ISTITUTO NAZIONALE DI FISICA NUCLEARE

ezione di Catania

INFN/AE-77/14
8 Novembre 1977

P. Caldirola, M. Pavšić and E. Recami: CLASSICAL QUARK CONFINEMENT FROM GENERAL RELATIVITY.

Servizio Documentazione
dei Laboratori Nazionali di Frascati

Istituto Nazionale di Fisica Nucleare
Sezione di Catania

INFN/AE-77/14
8 Novembre 1977

P. Caldirola(*), M. Pavšič(+) and E. Recami: CLASSICAL QUARK CONFINEMENT FROM GENERAL RELATIVITY(o).

ABSTRACT. -

By assuming covariance of physical laws under (discrete) dilatations, it seems possible to describe strong and gravitational interactions in a unified way. An Einstein-type equation with "cosmological" term is for instance suggested for strong field inside hadrons, which yields - among other things - a classical quark confinement in a very natural way. Further consequences are briefly discussed.

(*) Istituto di Fisica, Università di Milano, Milano, Italy.
(+) Institute "J. Stefan", University of Ljubljana, Yugoslavia.
(o) Work partially supported by M. P. I. (ex-art. 286 T. U.) and by CNR (Laboratorio Fisica Plasma).

When electromagnetic phenomena, besides the mechanical ones, were properly considered, it was necessary to pass from Galilei Relativity to Einstein's. One might now wonder whether - when investigating also nuclear forces - another generalization towards a new Relativity is necessary.

Let us observe that the symmetries of Maxwell equations have not been fully exploited by Special Relativity. Namely, Maxwell eqs. are known to be covariant (besides under Poincaré transformations) even under conformal transformations[1]. As a first step, let us fix our attention in particular on the dilatations.

$$x'_\mu = \varrho\, x_\mu \qquad \left[\mu=0,1,2,3\right] \tag{1}$$

and postulate that physical laws are covariant (= invariant in form) also under dilatations (1). We are supposing that in nature only <u>discrete</u> values of ϱ happen to have physical counterparts[2].

At this point, let us remember that:

1) For gravitational and strong interactions, respectively, we meet the dimensionless coupling-constant-squares

$$\frac{Gm^2}{\hbar c} \simeq 1.3 \times 10^{-40} \tag{2a}$$

$$\frac{Ng^2}{\hbar c} \simeq 15 \tag{2b}$$

where: (i) G and N are the gravitational and strong universal constants in vacuum, respectively; ii) quantities m and g represent the gravitational charge (=mass) and the strong-charge of a hadron (cf. the following). The value in eq. (2a) is calculated for the pion mass $m=m_\pi$; in eq. (2b) we typically used the value of $pp\pi$ coupling-constant square. Incidentally, with regard to the above expression "strong-charge of a hadron", let us consider <u>quarks</u> as the actual sources of strong field, i.e. the true carriers of strong-charge, and let us call "colour" the sign of quark strong-charges. Namely, the hadrons can be considered as carrying <u>zero</u> total strong-charge, each quark having a strong charge $g_i = s_i\,|g'|$ where $\Sigma_i\, s_i = 0$. Quantities s_i play the role of the strong-charge <u>signs</u>, but (instead of being + 1, or -1) they can e. g. correspond to the numbers

$$s_1 = -\frac{\sqrt{3}}{2} + \frac{i}{2}\,; \quad s_2 = +\frac{\sqrt{3}}{2} + \frac{i}{2}\,; \quad s_3 = -i\,; \tag{3}$$

in such a case, antiquarks would possess one the following strong-charge signs:

$$\bar{s}_1=s_2+s_3=+\frac{\sqrt{3}}{2}-\frac{i}{2}\ ;\quad \bar{s}_2=s_1+s_3=-\frac{\sqrt{3}}{2}-\frac{i}{2}\ ;\quad \bar{s}_3=s_1+s_2=+i \qquad (3')$$

as one can easily guess by depicting the strong-charge signs on the complex plane. Usual strong-interactions should then derive from forces of Van-der-Waals type[3]. In conclusion, in correspondence to quantity m of eq. (2a), in eq. (2b) we ought to have the quantity $g=|g'|+|\bar{g}'|=2|g'|$. Analogously, for a baryon we shall have $g\equiv 3g_o$, where g_o is the average modulus of the constituent-quark charges.

