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BLACK-HOLES AND TACHYONS(o).

ABSTRACT. - Given a Schwarzschild black-hole, we choose as reference-frame the frames σ at rest with respect to the Schwarschild metric. In these locally non-inertial frames, a free falling body is shown to reach the speed of light on the horizon and then to travel faster-than-light inside the horizon. The usual Szekeres-kruskal (SK) coordinates represent themselves frames that (with respect to the frames σ) travel at subluminal speed outside, at luminal speed on and at Superluminal speed inside the horizon (so that SK frames always describe any free falling body as a standard, slower-than-light object). At last, black-holes are shown to be possible sources of tachyons. Notice that the philosophy adopted in this paper is not the standard one of general relativity, but rather the one of "Extended Relativity"[20].

1. - BLACK-HOLES.

It is well known that Einstein eqs.[1 ÷ 5], in the case of a spherically symmetric mass distribution, allow for the exact solution in vacuum known as Schwarzschild solution[6]. The Schwarzschild metric reads (c = 1):

$$ds^2 = (1 - \frac{2GM}{r}) dt^2 - (1 - \frac{2GM}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{1}$$

which, in Cartesian coordinates, writes[2] ($G = c = 1$; $x_o \equiv ct$):

$$ds^2 \equiv g_{\mu\nu} \, dx^\mu \, dx^\nu = (1 - \frac{2M}{r}) dx_o^2 - \left[\delta_{ij} + \frac{2M}{r-2M} \cdot \frac{x_i x_j}{r^2}\right] dx^i dx^j , \quad \left[i, j = 1,2,3\right]. \tag{2}$$

For $r \longrightarrow \infty$, eqs. (1), (2) yield the flat metric.

The sphere with the Laplace-Schwarzchild radius[7]

$$R = 2M \qquad (c = G = 1) \tag{3}$$

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in the horizon[1÷8] of the spherically-symmetric mass M. Incidentally, any black-hole solution can be considered a "soliton", if we call soliton any confined-in-space solution of non-linear equations[9]. Moreover, notice that the use of Schwarzschild coordinates does not play any essential rôle in this work.

Let us suppose mass M to have a radius $r_o < R$. It is quite noticeable that solution (1)-(2)[10] holds for both $r < 2M$ and $r > 2M$. Sometime it is claimed that the coefficient g_{oo} of dt^2 must be positive since often eqs. (1, 2) are derived by putting $g_{oo} \equiv e^{\nu}$; but that position and that procedure are by no means necessary, and one can rather derive eqs. (1, 2) by following ref. (10).

At this point, eq. (1) or rather eq. (2) tells us that, for $r_o < r < 2M$, we should deal (after having e.g. chosen the metric signature (+ ---))with negative ds^2. To avoid this task, in the spherically symmetric cases usually recourse is made (in two dimensions) to the (unique) analytic and locally inextendible extension of the Schwarzschild solution[11] - which is called Szekeres-Kruskal extension[2,4,5] - so to have a positive ds^2 both for $r > 2M$ and for $r < 2M$.

In other words, in order to escape dealing with space-like intervals, they usually[12÷16] introduce two different sets of Szekeres-Kruskal coordinates, one for $r > 2M$ and one for $r < 2M$

$$(4)\qquad \begin{cases} u \equiv u_> \equiv (\frac{r}{2M} - 1)^{1/2}.\ \exp(\frac{r}{4M})\cdot \mathrm{Cosh}\,(\frac{t}{4M}); \\ v \equiv v_> \equiv (\frac{r}{2M} - 1)^{1/2}.\ \exp(\frac{r}{4M})\cdot \mathrm{Sinh}\,(\frac{t}{4M}); \end{cases} \qquad \text{for } r > 2M$$

and

$$(5)\qquad \begin{cases} u \equiv u_< \equiv (1 - \frac{r}{2M})^{1/2}.\ \exp(\frac{r}{4M})\,.\ \mathrm{Sinh}\,(\frac{t}{4M}); \\ v \equiv v_< \equiv (1 - \frac{r}{2M})^{1/2}.\ \exp(\frac{r}{4M})\,.\ \mathrm{Cosh}\,(\frac{t}{4M}), \end{cases} \qquad \text{for } r < 2M.$$

