black-hole.

b) Let us consider two different observers A and B.

_____

(*) We thus predict, incidentally, that the mean density in the cosmos is $\varrho \approx M/(c^3 t^3) \approx 10^{-26} \text{Kg/m}^3$ $\approx 10^{-29} \text{ g/cm}^3$.

If $\hat{BOA} = \beta$, then during the universe expansion they will appear to move each far away from the other along a straight line with the speed

(8) $u(t) \equiv \dfrac{d(\widehat{AB})}{dt} = \beta \dfrac{dR}{dt} = \beta (c_o - at)$,

which reads

(9$\underline{a}$) $u(t) = \beta c$

as soon as we $\underline{\text{adopt}}$ the physically self-clear identification:

(9$\underline{b}$) $c \equiv c_o - a t \approx \dot{R}$

Eq. (9b) tells us that the light-speed is the cosmos expansion-speed$^{(13)}$ in the abstract space. Since observer A considers only his own "tangent space" as $\underline{\text{physical}}^{(2)}$ (and necessarily "projects" onto it all his observations),he will find that his interval $dR_1 \equiv dR$ corresponds, in observer B frame, to $\lfloor d\tau_1 \equiv d\tau \rfloor$:

(10) $dR_2 = \dfrac{dR_1}{\cos\beta} = \dfrac{dR_1}{\sqrt{1-\sin^2\beta}}$; $d\tau_2 = \dfrac{d\tau_1}{\sqrt{1-\sin^2\beta}}$;

if, as usual, $\beta \ll 1$, then:

(11) $\begin{cases} d\tau_2 \approx \dfrac{d\tau_1}{\sqrt{1-\beta^2}} \; ; \\[3mm] \beta \equiv \dfrac{u(t)}{c} \end{cases}$ $(\beta \ll 1)$

Analogously, by defining

(12) $u(\tau) \equiv \dfrac{d(\widehat{AB})}{d\tau} = \beta \dfrac{dR}{d\tau} = \beta c_o$,

we $\underline{\text{would}}$ get

(13) $\begin{cases} d\tau_2 \dfrac{d\tau_1}{\sqrt{1-\beta^2}} \; ; \\[3mm] \beta \equiv \dfrac{u(\tau)}{c_o} . \end{cases}$ $(\beta \ll 1)$

In conclusion, by considering the "abstract coordinate" R/c_o as a "cosmological time" τ, we have derived that it actually transforms according to "Lorentz-transformations" when going from our frame to another frame in the relative motion due to cosmic expansion. If the angle β (see Fig. 4) is not small, then we are led to the "generalized Lorentz transformations"

(14) $\begin{cases} d\tau_2 = \dfrac{d\tau_1}{\sqrt{1-\sin^2\beta}} \quad , \\[3mm] \beta \equiv \dfrac{u(t)}{c} = \dfrac{u(\tau)}{c_o} . \end{cases}$

$A = A(t); \quad A' = A(t+dt)$
$B = B(t); \quad B' = B(t+dt)$

for passing from one galaxy to another, $\underline{\text{far}}$ galaxy.

FIG. 4

$R(t) = c_o t - \tfrac{1}{2} a t^2$
$\overrightarrow{BB'} = \overrightarrow{AA'} \doteq dR = c_o d\tau =$
$\quad = c_o dt - at\, dt$
$d\tau = \tfrac{1}{c_o}(c_o - at)dt$
$\beta = \dfrac{u}{c_o - at}$
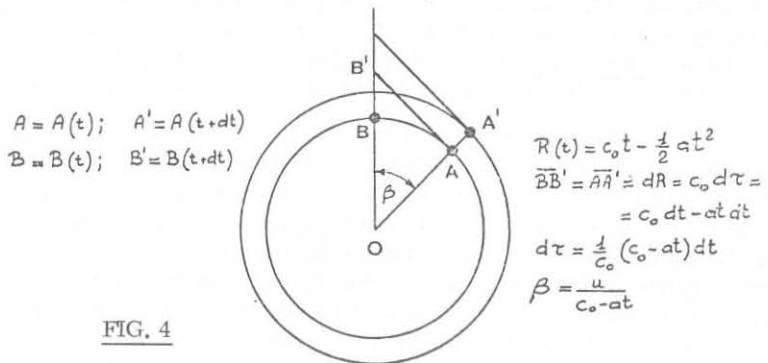
Of course, when reached the maximal expansion, the universe will start collapsing: the arrow of time will inverte and, in a certain sense, time will start flowing backwards to the starting point.

Before closing this section, let us emphasize that our simple model yields the "Hubble law" - as expected - with an "Hubble constant" close to the value usually derived by astrophysicists. Namely, from eqs. (8), (9):

$$u(t) \equiv u = \beta \cdot (c_O - at) = \beta \cdot c(t) \equiv \beta c,$$

and from the expression of the distance d(t) of two observers A, B

$$d(t) \equiv d = \beta \cdot R(t) \equiv \beta R,$$

one gets immediately the Hubble law:

(15a) $u = (\frac{c}{R}) d,$

with the Hubble constant

(15b) $H \equiv \frac{c}{R} \simeq \frac{2}{t} \simeq (10^{10} \text{ years})^{-1},$

as follows from eqs. (4).

6. - TIME-REVERSAL AND ANTI-MATTER.

If we well consider the becoming of physical reality, we realize that the "world" of our sensations initially refers - rather than to objects - to events, which need a temporal (besides spatial) localization. Nevertheless, the analysing activity of our 'mind decomposed the "space-time" distance between two events into a (3-dimensional) space-component and a (one-dimensional) time-component, mostly because of the fact that the structure itself of our senses and of our scientific devices "compels" the above four-dimensional "distance" to break in those two components.

Now, Special Relativity - following previous observations - has clarified that the space and time distances separating two events are not independent of the inertial observer considered, but vary according to his kinematical state.

Einstein, in 1905, overcame such a "relativity", by teaching us how to reconstruct the 4-dimensional distance, Δs, between the two events under consideration:

$$\Delta s = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2},$$

calculated by a generalization of the Phythagorean theorem, from the space and time measures taken by any inertial obesrver. The chronotopical "distance" Δs is an absolute quantity and does not depend any more on the observer, even if space separation and time separation between the two events do. (Incidentally, the Relativity theory should be better named "Absolutivity theory"!).

