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UNIFIED THEORY OF STRONG AND GRAVITATIONAL INTERACTIONS(o)

CONTENT

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"... Andererseits muss man zugeben, dass der Versuch, die unbezweifelbare atomistische und Quanten-Struktur der Realität auf dem Boden einer konsequenten Feld-Theorie zu begreifen, auf grosse Schwierigkeiten stösst, von deren Ueberwindbarkeit ich keineswegs überzeugt bin. Ich will dies kurz erläutern an der Theorie des asymmetrischen Feldes (so wie sie formuliert ist). Aus dem Bau der Feldgleichungen geht nämlich Folgendes unmittelbar hervor: Ist $g_{ik}(x)$ eine Lösung der Feldgleichungen, so ist auch $g_{ik}(x/\alpha)$ eine Lösung, wobei α eine positive Konstante ist ("ähnliche Lösungen"). Es möge das System der g_{ik} z.B. einen in einen flachen Raum eingebetteten Kristall von endlicher Ausdehnung darstellen. Es gäbe dann eine zweite "Welt" mit einem andern Kristall, der genau gleich konstituiert ist, dessen Linear-Dimensionen aber α mal grösser sind als die des ursprünglichen Kristalls.

Solange wir uns eine Welt denken, die nichts anderes enthält als eben diesen einen Kristall, so liegt hierin noch keine Schwierigkeit. Man sieht nur, dass die Ausdehnung eires solchen Kristalles ("Massstabes") durch die Feldgleichungen nicht bestimmt ist. Man denke sich aber nun, dass die von uns betrachtete "Welt" aus zwei solchen Kristallen bestehe, die gemeinsam in einen flachen Raum eingebettet sind und die voneinander beliebig weit entfernt seien. Für die Lösungen der Feldgleichungen gilt wegen deren Nicht-Linearität zwar nicht das "Superpositionsprinzip". Aber man ist doch wohl geneigt zu denken, dass es eine Lösung für das Gesamtfeld gebe, derart, dass das Feld innerhalb jedes der beiden Kristalle sich nur wenig unterscheidet von der Lösung für den Fall, dass dieser Kristall allein in der Welt vorhanden ist. Dann aber wäre dies eine Welt, in der es zwei körperliche Objeckte gäbe, die zu einander "ähnlich" aber doch nicht kongruent wären...

... Damit also die Theorie annehmbar wäre, wäre es nötig, dass selbst weit voneinander entfernte "ähnliche" Objekte auf Grund der Feldgleichungen so stark aufeinander einwirken, dass eine irgendwie dauernde Koexistenz "ähnlicher" (nicht kongruenter) Objekte nicht möglich ist. Wir sind weit davon entfernt zu sehen, wie aus den Feldgleichungen eine derartige Folgerung gezogen werden könnte... "

Princeton, 4 April 1955

A. EINSTEIN.(63)

"In jeden Quark begräbt er seine Nase"(91).

J. W. v. GOETHE, "Faust", 292.

ABSTRACT. - By assuming covariance of physical laws under dilatations, we succeed in describing strong and gravitational interactions in a unified way. In terms of the (additional, discrete) "dilatational" degree of freedom, our "cosmos" as well as hadrons can be considered as different states of the same system, or rather as similar systems.

Moreover, a discrete hierarchy can be defined of "universes", which are governed by force-fields with strenghts inversally proportional to the "universe" radii. Inside each "universe" an Equivalence Principle holds, so that its characteristic field can be geometrized there.

We can thus easily derive the whole usual "numerology", i. e. relations among numbers analogous to the so-called Weyl-Eddington-Dirac large-numbers. For instance, the "Planck mass" happens to be nothing but the (average) "strong charge" of the hadron-quarks. However, our "numerology" connects the (gravitational) macro-cosmos with the (strong) micro-cosmoses, rather than with the electromagnetic ones (as e. g. in Dirac's version).

Einstein-type equations (with "cosmological" term) are suggested for the strong interactions, which - incidentally - yield a classical quark-confinement in a very natural way and provide a priori a field theory of strong interactions.

PART A: HERISTICAL CONSIDERATIONS

1. - INTRODUCTION - WHY CONFORMAL RELATIVITY. -

1. 1. - INTRODUCTION (See Also Sect. 4. 1.):

It is well-known that, when enlarging the world of experience from classical Mechanics to Electromagnetism, it was necessary to abandon Galilei relativity in favour of Einstein's. One might now wonder whether, when in presence also of nuclear forces, another generalization towards a new Relativity is necessary.

Let us remember that the symmetries of Maxwell equations have not been fully exploited by Special Relativity. Namely, Maxwell eqs. happen to be covariant (besides under Poincaré transformations) even under all the conformal transformations[1].

We want in particular to fix our attention on the space-time (discrete) dilatations:

$$\boxed{x'_\mu = \varrho\, x_\mu ,} \qquad \left[\mu = 0, 1, 2, 3\right]. \tag{1}$$

As we told before, Maxwell eqs. are in particular covariant under transformations (1).

Let us observe at this point that — if we change our chronotopical measure-units, e. g. by dilatating them

$$\Delta' = \varrho^{-1} \Delta , \tag{2}$$

so that $x'_\mu = \varrho x_\mu$, — we should actually have no change in the form of our laws[2]. In other words, we would like to have all physical laws written in a form covariant under transformations (2).

Let us explicitly mention that a contraction (by a factor ϱ^{-1}) of our measure-units is completely equivalent to a dilatation (by the factor ϱ) of the observed world, and vice-versa. Let us stress that we prefer to consider unchanged measure-units (associated with a fixed frame, so as our own frame of reference), and therefore "dilatated" objects[4, 5].

For our purposes, it is mathematically convenient to choose one fixed set Δ_o of (chronotopical) units so that, when passing to dilatated units, we define:

$$\Delta \equiv K \; \Delta_o \; ; \; K \equiv \frac{\Delta}{\Delta_o} \; . \tag{3}$$

We can then introduce the "dilatationally invariant" coordinates[6]:

$$\boxed{\eta_\mu \equiv \alpha^{-1} x_\mu} \; ; \text{ with } \; \alpha^{-1} \equiv K. \tag{4}$$

As previously mentioned, we shall prefer to consider α as the "dilatation factor" - or the "dilatational coordinate" - of the observed physical system.

We are ready to explicity assume the postulate : "All physical laws must be covariant also under dilatations (1)". We are supposing ϱ to assume in nature only discrete values (see the following).

Therefore, in analogy to what historically done when building up Special Relativity, we have now to re-write the laws of Mechanics and Gravitation in a new form[2,5] covariant also under transformations (1), with the obvious condition that those physical laws must get their standard form for $\alpha = 1$.

Our task is made of course easier by the fact that we can refer ourselves to the (already developed) conformal relativity[7,2,5]. Since we are especially interested in gravitation (and strong interactions), then we'll refer ourselves to conformal relativity in curved (conformal) spaces[5].

For instance, let us initially consider a (small mass) test-body m,- as e.g. a pion or a proton, which can feel all the four fundamental interactions (see the following), - put in the gravitational field originated by the source[7] M; then, the following equation is in order[5,1]:

$$\boxed{\frac{d^2 \eta^i}{d\tau^2} = - \frac{G M_{oo}}{\eta^2} \cdot \frac{\eta^i}{\eta} \; ,} \qquad \left[i = 1, 2, 3 \right] \tag{5}$$

where $\eta^2 \equiv \eta_i \, \eta^i$; quantity $\tau \equiv \tau_m = \alpha_m^{-1} \; t \equiv K_m t$; and where M_{oo} is the conformally invariant mass[2] of the source[8]:

$$\boxed{M_{oo} \equiv \alpha_M \, M,} \tag{6}$$

quantities α_M, α_m being the "scale factors" (or dilatation-factors) of source and test-particle, respectively. If $\alpha_M = 1$, then eq. (5) reads:

$$\frac{d^2 \vec{\eta}}{d\tau^2} = - \frac{G M}{\eta^2} \cdot \frac{\vec{\eta}}{\eta} \; ; \tag{7}$$

which can be re-written in the form

$$\frac{d^2 \vec{r}}{dt^2} = - \alpha_m \frac{G M}{r^2} \cdot \frac{\vec{r}}{r} \; . \tag{8}$$

Of course, if it is also $\alpha_m = \alpha_M = 1$, then eq. (5) assumes its usual form.

Eq. (5) can be derived from Einstein equations in conformal space[1,5] when the test-particle has a conformally invariant mass m_{oo} satisfying the condition $m_{oo} \ll M_{oo}$ and a small speed $v \ll c$, and when the gravitational field is spherically symmetric and static. However, eq. (5) is dilatationally covariant (even if not generally-covariant in its present form).

If we want to eliminate the restriction $m_{oo} \ll M_{oo}$, so that $\vec{\eta}$ will now represent the relative (conformal) coordinates: $\vec{\eta} = \vec{\eta}_m - \vec{\eta}_M$, then - instead of eq. (5) - we must write (e. g. describing the motion of m with respect to M):

$$\frac{d^2\vec{\eta}_m}{d\tau_m^2} - \frac{d^2\vec{\eta}_M}{d\tau_M^2} = -\frac{G(m_{oo}+M_{oo})}{\eta^2}\cdot\frac{\vec{\eta}}{\eta} \qquad \left[\tau_M = \frac{t}{\alpha_M}\right], \tag{9}$$

as it comes from a straighforward extension of classical two-body problem; for instance, we can start from the Lagrangian

$$L = \frac{m_{oo}}{2}\left(\frac{d\vec{\eta}_m}{d\tau_m}\right)^2 + \frac{M_{oo}}{2}\left(\frac{d\vec{\eta}_M}{d\tau_M}\right)^2 - U(\vec{\eta}).$$

Of course, quantity t is the (usual) time as measured by the observer.

Eq.(9) can read, in terms of the usual, relative coordinates $\vec{r} \equiv \vec{r}_m - \vec{r}_M$,

$$\frac{d^2\vec{r}}{dt^2} = -\frac{G}{r^2}\left(\frac{m_{oo}}{\alpha_M} + \frac{M_{oo}}{\alpha_m}\right)\left(\frac{\alpha_M m_{oo} + \alpha_m M_{oo}}{m_{oo}+M_{oo}}\right)^2 \cdot \frac{\vec{r}}{r}. \tag{10}$$

In the particular, important case of two bodies with equal conformally invariant masses $m_{oo}=M_{oo}$, we get:

$$\ddot{r} = \frac{G}{4r^2}\cdot\frac{M_{oo}(\alpha_M+\alpha_m)^3}{\alpha_m\,\alpha_M}. \tag{11}$$

This case is important for the procedure we shall follow in the next Sections in order to compare the strengths of the different, four interactions (namely, we shall require to get two equal bodies, e. g. two protons, at the end of our dilatations). For instance, from eq. (11), if we put $\alpha_m = 1$ but $\alpha_M \ll 1$, we get:

$$\ddot{r} = \frac{G}{4r^2}\cdot\frac{M_{oo}}{\alpha_M}. \tag{12}$$

1. 2. - SOME HEURISTICAL CONSEQUENCES:

In other words, we can e. g. consider the two following cases:

a) Let us start from the case $\alpha_M^{(in)} = 1$, but in general $\alpha_m^{(in)} \neq 1$ so that initially $M_{oo} = M^{(in)} \equiv M$. Then, let us dilatate or contract (i. e. , let us "scale") the two bodies by the factors

$$\varrho_M \equiv \frac{\alpha_M^{(fin)}}{\alpha_M^{(in)}} = \alpha_M^{(fin)} \quad \text{and} \quad \varrho_m \equiv \frac{\alpha_m^{(fin)}}{\alpha_m^{(in)}}.$$

We shall get, from eq. (11):

$$\ddot{r}' = \frac{G}{4r^2}\cdot\frac{M(\varrho_M + \varrho_m\,\alpha_m^{(in)})^3}{\varrho_M\,\varrho_m\,\alpha_m^{(in)}}, \tag{13}$$

when $m_{oo} = M_{oo} = M$, and

$$\ddot{r}' = \frac{Gm_{oo}}{4r^2}\left(\frac{1}{\varrho_M} + \frac{N}{\varrho_m\,\alpha_m^{(in)}}\right)\left(\frac{\varrho_M + N\varrho_m\,\alpha_m^{(in)}}{N+1}\right)^2 \tag{14}$$

when $M_{oo} = M = N\,m_{oo}$.

For instance, from eq. (13), if we put $\varrho_m\,\alpha_m^{(in)} \equiv \alpha_m^{(in)} = 1$ and $\varrho_M \ll 1$, then we get (when $m_{oo} = M_{oo} \equiv M$):

$$\boxed{\ddot{r}' = \frac{G}{4r^2}\cdot\frac{M_{oo}}{\varrho_M} = \frac{G}{4r^2}\cdot\frac{M}{\varrho_M}.} \tag{15}$$

Analogously, from eq. (14), if we put as before $\varrho_m \alpha_m^{(in)} = 1$ and $\varrho_M \ll 1$, and if we consider $N \gg 1$, then we get (when $M_{oo} = M = N m_{oo}$):

$$\ddot{r}' = \frac{G m_{oo}}{4r^2} \left(\frac{1}{\varrho_M} + N \right). \tag{16}$$

b) Let us now start from the case $\alpha_M^{(in)} = \alpha_m^{(in)} = 1$, so that initially $M_{oo} = M^{(in)} = M$ and <u>also</u> $m_{oo} = m^{(in)} = m_{oo}$. Then, the last eq. (16) reads:

$$\ddot{r}' = \frac{Gm}{4r^2} \left(\frac{1}{\varrho_M} + N \right); \qquad \left[M_{oo} = M = Nm \right]. \tag{17}$$

Moreover, from eq. (13), if we now "scale" both objects by the <u>same</u> factor $\varrho_M = \varrho_m = \varrho$, we get (when $M_{oo} = M = m_{oo} = m$):

$$\boxed{\ddot{r}' = \frac{2GM}{r^2} \cdot \varrho ,} \qquad \left[M = m \right]. \tag{18}$$

* * *

In conclusion, let us investigate eqs. (15) and (16). Eq. (15) means that, under a dilatation of the (only) field-source M, we have

$$GM \longrightarrow \frac{GM}{\varrho_M} \tag{19}$$

if $M_{oo} = M = m_{oo}$; and

$$GM \longrightarrow GM \left(1 + \frac{1}{N \varrho_M} \right) \tag{20}$$

if $M_{oo} = M = N m_{oo}$.

With regard to eq. (19), we can say that, under a "source-dilatation" of that kind by the factor ϱ, the quantity $A = GM$ is divided by ϱ. One <u>could</u> say for instance that, under a <u>contraction</u> ($\varrho < 1$) of that kind, the "<u>universal constant</u>" G increases by a factor ϱ^{-1} (cf. Sect. 5).

Now, eq. (15) can be written $m_{oo} \ddot{r}' = \frac{G}{4r^2} \frac{M m_{oo}}{\varrho_M}$, so that eq. (19) reads:

$$G M m_{oo} \longrightarrow \frac{G M m_{oo}}{\varrho_M} , \qquad \left[M = M_{oo} \right] , \tag{21}$$

where $m_{oo} = m$ if initially we have <u>also</u> $\alpha_m^{(in)} = 1 = \alpha_m^{(fin)} = \varrho_m$.

It is interesting to observe since now that, for

$$\boxed{\varrho_M \approx 10^{-40} ,} \tag{22}$$

we can get transition from <u>gravitational</u> to <u>strong</u> interactions (for the problem of the exponential factors and of the ranges, see the following Sections).

<u>However</u>, if we want to compare (as we shall do) the final result of this procedure (for instance eq. (21)) with the behaviour of two <u>pions</u> or <u>protons</u>, which ones experience <u>all</u> the four fundamental forces, we must: (i) start from a source $M \equiv M_{oo}$ a radius $R_M \approx 10^{40} R(N) \approx 10^{26}$ m, of the order of the universe radius (so that, after contraction, it reduces e. g. to have the radius R(N) of a nucleon); (ii) attribute to the "<u>conformally invariant</u> mass" M_{oo} of the source M the value $M = M_{oo} = m = m_p$, quantity

$$m \simeq 1,6 \times 10^{-27} \text{ Kg}$$

being e. g. the nucleon mass (see also sect. 6).

In this way, we can pass from our test-hadron, experiencing a gravitational force, to a test-hadron experiencing a strong force, as is suggested by eqs. (21), (22) when we set $m \equiv m_{oo} = m_p$ and $M = m_p$.