Let us notice that the change of coordinates (4) $\longrightarrow$ (5) means a change of reference-frame when passing from the "ouside" to the "inside" region. We shall see the physical meaning of such a change of observers. Since now, let us however observe that eqs. (4, 5) mean interchanging the rôles of u and v when going inside the horizon, in the sense that:

$$(6)\qquad \begin{cases} u_< (r) = i v_> (r) \\ v_< (r) = i u_> (r), \end{cases}$$

so that:

$$(6')\qquad v_<^2 - u_<^2 = -(v_>^2 - u_>^2) = u_>^2 - v_>^2 .$$

Or rather, by defining

$$u_{||} \equiv \sqrt{\left|\frac{r}{2M} - 1\right|}\ .\ \exp(r/4M).\ \mathrm{Cosh}\,(t/4M);$$

$$u_{||} \equiv \sqrt{\left|\frac{r}{2M} - 1\right|}\ .\ \exp(r/4M).\ \mathrm{Sinh}\,(t/4M),$$

one gets:

$$\begin{cases} u_<(r) \equiv v_{||}(r); \\ v_<(r) \equiv u_{||}(r); \end{cases} \qquad \begin{cases} u_>(r) \equiv u_{||}(r); \\ v_>(r) \equiv v_{||}(r). \end{cases}$$

Moreover, the following one-to-one correspondence can be set ($R \equiv 2MG$):

$$(7)\qquad \begin{cases} u_<(R/r) \longleftrightarrow v_>(r/R); \\ v_<(R/r) \longleftrightarrow u_>(r/R), \end{cases} \qquad (\ r/R < 1\ ;\ R/r > 1\ ),$$

in the sense that[17] (with units such c = 2MG = 1):

$$
(8)\qquad \begin{aligned} v_{<}(1/r) &= \mathscr{S}\left[u_{>}(r)\right] \\ u_{<}(1/r) &= \mathscr{S}\left[v_{<}(r)\right] \end{aligned} \qquad (r \geqslant 1;\quad 1/r \leqslant 1),
$$

where $\mathscr{S}$ is the operator changing $r \longrightarrow 1/r$ and multiplying the whole function by the imaginary unit. Similar considerations could be made for other coordinates, as Finkelstein's[14].

In order to conclude this Section about the standard view on black-holes, let us remember that usually the horizon is considered only a coordinate dependent singularity (i. e. it is not considered a true singularity), since with coordinates eqs. (4)-(5) for $r = 2M$ we have no divergences. What we can say is that, e. g. for a free falling observer, the Riemann tensor components do not diverge on the horizon[5].

2. - BLACK-HOLE INTERIOR, AND KINEMATICS OF A FREE FALLING BODY.

Let us now consider the speed of a free falling body in the field of a black-hole as a function of its distance r from the center of the mass-distribution (that for simplicity wa shall assume to be practically concentrated in the space-coordinate origin 0). The speed dr/dt determined by the time t of a distant observer has no direct significance, as shown in ref. (18a). Let us therefore choose frames measuring a speed with direct physical meaning[3]: the best and simplest ones appear to be those frames f at rest (r, θ, ϕ constant) at the point which the particle is passing, and with the time coordinate orthogonal to the hyperplane of the space-coordinates.

Let us describe the speed $V \equiv dx/d\tau$ of the free falling body - for radial motion - from the frames f, following ref. (18b). Remember that, for instance, it is just the speed V that enters into the expression for the locally measured energy of the particle, $E \equiv E_{loc} = \sqrt{\delta_{oo}}\, m_o c^2 (1-V^2/c^2)^{-1/2}$. Notice that $x \neq x^1$, since x, τ are (orthogonal) coordinates in the local frame f. In ref. (18b) it is shown that

$$
(9)\qquad V \equiv \frac{dx}{d\tau} = \left[1 - \frac{1-2M/r}{1-2M/r_o}\right]^{1/2} ,
$$

where r_o is the radius at which the fall beings (i. e. $dr/dt = 0$).