It is really this very important fact that definitely suggests to us the 4-dimensional "kinematical space" as a more suitable framework for event ordering, a framework where space and time are merged and interdependent in a physically and mathematically clear way. Following F. Severi, we want to stress that the relativity of space and of time measures is possibly born from the "violence" done by our measure instruments to the chronotopical distance between two events so that it splits in one part measurable by clock and in another part measurable by rod. On the contrary, if we trace back - by means of reasoning - to the actual four-dimensional "distance", we find again "absolute" quantities. This fact, let us say, is analogous to what happens in Quantum Mechanics: Since, there, our measuring apparata "force" the unintuitive sub-atomic entities to behave either as an (intuitive) particle, or as an (intuitive) wave, here Heisenberg's "Uncertainty correla-

tions" originate. But if we, through our "uncertain"data, attain to build up the "wave-function" mathematically describing the unintuitive subatomic entity under consideration, then we get the determinism restored: In fact,   once the state function ψ (t_o) is known, we can find out the state function ψ(t) at any successive time t (provided that the "forces" acting on the subatomic object are known).

Even today, the best background for analysing the fundamental aspects of time is still that of Special Relativity (SR). This theory is based on three fundamental postulates:

1. The Principle of Relativity: "Physical laws of Mechaincs and Electromagnetism are co-variant (=invariant in form) when going from an inertial observer   to another inertial observer". This postulate is inspired to the observation that all the inertial frames, in uniform straight relative-motion, should be equivalent[*], since no one of them ought to be pri-vileged.

We can firstly observe[+] the following. In order the check the validity of the Principle of relativity for a certain physical law, we must first state the transformation formula performing the transition from an inertial frame s to another frame s', endowed with uniform, straight mo-tion relative to s; and then verify that - under application of the transformation formulae - the physical law considered maintains the same form (i. e. , is "covariant" with respect to those trans formations). In SR the transformation laws, for the passage from s to s', are known to be the "Lorentz-transformations", which substituted the Galilei-transformations holding in classical physics.

To establish the "Lorentz-transformations" some assumptions are of course necessary. For instance:

a) hypothesis of light-speed invariance: light signals in vacuum travel rectilinearly, with the same, constant speed c $\simeq$ 2. 997930 x 10^8 m/s at any instant of time, in all direction, for all inertial obser-vers;

b) hypothesis of motion reciprocity: given two inertial observers s, s', if s sees s'to move with ve-locity $\vec{u}$, then s'see s to move with velocity - $\vec{u}$.

From the conceptual viewpoint, it appears that the passage from a reference frame s to ano-ther frame s' is an operation that should better be independent of the assumption of the "princi-ple of relativity". But most Authors indeed prefer to get the Lorentz transformations (and in general the relativistc kinematics)also from the "Principle of Relativity" itself. In such a case - which we shall adopt in the following - the two previous hypotheses a) and b) about light-speed invariance and motion reciprocity can rather be substituted by the second postulate alone:

2. - "Space-time in homogeneous and space isotropic": From the second postulate the con-servation laws of energy, impulse (=kinetical momentum) and angular-momentum follow, which are well verified by experience.

Notice that, in this context, the hypothesis of light-speed invariance is no more necessary, since it can be derived[14] from the Principle of Relativity and the second postulate.

As we have seen, the natural framework of SR (=Special Relativity) is the four-dimensional, pseudo-Euclidean space-time. In such a geometrical description, the "Lorentz transformations" (bringing the physical quantities referred to the reference frame s into the corresponding quantities, referred to a reference frame s' in uniform, rectlinear relative-motion with speed u) have the geometrical meaning depicted in Fig. 5, where only two dimensions are for simplicity considered.

(*) For a formal definition of "equivalence" of two reference frames, by using symbolic logic, se Ref. (9), where also the distinction between laws and descriptions is discussed.

(+) On this point, we are for the moment following the philosophy by A. Palatini: in Enciclopedia del-le matematiche elementari (Hoepli Pub. , Milano, 1950). This phylosophy is developed in detail , e. g. ,by P. Caldirola: in ref. (1), (Part II).

Finally let us underline the following, important point.

If we want - as we do - to <u>avoid</u> information transmission into the past, a <u>Third Postulate</u> is however necessary:
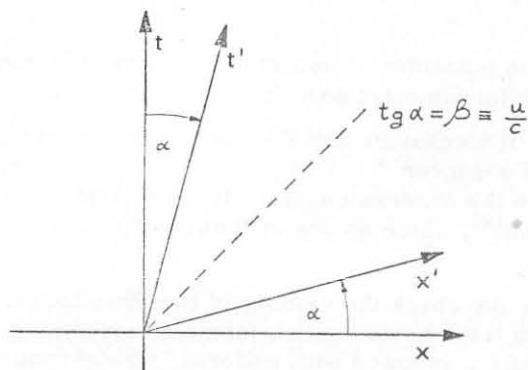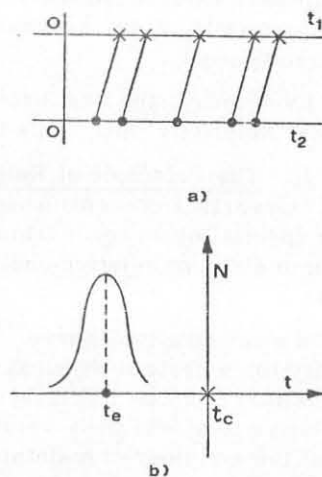

FIG. 5

FIG. 6

3. - <u>Principle of Retarded Causality</u>: For every observer, <u>causes</u> chronologically precede their own <u>effects</u> (for the definitions of "causes" and "effects", see the following). This "<u>Third Po-stulate</u> can be also called "<u>Principle of Reinterpretation</u>", for the reasons we shall see, and it will be shown to be equivalent to assuming that "negative-energy particles travelling forward in time <u>do not exist</u>; and physical signals are transported only by the objects that appear as carrying positive energy" (this last form being quite clear within information theory).