1.3. - FURTHER REMARKS:

We shall further develop these points in Section 3; In Section 2, however, we shall first see what are their consequences. Here, let us only add the following observations. In what precedes, we have been considering, for instance:

i) initially (before contraction): a test-proton with $\alpha = 1$ and $m_{oo} = m^{(in)} = m_p$; and a "source" with radius $R_M \approx 10^{40}$ R(N), with $\alpha = 1$ and with $M_{oo} = M^{(in)} = M = m_p$;
ii) finally (after contraction of the source by the factor $\varrho \approx 10^{-40}$): a test-proton, unchanged; and a "source" constituted by a proton, with R=R(N), $\alpha = 10^{-40}$ and $m'_{oo} = M_{oo} = m_p$; so that we got (as is implicit in eq. (21)):

$$\ddot{r}' = \frac{Gm_p}{4r^2 \varrho} = \frac{Gm_p}{4r^2} \cdot (10^{+40}) . \tag{23}$$

But, if the final proton (case (ii)) is to be associated to $\alpha = 10^{-40}$, the previous procedure is not symmetric. For instance, the test-proton itself ought to be associated (since the begining) with $\alpha = 10^{-40}$. Then, let us choose another procedure, so to have:

(i) initially (before contraction): a test-proton with $\alpha = 10^{-40}$ and $m_{oo} = m_p$; and a source-object with radius $R_M = 10^{40}$ R(N), with $\alpha = 1$ and with an a-priori unknown mass $M_{oo} = M^{(in)} = M$;

(ii) finally (after contraction of the source-object by the factor $\varrho = 10^{-40}$): let us use eq. (10), or rather eq. (14) which yields:

$$\ddot{r}' = \frac{GM_{oo}}{4r^2} \varrho = \frac{GM_{oo}}{4r^2} (10^{-40}); \tag{24}$$

therefore we have finally: a test-proton with $\alpha = 10^{-40}$ and $m_{oo} = m_p$; and an object with radius R = = R(N), with $\alpha = 10^{-40}$ and with the (a priori unknown) mass $M_{oo} = M$.

This second procedure must be equivalent to the previous one: thus eq. (24) must coincide with eq. (23). As a consequence, we get immediaty that

$$\boxed{M_{oo} = \varrho^{-2} m_p \approx 10^{80} m_p ,} \tag{25}$$

i.e. that the initial object (having the radius of our cosmos, $R_M \approx 10^{40}$ R(N) $\approx 10^{26}$ m) must possess the mass

$$M = M^{(in)} = M_{oo} \approx (10^{40})^2 \, m_p$$

which is well-known to be just the mass of our cosmos. Actually, eq. (24) was derived be setting $N + 1 \simeq N$. In other words, in this second procedure, the initial, "cosmological" object is naturally identifiable with a "cosmos" like our one. Relation (25) is known as EDDINGTON'S relation (see the following and ref. (17)).

2. - THE FOUR FUNDAMENTAL FORCES; AND A HIERARCHY OF "UNIVERSES" (AND OF ASSOCIATED "CHARGES")

2.1 - THE FUNDAMENTAL FORCES:

We essentially know about four "fundamental" forces in physics. In order to compare their relative strengths, let us consider for instance two equal particles, as two protons, which can interact both gravitationally, and weakly, and electromagnetically, and strongly. For instance, the ratio between the typical gravitational and electromagnetic "interaction strengths" s_G, s_{EM} can be derived[9] from the corresponding classical forces between two nucleons:

$$\frac{s_G}{s_{GM}} = \frac{F_G}{F_{EM}} \approx 10^{-36} .$$

Analogously, the ratio between the typical electromagnetic, and strong "strengths" can be inferred[9] from the corresponding typical interaction-durations:

$$\frac{s_{EM}}{s_S} = \frac{\Delta\tau_S}{\Delta\tau_{EM}} \approx \frac{10^{-23}\ s}{10^{-19}\ s} \approx 10^{-4} .$$

In conclusion, when dealing with nucleons (more generally with hadrons), the ratioes among typical (dimensionless) interaction "coupling-constant squares" can be represented by the following set of pure numbers[9]:

FORCE	RANGE	STRENGTH	
strong	short	$s_S \approx 1$	
electromagnetic	long	$s_{EM} \approx 10^{-4}$	(26)
weak	very short	$s_W \approx 10^{-13}$	
gravitational	long	$s_G \approx 10^{-40}$	

Incidentally, let us observe[10] that, - since the typical interaction-durations $\Delta\tau$ are inversally proportional to the interaction-strengths s, - then for the gravitational interactions we have for instance:

$$\Delta\tau_G \approx 10^{40} \times 10^{-23}\ s \approx 10^{17}\ s . \tag{27}$$

Since $10^{17}\ s \sim 10^{10}\ y$ we get that $\Delta\tau_G$ is of the same order of our universe age[10]. More generally, the quantity $\left[y \equiv yr \equiv year \right]$:

$$10^{17}\ s \simeq 3 \times 10^9\ y$$

can be considered as not far from a "characteristic time" of our cosmos evolution ("oscillation", or "decay").

Since the previous, heuristic considerations are not precise[11] enough, let us put the whole question on a more rigorous basis by carefully defining[9] dimensionless "coupling-constant squares" for the various forces. We know for instance that, in the electromagnetic case, we meet the dimensionless quantity[9]:

$$\frac{1}{4\pi\varepsilon_o} \cdot \frac{e^2}{\hbar c} \simeq \frac{1}{137} , \tag{28}$$

and in the gravitational case the dimensionless quantities

$$G \cdot \frac{m^2}{\hbar c} \simeq 6 \times 10^{-39} ; \quad G \cdot \frac{m_\pi^2}{\hbar c} \simeq 1,3 \times 10^{-40} , \tag{29}$$

where $(4\pi\varepsilon_o)^{-1}$ and G are the electromagnetic and gravitational "universal constants" (in vacuum), respectively. In eqs. (28,(29) we chose e and m equal to proton charge and mass, and m_π equal to the (charged) pion mass. As quantity e is called electric charge, so we can call m gravitational charge.

Analogously, we could introduce, for weak and strong interactions respectively, the dimensionless coupling-constant squares[9]:

$$N_S \frac{g^2}{\hbar c} \approx 15 \tag{30}$$

and

$$N_W \frac{g_W^2}{\hbar c} \approx 10^{-12} \tag{31}$$

where of course $g \equiv g_S$ and g_W are the strong and weak charges, respectively. Quantities N_S, N_W are then the universal constants a priori associated (in vacuum) with the strong and the weak "fields", respectively. Usually units are chosen so that numerically[13] $N_S=N_W=1$. For instance, in the strong case, it is usually written

$$\frac{g^2}{\hbar c} \approx 15 \tag{32}$$

which is in fact the standard expression for the square of the $pp\pi$ coupling constant[9].

The values of the (dimensionless) coupling-constant squares in eqs. (28 ÷ 31) forward the precise ratioes[14], approximately represented in eqs. (26).

In this paper, we shall (first) skip considering the case of weak interactions.

At this point, let us define as "universe" any (almost isolated) system whose (internal) constituents dominantly interact via one (and only one) of the four fundamental forces (see refs. (9), (10) and (15)). Of course, we shall have "gravitational universes", "electromagnetic universes", and "strong universes"[16]. We shall use that concept in order to explain e. g. the following, interesting relations[17]:

$$R(U):R(A):R(h) \approx s_S:s_{EM}:s_G = N_S g^2 : \frac{e^2}{4\pi\varepsilon_o} : Gm^2 \tag{33}$$

where R(U), R(A), R(h) are respectively the typical radii of our universe (that we shall call "cosmos" in the following, to avoid confusions), of the atoms, and of the hadrons. We shall call the above relations the "WEYL-DIRAC large numbers" relations (even if Dirac's ones were different).

2. 2. - GRAVITATIONAL AND STRONG "UNIVERSES":

Before going on, we ought to take into account what stated at the end of Sect. 4. 2.

Then, let us confine ourselves for the moment to gravitational and strong universes. In order to get $R(U)/R(h) \equiv N_S g^2/(Gm^2)$, we ought to choose $m = m_\pi$. In this paper, however, let us choose the nucleon (h≡N) as the "representative" of hadrons h, so that $m \equiv m(N)$. We want to compare the two fundamental forces, which govern the (internal and external) interactions of the two "universes", respectively, one with the other. First, let us observe that both gravitational and strong forces are always attractive, and associates to non-linear equations (since also their quanta are considered to be themselves field-sources); in terms of gauge-theories, we would eventually make recourse to non-Abelian gauge theories: and, in a sense, what we are attempting in this paper is just providing a geometrical interpretation — ante litteram — of the latters.

Moreover, we shall consider in the following our cosmos as a finite object embedded in a bigger universe (see what follows).

Let us here write down the espressions of the two "corresponding" potential energies (in the static limit) outside those "universes" (cf. end of Sect. 4. 2):

$$\phi \simeq -\frac{GMm}{r} \cdot \exp\left[-rm_G c/\hbar\right] ; \tag{36}$$

$$\Phi \simeq -\frac{N_S g g'}{r} \cdot \exp\left[-rm_S c/\hbar\right] , \tag{37}$$

where the gravitational potential-energy ϕ and the strong (Yukawian) potential-energy Φ do have the same physical dimensions: $[\phi] = [\Phi]$. Quantities m_G, m_S should respectively be the masses of ("external") graviton and the ("external") pion (where the "external graviton" mass is supposed to be small, but not zero): see the following. Expression (37), e. g. is the ordinary one for the scalar potential in the "strong case" (static limit). Eqs. (36, 37) will be derived from Einstein field-equations of General Relativity, containing however a non-zero (even if very small) cosmological term for the reasons we are going to see[17]. In order to show the connection between the cosmological term and the exponential factor entering e. g. eq. (36), let us anticipate (cf. Sect. 8) the following.

2. 3. - WHY A COSMOLOGICAL TERM:

Our cosmos will be considered as a finite object belonging to a bigger universe (cf. end of Sect. 4. 2.). Outside (see Sects. 2. 5, 3. 3, 4. 2) the gravitational universe (cosmos) let us assume - for the reasons we shall see - that Einstein-type equations (with cosmological constant Λ) hold in the "big universe" [cf. also Sects. 5 and 8] , which can be written in the form:

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - \Lambda g_{\mu\nu} = - \frac{8\pi G}{c^4} T'_{\mu\nu} , & [\Lambda > 0], \\ T'_{\mu\nu} \equiv T_{\mu\nu} - \frac{c^4}{8\pi G} \Lambda \left(f_{\mu\nu} - \frac{1}{2} g^{\alpha\beta} f_{\alpha\beta} g_{\mu\nu} \right), \end{cases} \tag{38}$$

or rather in the equavalent form:

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = - \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\varrho}_{\varrho} \right) + \Lambda f_{\mu\nu} . \tag{39}$$

Notice that, due to our conventions, $\Lambda > 0$ means attractive Λ . In eqs. (38) we have:

$$g_{\mu\nu} \equiv f_{\mu\nu} + h_{\mu\nu} \tag{40}$$

where $f_{\mu\nu}$ is a second metric-tensor representing the "infra-gravitational" metric of the "big-universe" (see also Sect. 8), and where quantities $h_{\mu\nu}$ vanish for large enough values of r, so that (for $r \gg R(U)$) they are first-order corrections to the components $f_{\mu\nu}$. Since tensor $f_{\mu\nu}$, in the surroundings of the considered cosmos (in the big-universe), practically coincides with the flat Minkowski[18] metric, $f_{\mu\nu} \cong \eta_{\mu\nu}$, we can write in suitable coordinates

$$g_{\mu\nu} \cong \eta_{\mu\nu} + h_{\mu\nu} \tag{40'}$$

where

$$\left| h_{\mu\nu} \right| \ll 1 \quad \left[\text{for } r \gg R(U) \right] . \tag{41}$$

In the same coordinates, we can substitute $f_{\mu\nu} \cong \eta_{\mu\nu}$ into eqs. (38, 39). Notice that the "gravitational" (external) metric-tensor $h_{\mu\nu}$ acts (in the big-universe) only on the bodies possessing "gravitational" charge", and not on the bodies possessing only "infra-gravitational" charge : cf. Sect. 8. By inserting eq. (40') into eq. (39) and under the condition eq. (41), we obtain the linearized Einstein eqs. with cosmological term[19] :

$$\begin{cases} \partial_\mu \partial^\mu h_{\alpha\beta} + 2 \Lambda h_{\alpha\beta} \simeq \frac{16\pi G}{c^4} \left(T_{\alpha\beta} - \frac{1}{2} \eta_{\alpha\beta} T^{\varrho}_{\varrho} \right) \\ \partial_\mu h^{\mu}_{\nu} = \frac{1}{2} \partial_\nu h^{\mu}_{\mu} . \end{cases} \tag{42}$$

One might have expected (e. g. by considering that the non-zero cosmological term enters our starting Einstein eqs. in an essential way) that we had rather to linearize our Einstein eqs. with respect to the Sitter metric,[20,19] as done in our Appendix A. Moreover, our phylosophy will always be the one of considering curved space-times (see Sect. 4, and Appendix B). However, not only we can linearize with respect to the flat metric (i. e. we can stick to eqs. (42), provided that we take account of condition (41), [see also Sect. 5]),but such a procedure seems to be the best one when considering - as we do - the "exterior" of a cosmos (cf. Sects 2. 5, 3. 2) and when the "bigger universe" has a very large curvature-radius (see what follows).

Eq. (42) is a relativistically-covariant, massive equation for a tensorial field, with[20]:

$$2\Lambda \equiv \left(\frac{m_G c}{\hbar}\right)^2, \tag{43}$$

quantity m_G being the field mass (in this case, the "external" graviton mass: cf. also eq.(50) in the following).

If we now restrict ourselves to the case of stationary sources, then the only non-vanishing component of $T_{\mu\nu}$ is $T_{oo} = \gamma c^2$, where γ is the mass-density; so that $T^\varrho_\varrho = T_{oo}$. Therefore (as previously done in eq. (36)), in the case of static field, i. e. when $\partial g_{\alpha\beta}/\partial t = 0$, we can confine ourselves only to the scalar field $\psi \equiv g_{oo}$; and eq. (42) will read[18]:

$$\nabla h_{oo} - 2\Lambda h_{oo} = \frac{8\pi G}{c^2}\gamma \ . \tag{44}$$

In the case of a point-particle M at rest at the origin of the space-coordinates, a spherically symmetric solution of eq. (42) is

$$\psi \equiv g_{oo} \simeq 1 - \frac{2GM}{c^2 r} \cdot \exp\left[-\frac{m_G c}{\hbar} r\right] , \tag{45}$$

provided that Λ is positive. Quantity $\psi \equiv g_{oo}$ is known to be[21, 22] essentially the scalar gravitational potential $V \equiv \phi/m$, so that for test-particle low speeds and for weak field:

$$g_{oo} \simeq 1 + \frac{2V}{c^2} \equiv 1 + \frac{2\phi}{mc^2} ; \tag{46}$$

we shall again discuss these points in Sect. 7. We can see immediately, however, that - within our theory - the previous approximations are practically equivalent to neglecting the spin-2 character of gravitons (cf. Sect. 9. 1).

Eqs. (45)-(46), in any case prove - under our assumptions - the connection between the exponential factor in eq. (36) and the presence of a non-zero cosmological term in the field equations.

At this point, let us go back to eqs. (36), (37). Let us observe that eq. (37) is known to hold - in the static limit - only outside hadrons, i. e. outside the "strong universes"; therefore, in our philosophy, eq. (36) was required to hold only outside our cosmos, i. e. outside the "gravitational universes" (see Sect. 2. 4 and Sect. 4). However, the previous derivation of eqs. (45), (46) suggests that eq. (36) might hold ("outside" any mass M) even inside our cosmos. [Actually, eq. (36) yields the usual GMm/r behaviour for not too large values of r, as shown in the next Sub-Section; contemporaneously, condition (41) limits the validity of eq. (45) for not too small values of r (i. e. for $r \gg 2GM/c^2$).]

But, strictly speaking, eqs. (36), (37) are required to hold - in our philosophy - only outside the gravitational and the strong "universes", respectively. As regards the interior part of those "universes", in Sect. 5 we shall derive the suitable, exact solutions of Einstein eqs. with cosmological terms. We shall find, by the way, that for very large values of r a term of the type Λr^{+2} dominates, which yields cosmos-costituent confinement (within the gravitational universes) as well as an interesting hadron-constituent confinement (within the strong universes): see Sect. 5.