In the case when $r_o = \infty$ (particle droppded from rest at $r = \infty$), then eq. (9) reduces to

$$
(10)\qquad V \equiv \frac{dx}{d\tau} = \sqrt{\frac{2M}{r}} \;;\; V^2 = \frac{2M}{r} \qquad (c = G = 1) ,
$$

which coincides with the classical, Newtonian expression. By the way, eq. (10) allows setting a one-to-one correspondence(*) between speed-sqares V^2 and radia r:

$$
(11)\qquad V^2 = 1/r \qquad\qquad (c = 2MG = 1),
$$

where $V = 0 \longleftrightarrow r = \infty$; $V = 1 \longleftrightarrow r = 1$; $V = \infty \longleftrightarrow r = 0$. Why we wrote also the last correspondence (i. e. we considered also $r < 1$ and $v > 1$) will be soon clear.

From eq. (9) it is immediate to derive that, for $r \longrightarrow 2M$, speed $V \longrightarrow c$. We can then ask ourselves which speed the free-falling body will have for $r < 2M$. Since eq. (9) seems to predict faster-than-light-speeds[19, 20] inside the horizon, let us check more carefully such a prediction.

(*) When the spherically symmetric mass-distribution is concentrated in the origin 0.

In order to see what happens when crossing the event-horizon, let us consider the following. Our previos frames f belong to the class Σ of the frames σ introduced e. g. by Saltzman & Saltzman (see ref. 21) (who called them frames S), where frames σ are defined as the coordinate-systems at rest with respect to the Schwarzschild metric, or rather as the coordinate-systems in which the Schwarzschild metric tensor is time-independent :

$$\frac{\partial g_{\mu\nu}}{\partial x^o} = 0 \tag{12}$$

Class Σ is therefore nothing but the set of the local, stationary observers[3] σ ; incidentally, we can call Σ also the set of transformations that relate the σ-frames one to the other. Let us remember that the total energy E for a test-particle motion is a constant of the motion only in σ-frames; moreover, the very value of E is σ-invariant (i. e. is the same in any σ-frames[21, 22] with $dT \perp dL$).

For simplicity, let us <u>confine</u> ourselves to the σ-frames with time element dT <u>orthogonal</u> to the hyperplane of the space elements dL. It can be shown[21, 23] that the quantities dT^2 and dL^2 <u>are</u> σ-<u>invariant</u>[23]:

$$dL^2 = \left(\frac{g_{oi}g_{oj}}{g_{oo}} - g_{ij}\right) dx^i dx^j = dx^2; \qquad (i, j, = 1, 2, 3); \tag{13}$$

$$dT^2 = g_{oo}\left(dx^o + \frac{g_{oi}dx^i}{g_{oo}}\right)^2 = d\tau^2. \tag{14}$$

Actually, since

$$E = m_o g_{oo} \frac{dx^o}{ds} = m_o \sqrt{g_{oo}} \frac{dT}{ds} ,$$

it is enough to normalize g_{oo} to 1 spatial infinity for getting that g_{oo} is a σ-invariant quantity, in the sense that:

$$g'_{oo}(x'^i) = g_{oo}(x^i) \tag{15}$$

where x^i and $x'^i = x'^i(x^i)$ are space-coordinates (of the same point) in any two σ-frames.