<u>The important point is that from "Postulate 3" existence of anti-matter can be (and will be) inferred.</u>

Before going on, let us notice incidentally that postulate 3 is a fundamental hypothesis of ours (in full accord with statistical termodynamics and with information theory), but a priori is <u>not</u> logically necessary[*][15]. In fact:(i) Let us suppose that a statistical correlation exists bet-ween two series of events, in the sense that e. g. each second-series event happens about 1 second before a first-series event (see Fig. 6). Such a statistical correlation will be called a "causal con-nection"; (ii) Let us now suppose that first-series events are the "independent" ones, in the sense that we make them occur, e. g., at instant chosen by consulting random-values tables (maybe pro-duced by a remote computer, having no reasonable relation with the events considered). Such events will be called the "causes"; (iii) The second-series events will be called the "dependent" ones in the causal correlation defined at point (i). They will be said to be the "<u>effects</u>"; (iv) One may therefore conclude, from the above definitions, that in this case effects <u>do</u> chronologically procede their own causes (Fig. 6). To conclude the present digression (<u>that has no relation at all with what follows!</u>), let us shed some light, on the possible nature of our difficulties in conceiv-ing effects chronologically preceding their causes, by reporting the following anectode[15], which does not involve present prejudices. For ancient Egyptians[15,9], who knew only the Nile its tri-butaries, which all flow from South to North, the meaning of the word "South" coincided with the one of "up-stream", and the meaning of the word "North" coincided with the one of "down-stream". When Egyptians discovered the Euphrates, which unfortunately happens to flow from North to South, they passed through such a crisis that it is mentioned in the stele of Tuthmosis I, which tells us about "<u>that inverted water that goes down-stream</u> (i. e. towards the North) in going up--stream"[15,9].

_____

(*) Notice explicitly that the following <u>digression</u> has nothing to do with what preceds and with what follows!

Let us now come back to our "Third Postulate", and consider Fig. 7 (where for simplicity a two-dimensional space-time id depicted). When we are in the position x = 0 at time t = 0 , we usually incline to consider as "existing" all the x-axis events. However, if another inertial observer, 0', moving along the positive x-axis, over us at x = 0, at the same time t = 0 he will tend to consider as "existing" all the x'-axis events. Therefore, if we want to be able to start discussing and exchanging informations with him, we must firstly consider all chronological events to "exist" (at least the ones outside the past/future zone of the light-cone). Then, nothing a priori prevents event A influencing event B backwards in time.
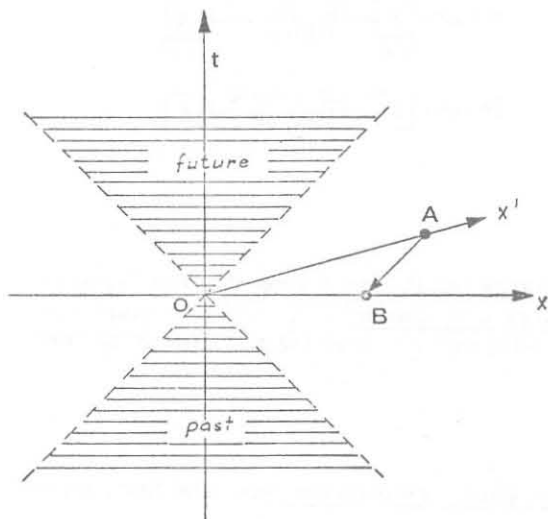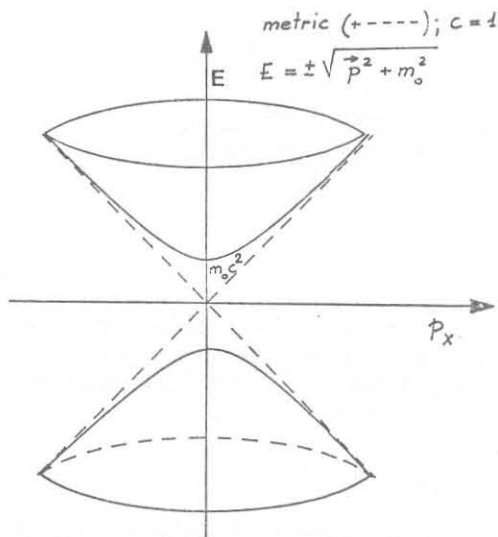

FIG. -7

FIG. - 8

Exactly to forbidding such a possibility, we introduce the "Third Postulate" (or "RIP" = Reinterpretation principle). One point is that, since we "explore" the Minkowski space-time going forward in time (along the direction determined by termodynamics and by cosmological evolution), any observer will see the event B as the first one and the event A as the last one. Moreover, it has been shown in ref. (9) that an object going backwards in time (Fig. 7) corresponds in the space dual of the chronotopical one, i. e. in the four-momentum space (Fig. 8), to an object carrying negative energy. And, vice-versa, changing the energy sign in one space corresponds to change the sign of time in the other (dual) space(*),(9).

Then, it is easy to convince ourselves that those two paradoxical occurrences (negative energy and motion backwards in time) will be reinterpreted in a quite orthodox way, by any observer, when they are - as they actually are - simultaneous.

Namely, let us suppose (Fig. 9) that a particle P, with negative energy and e. g. charge$^{(+)}$ - e, travelling backwards in time, is emitted by A at time t_1 and absorbed by B at time $t_2 < t_1$. Therefore, at time t_2, object A "loses" negative energy and charge - e i. e. gains negative energy and charge +e. And, at time $t_2 < t_1$, object B "gains" negative energy and charge -e, i. e. loses positive energy and charge +e. In fact, emission of negative quantity is equivalent to absorption of positive quantity, and vice-versa . The physical phenomenon here depicted will of course appear to be nothing but the exchange from B to A of a (standard) particle Q, with positive energy, charge +e, and travelling for-

---

(*) The same is true in Quantum field theory. For example, if

$$f(\vec{p}, E) = 1/(2\pi)^2 \int \tilde{f}(\vec{x}, t). \exp\left[i\, \vec{p}.\vec{x} - i\, E\, t\right]. d^4x , \text{ then:}$$

(16) $f(\vec{p}, -E)\; 1/(2\pi)^2 \int \tilde{f}(\vec{x}, -t). \exp\left[i\, \vec{p}.\vec{x} - i\, E\, t\right]. d^4x .$