Let us go back to eqs. (2) and call

$$\varrho\equiv\frac{Gm^2}{Ng^2}\simeq 0.9\times 10^{-41}. \qquad (4)$$

With regard to eqs. (2), (4), if we assume $g\equiv m$, then we get

$$N=\varrho^{-1}G\simeq 1.1\times 10^{41}G\simeq 7\times 10^{30}m^3kg^{-1}s^2\simeq 4\pi\frac{\hbar c}{m_\pi^2}\ ; \qquad (5)$$

conversely, if we choose units such that $[N]=[G]$ and $N=G=1$, then

$$|g'|=\frac{1}{2}g=\frac{m}{\sqrt{\varrho}}\simeq 5\times 10^{-33}cm\simeq 8\times 10^{-5}gr\simeq\pi\sqrt{\frac{\hbar c}{G}}\approx \text{Planck-mass}, \qquad (6)$$

where eq. (6) tells us, by the way, that Planck mass $\sqrt{\frac{\hbar c}{G}}\simeq m\sqrt{\varrho^{-1}}$ is nothing but the quark strong-charge (in suitable units). We don't expect, therefore, existence of small black-holes with mass of the order of Planck-mass, since we have already hadrons[4] with strong-charges of the order of Planck-mass, in suitable units;

2) If $R(U)\simeq 10^{26}$ m is our cosmos radius and r(h) is the hadron (pion) radius , then[2]

$$\frac{r(h)}{R(U)}=\varrho\simeq 10^{-41}. \qquad (7)$$

The previous heuristical considerations, contained in 1) and 2), suggest that our cosmos and hadrons (typically, pions), -both considered as finite objects (see the following) - can be systems internally governed by similar laws, differing only for the scale-factor ϱ which carries R(U) into r(h) and gravitational field into strong field. We are led to assume:

A) inside our cosmos, the Einstein eqs. (with cosmological term) $[G=1]$:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R^{\varrho}_{\varrho}-\Lambda g_{\mu\nu}=-\frac{8\pi}{c^4}T_{\mu\nu}; \qquad 2\Lambda\equiv\left(\frac{m_G c}{\hbar}\right)^2 , \qquad (8)$$

for the gravitational case; and

B) inside hadrons, the "scaled" Einstein eqs. $\left[N = G = 1\right]$:

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}R^{\varrho}_{\varrho} - H\tilde{g}_{\mu\nu} = -\frac{8\pi}{c^4}S_{\mu\nu}; \quad 2H = \left(\frac{m_S c}{\hbar}\right)^2, \tag{9}$$

for the strong case; where dimensional considerations (or, rather, conformal relativity(2,6)) easily show that $H \equiv \varrho^{-2}\Lambda$, and $m_G = m_S$. Moreover, the strong-charge tensor $S_{\mu\nu}$ is essentially $S_{\mu\nu} = \varrho^{-1}T_{\mu\nu}$, where $T_{\mu\nu}$ is the ordinary matter-tensor (containing e. g. the Dirac spinors, etc.). If we require gravitational interactions to have a range of the order of R(U), then(2,6) we get a graviton-mass $m_G \simeq 10^{-68}$ kg and a cosmological (attractive) constant $\Lambda \approx 10^{-56}\,cm^{-2}$. Furthermore, we can get the strong-quanta (gluon) mass to be $m_S \approx \varrho^{-1}10^{-68}$ kg $\approx m_\pi$ and the "cosmological" hadronic-constant H to be given by:

$$H^{-1} \equiv \varrho^2 \Lambda^{-1} \simeq 10^{-25}\ cm^2 . \tag{10}$$

It is also straightforward(2) to derive for the mass M of our cosmos

$$M = \varrho^{-2} m \simeq 10^{54}\ kg \tag{11}$$

in fair agreement with the astrophysical data (notice that the well-known Weyl-Eddington-Dirac "numerology" can be systematically derived - mutatis mutandis - within our "dilatational-covariant Relativity"(2)). Consistently with Einstein eqs. (8), - with attractive "cosmical (cosmological) term", - we can assume for the space-part of our cosmos (time aside) the simple model of the 3-dimensional hypersurface of a hypersphere(*). Analogously, eqs. (9) are consistent with the same model for hadrons too. We can extend Mach principle inside hadrons ("strong universes"), so that the inertia of each hadron-costituent (parton) will coincide with its strong-charge(+) : in this way, the Equivalence Principle results extended to the hadrons interior, justifying the present geometrization of strong field in hadrons.

Let us now find out the exact solution of eqs. (9) for a spherically symmetric strong-charge distribution. In the stationary (and small speed) case, the geodesic equation for a (small) hadron-constituent g_1 in vacuum is $\left[i, j = 1, 2, 3;\ N = 1\right]$:

(*) Embedded - if you like - in a "fictitious" four-dimensional space E^4. The problem of the intersections (which are 2-dimensional spherical surfaces) of hadrons with our cosmos will be considered later.

(+) And not with its gravitational mass (unlike what happens in the "gravitational universes").

$$\frac{d^2\vec{r}}{dt^2} = -\frac{c^2}{2}\left(1-\frac{2g_o}{c^2 r}+\frac{Hr^2}{3}\right)\left(\frac{2g_o}{c^2r^2}+\frac{2Hr}{3}\right)\frac{\vec{r}}{r} \tag{12}$$

where $g_o \simeq |g'| + |\bar{g}'| \equiv g$ is the (remaining) source strong-charge.

It is immediate to recognize that, for large distances ($r \approx r(h)$), in the case of "weak" fields ($g_{\mu\nu} \simeq \eta_{\mu\nu}$) we get from eqs. (9), (12), $[N=1;\ [N]=[G]]$:

$$\frac{d^2\vec{r}}{dt^2} \simeq \left(-\frac{g_o}{r^2} - \frac{c^2 Hr}{3}\right)\frac{\vec{r}}{r}; \qquad [r \simeq 1 \text{ Fermi}] \tag{13}$$

and therefore the confining force for any parton g_1:

$$F \simeq -g_1 \frac{c^2 H}{3} r \propto r \qquad \left[r \approx \left(\frac{6Ng_o}{c^2 H}\right)^{1/3} \simeq r(h)\right]. \tag{14}$$

We have thus in a natural way a confining potential $V \propto r^2$ of Nambu-Parisi type(7). Notice, however, that - since quarks are not small constituents - our eqs. (13),(14), and (16) in the following, hold only approximately for quarks.

If we eliminate the "weak" field condition, then for large enough values of r we get $[N=1]$:

$$-\frac{d^2 r}{ds^2} \simeq \frac{H^2 r}{9} + \frac{Hr}{3} + \ldots, \qquad [r > r(h)] \tag{15}$$

so that, when the hadron starts deforming (e.g. under the effect of high-energy collisions), the partons and the quarks - finding themselves with $r > r(h)$ - will suffer an even stronger confining-force:

$$-F \simeq g_1 c^2 \left(\frac{H^2 r^3}{9} + \frac{Hr}{3} + \cdots\right), \qquad [r > r(h)] \tag{16}$$

proportional to $-r^3$.