Therefore, also the test-particle speed

$$\frac{dL}{dT} = \frac{dx}{d\tau} \equiv V$$

<u>is</u> σ-<u>invariant</u>. From eqs. (13) and (14) it follows of course that

$$dT^2 - dL^2 = ds^2, \tag{16}$$

so that the proper-time element and energy E can be written:

$$ds^2 = d\tau^2 (1 - V^2); \tag{17}$$

$$E = \frac{m_o \sqrt{g_{oo}}}{\sqrt{1 - V^2}} \tag{18}$$

Eqs. (17, 18) yield:

(19) $$V^2 = 1 - \frac{(ds)^2}{(d\tau)^2} = 1 - (\frac{m_o}{E})^2 g_{oo}$$

The σ-invariant speed of light, for $m_o/E \longrightarrow 0$, is $V_1 = 1$

For our purposes, it is interesting to observe that, for any free-falling body, we get again (so as from eq. (9)) that:

(20) $$V \longrightarrow 1 \ \underline{\text{for}} \ \longrightarrow 0 \ \text{(i. e., as } r \longrightarrow 2M);$$

and again, eq. (19) seems to predict faster-than-light speeds for negative ds^2, i. e. for negative g_{oo} (or, further, for $r < 2M$). Notice that our starting philosophy is not the standard one of "General Relativity", but rather the one of "Extended Relativity"[20].

3. - ABOUT TACHYONS (AND NEGATIVE ds^2)

Due to the predictions got from eqs. (9) and (19), let us spend a few words on the possibility of introducing tachyons in relativity.

Let us start from the usual postulate[20] of (Special) Relativity, without assuming however $V^2 < c^2$.

Due to the fact that the light-speed is still invariant in the "extended relativity"[20], given a certain frame s_o the class (I) of the inertial frames can be exhaustively divided into the two non-intersecting classes (s), (S) of subluminal ($U^2 < c^2$) frames, respectively.

The (Generalized) Lorentz transformations (GLT) connecting two frames I_1, I_2 have been shown to be linear and such to preserve the quadratic forms excpet for the spin[20, 24]

(21) $$ds'^2 \equiv g'_{\mu\nu} \ x'^{\mu} \ x'^{\nu} = \pm \ g_{\mu\nu} \ x^{\mu} \ x^{\nu} \equiv \pm \ ds^2 \qquad (\underline{\text{for}} \quad U^2 \lessgtr c^2).$$

In fact, a usual object (bradyon) with respect to a frame s will appear as a tachyonic object with respect to any frame S, and vice-versa; so that under a Superluminal Lorentz transformation (SLT), time-like quantities must transform into space-like quantities, and vice-versa. By the way, the "equivalence principle" still holds since (even when in presence of both bradyons, B and tachyons, T), any particle will follow the same trajectory in a given gravitational field independently of its proper--mass: such a trajectory depending only on the particle (B or T) four-momentum [20].

If we confine ourselves for simplicity to Special Relativity, then the group G of GLT's is (where we represent by the 4 x4 matrices $\Lambda_<$ the usual, proper, orthochronous Lorentz transformations (LT))[20]:

(22) $$G \equiv (\Lambda_<) \cup (-\Lambda_<) \cup (+i\Lambda_>) \cup (-i\Lambda_>),$$

where $\Lambda_< \equiv \Lambda\ (\beta^2 < 1)$ and $\Lambda_> \equiv \Lambda\ (\beta^2 < 1)$, with $\beta \equiv U/c$. In other words, G is the extension[25] ε of the usual, proper, orthochronous Lorentz group $\mathscr{L}$ by the "discrete"[26] operations CPT and K:

(22bis) $$G = \varepsilon(\mathscr{L}, \text{CPT} \quad K)$$

where K is the operator[27] changing $\beta \longrightarrow 1/\beta$ and multiplying the whole transformation by the imaginary unit. Eqs. (22), (22bis) tell us that the SLT's are connected to the (both orthochronous and non-orthochronous) LT's along the same direction as follows[20]:

(23) $$\text{SLT}\ (1/\beta\ ) = K \Big[\ \text{LT}\ (\beta) \Big] ; \qquad (\beta^2 < 1\ ;\ 1/\beta^2 > 1\ )\ .$$

The last equation should be compared with eq. (8); and eq. 411) should be remembered. It is clear that the problem of considering objects inside horizon (in general relativity) is mathematically analogous to the problem of considering space-like objects (in Special relativity).