(+) Here and in the following, "charge" means any additive charge. Cf. Refs. (9), (16).
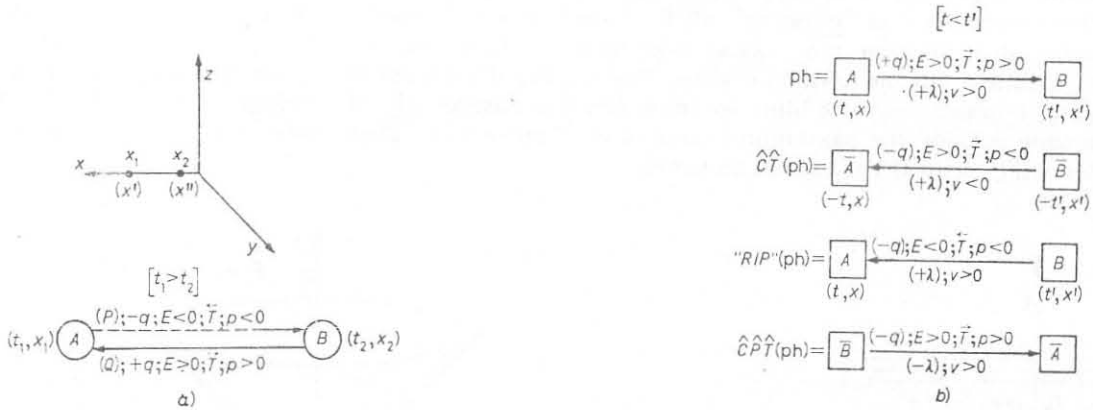
ward in time.

FIG. 9

We have, however, seen that Q has the opposite charge of P; this means that our "reinter pretation procedure" operates - among the other - a "charge conjugation", C. A closer inspection (see refs. 9, 16) of the "RIP" (reinterpretation principle) tells us that indeed Q will appear as the Antiparticle of P:

(17) $Q = \overline{P}$

We are meaning that the concept of antimatter is a purely relativistic one; and that, on the basis of the double sign (Fig. 8):

(18) $E = \pm \sqrt{\vec{p}^2 + m_o^2}$,   (c = 1)

the existence of antiparticles could have been predicted since 1905, exactly with the properties they actually showed to have when later discovered; provided that recourse to the "reinterpretation prin- ciple" had been made[*].

We therefore mean that the point of the lower hyperboloide-sheet in Fig. 8, - since they cor- respond not only to negative energy but also to motion backwards in time, - represent the kinematical states of the antiparticle $\overline{P}$ (of the particle P represented by the upper hyperboloid-sheet).

Notice explicitily that our "Third Postulate" ("RIP") not only asserts that we can reinterpret any negative energy object P (travelling backwards in time) in terms of its anti-object $\overline{P}$ going the apposite way (endoweed with positive energy and travelling forward in time), but also that we must apply that reinterpretation. In fact it requires, as we said, that "physical signals are transported only by objects travelling forward in time, or, equivalently, only by positive energy objects".

It is now clear that our Reinterpretation Principle, by eliminating any information transmis- sion into the past, implements the validity of the law of retarded causality ("causes happen before their own effects"). Let us observe that the reinterpretation procedure exchanges the rôles of source and detector, and that -with reference to Fig. 7 - every observer will deem B to be the source and A the detector of the (reinterpretet) antiobject $\overline{P}$.

Here we want to add the following: our "Third Postulate" allows of course solving also the para- doxes connected with the fact that many physical problems admit, besides standard, "retarted" solu- tions, also "advanced solutions": such "advanced solutions" merely represent antiparticles travelling the opposite way[9,16]. For instance, if Maxwell's equations admit solutions in terms of outgoing (polari

---

(*) The first interpretation of antiparticles as (lower hyperboloid) points moving backwards in time with negative energy was given by E. C. G. Stückelberg: Ref. (17), in 1941.

photons of helicity $\lambda = +1$, then they will admit also solutions in terms of <u>incoming</u> (polarized) photons[9] of helicity $\lambda = -1$.

All these considerations assume a more compact form when we allow room also for <u>Super-luminal</u> frames and for <u>tachyons,</u> so to consider all space-time rotations (for $0 < a < 2\pi$; see Fig. 5) as (<u>generalized</u>) Lorentz transformations. In this connection, let us explicitly emphasize that it is possible to extend Relativity so as to consider faster-than-light objects and reference frames <u>without violating the principle of retarded causality</u>[9,16]. Namely, it is enough to start from the aboveseen three Postulates, without assuming <u>a priori</u> $|v| \leqslant c$. The Reinterpretation Principle (i. e. our Third Postulate) will then be sufficient[9] to solve <u>all</u> causal paradoxes. Let us moreover remember that <u>also</u> the consideration of "extended Relativity"[9] - especially when applied to relativistic quantum mechanics - prompts us with a five-dimensional[18](at least) space-ce-time as the <u>best</u> background for Mechanics theories. These points will be possibly discussed on another occasion; presently, let us simply refer to refs. (9, 16); and mention the <u>following</u> point.

7. - ON TIME-REVERSAL (T), AND THE "CPT-THEOREM".

If we extrapolate usual Lorentz transformations (LT)for angles $|a| > 45^{\circ}$ (see Fig. 5, where for simplicity we considered the two-dimensional case), i. e. considering also tachyonic reference frames, we are led to a new group, G, of "Generalized Lorentz transformations" (GLT),which constituite <u>all</u> the rotations in Minkowski space-time for $0 \leqslant a \leqslant 360^{\circ}$. In particular, for $a = 180^{\circ}$, we get the total-inversion, $- \mathbb{1}$ (or strong-reflection). Requiring physical laws covariance under the generalized group G of Lorentz transformations implies in particular covariance of physical laws under the operation $\mathbb{1}$ By applying at this point the Third Postulate, it is possible to show (see refs. 18, 16, 9) that:

$$(19) \qquad - \mathbb{1} = CPT.$$

We can thus derive the <u>theorem</u> that: Physical laws, which do not violate the postulate of relativity, must be left invariant in form by changing time t into -t (<u>Time-Reversal</u>, T), any additive charge e into -e (<u>Charge-Conjugation,</u> C), and space-position $\vec{x}$ into $-\vec{x}$ (<u>Parity operation</u>, P). Actually, this theorem is already known within the relativistic <u>quantum</u>-theories (even if there derived under the restrictions of <u>local</u> field theories with the <u>usual</u> spin-statistics connection).

Now , in 1964, at the Brookhaven National Laboratory, while studyng the <u>weak</u> decays of the mesons K_L^0 into two pions, a phenomenon has been observed showing a violation of CP-invariance.