2. 4 - THE TWO SETS OF "UNIVERSES" AND OF ASSOCIATED "CHARGES":

Let us now start from eq. (36) and consider a contraction (by a factor $\varrho < 1$) of the system "gravitational source M plus its field" (leaving the second particle m unchanged); then, due to eqs. (21, 22), (6) and (13, 15), we shall get:

$$\phi' = -\frac{GMm}{\varrho r} \cdot \exp\left[-\frac{r m_G c}{\varrho \hbar}\right] \tag{36'}$$

It is interesting that, for $\varrho \approx 10^{-40}$, eq. (36') coincides with eq. (37) since it is

$$\frac{GMm}{\varrho} \equiv N_S\, gg', \tag{47}$$

and we can identify (see the following)

$$\boxed{\frac{m_G}{\varrho} \equiv m_S.} \tag{48}$$

Or, vice-versa, we can start from eq. (37) and consider a dilatation (by a factor $\varrho' > 1$) of the system "strong source g plus its field" (leaving as before the second strong-charge g' unchanged): we shall then get:

$$\Phi' \equiv -\frac{N_S gg'}{\varrho' r} \cdot \exp\left[-\frac{r m_S c}{\varrho' \hbar}\right] . \tag{37'}$$

It is interesting that, for $\varrho' = 1/\varrho \approx 10^{40}$, eq. (37') coincides with eq. (36) since it is (remembering eqs. (29, 30)):

$$\frac{N_S gg'}{\varrho'} \equiv GMm \tag{47'}$$

and we can identify (cf. eq. (6)):

$$\frac{m_S}{\varrho'} \equiv m_G. \tag{48'}$$

At this point, since m_S is the mass of the external strong-field quanta, it can be chosen to be of the order of the pion-mass:

$$m_S \approx m_\pi \ ; \tag{49}$$

we shall discuss later (see Sect. 9) the fact that gravitons and pions have actually different spins (see ref. (23)), at least "inside" and "outside" their correspondent "universes", respectively.

Then, we can predict for the "external" gravitation mass m_G a value(24) of the order of

$$m_G \approx 10^{-40}\, m_\pi \approx 10^{-65}\, g . \tag{50}$$

This very low value agrees with the upper-limits set in ref. (25). Incidentally, the value (50) predicted by us for the graviton mass is of the same order of the one predicted on a similar ground in ref (26). From eqs. (50) and (43) one gets for the cosmological constant a value of the order $\Lambda \approx 10^{-56}\text{cm}^{-2}$. Since this value can be considered slightly too high, we shall discuss this point in Sect. 9. 1 (although some authors(26) consider that value an acceptable one for closed, isotropic, homogeneous cosmoses).

In the case of gravitational universes, the cosmos-cosmos interaction potential, given by eq. (36), will practically vanish (due to an exponential factor of the order of e^{-10}) for distances d of the order of cosmos-radius itself:

$$d \approx 10^{10}\ \ell\text{-}y \simeq 10^{26}\ \text{m}. \tag{51}$$

In other words, its "effective range" r_G can be considered (in correspondence to a factor 1/e) to be of the same order:

$$r_G \approx d. \tag{52}$$

Quite analogously, in the case of strong universes, the hadron-hadron interaction is known to have an "effective range" of the order of 1 Fermi, which is given be the interesting ralation:

$$r_S \approx 10^{-40}\, r_G \simeq 10^{-13}\ \text{cm} . \tag{53}$$

The previous considerations lead us to write a "potential energy" in dilatation-covariant form. Let us start by writing it as follows:

$$\phi(\alpha) = \frac{GMm}{\alpha r} \cdot \exp\left[-\frac{r m_G c}{\alpha \hbar}\right], \tag{54}$$

where (see Sect. 1) it is: $M \equiv M^{(in)} = M_{oo}$; $m \equiv m^{(in)} = m_{oo}$; and $m_G \equiv m_G^{(in)} = (m_G)_{oo}$. Or rather, using explicitly dilatation-invariant quantities:

$$\boxed{\phi(\alpha) = \frac{GM_{oo} m_{oo}}{\alpha r} \cdot \exp\left[-r\frac{(m_G)_{oo} c}{\alpha \hbar}\right],} \tag{54'}$$

which should be associated with the initial (dilatation-covariant) eqs. (5, 10). Eqs. (54, 54') are already dilatation-covariant, but they can assume a more suitable form. In fact, by comparing eq. (54') with eqs. (36, 37), it is immediate to clarify the physical rôle of the dilatation-invariant mass for $\alpha \neq 1$. Namely, we have for $\alpha \approx 10^{-40}$, if we set $G \equiv N_S = 1$:

$$\frac{M_{oo}}{\sqrt{\alpha}} = g; \quad \frac{m_{oo}}{\sqrt{\alpha}} = g', \qquad \left[G \equiv N_S = 1\right], \tag{55}$$

so that the quantities $M_{oo}/\sqrt{\alpha}$ and $m_{oo}/\sqrt{\alpha}$ play the rôle of strong charges g, g' for $\alpha \approx 10^{-40}$ (i.e., when we pass from a field with the mass m_G of the "graviton" to a field with the mass $m_S = m_G/\alpha \approx 10^{40} \times m_G \approx m_\pi$ of the "pion"). By the way we have thus answered the problem set at the end of our footnote (13).

Conversely, as mentioned in Sect. 1.2 and in footnote (13), we could set $N_S = \alpha^{-1} G \approx 10^{40} G$ and then

$$M_{oo} = g; \quad m_{oo} = g', \qquad \left[N_S = \alpha^{-1} G\right]. \tag{56}$$

Let us stress that, on many respects (as we shall mention), this choice would be simpler.

With the choice (55), we can write down the same formal expression:

$$\phi = \frac{qq'}{r} \exp\left[\frac{-r m_{exch} c}{\hbar}\right], \quad \left[G = N_S = 1\right], \tag{57}$$

for both gravitational and strong cases, where charges q, q' can be either gravitational or strong charges and contemporaneously the field-mass m_{exch} represents the mass either of the gravitation quanta or of the strong-field quanta.

With the choice (56), on the contrary, we can write down the analogous, "unified" expression:

$$\phi = N \frac{M_{oo} m_{oo}}{r} \exp\left[-\frac{r m_{exch} c}{\hbar}\right], \quad \left[N_S = \alpha^{-1} G\right] \tag{57bis}$$

where N can be either G or N_S and, contemporaneously, the field-mass m_{exch} can be either the gravitation or the strong-field quantum.

Let us explicitly notice that the form of eqs. (54, 54') is dilatation-covariant only if we do not scale the distance r, which is actually the case considered by us (remember our procedure in constructing relations (26)).

2. 5. - CONCLUSIONS OF THE SECTION:

In a sense, we thus succeeded in describing in a unitary way gravitational and strong forces. In particular, the same eq. (57bis), will hold for interactions in the static limit both among gravitational universes (like our cosmos) and among strong universes (the nucleons, and more generally the hadrons). In Sect. 5 we shall see in a more rigorous way that the same fact is true also for the interactions among constituents of the two sets of "universes".

As it is clear, we are considering (and we can[27, 28] consider) our cosmos as a "gravitational universe" belonging to a larger entity[9, 10, 27] (or "big universe"), as suggested also by HOYLE (see refs. (28), (29)). Here we are speaking of interior and exterior of our cosmos so as we speak of interior and exterior of a hadron; however, cf. also Sects. 7 and 4. 2.

Moreover, let us mention that the interactions between two "universes" of the same kind might be possibly derived also as due to Van-der-Waals-like forces[30]; cf. Sect. 3. 2.

An interesting result of our "dilatationally-covariant" procedure is that -through our dilatations or contractions - we have associated to our cosmos and to hadrons (nucleons), respectively, radii R(U) and R(N) which are connected by the relation [R(N) ≡ nucleon radius] :

$$\boxed{\frac{R(U)}{R(N)} \approx 10^{40} \approx \frac{s_S}{s_G}} , \tag{58}$$

thus actually explaining a relation similar to one of the so-called (heuristical) "Dirac large-numbers" relations[17]. Eqs. (54') and (57) will be investigated more in detail in the following.

Here, let us repeat that in eq. (54') the quantity $s(\alpha) \equiv GM_{oo}m_{oo}/(\alpha \hbar c)$ can be identified with $[G = 1]$:

$$s(1) \equiv \frac{GM_{oo}m_{oo}}{\hbar c} = \frac{Mm}{\hbar c} \qquad \left[\alpha = 1\right] \tag{59}$$

in the case of gravitational universes (e. g. our cosmos) associated to the radius $R \gtrsim R(U)$; and with $[N_S = 1]$:

$$s(10^{-40}) \equiv \frac{N_S gg'}{\hbar c} = \frac{gg'}{\hbar c} \qquad \left[\alpha \approx 10^{-40}\right] \tag{60}$$

in the case of the strong universes (e. g. nucleons), associated to the radius $R \approx 10^{-40} R(U) \equiv R(N)$.

In the case of two equal neutrons - or protons, as considered in Sect. 1, - we have:

$$\begin{cases} s(1) = s_G; \\ s(10^{-40}) = s_S. \end{cases}$$

Therefore, let us repeat, the quantity

$$q = \frac{m_{oo}}{\sqrt{\alpha}} , \qquad \left[G \equiv N_S = 1\right] ,$$

can represent in general the (gravitational, strong, ...) charge of the considered body [proton] in the (gravitational, strong, ...) field characterized by factor α .

The connections of the present theory with the strong-gravity one[31] are evident (see Sect. 9. 1)

A problem left open here - of course - is the one of explaining why nature did not realize a "continuity" of universes, but only a discrete hierarchy of "universes" as:
..... cosmos ($\alpha \approx 1$); atom ($\alpha \approx 10^{36}$); hadron ($\alpha \approx 10^{-40}$);.....

The discreteness of the scale-factor α should follow from the proper "quantization" of this theory, for example through the methods indicated in ref. (32).

3. - EQUIVALENCE AND MACH PRINCIPLES EXTENDED TO ALL "UNIVERSES"; EDDINGTON NUMBER; INTERACTION BETWEEN TWO "UNIVERSES".

3. 1 - INTRODUCTION:

From the previous Sections, it follows that we can now consider that the interactions acting inside the gravitational universe (i. e. among the cosmos-constituents) and inside the strong universes (i. e. among the hadron-constituents) are governed by the same dilatation-invariant laws. In other words, strong forces - either among hadron constituents or among hadrons - can be derived from gravitational forces - either among cosmos constituents or among cosmoses - by a contraction (see, also,the following). Merely for briefness' sake we shall sometimes write that "any hadron can be considered as deriving from a contraction of the cosmos" instead of writing that dilatation-transformations bring the physical laws holding inside (outside) the hadrons into the physical laws holding inside (outside) the cosmos. That will be a shorthand, without further meanings. Cf. eqs. (54, (55), (57). We shall explicitly show the above connection between cosmos and hadron interiors in Sect. 5.

We said (as we showed in the previous Sections) that also test-hadrons interact strongly with other source-hadrons. We can understand this fact by considering that also the constituents of the first hadron are able to interact strongly with the constituents of the second one. This has been interpreted (at the end of Sect. 1) as meaning that even the test-hadron itself must be considered as deriving from the "collapse" (with $\varrho \approx 10^{-40}$) of another object like our cosmos.

Let us then

a) first, summarize the demonstration of what claimed in this Section with regard to the interior of hadrons and of the cosmos;

b) second, re-derive the results of Sect. 2 (with regard to the external interactions of hadrons and of "cosmoses") by starting directly from two universes like our cosmos, and then by contracting both of them.

3. 2 - COSMOS AND HADRON INTERIOR:

Let us clarify what we did in Sect. 2 and what we are going to do with respect to cosmos and hadron interior (case (a)).

The heuristical considerations that previously guided us require that eqs. (36), (37) - with exponential terms - hold (only) outside hadrons and outside our cosmos, in the static limit.

Actually, in Sect. 7 we shall consider the exact solution of the "Einstein equations" (with cosmological terms) for spherically-symmetric (gravitational or strong) sources inside cosmos or hadrons; and we shall find the constituent-confining metric-component $[c = G = N_S = 1]$:

$$g_{oo} \approx - \frac{2\varrho M_{oo}}{r} + \frac{\Lambda_{oo}}{3\varrho^2} r^2 \qquad (61)$$

in spherical coordinates, which yields a potential of the type $V \approx -(\Lambda_{oo}/6\varrho^2)\, r^2$ when **r** is large. [The last addendum in eq. (61) is known to correspond to the De-Sitter correction to flat metric, since our "cosmological" models tend for larger r to be De-Sitter spaces(19, 21). Therefore, one expects that, by linearizing our Einstein equations with respect to the De-Sitter metric, only a potential of the type $V \approx -\varrho M_{oo}/r$ remains. In fact, if we confine to the universes-interior and refer to frames in which the metric appears time-independent, then(33) we just get that potential $V \approx -\varrho M_{oo}/r$, for $g_{oo} \approx 1$.]

In other words, let us consider e. g. two cosmos constituents, one of them being for simplicity a test-particle (with gravitational charge $m = m^{(in)} = m_{oo}$) and the other one a source $M = M_{oo}^{(in)}$. Then under the contraction by a factor $\varrho \approx 10^{-40}$, we pass from the potential-energy (54)-(54') with $\alpha = 1$, $[r \ll 10^{+26}\ \mathrm{m}]$:

$$\phi(1) = \frac{M_{oo} m_{oo}}{r} = \frac{Mm}{r} , \quad \left[G = c = 1 \right] , \qquad (62)$$

to the potential-energy $\left[c = N_S = 1\right]$:

$$\phi(10^{-40}) = \frac{M_{oo} m_{oo}}{r} \times 10^{40} = 10^{40} \frac{Mm}{r}, \quad \left[r \ll 10^{-13} cm\right], \qquad (63)$$

where the esponential terms are absent in the universes interior, in this paper(34). In any case, with regard to point (a), we are going to show that - under a contraction by the factor $\varrho \approx 10^{-40}$ - the cosmos reduces to have the radius of a nucleon, and simultaneously the gravitational forces (acting inside the cosmos) transform into strong (acting inside the nucleon)(34).

Before going on, however, let us emphasize once more that the equations in Sect. 2 with exponential terms (strictly) hold only when M represents the mass of the whole cosmos and g represents the strong-charge of a whole hadron (nucleon). Let us moreover underline that, when we consider the interactions between two hadrons (or two cosmoses), we must linearize Einstein-equations with respect to flat metric, since in such a case we are outside the Einstein-De Sitter "universes" under consideration. Only in that case we get the exponential term shown in Sect. 2. 3.

Now, following eq. (55), we can write eq. (63) - according to our postulate of dilatational covariance - in the same form of eq. (62):

$$\phi(10^{-40}) = \frac{gg'}{r}, \qquad \left[r \ll 10^{-13} cm\right], \qquad (64)$$

where units are chosen such that $N_S = 1$, provided that we define the strong charges as $g \simeq 10^{20} M$; $g' \simeq 10^{20} m$. Let us explicity notice that, under the considered contraction, the cosmos-radius $R(U) \approx$ $\approx 10^{26}$ m goes into the nucleon-radius $R(N) \approx 10^{26} \times 10^{-40} \approx 10^{-13}$cm; we already noticed that such a relation is better satisfied in the "pion case"(19), i. e. when we choose the second eq. (29), so that $\varrho \approx 10^{-40}$ becomes $\varrho \approx 10^{-41}$.

At this point, let us furthermore notice the following:

i) If we scale only the (initial) cosmos (and not the measure units) - as done before, - then we pass from gravitational interactions to strong interactions;

(ii) If we however scale not only the initial cosmos, but even the measure-units, then - of course - nothing will change: in other words, for the observer, who contracts together with the cosmos, eq. (62) is not only covariant but even invariant. It means that a small "Lilliputian"(35), inside a nucleon, using units contracted by the same factor $\varrho = R(N)/R(U)$, will describe the forces acting inside its universe (the nucleon) exactly so as we describe the gravitational forces inside our cosmos: that is to say, he will feel as gravitational the interactions that we call strong. (Incidentally, we expect also the corresponding quantum theories to be dilatation-covariant; i. e. we expect quantization to be present both inside hadrons and inside cosmos, but with properly dilatated "Planck-constant". Cf. the following, especially Sects. 5 and 6). See Fig. 1.

It is therefore possible to generalize the Mach Principle(36) to the interior of hadrons (nucleons) by saying that - for an internal observer - the "inertial mass" of any nucleon-constituent is originated by its interactions(37) with all the other constituents of that "universe" (nucleon). Therefore, the inertial mass m_I possessed by nucleon constituents inside the nucleon coincides - in our language - with their strong charge (see the following). This is analogous to the fact that, in our cosmos, the cosmos-constituents show to us (who are "inside-observers") an inertial-mass coinciding with their gravitational charge. We have thus generalized even the Principle of Equivalence to the interior of nucleon. It means that, in our language, and in our approximations, inside the "strong universes" (hadron, or nucleons) it holds:

$$F = m_I a = g' a = N_S \frac{gg'}{r^2} \qquad \left[r \ll 10^{-13} cm\right] \qquad (65)$$

so that

$$\boxed{a = g/r^2}, \qquad \left[N_S = 1\right] \qquad (66)$$

Eq. (66) is obviously analogous to the gravitational relation $a = M/r^2$, $\left[G = 1\right]$. It is then clear that we can geometrize also the strong field inside hadrons[9] since - let us repeat - inside the hadrons the rôle of inertial mass is played by the strong charge.