4. - CROSSING THE EVENT-HORIZON. PHYSICAL MEANING OF SZEKERES-KRUSKAL COORDINATES. -

Since in Extended Relativity (ER)[20] a meaning was given (with the signature (+ ---)) also to negative ds^2, - as recently clarified e. g. in ref. (24), - now we are ready to interpret the Sxhwarzschild geometry for $r < 2M$ and what eqs. (9), (19) predict about the speed inside the horizon of a free falling body. First remember, however, that ER, through its "Third Postulate" (the "Reinterpretation Principle"), allows eliminating any motion backwards in time and any negative energy by reinterpreting them in terms of antiobjects (moving with positive energy forwards in time): so that no causality problem was left open[28].

We easily see from eq. (2) that (in the frames σ) if the Schwarzschild metric describes a bradyon (tachyon) for $r > 2M$, then it describes a tachyon (bradyon) for $r < 2M$.

Moreover (with respect to the frames σ) eqs. (9), (19) yield that a free falling body with arbitrary initial radial-speed reaches the light-speed for $r = 2M$ and then travels faster-than-light[20] for $r < 2M$. It should be noted that even if $V \longrightarrow c$ when $r \longrightarrow 2M$, the total energy E is finite, since $g_{oo} = 0$ for $r = 2M$.

Let us remember that (in the frames σ) the total energy E is a constant of the motion for a free falling body. For $r < 2M$, in eq. (18) both g_{oo} and $1 - V^2$ become negative, and E remains real; if you like, you could e. g. write:

$$E = \frac{m_o\sqrt{|g_{oo}|}}{\sqrt{|1-V^2|}} \qquad \text{(for } r \lesseqgtr 2M\text{)} . \tag{18bis}$$

In the case of free fall from infinity, eq. (10) forwards a result identical to the Newtonian one (except that in general relativity $V = 1$ can be got only at the horizon)(*).

To explain[21] way in the frames σ no divergent energies are associated with light-speed, let us observe, that frames σ are locally inertial only when their origin is at asympotic distances. In other words, the frames σ considered by us are locally-flat, but not locally inertial. This fact explains way the frames σ on the horizon would observe V=c but finite energies, provided that one takes into account that inertial (free falling) frames would themselves reach just the speed of light on the horizon with respect to the frames σ, which are at rest relative to the horizon(+).

In fact, it should be explicity remembered that the well-known (G-covariant) expression[20] $(c^2-V'^2)/c^2=(c^2-V^2)(c^2-U^2)/(c^2-UV)^2$, which holds when $\overline{U} \,||\, \overline{V}$, in the limiting case $V=c - \varepsilon \longrightarrow c$ and $U = c - \varepsilon' \longrightarrow c$, with $\varepsilon' = \varrho\varepsilon$, yields:

$$V' = c\,\frac{1-\varrho}{1+\varrho} \,, \tag{24}$$

(*) To satisfy our intuition (with regard to the assertion that V=c for r=2M), let us remember that, even with a (fixed) point-like charge, we would get that any (point-like) test-charge reaches the light speed at (and uniquely at) the "Coulomb singularity". (In that case, a way to avoid infinities was considering the finite dimensions[29] of elementary particles).

Moreover, if we remember the correspondence $V^2 \longleftrightarrow 1/r$ in eq. (11), then we might exclude the frames σ with their origin just on the horizon, so as we excluded in ER the "luminal" (U = c) frames as "unphysical"[20].

(+) If you want, you might say that in such non-inertial (N. I.) frames the formulas of SR can be forced to hold only when redefining $m_o^{N.I.} \equiv \sqrt{g_{oo}}\; m_o$. Moreover, notice that the free-falling bodies do not become "photons" on the horizon, even if their speed is V=c there.

which a priori forwards any possible real value; so that, for instance, if $\varrho = 0$, then $V' \longrightarrow c$; but if $\varrho \neq 0$ (as in our hypothesis) then $\lim V' \neq c$. For example, if $\varrho = 1$, then $V' \longrightarrow 0$.