We have seen that Relativity requires covariance only under CPT, and <u>not</u> under T(in fact, the T-operation is <u>not</u> a space-time rotation). Thus, we ought <u>not</u> to wonder about that violation of time-reversal symmetry! However, let us <u>remember</u> that in classical and quantal macro-physics irreversibility is essentially brought in by the fact that any macroscopical body is composed of a very large number of micro-components. Therefore, if we - as a working hypothesis - extrapolate the validity of that statement also for quantum-microphysics, then the fact that some sub--nuclear reactions (known as <u>super-weak</u> interactions) are <u>not</u> time-reversible, even at their <u>elementary</u> level, could be explained in the following way. We could think that those "time-reversal" violating "elementary processes", - as the above mentioned decays of K_L^0 - are <u>actually non-elementary</u>[9]; where "elementary" means here "without parts" and "without inner structure" (or without <u>intervention</u> of inner structure in the considered process). On the contrary, in those processes the very <u>interior</u> of the so-called "elementary" particles might be concerned, so that the T-covariance can result to be violated. And such an "interior" should consist of so many constituents as required for explaining the experimentally observed <u>small</u> time-irreversibility of super-weak processes. Roughly speaking, when the number of constituents is of the order of Avogadro's Number (10^{23}), then the irreversibility is practically 100%; on the contrary, the constituents number in the case of super-weak interactions should be evaluable from the fact that the irreversibility is <u>only</u> of the order[19] of 0. 3%. Let us call "<u>partinos</u>" such internal constituents of subnuclear particles and possibly of <u>quarks</u> themselves. As you see, one might derive informations about the

structure of subnuclear particles just from considerations of time reversibility or irreversibility[9]. If any proton (and, more generally, any strongly interacting particle) is constituted by leptons (i. e. by weakly-interacting particles, as electrons and neutrinos), then the number N of constituent leptons, or antileptons, should satisfy the relation[*] ($s \equiv$ strong; $w \equiv$ weak):

$$(20) \qquad \sigma_s \sim |A_s|^2 \sim |N^2 A_w|^2 \sim N^4 |A_w|^2 \sim N^4 \sigma_w .$$

Since the ratio σ_s/σ_w is of the order of 10^{13}, as we saw before, then it would follow:

$$N \simeq 2000,$$

so that the average mass of any bound lepton would come to be about the electron mass; and we could have an elementary picture of the reason why the strength of "strong interactions" is 10^{13} times the strength of "weak interactions".

To support this picture, let us moreover remember[+] that in nuclear physics the reactions may be roughly divided into two classes: the fast processes (i. e. the direct reactions), in which essentially two particles - two nucleons - come into the game; and the delayed ones (i. e. the "compound-nucleus" reactions), to which all nucleus-particles take part. Analogously, in sub-nuclear physics we meet fast processes (the strong reactions, as we know, which involve a few internal "degrees of freedom"), and the slow ones (namely the weak reactions, as we saw), in which presumably a lot of internal degrees-of-freedom are involved[9].

Following a similar phylosophy, e. g., Kreuzer and Kuper succeeded in explaining the T--violating decay of the meson K_L^o (into two pions) as due to the interaction of the sub-nuclear particle K_L^o with a termodynamical bath of pions[20].

8. - ABOUT THE TWIN PARADOX.

Let us go back to our cosmological model (Figs.3, 4). It is clear that every body (every galaxy, let us say), besides the "cosmological motion" due to the expansion of the spherical hypersurface representing the universe, will show, with respect to an observer, also a "local motion" (i. e. a motion that the galaxy would keep even if we "froze" the universe expansion). Let us assume - following Special Relativity (SR) - that, if our galaxy is an inertial frame, then we cannot determine its "absolute" state of motion. In fact, in our model (as well as in the hypotheses of SR) there is no privileged reference point , and, chosen a particular inertial frame f_0, all the other inertial frames f will be equivalent[21], they being at rest or endowed with constant straight motion with respect to f_0. In particular, in all inertial frames the observers will "move in time" with the same "speed" (by the way, the maximal possible "speed").

However, we can realize if a frame is not inertial. For instance, in a non-inertial frame the observer will "move in time" (compared with the class of inertial frames) with a slower "speed"! This has been experimentally verified in the following way. The (experimentally checked)identity of inertial mass, entering Newton's fundamental law of Mechanics

$$\vec{F} = \frac{d}{dt} (m \vec{v}),$$

and of gravitational mass, entering e. g. the classical gravitation-law

$$F = \frac{G m M}{r^2},$$

(*) In relation (20), quantities σ and A are the typical "cross-sections" and "amplitudes", respectively.

(+) Cf. second ref. (31).

tells us that the gravitation field is equivalent to a (mechanical) acceleration-field. This is, roughly speaking, the "Equivalence principle", the starting point of General Relativity.

And in 1960 Pound and Rebka[22] experimentally verifield that two clocks run with different speeds when subjected to different gravitational fields. Precisely, Pound and Rebka - by making recourse to the Mössbauer effect - revealed that the gamma rays present different frequencies (cf. Fig. 10) when emitted by radioactive nuclei of the same element, but put at a different heigth in the Earth gravitational field.
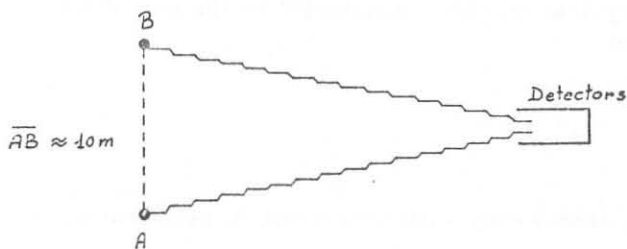

$\overline{AB} \approx 10\,m$

FIG. 10

In other words, a frame which undergoes accelerations is physically different from a frame which remained inertial; this difference manifests itself in clocks having different speeds in the two frames.

Let us explicity observe, however, that if we consider only inertial frames (always in uniform, straight relative-motion), then the time dilatation predicted by SR is obviously only a relative effect, in the sense that each observer will deem the other observer's clock to go slower than his own. In such a case, comparing the two clock speeds has no (absolute) meaning; in fact, the two observers are unable to meet each other twice, as necessary for comparing time-intervals "in an absolute way"[7].