The problem of strong interactions between two hadrons requires however considering the intersection of hadrons with our cosmos: such intersections being 2-dimensional spherical surfaces. The modified Einstein equations (in our cosmos) representing - within a "bi-scale theory" - the deformed space-metric in the surronding of a hadron will be considered elsewhere, when more details will be given also about the content of this letter. Here, let us anticipate only the following: (i) if we put $g_{\mu\nu} = g_{\mu\nu}^{Grav} + h_{\mu\nu}$, where $g_{\mu\nu}^{Grav} \simeq \eta_{\mu\nu}$ and $h_{\mu\nu} \to 0$ for $r \gg 1$ Fermi, then we shall get in the static limit(2) the Yukawian behaviour $h_{oo} \simeq -(2g/c^2 r)\exp[-(m_S c/h)\, r]$; (ii) if we consider the intersections of hadrons with our space (which are what we call "hadrons" tout court), in the case of spherically-symmetric strong-charge distributions the calculated[2] "strong Schwarzschild radii" appear related to the experimental hadron-radii in strong interactions: for instance $r_S^{(s)}=0.8$ Fermi for nucleons and $r_S^{(s)}=1.4$ Fermi for pions. In such a context the "strong event-horizon"[5, 2] plays for hadrons the same role of the MIT "bag"(*).

At this point let us add that our classical confinement can be violated by quantum effects so as e. g. Hawking's (the "Hawking temperature" for a "strong black-hole"(*) can be of the order[2] of $T=2\times10^{11}$ °K, corresponding to an evaporation time of $\Delta t \simeq 10^{-23}$s, unless stability is imposed by Bohr-type conditions[2]).

In any quantum theory, however, quarks can be again "totally" confined by associating to their classical (Schwareschild) horizon a suitable barrier of selection-rules and conservation-laws.

One of the authors (E. R.) is grateful to V. De Sabbata, A. Papapetrou, D. W. Sciama, J. A. Wheeler and particularly to B. Bertotti, C. Berritta and P. Castorina for stimulating discussions. He moreover acknowledges a fellowship Accademia Lincei / Royal Society and thanks D. E. Blackwell for the hospitality received at the Department of Astrophysics, University of Oxford, during the preparation of this work.

(*) inside our cosmos (i. e. in our space) hadrons can be considered as "strong black-holes"[5, 2]. It has been recently shown that black-holes can carry further quantum numbers (besides mass, charge, spin).

REFERENCES.

(1) E. Cunningham, Proc. Lond. Math. Soc. 8, 77 (1910); H. Bateman, ibidem 8, 223 (1910); T. Fulton, F. Rohlish and L. Witten, Rev. Mod. Phys. 34, 442 (1962).

(2) P. Caldirola, M. Pavšić and E. Recami, Report INFN/AE-77/10 (1977).

(3) See e. g. A. O. Barut, Phys. Rev. D3, 1747 (1971); A. O. Barut and J. Nagel, Phys. Letters 55B, 147 (1975); T. Sawada, Nuclear. Phys. B71, 82 (1974); see also P. Caldirola, M. Pavšić and E. Recami, ref. (2); L. Van Hove: invited lecture at the SIF meeting (Como, 1977). A stimulating discussion with P. Castorina on the above point is ackknowledged.

(4) Hadrons can be considered as "strong black-holes", cf. ref. (2) and refs. (5).

(5) See e. g. E. Recami and P. Castorina, Lett. Nuovo Cimento 15, 347 (1976); R. Mignani, Lett. Nuovo Cimento 16, 6 (1976).

(6) See e. g. C. Sivaram and P. K. Sinha, Phys. Letters 60B, 181 (1976); A. O. Barut and R. B. Haugen, Ann. of Physics 71, 519 (1972); H. A. Kastrup, Ann. Phys. (Lpz.) 7, 388 (1962); M. Pavšić, Int. Journ. Phys. 14, 299 (1975); Nuovo Cimento 41B, 397 (1977); F. Hoyle and J. V. Narlikar, Nature 233, 41 (1971). See also ref. (2).

(7) G. Parisi, Phys. Rev. D11, 970 (1974); Y. Nambu, Phys. Rev. D10, 4262 (1974); H. B. Nielsen and P. Olesen, Nuclear Phys. B61, 45 (1973).