Let us notice that, when a hadron-constituent comes outside (from inside[38]), then its "inertial mass" decreases by a factor $\varrho^{-1/2} \gtrsim 10^{20}$ since we have $\left[m = \text{gravitational charge}\right]$:

$$m_I = g' \longrightarrow m_I = m, \qquad \left[g' = 10^{20} m\right]. \tag{67}$$

However, it is necessary to stress that the previous consideration (in the previous form) hold only if we make recourse - as usually done - to the equation:

$$ma \approx \frac{gg'}{r^2} \quad , \tag{65'}$$

where, in the outside case, for simplicity we have assumed $\exp\left[rm_\pi c/\hbar\right] \simeq 1$. The use of eq. (65') is standard when considering the interaction (in our cosmos) of two hadrons; we shall call eq. (65') the "outside" equation, and such a use the "outside view-point". We adopted till now the "outside viewpoint" only for comparing our theory with the experimental results, which are commonly interpreted just on the basis of eq. (65').

But, especially in the interior of hadrons, we ought to substitute eq. (65') with the "inside" equation:

$$m_I\, a \equiv g'\, a = gg'/r^2 \Longrightarrow a = g/r^2. \tag{66'}$$

It is clear that, if we use (more correctly) eq. (66') instead of eq. (65'), we get that the strong-charges g' of hadron-constituents are related (inside hadrons) to their gravitational-charges m by equation:

$$\boxed{g' \gtrsim 10^{40}\, m\,.} \tag{68}$$

Within this second (more correct) "inside viewpoint", we can say that when a hadron-constituent comes outside (from inside[38]), then its inertial mass decreases by a factor $\varrho \gtrsim 10^{40}$, since we pass

$$\boxed{\text{from } m_I \equiv g' \quad \text{to} \quad m_I \equiv m,} \qquad \left[g' \approx 10^{40} m\right]. \tag{67'}$$

In the following, let us go back to the "outside viewpoint", for practical reasons.

For instance, as usually each quark[39] is conventionally attributed a barionic number B=1/3, so we can tentatively assign to each quark the "(average) strong-charge" $g' \gtrsim (1/3)g$, quantity g being the outside "nucleon-strong-charge," whose order of magnitude can be derived from eq. (32): $g^2/\hbar c \approx 15$; $\left[N_S=1\right]$.

We are thus led to claim that, if (inside the nucleon) the inertial mass of the quark is considered - within the outside viewpoint - to be:

$$m_I = g' \approx \frac{g}{3} \quad , \qquad \left[G = N_S = 1\right], \tag{69}$$

then outside the nucleon that quark will have the inertial mass (now coinciding with its gravitational charge)

$$m_I' = m \approx \frac{10^{-20} g}{3} \quad . \qquad \left[G = N_S = 1\right]. \tag{69'}$$

Notice that such considerations are different from the known ones[31], based on "Archimedes effect".

Let us observe that, in order to do explicit calculations, we must pay attention to use the suitable units[40], so that $G=N_S=1$. For instance we have, from eq. (32),

$$g \approx 6 \times 10^{-33} \text{ cm} \approx 0.5 \times 10^{20} m_p, \quad (69'')$$

of course in agreement[41] with our eqs. (55), as well as with eqs. (29), (30), and always within the "outside viewpoint".

Eq. (69') then tells us that - under our hyphotheses - we should expect gravitational and inertial mass of a quark (outside hadrons) to be

$$m_I' \approx \frac{m_p}{3} .$$

Vice-versa, if we assume the last equation for quarks, then we immediately derive that (inside hadrons) quarks are expected to have an inertial mass (identical to their strong-charge) given within the outside viewpoint by eqs. (69'), (69''):

$$\boxed{m_I = g' \approx 2 \times 10^{-33} \text{cm} \approx 3 \times 10^{-5} \text{g} \approx 1.3 \sqrt{\frac{\hbar c}{G}} \approx \text{Planck-mass.}} \quad (70)$$

This interesting result tells us that, if we assume either (outside) $m_I' \approx m_p/3$, or (inside) $g' = m_I \approx g/3$, then the inertial mass of quarks inside hadrons is nothing but the "Planck mass". In other words, Planck mass can represent the quark (average) strong-charge, within the "outside viewpoint". And we do not expect to find new particles with rest-mass equal to the Planck mass!

Our model, moreover, explains also the reasons why[42] (within the "outside viewpoint"):

$$\text{Planck-mass} \equiv \sqrt{\frac{\hbar c}{G}} = \varrho^{-1/2} m_p \approx 10^{20} m_p . \quad (71)$$

In fact, we have that $\left[\text{cf. eqs. (29), (30), so that } \varrho \approx \frac{6 \times 10^{-39}}{15}\right]$:

$$15 \approx \frac{g^2}{\hbar c} \approx \frac{G m_p^2}{\varrho \hbar c} , \qquad \left[N_S = 1\right] \quad (71')$$

wherefrom (putting $\varrho^{-1} \approx 10^{40}$):

$$\boxed{\sqrt{\frac{\hbar c}{G}} \simeq \frac{m_p}{\sqrt{15}} \sqrt{\varrho^{-1}} \approx 10^{20} m_p .} \quad (72)$$

(Following a slightly different philosophy[39], the Planck-mass may be considered as the strong-charge of the whole nucleon).

Incidentally, we can get the definition of G in terms of other constants[42]:

$$G \equiv \varrho \frac{g^2}{m^2} = \frac{g^2}{m\sqrt{mM}} = \frac{\bar{r}}{\bar{R}} \frac{g^2}{m^2} , \qquad \left[g \approx \hbar c \text{ if } N_S = 1 \right], \quad (73)$$

where $m \equiv m_p$; $M \equiv M(U)$; $\bar{r} \equiv R(N)$; $\bar{R} \equiv R(U)$: cf. ref. (51) and Sects. 7, 8 in the following. Of course, in suitable units $G = 1$.

Eqs. (68) and follows are based on a very naïve assumption. However, according to our previous (naïve) approach, we can conclude that Planck-mass $\sqrt{\hbar c/G} = 2.1 \times 10^{-5}$g can be considered as about 1/3 of the "nucleon strong-charge" and it can therefore be associated to quark strong-charge within the outside viewpoint.

We shall discuss the problem of quark binding-energy[39] later.

Less naïvely, the hadrons can be e.g. considered with total strong-charge zero, each quark having a strong charge $s_i g$ where $\Sigma_i s_i = 0$. Quantities s_i play the rôle of the strong-charge signs, but (instead of being +1, -1) they can e.g. correspond to the numbers $-\sqrt{3}/2 + i/2$; $\sqrt{3}/2 + i/2$; $-i$. In such a case, antiquarks would possess one of the following strong-charges: $+ig$; $(\sqrt{3}/2 - i/2)\,g$; $(\sqrt{3}/2 - i/2)\,g$, as one can easily guess by depicting the strong-charge signs on the complex plane.

It is self-evident that the sign of quark-strong-charge can a priori be identified with colour. Usual "strong" interactions should then derive from forces of Van-derWaals type. (We shall touch again these problems in Sects. 5 and 7).

[Before passing to the new Sub-section, let us notice that, if an hadron-constituent possesses on the contrary (inside the hadron) only an inertial mass $m_I \approx m_p/3$, then it would possess outside the hadron the inertial mass $m_I \approx 10^{-20}\, m_p/3 \approx 6 \times 10^{-45}$g. In such a case, that hadron-costituent would not be easily detected (when possibly emitted by the hadrons). The last considerations, of course, are still within the "outside viewpoint".] We are left with the problem of investigating what the hadron-constituents exchange among themselves when (strongly) interacting. Let us remember that the internal, Lilliputian[35] observer should see that they exchange nothing but gravitons; we must translate this in our language (without forgetting that, inside hadrons, the correct equation should be the eq. (66')).

We might suppose that (inside our cosmos) gravitational interactions are mediated by spin-two gravitons having about the same gravitational charge as the spin-zero "gravitons" which must carry the gravitational interaction in the surroundings of the cosmos itself (cf. eq.(50)) in the static limit:

$$\boxed{m_G(j=2)^{int} \approx m_G(j=0)^{ext} \approx 10^{-40} m_\pi \approx 10^{-120} M,} \qquad (74)$$

where M is the cosmos-mass.

In our theory, one might analogously say that "strong quanta" inside hadrons (let us a priori call them "strong gravitons" or "spin-2 gluons"[39]) are expected to possess conformally invariant masses equal to 10^{-120} times the conformally invariant mass of their "universe" (i. e., of the hadron as seen from inside).

It means that hadron-constituents (e. g., quarks) might interact via spin-2 "gluons" with strong-charge $[G=N_S=1]$:

$$g_{SG} \approx 10^{40} m_\pi \qquad (75)$$

where SG means "strong-graviton"[42bis]. Notice that we need here a factor 10^{40} (instead of 10^{20}, as in eqs. (55)) since, in the interior of the hadrons, we must substitute the "outside" equation (65') with equation (66'), strictly speaking.

Even in the immediate surroundings of the hadron, when dealing with strong charges, we ought to use eq. (66') rather than eq. (65'): in such a case, we ought to say that even the strong-charge of the "external" strong-quanta is 10^{40} times m_π. In order to comply with the usual, physical procedures, we can however go on writing (so as in Sect. 2. 4) that within the "outside viewpoint":

$$g_\pi \approx 10^{20}\, m_\pi\,, \qquad \left[\text{outside hadrons}\right].$$

Let us repeat that the theory would be simpler, even from the viewpoint of physical-dimension theory, by adopting the choice in eq. (56).

3. 3. HADRON AND COSMOS EXTERIOR:

Let us now come to point (b) of Sect. 3. 1. Namely, let us consider two cosmoses, i. e. two objects with the same size and mass of our cosmos[43]. Since we - for simplicity - are assuming them to have the same mass $M_{oo}=M^{(in)}=M$, we are supposed to use eq. (11). Actually, let us consider the gravitational interaction between the two cosmoses when they are close one another (in the same way as when considering the strong interaction between two hadrons). Initially we have $\left[\alpha_M^{(in)}=\alpha_m^{(in)} \equiv \alpha = 1\right]$:

$$\ddot{r} = \frac{8GM_{oo}}{4\,r^2} = \frac{2GM_{oo}}{r^2}\,, \qquad \left[\alpha_M = \alpha_m \equiv \alpha = 1\right]. \qquad (11')$$

Then, if we scale both cosmoses by the factor ϱ, from eq. (18) we get:

$$\ddot{r} = \frac{2GM_{oo}}{r^2}\,\varrho\ , \qquad \left[M_{oo} = M^{(in)} = M\right]. \tag{18'}$$

If $\varrho \approx 10^{-40}$, then the two cosmoses reduce to nucleon size, and at the same time we have:

$$\ddot{r} \approx \frac{2GM}{r^2} \cdot 10^{-40}.$$

But, at the end of Sect. 1, we have already seen that such a contraction (by the factor $\varrho \approx 10^{-40}$) transforms gravitational interactions into strong interactions. So that it must be $[m_p \equiv \text{nucleon mass}]$:

$$\frac{2GM}{r^2}\,\varrho = \frac{2GM_p}{r^2\,\varrho}\ , \qquad \left[\varrho \approx 10^{-40}\right] \tag{76}$$

wherefrom

$$M = \varrho^{-2} m_p\ , \tag{77}$$

that is to say:

$$M \approx 10^{80} m_p\ ; \qquad \boxed{m_p \approx 10^{80}\ M}\ . \tag{25}$$

We have thus demonstrated Eddington's relation[17], which expresses the relation (in the past noticed only heuristically) between cosmos-mass and nucleon-mass. Namely, we have derived, within our conformal theory, that cosmos-mass M must equal about $(10^{40})^2\, m_p$. We shall re-derive again this relation in the following, within a more detailed model.

At this point we might ask ourselves the following: the nucleon strong-charge and interactions can be explained as deriving from the postulate of dilatation-covariance applied to the cosmos characteristics and laws (i. e., briefly speaking, by contracting a "gravitational universe" like our cosmos); then, why nucleons possess also a gravitational-charge m_p? The answer relies on the fact that nucleons, besides being (strong) universes, can belong themselves to a "higher-order" universe, i. e. to our own cosmos (that we see governed by gravitational interactions, and in which inertia coincides with gravitational-charge. In our cosmos, moreover, inertia comes from gravitational interaction with all the other bodies of the cosmos, according to Mach principle).

Analogously, we can start from two nucleons, and then dilatate both of them[43] by a factor $\varrho' \approx 10^{40}$, thus obtaining two "cosmoses". Such cosmoses will be governed (inside themselves) by gravitational interactions, if the two initial nucleons are governed, inside themselves, by strong interactions. Moreover, those two cosmoses will mutually interact (when "close" to each other) through eq. (36), in the static limit. Such a gravitational interaction between the two cosmoses, which corresponds to the potential-energy (36), is completely analogous to the strong-interaction of two nucleons, which corresponds to the potential energy (37). Eqs. (36) and (37) were written in the static limit, but these statements should hold also in the non-static cases.

In other words, for an observer who dilatates together with the two initial nucleons, the two final cosmoses interact via short-range strong-interactions.

Let us notice that the characteristic time of such two-cosmos interactions (e. g. with subsequent gravitational decays) would be given by eq. (27):

$$\Delta\tau_G \approx 10^{17}\ \text{s} \approx 3 \times 10^9\ \text{y} \tag{27'}$$

so that we should very scarcely realize interactions of such a kind of our cosmos with other (possible) cosmoses.

However, if the two cosmoses belong themselves to a "universe" of even higher order, then they will posses (besides the gravitational-charge) also a new infra-gravitational charge (or mass), due to their interactions -via an infra-gravitational field - with all the bodies of the "big-universe" (always in accord with Mach principle). See also the end of Sect. 4. 2.

We are therefore led to enlarge, a priori, the possible hierarchy of "universes", with their associated fields and characteristic "charges": we might call the hadron (and strong-charge) the zeroth order universe (and charge); our cosmos (and gravitational-mass) the first-order universe (and charge); the "big-universe" (and infra-gravitational mass) the second order universe (and charge). If we start from a cosmos (containing very many nucleons, according to eq. (25)) and contract it by the factor $\rho \approx 10^{-40}$, then each initial nucleon will go into a universe of minus-one order (-1), associated to a hyper-strong charge (of order -1). And so on. We can add the observation that we are much easily able to discover the fields stronger than the gravitational one, rather than the fields associated to possible universes of order larger than one.

Therefore, we may even further generalize both Mach principle and Equivalence principle, in the sense that - briefly speaking - : Inside a universe of order n, $(n=0, \pm 1 \pm 2, \ldots)$, the inertia coincides with the charge of the same order n, so that (only) the same-order field is geometrizable.

Let us now go back to eqs. (33), which suggest to us that, if we now contract a cosmos by the new factor $\bar{\rho} \approx 10^{-36}$ (so to pass from the cosmos-radius to about the atom-radius), then we pass from the gravitational interaction-strength to the electromagnetic interaction-strength. This looks true, but the correspondence strong $\rightleftarrows$ electromagnetic interactions cannot be developed much further (at least at the present level) for the following reasons:

a) gravitational and strong interactions correspond to non-Abelian gauge theories (see e.g. ref. (39)), since those fields act as sources of themselves (and even their quanta feel their corresponding fields). On the contrary, Maxwell's theory can be an Abelian gauge theory: for instance, photons do not carry electric charge;

b) gravitational and strong interactions seem to be always attractive, differently from electromagnetic interactions.

For these reasons it appears difficult to define a "universe" inside which the electromagnetic field is geometrizable; so that we confined ourselves to gravitational and strong interactions. Of course, other approaches are however possible[44].

4. - DIGRESSION: THE SIMPLEST COSMOLOGICAL MODEL (AND MACH PRINCIPLE).

4. 1 - OUR PROGRAM:

On the basis of what previously explored, we are now in the condition to be able to derive, for instance, the value of many physical quantities from a few input-data: (i) the experimental value $G \approx 7 \times 10^{-11}$ Joule x m/kg^2 ; (ii) the experimental value of the ratio gravitational-interaction strength over strong-interaction strength $s_G/s_S \approx 10^{-40}$; and (iii) the values of the age $t \approx 10^{10}$y of our universe and (at a certain extent) of the light-speed $c \approx 3 \times 10^8$m/s. Moreover, we shall assume as known essentially: (a) the Newton gravitation-equation; (b) the experimental behaviour of Yukawa potential; (c) Einstein equations[20] with cosmological term.

Then, our "dilatation-covariant" theory allows us to derive for instance: 1) radius and mass of the cosmos; 2) the nucleon mass; 3) the radii of nucleons and other hadrons; 4) the strength of Yukawa potential; 5) the value of quark strong-charges; 6) the graviton mass; and so on.

In order, to accomplish the previous program, we want now to "particularize" our previous theory, by choosing specific models for the gravitational and the strong universes. Actually, we are going to consider Einstein-type equations associated to both gravitational and strong interactions, together with their Schwarzschild solutions. We shall thus deal with (gravitational) black-holes and with "strong-black-holes"[9], instead of dealing with generical universes.