On the contrary, let us consider two inertial observers $0_1, 0_2$, who meet each other once, at a certain point. Then, after some time, we make observer 0_2 to abandon his inertial motion, to accelerate and to go back towards 0_1, so that they meet a second time. If the two observers, on the first meeting, were the same age, then on the second meeting they will be differently aged: the younger one being the observer 0_2 who left the inertial motion; and the older one being the observer 0_1, who always remained inertial.

This is the so-called twin-paradox, definitely solved since long. As we have seen, there is no paradox. When two twins separate, and then meet again, they will result differently aged on the second meeting only if they have undergone physically different experiences: for example, if 0_1 remains in an inertial frame (and without gravitational fields acting on him), and on the contrary 0_2 accelerates (or is subjected to gravitational fields).

From the formal viewpoint, calculations can be performed both within SR (since observer 0_1 always remains inertial, and therefore may describe any process by SR), and within General Relativity (since observer 0_2 has to come back, and thus is an accelerating frame, well describable in the General relativistic framework) :

i) Within SR, it has been clearly shown, e. g. by Chevalier[23], the following. Let 0_1 remain inertial (and everything be referred to it); let moreover 0_2 overtake 0_1 at 0, fly with speed +u from 0 to A (see Fig. 11) and then - abruptly accelerating - fly back with speed -u from A to 0. The two time-durations, between the two meetings of 0_1, 0_2 at 0, measured in the two reference frames, will be connected each other through the formula

(21) $\Delta t_2 = \Delta t_1 \sqrt{1 - (\frac{u}{c})^2}$,

where

$\Delta t_2 > \Delta t_1$.

FIG. 11

The ratio between the two times (or time-speeds"), i. e. quantity
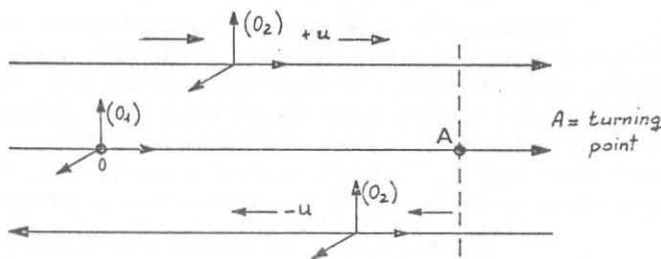
$\sqrt{1 - (\frac{u}{c})^2} \neq 1$,

is directly due to (and only to) the acceleration of 0_2 at A.

ii) Passing now to general relativity (GR), and following e. g. Fock[24], we can again repeat that on the two instants when 0_2 overtakes 0_1 (with speeds $+u$ and $-u$, respectively), their two clocks can be directly compared, practically without any mediation of lightsignals, i. e. without any relativistic effect. In other words, reading time on two clocks at the same space-time point is an absolute, "objective" fact. There follows that all correct procedures must forward the same values for Δt_1 and Δt_2. If we suppose, for simplicity, that we can calculate ds^2 by the so-called Newtonian approximation, then in the 0_1 rest-frame:

$$(22\underline{a}) \qquad \Delta t_1 = \int_O^T dt = T,$$

where O and T ate the two clock-meeting instants. Analogously, and still in the 0_1 rest-frame, it will be:

$$(22\underline{b}) \qquad \Delta t_2 = \int_0^T \left[1 - \frac{1}{c^2} (U + \frac{u^2}{2}) \right] dt ,$$

quantity U being - thanks to the "Principle of equivalence" - the gravitational potential corresponding to the acceleration supported by 0_2 when turning his motion at A. Even without performing the calculations, it is again immediately clear that

$$(23) \qquad \Delta t_2 > \Delta t_1 ,$$

so that time ran in 0_2 more slowly than in 0_1.

Of course, even performing calculations in the 0_2 rest-frame we'ld get the same result, since we would have nothing to do but evaluating the same final integral (merely expressed in terms of different variables!)[7, 24].

Before going on, let us mention that, - after the discovery that universe is filled with a "fossil" radiation, corresponding to the emission of a 3° K black-body, and possibly a remnant of the big-bang initial explosion, - the philosophy that absolute reference frames can exist started growing up again in popularity. For instance, the absolute frame can be defined as the one in which that 3°K radiation comes from all space isotropically. It seems that the Earth absolute speed might be[25] about 300 Km/s.

9. - IS TIME CONTINUOUS OR DISCRETE?

As far as we know, already during the old civilization of India the idea spread over that time is a quantized - i. e. discrete - quantity, constituted of indivisible "present moments". Subsequently, Greeks also about one century B. C. extended the atomistic theory to time, thus considered as discontinuous (every object was deemed to be a series of successive, instantaneous "existences"). Later, the Arabs formulated a theory according to which also space and time were made of "atoms". In recent times, e. g. Heisenberg seriously advocated existence of a fundamental length, λ , and therefore of a fundamental time (the chronon): $\tau = \lambda / c$.

The same philosophy we already expressed at the beginning, in our Sect. 2. Actually, present-time theoretical physicists show interest in the structure of "vacuum", or in a possible "lattice-structure" of space-time (following the modern, so-called "gauge theories", or after the "non-local" interactions and fields). Since we don't know if the fundamental length λ must be searched at the level either of 10^{28} cm (universe radius), or of 10^{-13} cm (electron radius), or of 10^{-33} cm (General relativity constant), then we don't know yet the possible value of the chronon. If we assumed $\lambda = 10^{-13}$ cm, then we'ld get $\tau = 10^{-24}$s, which is the characteristic time of the

fastect interactions, the strong ones. But probably the value of one "step" in the "time-lattice" is much smaller (if it exists). We previously mentioned another view, according to which different chronon-values are needed at different levels of theoretical analysis.

A different approach was followed by one of us[26] who - abandoning the "different equations", clearly suited only for the continuum theories - introduced a "finite-difference" equation for a subnuclear particle like the electron. In such a way the known paradoxical motion of a radiating, classical electron (pre-acceleration; run-away solutions etc.) are easily eliminated; and the muon can be explained as being an excited state of the electron[27]. These results are obtained without recourse to a true space-time lattice, but merely quantizing the electron trajectory (for instance, here the chronon may depend on the characteristics of the particle considered). For an extensive discussion of some peculiar aspects of this theory, limited to the case of the classical radiating electron, let us quote ref. (28). Cf. also the first ref.(13).