We have also to add that, when assuming existence of both bradyons (= slower-than-light particles) and tachyons, it is meaningfull to consider also what happens inside the horizon, since we can get informations from the interior part of the horizon (see also the following).

We want now to explain why it is usually maintained that bradyons ramain bradyons both inside and on the horizon. In fact, e.g. from the definitions eqs.(4),(5) of the Szekeres-Kruskal coordinates[14÷16], it is immediate to get that:

(1) eqs. (4) define a time-coordinate v and a space (radial)-coordinate u which constitu that, with respect to frames σ, is moving with slower-than-light speed for $r > 2M$ and with the light-speed for $r = 2M$. In fact, let us consider[21] a fixed point of the Szekeres-Kruskal frame, so that:

$$\frac{du_>}{dv_>} = 0 . \tag{25}$$

Let us then consider, in general, a particle moving radially with respect to frames σ ; its radial speed $V_r \equiv V$ is given by eq. (9), which can be also written

$$V = (1 - \frac{2M}{r})^{-1} \frac{dr}{dt} \quad ; \tag{9'}$$

on the contrary, its speed relative to the (u, v) coordinates of Szekeres-Kruskal[14÷16] (SK) is, by straightforward calculations,

$$\frac{du_>}{dv_>} = \frac{V - w}{1 - Vw} \; ; \qquad w \equiv - v_> / u_> \tag{26}$$

For simplicity, let us choose any radial direction (θ = constant; ϕ = constant, in both σ frames and SK frames); then[21] eq. (26) assumes just the form of the velocity composition law of SR. The relative speed w of the Szekeres-Kruskal (SK) observer with respect to frames σ is however a function of the coordinated $u_>$, $v_>$ (or of the Schwarzschild time t)[21]. Actually, if we insert eq. (26) into eq. (25), we get that any fixed point of the ($u_>$, $v_>$) frames has the σ-invariant (radial)-speed

$$V = w \equiv - \frac{v_>}{u_>} = - \mathrm{Tanh}(\frac{t}{4M}), \tag{25bis}$$

and that $|w| \leq 1$. It this formalization, the sign minus means inward speeds;

(2) eqs. (5) define a time-coordinate $v_<$ and a space (radial)-coordinate $u_<$ which constitute a frame that, with respect to the frame defined by eqs. (4), moves with the light-speed for r=2M and with faster-than-light speed for $r < 2M$.

This is immediately got by comparing eqs. (8) with eqs. (23). For instance, we can proceed as follows. Let us considere for simplicity a free-falling body starting from infinity and moving radially. Then eq. (11) holds, and eqs. (8) can be rewritten (c=2MG=1):

$$\begin{cases} v_<(1/V^2) = K \left[u \;\; (V^2) \right] \; ; \\ u_<(1/V^2) = K \left[v \;\; (V^2) \right] \; ; \end{cases} \qquad (V^2 < c^2;\; 1/V^2 > c^2) \tag{8bis}$$

where we wrote now K instead of $\mathcal{S}$ since the exchange $r \longrightarrow 1/r$ transformed into the exchange $V^2 \longrightarrow 1/V^2$ (compare eq. (22bis))[27]. Briefly, eqs. (8bis) can be written (cf. eqs. (23) in ref. (20)).

$$v_< = K u_> \; ; \quad u_< = K\, v_> \; , \tag{27}$$

where K is the tipical[30, 27] Superluminal Lorentz transformation[20,24] as shown in eq. (23). In other words, eq. (23) tells us that K transforms observations made by subluminal frames into observations by Superluminal frames. Thus eq. (23) tells us that the "internal" ($r < 2M$) SK-frames move with faster-than-light-speed with respect to the "external" ($r > 2M$) SK-frames (and therefore with respect to frames σ).