4. 2. - DIGRESSION: A VERY SIMPLE COSMOLOGICAL MODEL:

Before performing that program, let us however introduce in this Section a very simple cosmological model (which apparently accords with the big-bang theory[45]), so to fix our ideas - first of all - with regard to our own cosmos. As a first, elementary result we shall calculate radius R and mass M of our cosmos. Notice, however, that such a model is not essential to the economy of the present work.

If we accept the reasonable philosophy that our 3-dimensional cosmos is finite and, roughly speaking (i. e. apart from local deformations), with constant curvature, then such a curvature must be positive. Namely, we are led to a 3-dimensional spherical hyper-surface, embedded in a four-dimensional (Euclidean), outer, "abstract" space whose fourth Cartesian axis may be called the "abstract-coordinate" axis. Cf. also Appendix B.

In order to explain Hubble law, our cosmos can thus be imagined as the "surface" of a hyper-balloon[46,47] which started with a radius[48] $R_o \approx 0$, is expanding untill a maximal radius $\bar{R}$, and then will contract again to $R_o \approx 0$. For instance if galaxies are like dots on the balloon hyper-surface, then during the universe expansion they will recede far away from each other. All the points of the cosmos are equivalent (the "center" of the cosmos belongs to the abstract space, and not to the cosmos itself!). Moreover, the fact that the older the detected galaxy-image is, the faster the galaxy appears to move, suggests that speed $\dot{R}(t)$ is decreasing with time. For further details, cf. refs. (10, 49); here let us remember that any observer P will see everything "projected" onto his tangent space (extrapolation of his local, flat space)[26]. See also Appendix B.

Since in its expansion the universe is slowed down by its own gravitation (that acts something like the surface tension in a bubble), roughly speaking we can assume its radius R to change with time as follows:

$$R \simeq v_o t - \frac{1}{2} at^2, \tag{78}$$

where the initial speed v_o is, and remains, the maximal speed (in the abstract space), and can be assumed to be the light-speed c_o at that time. We shall assume moreover c_o to be not far from the present-time light-speed c (cf. eq. (89) in the following).

The maximal radius $\bar{R} = R(\bar{t})$ will correspond (if $c_o \approx c$) to

$$\dot{R} \simeq c_o - \bar{a}\,t \approx c - \bar{a}\,t = 0, \tag{79}$$

whence:

$$\bar{a} \simeq \frac{c_o}{t} \approx \frac{c}{\bar{t}} \approx 10^{-9}\ \mathrm{m/s^2}, \tag{79'}$$

so that

$$R \simeq \frac{c_o \bar{t}}{2} \approx \frac{c\,\bar{t}}{2}\ ;\ \bar{t} \approx \frac{2\bar{R}}{c}. \tag{80}$$

Stictly speaking, the negative acceleration $-\underline{a}$ is however a function of time. As we shall see, we can assume our cosmos to be not far from its maximal expansion ($R \lesssim \bar{R}$), and eq. (78) to hold at least in a certain range $\bar{R} - \Delta R \leq R' \leq \bar{R}$ of values $R' \equiv R(t')$, where now $v_o = v_o(t') \simeq c$. In this last case, we are fully authorized to assume that $v_o(t')$ is not far from present-time light speed.

Within our approximations, if the negative acceleration, $-\underline{a}$, of $R = R(t)$ in the abstract space is due - as previously said - to the gravitational effect of the cosmos-mass M on itself, then[50]:

$$\bar{a} \simeq \frac{GM}{\bar{R}^2}. \tag{81}$$

But, since $\bar{a} \simeq 1/2\,(c^2/\bar{R})$, then it is[51]:

$$\boxed{M \simeq \frac{1}{4}\frac{c^3\bar{t}}{G} \simeq \frac{1}{2}\frac{c^2\bar{R}}{G}}\ ; \tag{82}$$

the fact that, from the value of G, we can calculate the cosmos-mass M is in full egreement with Mach's principle. Vice-versa: $G \simeq \frac{1}{2} c^2 \bar{R}/M \simeq \frac{c}{2} \mathscr{H}/M^2$, (see eq. (115'), Sect. 6).[51]

Our model, though very rough, forwards acceptable results. For instance, using as input-datum only the cosmos expansion-time $\bar{t}$, we can derive both the cosmos expansion-radius $\bar{R}$ and the cosmos-mass M:

$$\boxed{\bar{R} \simeq 10^{26} \text{ m}} \tag{83}$$

and

$$\boxed{M \simeq 10^{53} \text{ Kg}} . \tag{84}$$

Moreover, let us underline that eq. (82) can read(in full accord - as we see - with the standard estimations of modern astrophysics for cosmos radius and mass):

$$\boxed{\bar{R} \simeq \frac{2GM}{c^2}} \tag{85}$$

which yields for the cosmos maximal radius $\bar{R}$ exactly the cosmos "Schwarzschild radius" $2GM/c^2$; cf. the following[52].

We thus predict, incidentally, that the mean density in the cosmos is $\varrho \approx M/c^3\bar{t}^3 \approx 10^{-26} \text{ Kg/m}^3 \approx$ $\approx 10^{-29} \text{g/cm}^3$; moreover let us notice that the condition, set recently by Cook within the Mack-Einstein-Sciama-Dicke explanation of inertia[53],

$$R = \frac{c}{\sqrt{2\pi G}} \approx \frac{3}{2} \frac{GM}{c^2} \approx \frac{3}{4} \bar{R} \tag{86}$$

(where R is the present-time cosmos-radius) is quite natural in our model, - and consistent with our own assumptions.

Owing to the fact that, during expansion, $R=R(t)$ is an increasing function of t, we could choose the axis R as the axis of a certain "cosmological time" $\tau \equiv R/c_o$. One <u>might</u> thus <u>interpret</u> why we can stop our movement in space but not our "movement" in time (i. e. along the "abstract" radial axis). Such a suggestion to consider the "abstract", fourth dimension of our model as a time-coordinate (except for a multiplicative constant with the physical dimensions of a speed) is in agreement also with the considerations in notes[54, 55].

Let us emphasize that our simple model yields the "Hubble law" - as expected - with an Hubble constant not far from the usually accepted value. Namely, let us start by considering two different observers A and B, and call $B\hat{0}A \equiv \beta$, where 0 is the "center" (belonging to the abstract space!) of the hyper-balloon. Then, during the cosmos expansion they will appear to move each far away from the other along a straight-line with the speed

$$\mathcal{U}(t) \equiv \frac{d(\widehat{AB})}{dt} = \beta \frac{dR}{dt} = \beta(c_o - at), \tag{87}$$

which reads

$$u(t) = \beta c \tag{88}$$

<u>as soon as we adopt</u> the physically self-clear identification:

$$\boxed{c \equiv c_o - at \simeq \dot{R},} \tag{89}$$

where $\dot{R} \equiv \dot{R}(t)$ means the value of $\dot{R}$ at the present time t. Eq. (89) tells us also the interesting result that the ligh-speed is connected to the expansion speed of the cosmos in the abstract space[55bis]. Actually this result is consistent with the fact that Relativity seems to predict (by extrapolating the usual "Dragg-effect" formula)[56, 57] what detailed in note[57]. In other words, this result is consistent with a model where our cosmos moves with the light-speed relative to the (four-dimensional) abstract-<u>space</u>, such a motion introducing <u>no</u> anisotropy in our cosmos since the speed $\dot{R} = c$ is directed <u>orthogonally</u> to our 3-dimensional space. This seems to explain why nature suggests[17], as notices e. g. by Minkowski, that 1 second ≡ 299, 792, 500 meters, or rather: 1 second≡i(299, 792, 500) meters.

In our approximations, one could write, remembering eq. (79'):

$$c_o \approx c(1 + t/\bar{t}) \approx 2c; \tag{90}$$

notice that the alternative approximation $c_o \approx (1 - t/\bar{t})^{-1}$ yields e.g. $c_o \approx 3c$ for[53] $t \approx (2/3)\,\bar{t}$. Moreover, we connected by eq. (80) the light-speed c with quantities $\bar{t}$, $\bar{R}$; if we (instead of deriving the value of $\bar{R}$ from our theory) borrow or extrapolate the values of both $\bar{t}$ and $\bar{R}$ from experience, then we can heuristically calculate even the value of the light-speed. In fact, from eqs. (80), (85) and (90) one can write

$$c \approx \bar{R}/\bar{t} \approx 2GM/(c^2\,\bar{t})$$

and then calculate

$$\boxed{c \approx \left(\frac{2GM}{\bar{t}}\right)^{1/3} \approx \frac{\bar{R}}{\bar{t}} \approx \frac{10^{26}\,\mathrm{m}}{3\times10^{17}\,\mathrm{s}} \approx 3\times10^{8}\,\mathrm{m/s},} \tag{91}$$

In conclusion, from eqs. (85), (88) and (89) we get

$$u(t) \equiv u = \beta\cdot(c_o - at) = \beta\cdot c(t) \equiv \beta c,$$

and from the expression $d(t)\equiv d = \beta\cdot R(t) \equiv \beta\cdot R$ of the distance d(t) of the two observers A, B one gets immediately the Hubble law:

$$\boxed{u = (c/R)\,d,} \tag{92}$$

with the Hubble constant

$$H_o \equiv \frac{c}{R} \cong \frac{\dot{R}}{R} \approx (10^{10}\ \mathrm{years})^{-1}, \tag{92'}$$

as follows from eqs. (83) and (89).

The present model is essentially a Friedman model, or rather an Einstein-De Sitter model with non-zero cosmological constant but nevertheless non-static.

For the following of this work we have to retain from this Section 4. 2 only these points: (i) that we got cosmos radius and mass from the age of the cosmos (and from the value of G); (ii) that our cosmos may be considered as a "gravitational black-hole[27] embedded in the "big-universe"[28]. For simplicity, the previous considerations about hyper-space can be forgot in most cases, in the following, and we shall often be able to refer to "Newtonian" models (in three space-dimensions); they are however important to satisfy the Copernican principle.

Here, let us explicitly clarify that the point (ii) above means merely that: (a) according to the previous model, our cosmos can reach (in the "big-universe") the maximal radius $\bar{R}$ given by eq. (85); (b) inside the "surface" corresponding to $\bar{R} = 2GM/c^2$ the radius R of our cosmos oscillates periodically from R_o to $\bar{R}$, and then from $\bar{R}$ back to R_o, and so on.

Before going on, let us clarify that, - if we assume for the spatial part of our cosmos the model of a spherical hyper-surface, - then the same model must be adopted for the "big universe" (as well as for hadrons), and so on. Therefore, the intersection Z of our cosmos with the "big universe" will be a two-dimensional spherical-surface (as well as the intersections z of hadrons with our cosmos: see Sect. 7). As a consequence, the expression "inside a hadron" will have a meaning analogous to "in our cosmos". On the contrary, the expression "outside our cosmos" will have the meaning "outside Z, in the 'big universe'", (and will be quite analogous to "outside z, in our cosmos"); cf. also Wheeler, ref. (41). As we adopt Einstein equations - with cosmological term - in our cosmos (for gravity), so we adopt scaled ("contracted") Einstein equations in hadrons (for strong-"field and scaled ("dilatated") Einstein equations in the "big universe" (whose spatial part is still 3-dimensional!), for infra-gravity. For instance, we can have a priori (gravitational) black-holes in our cosmos strong black-holes in hadrons, infra black-holes in the "big-universe", etc. Notice - however - that, even if we consider each whole cosmos (intersecting the "big universe") as a "gravitational black-hole" and each whole hadron (intersection our cosmos) as a "strong black-hole", for the surroundings of such particular "black-holes" (in the "big universe" or in the cosmos, respectively) we ought to make recourse to equations of the type of eqs. (38), (39), - yielding an exponential damping - due to the interference of their characteristic (strong, gravitational) field with the gravitational, infra-gravitational) "field" of the "embedding", higher-order universe (see Sect. 5).

PART B: FORMALIZATION OF THE THEORY

5. - REFORMULATION OF THE THEORY.

Within our philosophy (cf. e. g. Sect. 4. 1), we wrote Einstein-type field-equations in correspondence to gravitational or to strong "fields", respectively. Generalizing it, we shall write Einstein-type equations in correspondence to any one of our n-order fields (i. e. fields associated to n-order universes). Such equations will admit black-hole-type solutions, so that we can deal with gravitational (or first-order) black-holes, strong (or zeroth-order) black-holes, and so on. Before going on, we ought to remember what stated at the end of Part A.

From Sect. 4, we can infer that our cosmos itself may be considered not only as a (generical) gravitational-universe, but more particularly also as a gravitational "black-hole"(27), intersecting the "big-universe"(28): see end of Sect. 4. 2.; so as hadrons themselves can be considered as "strong black-holes" intersecting our cosmos.

With regard to the gravitational case, let us base ourselves on General Relativity. Let us start from Einstein field-equations(58) with cosmological term, which read (when only gravitational interactions are present):

$$R_{\mu\nu} - g_{\mu\nu} R^{\varrho}_{\varrho} - \Lambda g_{\mu\nu} = - \frac{8\pi G}{c^4} T_{\mu\nu} , \qquad (93)$$

where(58) $T_{\mu\nu} \equiv (P+\varrho_M c^2) u_\mu u_\nu - P g_{\mu\nu}$ is different from zero only inside the source mass-distribution; and where $2\Lambda \equiv (m_G c/\hbar)^2$, quantity m_G being the graviton-mass (see eq. (43)) and quantity ϱ_M being the mass-density.

Eq. (93) characterizes(21, 58) incidentally a gravitational universe with radius $R = \frac{1}{\sqrt{\Lambda}}$.

A priori, eq. (93) holds inside, rather than outside, our cosmos (see Sects. 4. 2. and 2. 5). With regard to the "inside" case, for every spherically-symmetric mass distribution eq. (93) yields a "Schwarzschild solution" with cosmological term (see Sects. 6 and 7). As we shall see, such a solution (for bodies gravitating inside our cosmos) has a quite good behaviour, since it gives the potential Gm/r for not too-large distances and the "confining" potential $c^2\Lambda r^2$ for very large distances (see refs. (59÷61)). However, the situation becomes different when we consider the whole cosmos as a "black-hole", because our cosmos is then embedded in a higher-order universe (the "big-universe"(28)) and no more in a gravitational universe: let us call "infra-gravitational" the field characteristic of the "big-universe". To get the gravitational potential nearby our cosmos (in the big-universe), we must proceed as in Sect. 2. 3 (where we had to refer to a flat background); we can explain the origin of the additional terms in eqs. (38), (39) of Sect. 2. 3 either in geometrical terms [as we did, and shall do (Sect. 8)], or as due to the interference between the gravitational field (present nearby the cosmos) and the "infra-gravitational" field $g_{\mu\nu}^{(INFRA)} \simeq \eta_{\mu\nu}$ (typical of the "big-universe", and representing itself a De-Sitter infra-background deviating from the flat space only over big distances in the big-universe). The last explanation is analogous to the procedure recently followed by Salam (see ref. 42bis) when considering the same problem at the next lower "hierarchical" order (i. e. when considering the surrounding of a hadron in the gravitational universe).

At this point let us postulate that the field equations for gravitational and for strong interactions are respectively given by the corresponding Einstein eqs. with suitable cosmological constants, as follows $[G = N_S = 1]$:

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - \Lambda g_{\mu\nu} = - \frac{8\pi}{c^4} T_{\mu\nu}; \quad 2\Lambda \equiv \left(\frac{m_G c}{\hbar}\right)^2,} \qquad (94)$$

for the gravitational case; and

$$\boxed{\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\varrho}_{\varrho} - H\tilde{g}_{\mu\nu} = - \frac{8\pi}{c^4} S_{\mu\nu}; \quad 2H \equiv \left(\frac{m_S c}{\hbar}\right)^2} \qquad (95)$$

for the strong case; where eq. (94) goes into eq. (95) under a contraction[62, 63] by a factor $\varrho \simeq 10^{-40}$. Quantities m_G, m_S are the masses of the gravitational quanta and of the strong quanta, respectively (see Sect. 3. 2 and 9. 1). Analogously, let us assume eqs. (38÷46) outside our cosmos (in the "big--universe") and the corresponding, scaled equations outside hadrons (in our cosmos) : cf., however, also Sect. 8.

In particular, we shall call Λ the "cosmic" (rather than cosmological) constant, and

$$\boxed{H = \varrho^{-2}\Lambda \approx (10^{40})^2 \; ; \quad H^{-1} \approx 10^{-25}\ \mathrm{cm}^2 = 0{,}1\ \mathrm{barn}} \tag{96}$$

the "hadronic" constant. Tensor $S_{\mu\nu}$ is connected to the strong charge distribution. For instance, in the "static" case, we have

$$T_{oo} \approx \varrho_G c^2 \equiv \varrho_M c^2 \; ; \; S_{oo} \approx \varrho_s c^2 \tag{97}$$

where $\varrho_G \equiv \varrho_M$ and ϱ_S are respectively the gravitational and the strong charge-densities. In other words, the "strong-matter" tensor $S_{\mu\nu}$ is essentially $S_{\mu\nu} = \varrho^{-1} T_{\mu\nu} \simeq 10^{40} T_{\mu\nu}$, where T is a priori the ordinary matter tensor (containing e. g. the Dirac spinorial functions). Let us remember that, owing to what precedes, we shall use eqs. (94) and (95) in connection to purely gravitational interactions or purely strong interactions, respectively. Let us observe that, as in our cosmos we have $E = mc^2$, where m is the gravitational charge, so inside "strong universes" (hadrons) we shall have (within the "inside viewpoint") $E = gc^2 \approx 10^{40} mc^2$, quantity g being the strong charge evaluated within the "inside viewpoint" ($g \approx 10^{40} m$), for all objects having a strong charge (i.e with scale factor $\alpha = \varrho \simeq 10^{-40}$).