10. - TIME IN QUANTUM-MECHANICS.

Quantum theory taugth us - on the basis of a large experimental evidence - that the possible output of measurements on micro-systems are, generally speaking, discrete (i. e. quantized) values. Standard quantum mechanics (Q. M.), however, does not assume any discontinuity for time. Nevertheless, there are problems with the operator for time.

Let us remember that, in Q. M., a (mathematical) operator corresponds to any quantity-measurement. However for time difficulties are met, both in the relativistic (QFT) and non-relativistics (Q. M.) cases.

A) In the relativistic case[29] (Quantum field theory), the usual form for the space-time operator

$$(24) \qquad x_\mu = i\hbar \frac{\partial}{\partial p^\mu}$$

is non-Hermitian, i. e. admits non-real, but complex values. This fact itself can be interpreted in the following way; the real parts give the average space-time position, and the imaginary parts the particle-spread around that "central" point. It therefore seems that in Relativity point-like chronotopical positions are meaningless (in fact, e. g. pair creation precludes a point-like space-time localization). We are thus led to accept an extended-type localization[29] of sub-nuclear particles, both in space-time and in time. For instance, in the better known case of space-position, operator (24) can be split into its Hermitian part plus its anti-Hermitian part;

$$(25) \qquad \vec{x} \equiv i\hbar \frac{\partial}{\partial \vec{p}} = \frac{i\hbar}{2} \frac{\overleftrightarrow{\partial}_-}{\partial \vec{p}} + \frac{i\hbar}{2} \frac{\overleftrightarrow{\partial}_+}{\partial \vec{p}} ;$$

$$\psi^* \frac{\overleftrightarrow{\partial}_\pm \varphi}{\partial_\pm} \equiv \psi^* \frac{\partial \varphi}{\partial \vec{p}} \pm \varphi \frac{\partial \psi^*}{\partial \vec{p}} .$$

Well, the Hermitian part can be easly shown to be nothing but the usual Newton-Wigner operator[30]:

$$(26\underline{a}) \qquad \frac{i\hbar}{2} \frac{\overleftrightarrow{\partial}_-}{\partial \vec{p}} \equiv i \frac{\hbar \partial}{\partial \vec{p}} - \frac{i}{2} \frac{\hat{\vec{p}}}{\hat{p}^2 + m_0^2 c^2} ,$$

and the anti-Hermitian part will of course be:

$$(26\underline{b}) \qquad \frac{i\hbar}{2} \frac{\partial_+}{\partial p} = \frac{i}{2} \frac{\hat{\vec{p}}}{\hat{p}^2 + m_0^2 c^2}$$

In other words, we are led once more by Relativity not only to an extended-type localization in space and time, but even to a complex space-time.

B) In the non-relativistic case[31] the anti-Hermitian parts go to zero, and we _can_ deal with point-
-like localizations. For time operators, we are left with the forms $(\overleftarrow{\partial} \dot{=} \overleftrightarrow{\partial}_-)$:

$$(27) \qquad \dot{t}_1 = i\hbar \frac{\overleftarrow{\partial}}{\partial E} \; ; \quad \text{or:} \quad \widehat{t}_2 = -\frac{i\hbar}{2} \frac{\overleftrightarrow{\partial}}{\partial E} \quad .$$

Since the latter is not a standard operator (but a _bilinear_ "derivation"[29]), let us for simplicity analy-
ze the former. It is Hermitian and admits a priori real values. Nevertheless various difficulties
are still present, due to the fact that (in the non-relativistic case) the functions F on which the ope-
rator $\widehat{t} \equiv \widehat{t}_1$ acts[31] can be functions only of the _positive_-values of energy E; i. e. the F's are de-
fined only over $0 \dot{\le} E < \infty$. This is analogous to the problem of considering the momentum $p_x = -i\hbar \frac{\partial}{\partial x}$
for a particle confined in a semispace bound by a rigid wall, so that $c < x < \infty$. In this latter case,
as well as in the former, the operators under exam are _not_ self-adjoint (even if Hermitian), and
do _not_ admit true eigenfunctions and true eigenvalues.

For that reason, in Q. M. , physicists remained with an operator for space-position but
without a standard operator for time-position. However, we have seen that operator $\widehat{t}$ _can_ really
be adopted (eqs. (26)), since we are able to evaluate at least its _mean values_ over the "physical
states" (_wave-packets_) of the considered particle. In conclusion, by operator $\widehat{t}$ we _can_ derive the
well-known Heisenberg's "uncertainty correlation"

$$(28) \qquad \Delta t \; . \; \Delta E \gtrsim \hbar/2$$

connecting the error Δt necessarily made when measuring the time-position with the error ΔE
necessarily made when measuring the energy of a particle. In particular a good energy determi-
nation requires[31] a large error in time (practically, requires a long duration of the measure-
ment-interaction itself).

In conclusion, let us observe the following. Statistical termodynamics taught us that the
macro-system becoming happens in the direction of increasing disorder, i. e. of increasing en-
tropy, as a total result. But this law does not say enough to us. In fact, the formation in the uni-
verse of galaxies and then of stars, or the formation on the Earth of cristals, tell us that nature
has also a clear tendency to _locally_ produce "organization" and "regions of order".

Since from cristals we can ideally pass to viruses and to organic macro-molecules and
then to living bodies, which in a sense are regions of highly ordered structures, the abovemen-
tioned _tendency_ can possibly imply _new laws_, able to explain the phenomenon of life within a
"corpus" of laws explaining coherently all nature phenomena. All these problems seem connec-
ted with the arrow of time.

REFERENCES.