It is than clear why with SK coordinates, one describes always bradyons. In fact, as already mentioned, a tachyon (with respect to an s frames) appears as a bradyon to any Superluminal frame S. Analogously, if we pass from s frames to S frames when passing from considering bradyons to considering tachyons, then even in SR we shall always to describe bradyons (see refs. 20, 24).

Of course, the transformation[27, 30] K operates in two dimensions the transitions[30]

$$
(28) \qquad \begin{array}{l} x(\beta) \longrightarrow t(1/\beta); \\ t(\beta) \longrightarrow x(1/\beta). \end{array}
$$

5. - CONCLUSIONS.

Therefore, the so-called "exchange of the rôles of time-coordinate and radial-coordinate", that we meet in eqs. (26), merely means - according to Extended Relativity - that (with respect to frames σ) any "outside" bradyon become a tachyon when crossing the horizon and any "outside" tachyon become a bradyon when crossing the horizon.

We don't need, therefore, speaking about any collapse of the Schwarzschild geometry[5] inside the horizon. On the contrary (with respect to frames σ) we shall interpret those usual considerations (see e. g. ref. (5)) in the following way (B ≡ bradyon; T ≡ tachyon):

a) any infalling B transforms into a T for $r < 2M$; tachyons T, of course, must go on moving along their radial direction towards 0 (since tachyons by definition are never at rest). Moreover, "inside" tachyons (or antitachyons) T cannot[5] come out from the horizon;

b) any infalling T transforms into B for $r < 2M$; such inside bradyons (or anti-bradyons) B can come out from the horizon, then transforming again into T's. Therefore, black-holes are expected to radiate out tachyons (with respect to frames σ).

However, with respect to any internal SK frame, we can say that:

c) any inside bradyon B cannot[5] go beyond the horizon;

d) any inside tachyon T can go beyond the horizon (transforming into a B, if we go on refering everything to the initial, internal SK frame).

To sum up, for frames σ the horizon behaves as a (out $\longrightarrow$ in) one-way-membrane with respect to outside B's only. Conversely, for internal SK frames, the horizon behaves as a non-permeable membrane only for inside B's. The same holds for the respective anti-particles (where antiparticles have been shown since long to be equivalent to "negative energy particles moving backward in time")[31, 28].

All what precedes demonstrates that the event-horizon is, in a sense, a "singular" surface, even if there no divergence of Rieman tensor is met. This agrees with what suggested about existence of coordinate-independent "singular surfaces" e. g. in refs. (32) when considering perturbed Schwarzschild problems. We have shown that crossing the horizon transforms B $\rightleftarrows$ T (and photons $\rightleftarrows$ photons): such a crossing, therefore, can be considered mathematically as a "catastrophe"[24]. In our theory, the type of the geodetics does change on the horizon.

Moreover, it is known that - considering a "classical" space-time coupled to quantum mechanical matter-field - Hawking[33] showed that the particle-creations near the event-horizon cause position and concept of horizon to be somewhat indeterminate; namely, the wave-packets (p_{jn}) and (q_{jn}) provide a complete basis for the solutions of the wave-equation everywhere except on the horizon. In any case, the horizon actually behaves as an irregular surface in space-time.

Let us conclude with the following observation. Firstly, let us remember that what we said in this paper holds not only in two dimensions, but also in four-dimensions (compare e. g. eqs. (2) and folls.). On the contrary, it is easy to interpret within Exended Relativity[20] the meaning[34] of the spin-change $ds^2 \longrightarrow -ds^2$ only in two-dimensions, due to the difficulty in interpreting the imaginary units[34, 24] entering some SLT-components.

However, we have shown that exactly the same problems will be met when considering non-radial motion or even more non-spherical black-holes. We think, therefore, that once solved such problems in general relativity, also the problems of Extendend (Special) Relativity will be solved too. In fact, our feeling is that we have not yet well understood the rôle of space and time (see ref. 35), so that possibly a complex space would be useful[35].

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