In all cases, i. e. in all "universes", we can write for all bodies possessing a non-zero charge of the corresponding order:

$$\boxed{E = m_I c^2} \tag{98}$$

where m_I is the inertial mass in the universe considered (cf. eqs. (66'), (67')).

Let us first consider the gravitational case, i. e. usual Einstein eqs. with cosmological constant[64] (or rather with "cosmical constant"). For a stationary, spherically symmetric mass distribution M, we get[21] in vacuum the Schwarzschild metric $[G=c=1]$:

$$ds^2 = (1 - \frac{2M}{r} + \frac{\Lambda r^2}{3}) dt^2 - (1 - \frac{2M}{r} + \frac{\Lambda r^2}{3})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \cdot d\phi^2). \tag{99}$$

Let us now write, in an explicitly dilatation-covariant way (see Sect. 1):

$$d\sigma^2 = (1 - \frac{2M_{oo}}{\eta} + \frac{\Lambda_{oo}}{3} \eta^2) d\tau^2 - (1 - \frac{2M_{oo}}{\eta} + \frac{\Lambda_{oo}}{\eta} \eta^2)^{-1} d\eta^2 - \eta^2 \, d\Omega , \tag{100}$$

where $d\sigma^2 \equiv d\eta^\mu \, d\eta_\mu$; eq. (100) can read (cf. Sect. 1):

$$\boxed{ds^2 = (1 - \frac{2M_{oo}}{r} \varrho + \frac{\Lambda_{oo}}{3\varrho^2} r^2) dt^2 - (1 - \frac{2M_{oo}}{r} \varrho + \frac{\Lambda_{oo}}{3\varrho^2} r^2)^{-1} dr^2 - r^2 \, d\Omega} \; , \tag{101}$$

where $\varrho = 1$ in the gravitational case, and $\varrho \approx 10^{-40}$ in the strong one.

Then, in the strong case, for a stationary, spherically symmetric distribution of strong-charge, we have in vacuum the "strong Schwarzschild metric" $[N_S = c = 1]$ given by eq. (101), where $\Lambda_{oo} / \varrho^2 \equiv H$.

Before going on, let us once more underline - with respect to eq. (101) - that we started from a spherical, gravitational mass-distribution and then we "collapsed" the whole system "source plus test-object", so that we proceeded as in eqs. (10), (18). However, let us remember that we could have "collapsed" only the source (plus its field), with the convention of testing both the initial and the final field by the same test-object (like a hadron, i. e. an object sensitive to both gravitational and strong fields)(65). In this case we would proceed as in eqs. (15), (20). By comparing those procedures, in the case when one starts from the whole cosmos, - eqs. (38÷46), - with mass $M=M^{(in)}=M_{oo}$, and then uses as test-particle a nucleon, with mass $m(p)\equiv m_p$ and strong-charge $m(p)/\varrho$ within the "inside viewpoint", we got in Sect. 1 that $m(p) \approx 10^{-80} M$. Since in Sect. 4 we calculated the value $M \approx \approx 10^{53}$ Kg, then our theory yields also the nucleon mass, as already claimed: $m(p)\equiv m_p \simeq 10^{-27}$ kg.

Another way to get this result has been put forth by us in connection with eq. (77).

Since the present point looks to be important, here let us briefly reformulate the related procedure, remembering eq. (19) and eq. (54'). Namely, we can consider the initial, gravitational masses as actually invariant:

$$M \equiv M_{oo}; \qquad m \equiv m_{oo}; \ldots,$$

and call them the "charges" (or the "masses") tout court of the initial bodies. Then, after a dilatation, we can include the factor ϱ entering the expression GM_{oo}/ϱ (cf. eq. (19)) or $GM_{oo}m_{oo}/\varrho$ (cf. eq. (54')) into the universal constant G, so that:

$$G \longrightarrow G' \equiv \frac{\mathbb{G}}{\varrho} , \tag{102}$$

as already suggested at the end of Sect. 2. 4 (for an alternative convention, see eq. (56)). For instance, with $\varrho \approx 10^{-40}$ we should pass from the gravitational potential-energy $GM_{oo}m_{oo}/r$ (let us for simplicity forget about the exponential factor appearing in the "external" case) to the strong potential-energy

$$\frac{N_S M_{oo} m_{oo}}{r}; \left[N_S \equiv \frac{G}{\varrho} \approx 10^{40} G \right], \tag{103}$$

where we do not introduce any more both gravitational and strong charges, but only the charges (or masses) m_{oo}, M_{oo}. Of course, this procedure assumes $G \neq N_S \neq 1$, in agreement with the "Generalized Theory of physical dimensions"(66)(and with our Sect. 1. 2, Sect. 2. 4 and footnote(13)). In such a new context, we may call G the gravitation universal-constant and $G_S \equiv G' \equiv N_S$ the "strong gravitation" universal-constant; for instance, in I. S. units:

$$\boxed{G_S = N_S \approx 10^{30} \mathrm{m}^3 \mathrm{Kg}^{-1} \mathrm{s}^{-2} \; \frac{\hbar c}{m_\pi^2}} \tag{104}$$

where(67) of course $[G_S] = [G]$ and the strong field is considered as a "strong" gravitational-field acting (through the constant G_S) on the usual masses (or charges) $m_{oo}=m$; $M_{oo}=M$;...

Going back to the "inside" case, in this new formalism ($G \neq G_S \equiv N_S \neq 1$; $[G_S] = [G]$), for the strong case we get the metric:

$$ds^2 = (1 - \frac{2G_S m_{oo}}{c^2 r} + \frac{Hr^2}{3})dt^2 - (1 - \frac{2G_S m_{oo}}{c^2 r} + \frac{Hr^2}{3})^{-1} dr^2 - r^2 d\Omega , \tag{105}$$

where m_{oo} refers to any hadron-constituent, in analogy to the gravitationa equation (101) which read

$$ds^2 = (1 - \frac{2GM_{oo}}{c^2 r} + \frac{\Lambda_{oo} r^2}{3})dt^2 - (1 - \frac{2GM_{oo}}{c^2 r} + \frac{\Lambda_{oo} r^2}{3})^{-1} dr^2 - r^2 d\Omega , \tag{106}$$

where M_{oo} refers to any cosmos-constituent.

The new formalism is preferable also because it allows assuming for instance $c=\hbar=1$, so to have $[M] = [L^{-1}]$ in agreement with eq. (6); on the contrary, when assuming (as previously) $G=c=1$, then we have $[M] = [L]$, which is a dimensional relation that does not fit elegantly into our "dilatation-covariance" philosophy.

6. - GRAVITATIONAL AND "STRONG" BLACK-HOLES AS PARTICULAR REALIZATIONS OF "UNIVERSES".

In Sect. 4 we saw how our cosmos can be considered (at least under some approximations) as a gravitational "black-hole" (cf. end of Sect. 4. 2) with radius $r_s^{(G)}(U) \equiv R \simeq 2GM/c^2 \approx 10^{26}$m. By extrapolating for a moment eqs. (99) or (106) to the whole cosmos, in correspondence to $M \approx 10^{53}$ Kg; $m_G \approx 10^{-68}$Kg; ($\rho \approx 10^{-40}$), also our reformulated theory yields the unique solution

$$r_s^{(G)}(U) \equiv R \approx 10^{26} \text{ m}, \qquad \Big[(G) \equiv \text{gravitational} \Big], \tag{107}$$

in agreement with eq. (83). Of course eqs. (99), (106) are not to be used for deriving the metric both inside[55] and outside our cosmos (in the "big-universe"[28]: but they can be here enough for our purposes. On the contrary it is always possible (by using suitable coordinates)[55] to describe in an "orthodox" way the Einstein-De Sitter space constituting in our philosophy the "interior" of the cosmos.

With regard to hadrons (e. g. nucleons), they can be considered as "strong black-holes" (see eq. (101)), e. g. extrapolating to the whole "strong universe" the eq. (101) and the Einstein-type equations (95). Namely, we can look for (strong) "black-hole" solutions of eq. (95), that is so say of eq. (101) or (105). The "(strong") Schwarzschild radii), that we find out in this way, will represent the nucleon radius (more generally the hadron radii), as shown by hadrons in strong interactions, in the limit when we choose $m_{oo} = m(p) = m_p$.

Let us start by fixing our attention on eq. (106) for the gravitational case, in the limiting case when $M_{oo} = M =$ cosmos mass. To get the Schwarzschild radii, we need essentially solving the equation

$$1 - \frac{2GM}{c^2 r} + \frac{1}{6} \left(\frac{m_G c}{\hbar}\right)^2 r^2 = 0, \tag{108}$$

which always admits only one solution (cf. eq. (107)). In dilatation-covariant form, eq. (108) writes

$$\eta^3 + 6 \left(\frac{\hbar}{m_G c}\right)^2 \eta - 12 \frac{GM}{c^2} \left(\frac{\hbar}{m_G c}\right)^2 = 0, \tag{109}$$

where as usual $\eta \equiv r/\rho$. In the strong case, it is $\rho \approx 10^{-40}$; so that - if we remember that eq. (107) is the solution of eq. (108) - one might expect the nucleons to have radii $r \approx 10^{-40} \times 10^{+26}\text{m} \approx$ $\approx 10^{-14}$m. However eqs. (108), (109) were of course derived in the case when the fields are tested by small test-objects: in other words, they hold for cosmos-constituents and (only) for the possible, small constituents of nucleons, respectively. On the contrary, when considering quarks - which in any case are not a negligible part of the nucleon - the eqs. (101), (105) are only approximate; and we could heuristically proceed as follows. When quark consideration is essential, then in eqs. (105), (106) - for instance - we should have to compare, with the gravitational term $2GM_{oo}m/c^2r$ (where m is the negligible mass of the test-object), strong terms as $2G_S(m_p - m_q)m_q/(c^2 r)$ (where $m_p \equiv m(p) \equiv m_{oo}$ and $m_q \approx (1/3) m_p$ is the quark-mass) rather than as $2G_S m_p m/(c^2 r)$.

Roughly speaking, we thus get an extra factor of the order of $2/3 \cdot 1/3 \approx 0.2$, with respect to the "negligible-test-object" case, which enters the second term of eq. (109). For the nucleon-radius $r_s^{(S)}(N)$, the eq. (109) then yields - also in our reformulated theory - the unique solution

$$\boxed{r_s^{(S)}(N) \approx 10^{-15}\text{m} = 10^{-13} \text{ cm}}, \qquad \Big[(S) \equiv \text{strong} \Big]. \tag{110}$$

More detailed calculations will follow (they partially appeared in ref. (9)).

In this model (which, incidentally, is a realization of the MIT bag model[68]), the quark confinement is an automatical consequence of considering hadrons as "strong black-holes"- at least when neglecting hadron spin and electric charge. Notice however, that our classical confinement inside the black-hole horizons (which, incidentally, acts only on objects with scale factor $\alpha = \varrho \simeq 10^{-40}$, i. e. feeling the strong field), can moreover be partial, a priori, due to possible quantum effects[60].

Here let us add that within our model the hadrons can be considered - loosely speaking - as being something like cosmoses (which are gravitational black-holes), but with much smaller radii because they possess in their interior much stronger forces. But let now start from a nucleon with its radius given by eq. (110) and endowed with its (internal) strong-field. If the nucleon (let us call $m_p \equiv m(p)$ its unknown mass) had to be a black-hole internally governed by the gravitational field, then its internal distances ought to be reduced by another factor $\varrho \approx 10^{-40}$ in order to obtain the same effects as by the strong field. It means that the gravitational Schwarzschild radius of the nucleon should be

$$r_s^{(G)}(N) \approx 10^{-40} \times 10^{-40} \; r_s^{(G)}(U) \; . \tag{111}$$

By comparing eqs. (107) and (111), since in the gravitational case the Schwarzschild radii are proportional to be masses:

$$r_s^{(G)} \approx \frac{2GM}{c^2} \; ,$$

then it again follows that it must be

$$\boxed{m(p) \approx (10^{-40})^2 \; M(U) \approx 10^{-80} \times 10^{+53} \; Kg \simeq 10^{-27} Kg.} \tag{77}$$

We have therefore derived eq. (77) also within the reformulated theory, and in particular within our "black-hole" model.

If we want to imagine hadrons as produced by contraction (by different factors) of the same "reference hadron", then eqs. (4) and (6) yield:

$$\boxed{r_i \approx \frac{\bar{r} \; \bar{m}}{m_i}} \tag{112}$$

where r_i, m_i and $\bar{r}$, $\bar{m}$ are respectively radii and masses of the hadrons considered and of the "reference hadron". The "reference hadron" can be e. g. the neutron. If $\bar{r} \approx 10^{-15}$ m and $\bar{m} \approx 10^{-27}$ Kg (as for nucleons), then:

$$\bar{r} \; \bar{m} \approx 10^{-42} \; m \; Kg. \tag{113}$$

If we identify $\bar{r} \, \bar{m} \equiv \hbar/c$, so to be able to write the Compton wave-lenght-relation of Quantum Mechanics

$$r_i \approx \frac{\hbar}{m_i c} \; , \tag{112'}$$

then we are able to calculate that:

$$\boxed{\hbar \approx 10^{-34} J \; s.} \tag{114}$$

Analogously, we could proceed in a similar way when considering various cosmoses. In such a case, we should have:

$$R_i \approx \frac{\bar{R} \; \bar{M}}{M_i} \tag{115}$$

where $\bar{M}$, $\bar{R}$ refers e. g. our cosmos (the "reference cosmos"), so that $\bar{M} \, \bar{R} \approx 10^{120} \bar{m} \, \bar{r}$, and we can write $[\mathcal{H}/c \equiv \bar{M} \, \bar{R}]$:

$$\boxed{R_i \approx \frac{\mathcal{H}}{M_i c}} \tag{115'}$$

If we assume that the same c enters both eqs. (112') and (115') - since under dilatation the speeds do not change - then we get for the cosmological correspondent of Planck-constant (i. e. for the "cosmical quantum of action") the value:

$$\mathcal{H} \approx 10^{120} \hbar. \tag{116}$$

If we consider our own cosmos as "corresponding" not to a generical hadron, but just to a spin 1/2 baryon (e. g. nucleon), then according to our model it would be expected to have an angular momentum

$$\Sigma = \frac{\mathcal{H}}{2} \simeq 10^{120} \frac{\hbar}{2} \tag{117}$$

wherefrom it would follow that our cosmos rotates with the angular frequency $\omega \approx 10^{-20} s^{-1}$. This attributes to the cosmos the following rotation-period and frequency:

$$\begin{cases} T \approx 10^{20} s; \\ \nu \approx 10^{-3} (\text{cosmos age})^{-1}. \end{cases} \tag{118}$$

These results, - derived in correspondence with the analogous, semi-classical evaluations of period and frequency of the hadrons themselvers, - are only indirectly associated with its upper limit, evaluated to be very low[69]. Nevertheless, if we disliked results (118), we might then "associate" - within our model - the cosmos to a spin-0 meson (as the pion[19]), rather than to a nucleon (model of the Super-pion). In such a case, we should meet the nice feature of considering our cosmos as having a structure similar to the "quark-antiquark" structure of mesons. In other words, our theory - when taken seriously - would possibly lead to the known model where our cosmos is essentially consituted by two "sub-cosmoses" or "Meta-galaxies", one of matter and the other of antimatter. Moreover let us recall - as already mentioned - that many "numerological" relations are well satisfied just by the pion-mass; for instance, from our eq. (25), the second eq. (29), eqs. (85), (86) and eqs. (92),(93), one can derive Weinberg's relation[19]:

$$m^3 \simeq \frac{\hbar^2 H_o}{c\,G} .$$

It must be explicitly remembered, however, that the "quantum version" of our present model is still an open question. Again, we shall only remember that, due e.g. to the celebrate Hawking effect (see refs. (60) and (70)), Schwarzschild black-holes are predicted within quantum field theory to "evaporate"[70] by emitting particles with a thermal spectrum corresponding to the Hawking temperature:

$$T = \frac{\hbar c^3}{8\pi G M k} = \frac{\hbar}{c} \frac{\chi}{2\pi k} \approx \frac{10^{23}}{M}\ {}^{o}K, \tag{119}$$

quantity k being the Boltzmann constant and M the black-hole mass. In other words, temperature T is proportional[60] to the "surface gravity" χ of the black-hole.