(1) - See P. Caldirola, "Istituzioni di Fisica Teorica" (Viscontea, Milano, 1965); E. Recami and
       C. Spitaleri, Scientia 110, 205 (1975); E. Recami and R. Mignani, Rivista Nuovo Cimento
       4, 259 (1974).
(2) - See e. g. G. Arcidiacono, "Relatività e Cosmologia"(Veschi, Roma, 1973).
(3) - See e. g. P. Caldirola, "Teoria Quantitistica Relativistica"(Viscontea, Milano, 1966);
       D. Y. Kim, Canad. Journ. Phys. 51, 1577 (1973); E. A. Raucher, Lett. Nuovo Cimento 4, 757
       (1972); T. G. Pavlopoulos, Nuovo Cimento 60B, 93 (1969).
(4) - See e. g. D. I. Blokhintsev, Nuovo Cimento 16, 382 (1960); E. A. Raucher, Lett. Nuovo Cimento
       7, 316 (1973).
(5) - E. Recami and C. Spitaleri, Scientia 110, 204 (1975); G. Arcidiacono, ref. (2), pag. 233;
       Th. Kaluza, Sitzunsber. Preuss. Akad. Wiss. Berlin, Math. Phys. Kl. (1921), p. 966; O. Klein,
       Zeits Phys. 37, 895 (1926); A. Einstein and P. Bergmann, Ann. Math. 39, 683 (1938):
       E. Leibowitz and N. Rosen, General Relat. Gravit. 6, 449 (1973). For the description of a
       particle with an arbitrary number of "degrees of freedom" in a n-dimensional geometrical
       variety, see P. Caldirola, Rend. Acc. d'Italia 7, 19 (1939). For the interpretation of the
       fifth axis, see P. Caldirola, Nuovo Cimento 19, 25 (1942); E. Recami and G. Ziino,
       Nuovo Cimento A33, 205 (1976);
       R. L. Ingraham, Nuovo Cimento 12, 825 (1954); A. Maduemezia, Report IC/68/78 (ICTP,
       Trieste, 1968); J. D. Edmonds jr., Found of Phys.
(6) - M. Paty, Scientia, 107, 1027 (1972).
(7) - P. Caldirola and E. Recami, Giornale di Fisica 9,163 (1968).
(8) - P. Caldirola, "Dalla Micro alla Macrofisica" (Mondadori, Milano 1975), Cap. III, p. 121.
(9) - E. Recami and R. Mignani, Rivista Nuovo Cimento 4, 209,398 (1974).
(10) - P. Caldirola and L. Lugiato, "Irreversibility in Physics" Progress in Scientific Culture
       (2nd issue), in press; see also L. Rosenfeld, in "Irreversibility in the many body Problem"
       (Plenum Press; New York, N. Y.; 1972).
(11) - See e. g. E. Recami and P. Castorina, Lett. Nuovo Cimento: 15, 347 (1976).
(12) - See e. g. R. Mirman, Found of Phys. 5, 491 (1975); D. Layzer, Scient. American
       T. Gold and D. L. Schumacher (editors), "The Nature of Time"(Cornell Univ. Ithaca; 1967);
       T. Gold, in "Recent Developments in General Relativity" (Pergamon Press, 1962); B. Galor,
       Science, 176, 11 (1972).
(13) - P. Caldirola, M. Pavsic and R. Recami, (in preparation); see also C. W. Misner, K. S. Thorne,
       and J. A. Wheeler, "Gravitation" (Freeman, 1973); D. M. Eardley, Phys. Rev. Letters 33
       442 (1974).
(14) - W. V. Ignatowski, Phys. Zeits 21, 972 (1910); P. Frank and H. Rothe, Ann. der Phys. 34, 825
       (1911); E. Hahn, Arch. Math. Phys. 21, 1 (1913).
(15) - R. Newton, Phys. Rev. 162, 1274 (1967); P. L. Csonka, Nucl. Phys. 21B, 436 (1970); E. Recami,
       in "Annuario 73, Enciclopedia EST-Mondadori" (Mondadori, 1973),page 85; P. L. Csonka, Phys.
       Rev. 180, 1266 (1969).
(16) - R. Mignani and E. Recami, Nuovo Cimento A24 438 (1974); Lett. Nuovo Cimento 11, 421 (1974);
       Int. Journ. Theor. Phys. 12, 299 (1975).
(17) - E. C. G. Stückelberg, Helv. Phys. Acta 14, 584 (1941). See also P. A. M. Dirac, Proc. Roy Soc.
       A. 126 360 (1930); R. P. Feynman, Phys. Rev. 76, 749, 769 (1949); P. Caldirola, Ref. (26),
       Sect. 11; O. M. P. Bilaniuk, V. K. Deshpande and EC. G. Sudarshan, Amer. Journ. Phys. 30, 718
       (1962); E. Recami, Giornale di Fisica 10, 195 (1969), ref. (15), and ref. (9).
(18) - E. Recami and G. Ziino, Nuovo Cimento A33, 205 (1976).
(19) - G. Steinberger (private communication)
(20) - H. J. Kreuzer and C. G. Kuper, Journ. of Phys. G2, 9 (1976).
(21) - For a definition by symbolic-logic of equivalence, see e. g. E. Fabri, Nuovo Cimento 14, 1130
       (1959); A. Agodi (unpublished); E. Recami and R. Mignani, ref. (9). Pag. 249.
(22) - R. V. Pound and G. A. Rebka, Phys. Rev. Lett. 4, 337 (1960).
(23) - J. Chevalier, Helv. Phys. Acta 46, 161 (1973).
(24) - V. Fock "Theory of Space, Time and Gravitation" (London, 1959).
(25) - D.W. Sciama, Preprint IC/74/10 (ICTP, Trieste, 1974).
(26) - P. Caldirola, Suppl. Nuovo Cimento 3,297 (1956); Lett. Nuovo Cimento 15, 486 (1976).

(27) - P. Caldirola, Lett. Nuovo Cimento 16, 151 (1976);17, 461 (1976);18, 465 (1977).

(28) - H. Arzelies, "Rayonnement et Dynamique du Corpuscole Chargé fortment accéleré"
(Gauthier-Villars, Paris, 1966), III part. chap. XVIII; particularly pags. 387-391)

(29) - V. S. Olkovsky and E. Recami, Lett. Nuovo Cimento 4, 1165 (1970) and references
therein; E. Recami, Acc. Naz. Lincei, Rend. Sc. 49, 77 (1970).

(30) - M. Baldo and E. Recami, Lett. Nuovo Cimento 2, 643 (1969).

(31) - V. S. Olkhovsky, E. Recami and A. I. Gerasimchuk, Nuovo Cimento A22, 263 (1973);
V. S. Olkhovsky and E. Recami, Nuovo Cimento 53A, 610 (1968); E. Recami: in "The Uncerta-
inty Principle and Foundamentions of Quantum Mechanics"; W. C. Price and S. S. Chissik Edi-
tors (J. Wiley; London, 1977), p. 21 .