In the case of strong black-holes we should have to deal with the much higher temperatures [since, e. g., quantity χ has to be substituted by the "surface strong-gravity"]:

$$\boxed{T' = \frac{T}{\varrho}}, \qquad \left[\varrho \approx 10^{-40}\right], \tag{120}$$

and therefore with much higher evaporation rates. For instance, if an unstable cosmos ($M \gtrsim M(U)$) has an "evaporation" time of the order of $\tau \approx 10^{17}$s, then an unstable hadron ($\varrho \approx 10^{-40}$) is expected to "evaporate" in a time of the order of

$$\tau' \approx (10^{17}\ s) \times 10^{-40} = 10^{-23} s, \tag{120'}$$

in agreement with the experimental life-times of unstable hadrons in strong interactions. Or rather: for $M \approx M(U)$ from eq. (119) we derive $T \approx 10^{-30}$ °K; therefore for hadrons we immediately predict a temperature $T' \approx 10^{10}$K.

More precisely, from the relation:

$$\boxed{T = \frac{\hbar c}{4\pi k r_s}}, \qquad \left[r_s \equiv \frac{2GM}{c^2} \right], \tag{119'}$$

we immediately get in the case of hadrons ($r_s \approx 10^{-13}$cm):

$$\boxed{T \approx 2 \times 10^{11}\ {}^{\circ}K} \tag{121}$$

which corresponds to an evaporation-time of the order of the strong-interactions decay time (see refs. (60)).

We meet then the problem: if cosmoses and hadrons are (gravitational or strong) black-holes(71), why some hadrons - and possibly some cosmoses - do not evaporate and are stable? A possible answer is that the "quantized" version of this theory must contain some assumptions analogous to the one set by Bohr with regard to electron-orbits in atoms. For instance, Salam(42bis) put forth that the Hawking temperature becomes zero (no thermal radiation) if a certain Regge-like relation holds between spin and masses of hadrons.

By a quantization-condition of that kind we will get not only discrete (stable) hadrons, but also discrete (stable) cosmoses.

With regard to hadrons, let us add that they should correspond to Kerr-Newmann (strong) black-holes(47), rather then to Schwarzschild's. In that case, we shall meet naked singularities(47) : see the following. Moreover, stationary black-holes are generally believed to be characterized only by mass, angular momentum and electric charge, but they can actually be associated even to other quantum numbers(72).

7. - ON QUARK CLASSICAL CONFINEMENT. THE "INSIDE" CASE.

As already mentioned in the previous Sections, we need considering eq. (99)÷(101), (105,106) and (108,109) inside the (gravitational or strong) universes, i.e. inside the "cosmological" (gravitational or strong) "black-holes". (For instance, we are considering our whole cosmos as a "black-hole"). An interesting point arises in this connection, since crossing the Schwarzschild horizon (in General Relativity) seems to be a problem mathematically very similar to the one met in Special Relativity when "crossing" the light-cone in four-momentum-space(73). See also Sect. 4. 2.

Let us start considering eq. (106), for the gravitational case, keeping into mind that similar results will hold for eq. (105), i.e. in the strong case. In the stationary (and small speeds) case, the goedesic equation is $[i, j = 1, 2, 3]$:

$$\frac{d^2 x^i}{dt^2} = \frac{c^2}{2} g^{ij} g_{oo,j} \Longrightarrow \frac{d^2 \vec{r}}{dt^2} = -\frac{c^2}{2} \left(1 - \frac{2mG}{c^2 r} + \frac{\Lambda r^2}{3}\right)\left(\frac{2mG}{c^2 r^2} + \frac{2\Lambda r}{3}\right)\frac{\vec{r}}{r} \tag{122}$$

where m is now any source-mass (not the cosmos mass!), and the second eq. (122) holds in the spherically symmetric case.

In the case of "weak fields", i.e. when we can assume $g^{rs} \approx \eta^{rs}$, then we simply get

$$\frac{d^2 \vec{r}}{dt^2} \approx -\frac{c^2}{2} \vec{\nabla} g_{oo} \approx \left(-\frac{mG}{r^2} - \frac{c^2 \Lambda r}{3}\right) \frac{\vec{r}}{r} \tag{123}$$

so that a test-particle m', for very large values of r, will feel an attractive, confining force proportional to r:

$$F \approx -m'\Lambda \frac{c^2}{3} r, \qquad \left[r \approx \left(\frac{6mG}{c^2\Lambda}\right)^{1/3} \approx R(U) \right]. \tag{124a}$$

In the strong case, we are therefore finding the same kind of confining forces as found e. g. by Nambu and Parisi within the quark-monopole theory[74a]. Let us remember, however, that a whole quark cannot be considered as a test-particle inside hadrons; so that the result (if we define $\vec{F}=g'\vec{a}$, according the "inside viewpoint")

$$F \approx -g'H \frac{c^2}{3} r \qquad \left[r \approx \left(\frac{6N_S g}{c^2 H}\right)^{1/3} \approx r(N) \right] \tag{124b}$$

holds a priori for the possible quark-constituents[10], but it is only approximate for quarks.

If we eliminated the weak-field condition, then from eq. (122) we would have that, for large enough values of r:

$$-\frac{d^2 r}{ds^2} \approx \frac{\Lambda^2 r^3}{9} + \frac{\Lambda r}{3} + \ldots . \tag{125a}$$

so that we would get an even stronger confinement. Analogously, in the strong case we'ld have

$$-\frac{d^2 r}{ds^2} \approx \frac{H^2 r^3}{9} + \frac{Hr}{3} + \ldots \tag{125b}$$

but in our model the conditions ($r > R(U)$); ($r > r(N)$) for the validity of eqs. (125a, b) are almost never satisfied, neither inside the cosmos, nor inside the hadrons; and therefore we are mainly left with eqs.(124a, b), except when the hadrons - for instance - start deforming due to very high energy collisons.

As previously mentioned, eq. (124b) should hold for small hadron-constituents[10] (let us call them partinos), strongly interacting with the other hadron-constituents, rather than for quarks. Analogously, eq. (124a) should hold for small cosmos-constituents (like galaxies, stars and usual bodies), gravitationally interacting with the other cosmos-constituents, rather than for big portions of our cosmos[74b].

Let us notice that eqs. (124a, b) yield a classical confinement which however may be partial, due for instance to quantum effects (cf. also ref. (73)). We should moreover remember that only strongly-interacting objects are a priori expected to be (partially) confined inside hadrons (so as only gravitationally-interacting objects ought to be deemed as confined inside our cosmos).

If we particularize our philosophy by considering our Sect. 6, then - when describing our cosmos and hadrons as "black-holes"- the confinement of their constituents can be understood as due to the horizon properties. Even in this case, however, the confinement can be partial, e. g. due to quantum effects; in any quantum theory, however, quarks can be again "totally" confined by associating to the classical horizon a suitable "barrier" of super-selection rules and of super-conservation laws.

The fact that hadrons are "colour singlets" may then be explained as in Sect. 3. 2, by identifying quark-colour with the sign of quark strong-charge (cf. Sect. 3. 2). Hadrons would then have zero total, net strong-charge (nevertheless, they would strongly interact, so as atoms can electromagnetically interact). Something similar should then happen for the "Meta-galaxies" (sub-cosmoses) constituting our cosmos (remember e. g. the model of the "Super-pion").

We already mentioned (Sect. 6) that, since hadrons can bear e. g. angular momentum (spin, J) and electric charge, e, we ought to deal with Kerr-Newmann[47] "strong black-holes", rather than with Schwarzschild's. But electric charge e of hadrons is very large in comparison to their mass (see ref. (75)), so that the corresponding (axially symmetric) solutions of Einstein-Maxwell equations in the Kerr-Newmann case seem to show - if we accept them - that hadrons should be "naked singularities" rather than Schwarzschild strong black-holes. In such case, we'ld still have quark (partial) confinement due to eq. (124b), and the ("dilatation-invariant") spirit of our theory would not be affected; but we'ld have to modify our model accordingly. However, since hadrons carry futher quantum-numbers(72), it might be convenient (even if not necessary),(72) to generalize our model by looking for (solitonic) solutions(71) in Yang-Mills theories, e. g. following Salam(42bis). In such a new approach, hadrons still behave like strong black-holes (or rather "strong solitons"), - in most cases, - even when endowed with spin, electric charge, etc. And, as we anticipated, stable hadrons might correspond to zero Hawking "temperature", e. g. to a Regge-like relation of the type(42bis):

$$\frac{J(J+1)}{M^2} + N_S\, e^2\, I(I+1) = N_S\, M^2 \tag{126}$$

where I is the isospin. Relations of the kind of eq. (126) look a priori to be able to play in hadron-structure-understanding the same rôle played by Bohr conditions in atomic structure.

At this point, let us add the following. If we consider the hadron interior as being an Einstein-De Sitter-type universe, or rather as having a geometry of Schwarzschild or Kerr-Newmann type, then the SU(3)-symmetric nature of hadrons can possibly be derived just from their internal geometry. In fact, the Schwarzschild and Kerr-Newmann geometries (even with cosmological constant) correspond(21) to Petrov D spaces; and it seems that Petrov D spaces automatically lead(76) to SU(3) symmetry. The same should then be translated in "gravitational" language, for the cosmoses.

Let us moreover notice that, if the space-parts of both our cosmos and the hadrons are (3-dimensional) hyper-surfaces embedded in a four-dimensional space, i. e. if they are the surfaces of four-dimensional spheres, then the intersections of hadrons with our cosmos would be two-dimensional spherical-surfaces. Such ordinay spherical surfaces should just be what we see of hadrons(77). Let us moreover observe that, if we accept that our cosmos expands untill it reaches the maximal radius $\bar{R} = 2GM/c^2 \approx 10^{26}$m (cf. Sect. 4. 2, eq. (85)) and then recontracts, the whole period being of the order of 4×10^{10} years, then each hadron would undergo a similar internal oscillation with maximal radius $r = 2N_S m_p/c^2 \approx 10^{-13}$cm (cf. eq. (53)) with a period of the order of 10^{-22} seconds only. Of course, by usual experiments we should measure the average radii of hadrons. The possible rôle of such a "zitterbewegung", predicted by our theory, with regard to the properties and the quantum behaviour of elementary particles will be considered elsewhere. Here let us simply observe that our theory seems to support some "extended-type" models of elementary particles(78), particularly the ones where the particle-trajectory is defined only by discrete points (connected by a periodic function(78)), so as in Caldirola's model.

To conclude this Section, we have to consider the behaviour of a "universe-constituent" for small values of r. Let us e. g. consider the case of a "partino"(10): the geodesic equation in the strong case writes $[N \equiv N_S]$:

$$a = -\frac{Ng}{r^2} - \frac{c^2 H r}{3} + \frac{N^2 g^2}{c^2 r^3} + \frac{NHg}{3} - \frac{c^2 H^2 r^3}{9} \tag{122'}$$

where a is the radial acceleration. We already discussed in eqs. (124b) and (125b) the yield of eq. (122') for very large values of r. For small values of r, on the contrary, the attractive term $\propto -1/r^2$ dominates (as in the gravitational case). Notice that the repulsive term $\propto +1/r^3$ effectively works only at extremely small values of r, so that $a \simeq 0$ only for $r \simeq 10^{-33}$ cm (and, in the gravitational case, we'ld get $a \simeq 0$ only for $r \simeq Gm/c^2$!). However, we can comply with the requirements of the so-called asymptotic freedom by attributing a kinetic-energy (and an angular-momentum J with respect to 0) to the considered partino(10), i. e. by adding the "kinetic-energy term" to the potential corresponding to eq. (122'); so that for small r, $[r \ll r(N)]$, and with the choice (56):

$$V \simeq \frac{(J/g')^2}{r^2} - \left(\frac{Ng}{r} - \frac{N^2 g^2}{2c^2 r^2} - \frac{c^2 H}{3} + \cdots\right) \simeq -\frac{Ng}{r} + \frac{(J/g')^2}{r^2} ,$$

where g' is the strong-charge of the "partino". Of course the same approximately holds when considering a quark (and its required "asymptotic freedom"), instead of a partino. And in the quark case ($g' \simeq (g-g')/n'$, with n' = 1, 2), one gets $V \approx 0$ for $r \approx 10 \times J^2/(Ng^3)$. If we e. g. borrow from quantum theory the suggestion that $J \approx n\hbar$, then we obtain:

$$V \simeq 0 \Longleftrightarrow r - \frac{n^2 \hbar^2}{N g^3} \times 10 \simeq n^2 \times 0.1 \text{ Fermi}, \qquad \left[n = 1, 2, \ldots \right]$$

so that quarks are expected (in the more "stable" states) to stay at relative distances $r \lesssim 10^{-14}$ cm from each other, and in such conditions to behave as practically non-interaction (free) objects. Notice that the the assumption $J \simeq \hbar$ corresponds to attributing a revolution-speed $v \simeq c$ to the moving quark.

Our eq. (122') seems therefore to give account both of quark confinement "infrared divergency") and of their asymptotic freedom.

Static "partinos", on the contrary, would tend - as we have seen - to a relative position with $r \approx 10^{-32}$ cm

At last, let us consider e. g. the baryons' case ($N \simeq 10^{40}$ G). If we assume the quark "stability radius" to be of the order of 1/100 of the "strong Schwarzschild radius" $r_o \equiv 2Nm/c^2$ of our hadron (considered as a strong black-hole), then - after Castorina - we get the Regge-like relation $J \approx Nm^2/(100c)$ where m is the baryon-mass in Kg; this relation also reads, with m now measured in GeV/c^2,

$$J/\hbar \approx m^2 .$$

8. - THE "OUTSIDE" CASE; AND HADRON RADII.

In Sects. 5 and 7 we have been considering the field equations holding inside hadrons and inside cosmoses, respectively. We are left with the more difficult problem of writing down the fields equations describing (for instance) the metric of our cosmos in the neighborhood of hadrons (i. e. in the neighborhood of their "intersections" with our cosmos, that we call "hadrons" tout court: cf. end of Sect. 4. 2 and Sect. 7). Such field-equations will hold for the bodies with "scale factor" $\alpha' = \varrho \simeq 10^{-40}$; i. e. such a metric will be "felt" by objects possessing both strong-charge and gravitational charge. We already approached that problem in Sect. 2. 3, where we tackled the analogous, "next-higher-order" case of the neighborhood of our "cosmos" in the big-universe: see eqs. (38), (39).

To describe the space-time metric of our cosmos both afar and in the surroundings of hadrons we need a "bi-scale" theory (rather than a "bi-metric" theory), since the metric deformation caused in our cosmos by the hadron-"intersections" has to act strongly — as we already mentioned also in Sect. 2. 3 — only on the bodies with "hadronic charge" (besides the gravitational charge "due to" the Mach principle). That deformation, on the contrary, will act only "gravitationally" on the possible particles possessing only "gravitational- mass" (i. e. with $\alpha = 1$). Incidentally, let us observe that, by analogy, even our cosmos might be crossed by objects - entering it "from outside" - devoid of gravitational charge (and possessing only the "higher-order" infra-gravitational charge).

We want here to exploit a little the philosophy followed in Sect. 2, and particularly in Sect. 2. 3. Before going on, let us remember that according to our theory in the cosmos the Einstein (gravitational field) equations hold:

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = - \frac{8\pi}{c^4} \left[T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\varrho}_{\varrho} \right], \qquad \left[G=1 \right]$$

which can read - as wellknown -

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{94'}$$

since $R^{\varrho}_{\varrho} \equiv R = (8\pi G/c^4) T^{\varrho}_{\varrho} - 4\Lambda$. Let us moreover remember that inside a hadron the strong field-equations hold $\left[N_S \equiv G \varrho^{-1} \simeq 10^{40} G \right]$:

$$\tilde{R}_{\mu\nu} + H \tilde{g}_{\mu\nu} = - \frac{8\pi}{c^4} \left[S_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} S^{\varrho}_{\varrho} \right], \qquad \left[S_{\mu\nu} \equiv N_S T_{\mu\nu} \right]$$

which can read, since $\tilde{R}^{\varrho}_{\varrho} \equiv \tilde{R} = (8\pi/c^4) S^{\varrho}_{\varrho} - 4H$, as follows:

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R}^{\varrho}_{\varrho} - H \tilde{g}_{\mu\nu} = - \frac{8\pi}{c^4} S_{\mu\nu} \tag{95'}$$

where, with the choice (56), one has $S_{\mu\nu} = N_S T_{\mu\nu}$, $\left[N_S \neq G \right]$.

In the surroundings of a hadron we can assume $\tilde{f}_{\mu\nu} \cong \eta_{\mu\nu}$ for the gravitational metric-tensor, in suitable coordinates. However – let us repeat – around a hadron (in our cosmos) we have also the "strong-gravity", acting there (only) on the particles with $\alpha = \varrho \simeq 10^{-40}$. In such a region, when the test-object possesses (also) strong-charge, we can neglect $T_{\mu\nu}$ in comparison with $S_{\mu\nu}$, and possibly the gravitational-field energy-momentum pseudo-tensor(79) $t_{\mu\nu}$ in comparison with $t^{(strong)}_{\mu\nu} \equiv s_{\mu\nu}$.

In the surroundings of any hadron in our cosmos, therefore, when taking account of both gravitational and strong fields, we can assume (in suitable coordinates: cf. eq. (40)):

$$\tilde{g}_{\mu\nu} \equiv \tilde{f}_{\mu\nu} + \tilde{h}_{\mu\nu} \cong \eta_{\mu\nu} + \tilde{h}_{\mu\nu} \qquad (127)$$

where the components of the strong metric tensor $\tilde{h}_{\mu\nu}$ have to vanish for $r \gg 1$ Fermi. The total deformation of the cosmos-metric due to the "intersection" with a hadron can be represented by the superposition (around any hadron) of the two abovementioned fields. Such two fields, of course, can also interfere each other. The terms containing Λ can in any case be neglected, and only the "hadronic (cosmological) constant" $H = \varrho^{-2}\Lambda \simeq 10^{25}$ cm^{-2} will enter. Let us moreover observe that the strong field (completely geometrizable inside hadrons), can still be geometrizable in their neighborhood – in our comsos – provided that we go on attributing to the (hadronic) test-object an inertial mass coinciding with its "strong-mass" or "strong-charge" (see what precedes, particularly Sect. 3. 2). Of course, the strong-mass of objects as photons or leptons is zero (or practically zero) there.

The previous considerations lead us to assuming in the surroundings of a hadron – in our cosmos – the following field-equations valid for test-objects possessing both gravitational and strong charges [remember eqs. (127), and $N_S = G\varrho^{-1}$;$S_{\mu\nu} = N_S T_{\mu\nu}$] :

$$R_{\mu} + H\tilde{h}_{\mu\nu} \simeq -\frac{8\pi}{c^4}(S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho}) , \qquad (128)$$

where the "cosmological (strong) term" with the hadronic constant H takes care of the geometric properties of the strong field around the "source hadron" (and has to be effective in a region with linear size of the order of 1 Fermi). Eq. (128) writes in suitable coordinates:

$$R_{\mu\nu} + H(g_{\mu\nu} - \eta_{\mu\nu}) \simeq -\frac{8\pi}{c^4}(S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^{\varrho}_{\varrho}) , \qquad (39')$$

which is essentially eq. (39) of Sect. 2. 3, re-written for the case of strong interactions. This – therefore – justifies also eq. (38) assumed in Sect. 2. 3, which in the next-lower hierarchical case reads (remember eqs. (94'), (95')):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - H(g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta}\eta_{\alpha\beta}) \simeq -\frac{8\pi}{c^4} S_{\mu\nu} , \qquad (38')$$

and which can be considered as a particular case (i. e. resulting after elimination of the terms negligible in the present case) of the general ("bi-scale") equation that describes the simultaneous presence in the neighborhood of the source-hadron of two fields of different hierarchical orders, provided that we remember the relation $\tilde{f}_{\alpha\beta} \cong \eta_{\alpha\beta}$ in suitable coordinates. The quantity $(1/2)Hg_{\mu\nu} g^{\alpha\beta}\eta_{\alpha\beta}$ appears in our approximations to play the rôle of the interference (mixing) term.

Since $|\tilde{h}_{\mu\nu}| \ll 1$ for $r \gg 1$ Fermi, by following a procedure similar to the one in Sect. 2. 3 in the static limit (and for the strong case) we get $[N_S = G = 1]$:

$$g_{oo} \simeq 1 - \frac{2\,g}{c^2 r} \cdot \exp\left[-rm_S c/\hbar\right] \qquad (45')$$

where actually $g_{oo} \longrightarrow 1$ for $r \gg 10^{-13}$ cm.

Before going on, let us observe that eqs. (39') and (38') can provide a classical field theory of strong interactions, where the strong field is the second-rank tensor $(1/g)\Phi_{\mu\nu} \equiv (1/2)\tilde{h}_{\mu\nu} \cong 1/2 \times (g_{\mu\nu} - \eta_{\mu\nu})$.

Let us now introduce also an alternative approach, following the spirit of refs. (9, 80), even if we shall have to deal with approximations. For test-objects with $\alpha = \varrho \cong 10^{-40}$, and therefore possessing both strong and gravitational charges, in the surroundings of a hadron, in our cosmos, and in suitable coordinates, we can write the field-equations $\left[N_S = \varrho^{-1} G\right]$:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} \simeq -\frac{8\pi}{c^4} \left(N_S T_{\mu\nu} + t^{(strong)}_{\mu\nu}\right) \tag{129}$$

where: (i) we eliminated the negligible terms; (ii) $N_S T_{\mu\nu} \equiv S_{\mu\nu}$ represents the "strong-mass" tensor; and (iii) in our approximations the quantity $t^{(strong)}_{\mu\nu}$ is the energy-momentum tensor of the strong field[79]. By comparing eq. (129) with eq. (38'), we get that it must be

$$t^{(strong)}_{\mu\nu} \cong -\frac{c^2 H}{8\pi} \left(g_{\mu\nu} + \eta_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \eta_{\alpha\beta}\right).$$

If we remember that $\eta_{\mu\nu}$ should rather be substituted by $\tilde{f}_{\mu\nu}$, we realize that $t^{(strong)}_{\mu\nu}$ can be considered as a tensor (and not a pseudo-tensor[79]). Moreover, let us notice that, when approaching the source-hadron surface, $g_{\mu\nu} \longrightarrow \tilde{h}_{\mu\nu}$ (and that "asymptotically" $g_{\mu\nu} \longrightarrow \tilde{f}_{\mu\nu}$, as we already know). As a consequence, if we put:

$$N_S T_{\mu\nu} + t^{(strong)}_{\mu\nu} \equiv S_{\mu\nu} + t^{(strong)}_{\mu\nu} \equiv S'_{\mu\nu} \quad ,$$

then we may suppose to be entitled to write eq. (129) not only in the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} \simeq -\frac{8\pi}{c^4} S'_{\mu\nu} \tag{129'}$$

but also in the equivalent form:

$$R_{\mu\nu} = -\frac{8\pi}{c^4} \left(S'_{\mu\nu} + \frac{1}{2} g_{\mu\nu} S'^{\varrho}_{\varrho}\right), \tag{130}$$

to be compared with eq. (128). However, now we must heuristically introduce "from the outside" the supplementary notion that

$$\left| t^{(strong)}_{\mu\nu} \right| \ll 1 \text{ for } r \gg 10^{-13} \text{ cm} \quad ,$$

since $S'_{\mu\nu}$ now contains both the usual $S_{\mu\nu}$, and the "hadronic (cosmological) term", and the interference term.

Let us for simplicity consider the case of a spherically-symmetric distribution of strong charge. Then, in the vacuum and for a static (strong) field, we can evaluate $t^{(strong)}_{oo}$ by an iterative procedure, following refs. (9, 80):

$$t^{(strong)}_{oo} = \left(\Phi_{oo}/g'\right) \cdot u(r) \simeq \frac{1}{2} \left(g_{oo} - 1\right) \cdot u(r) \ , \tag{131}$$

where $\left[\Phi \equiv \Phi_{oo} \ ; \ N_S = 1; \ \check{\mu} \equiv m_S c/\hbar\right]$:

$$u(r) = \frac{1}{8\pi g'^2} \left[\left|\vec{\nabla}\Phi\right|^2 + \tilde{\mu}^2 \left|\Phi\right|^2\right] \simeq \frac{1}{32\pi} \left[\left|\vec{\nabla}(g_{oo} - 1)\right|^2 + \tilde{\mu}^2 \left|g_{oo} - 1\right|^2\right] . \tag{131'}$$

In the static limit, for the first iteration we can take g_{oo} equal to its zero-order approximation (solution of eqs. (38) after linearization), and write $\left[N_S = c = 1\right]$:

$$\frac{1}{2}\left(g_{oo} - 1\right) \simeq \Phi/g' \approx -\frac{g}{r} \cdot \exp\left[-r\tilde{\mu}\right] \quad ,$$

where such a solution — let us repeat – is valid only "asymptotically" (i. e. for $r \gtrsim 1$ Fermi). Therefore:

$$u(r) \approx \frac{1}{8\pi} \frac{g^2 \cdot \exp\left[-2\tilde{\mu} r\right]}{r^2} \left(\frac{1}{r^2} + \frac{2\tilde{\mu}}{r} + 2\tilde{\mu}^2\right) . \tag{132}$$

As usual $\left[\Phi_{oo}/g' \equiv \frac{1}{2}\, \tilde{h}_{oo} \cong \frac{1}{2}(g_{oo}-1) \equiv \exp\left[\nu(r)\right] \right]$:

$$\check{h}_{\mu\nu} \cong g_{\mu\bar{\nu}}\eta_{\mu\nu} = 2 \begin{pmatrix} e^{\nu(r)} & & & \\ & e^{\lambda(r)} & & \\ & & -r^2 & \\ & & & -r^2 \sin^2\theta \end{pmatrix} \tag{133}$$

where ν, λ are functions still to be determined. Notice that we are essentially looking for "strong black-hole" solutions, considering of course the horizon exterior, in our cosmos.

By insterting eqs. (131), (133) into eq. (129), in the vacuum we get – among the others – the equation (see refs. (80, 9)):

$$-\frac{8\pi g^2}{m'^2 c^2}\, u(r) \approx \exp\left[-\lambda(r)\right] \cdot \left(\frac{1}{r^2} - \frac{1}{r}\,\frac{d\lambda(r)}{dr}\right) - \frac{1}{r^2} \tag{134}$$

where for instance $g^2/\hbar c \approx 15$, and where the test-particle mass m' can be choosen to be $m' \approx$ $\approx m_q =$ quark-(average)-mass, (the test quark being considered a priori as situated outside the horizon). Notice that, as expected, in eq. (134) the "strong" quantity g^2/m'^2 substitutes the gravitation constant G (remember that, when $N_S=1$, then $[G] = [g^2 M^{-2}]$). The exact solution of eq. (134) is[80, 9].

$$\exp\left[-\lambda(r)\right] = 1 - \frac{2\ell}{r} + \frac{\tilde{\mu}k}{r} \cdot \exp\left[-2\tilde{\mu}r\right] + \frac{k}{r^2} \cdot \exp\left[-2\tilde{\mu}r\right], \tag{135}$$

where $k \equiv g^4/c^4 m'^2$ and ℓ is an integration constant with the dimensions of a lenght[80, 9]: $\ell \equiv g^2 m/c^2 m'^2$, quantity m being the hadron mass (e. g., $m \gtrsim m(p) \equiv m_p$).

Obviously, in correspondence with the strong "Schwarzschild radius" $r_s \equiv r_s^{(S)}$ of our strong Schwarzschild geometry, we shall have:

$$\exp\left[-\lambda(r)\right] = 0. \tag{136}$$

Eq. (136) yields values of r_s slightly depending on $\tilde{\mu}$. If we assume $\tilde{\mu} \gtrsim m_\pi c/\hbar$ or $\tilde{\mu} \approx 0$ we get almost the same results. In the simple case $\tilde{\mu} \approx 0$, we arrive at the equation

$$r_s^2 - 2\ell\, r_s + k = 0 \tag{137}$$

which for the nucleon yields e. g. the solutions

$$r_s(N) = \ell \pm \sqrt{\ell^2 - k} = \begin{cases} r_1 \approx 10^{-15}\,\text{cm}; \\ r_2 \gtrsim 0.8 \times 10^{-13}\,\text{cm}. \end{cases}$$

Many alternative interpretations might be suggested for the smaller value r_1. But, since our theory (Sect. 6) yielded only one solution (of the order of 10^{-13}cm) when "extrapolating" the inside case, we must rather neglect the value r_1 as possibly due to the approximations of the method used at the end of this Section; so that we remain with:

$$\boxed{r_S(N) \gtrsim 0.8 \times 10^{-13}\,\text{cm}.} \tag{138}$$

In the case of pions, eq. (137) yields (with m' $\gtrsim$ average effective mass of pion-quarks):

$$r_s(\pi) \approx 1.4 \times 10^{-13} \text{cm}, \tag{139}$$

and, in general $\left[N_S=1\right]$:

$$r_s(\text{hadron}) = \frac{g^2 m_h}{c^2 m'} + \sqrt{\left(\frac{g^2 m_h}{c^2 m'}\right)^2 - \frac{g^4}{c^4 m'^2}} \approx$$

$$\approx \frac{2g^2 m_h}{c^2 m'^2} \approx \begin{cases} \boxed{4g^2/(c^2 m_h)} & \underline{\text{for}} \text{ mesons;} \\ \boxed{6g^2/(c^2 m_h)} & \underline{\text{for}} \text{ baryons;} \end{cases} \tag{140}$$

where m_h is the hadron-mass and m' the average, effective mass of quarks in the hadron considered.

From eq. (133), let us write the line element in the strong case:

$$ds^2 = \exp\left[\nu(r)\right] c^2 dt^2 - \exp\left[\lambda(r)\right] dr^2 - r^2 (d\theta^2 + \sin^2\theta \cdot d\phi^2). \tag{133'}$$

The Laplace-Schwarzschild radii[(81)] previously calculated have been derived from the conditions $\exp\left[\lambda(r)\right] = \infty$; we ought now to verify that on the Laplace-Schwarzschild horizon it is also $\exp\left[\nu(r)\right] = 0$, but the calculation[(80)] of function $\nu(r)$ can be performed only with further approximations. Therefore, we limit ourselves to verify that, in the present case of hadrons, we actually have, as required, that

$$\exp\left[\nu(r)\right] \approx \left\{\exp\left[\lambda(r)\right]\right\}^{-1} \approx 1 - \frac{2g^2 m_h}{rc^2 m'^2} , \tag{141}$$

It is worth while to notice that our "strong" metric (133') together with eqs. (135) and (141), has been shown by Mignani[(9)] to be identifiable with t'Hofft monopole metric (in curved space-times (see refs. (82),(83)).

9. - COMPLEMENTARY REMARKS.

9.1 - AGAIN ON STRONG AND GRAVITATIONAL QUANTA:

We already noticed many times that our eq. (50) is approximate, since we ought to relate "internal" ("external") gravitational-quanta with "internal" ("external") strong-quanta, rather than "internal" gravitons with "external" pions.

The settlement of this problem in our theory is as follows: (i) both gravitation and strong fields are represented by tensorial quantities: cfr. eqs. (94), (95); (ii) the Yukawian field is merely the Φ_{oo} component (in the static limit) of the strong-field tensor $\Phi_{\mu\nu}$. (Cf. also Appendix A).

This means (as previously remarked) that our theory predicts hadron-constituents - let us call them "partinos": cf. Sect. 7 - to interact by exchanging spin-2 strong-quanta, corresponding to the spin-2 gravitons[(84)]. Such ("internal") spin-2 strong-quanta can be identifield with spin-2 "gluons", or even with f^o-mesons. Such an identification shows the connection of our theory with the "strong-gravity" theories[(85)]. Of course, things can – however – be generalized in the spirit of Super-gravity.

Let us remember, at this point, that the "dual theories" of elementary particles have brought to understanding hadron structure in terms of "strings", and that "closed strings" can be associated[(83)] to spin 2.

Conversely, we can relate the (spin-0) pions with ("external") spin-0 "gravitons" transmitting - in the static limit - the gravitational interaction[58] between two close cosmoses in the "big-universe", and associated to the ϕ_{oo} component of the gravitational-field tensor $\phi_{\mu\nu}$.

At the level of numerical evaluations, therefore, our eqs. (30), (32) are only approximate; as a consequence, we could frequently calculate only the orders of magnitude. In order to perform more precise evaluations, we ought to choose for the quantity g, instead of the value $g_{pp\pi}$, the value of "coupling constants" as the "partino-partino-(spin-2)'gluon'" one[86]. Analogously we may have got in Sect. 2. 4 a slightly too high ($\Lambda \approx 10^{-56}$ cm^{-2}) value for the cosmological constant Λ, (although – let us repeat – that value to some authors[26] seems a good one for closed, homogeneous, isotropic models).

It is clear that we can easily comply with the possible requirement that "internal" (spin-2) gravitons - and internal (spin-2) gluons - have exactly zero rest-mass; in fact, our eqs. (43), (48), (49) have to hold only for the "external" quanta, i. e. for "external gravitons" and for pions: cf. eqs. (36) and (37).

At last, let us mention the following problem. When "quantizing" our theory, we might be worried by the fact that the forces mediated by spin-2 quanta are always attractive between like particles. In the strong case, and precisely inside hadrons, this can be accepted for "partinos" but does not seem to be true for quark-quark interactions. However, a solution is offered by the fact that hadrons may be constituted (besides of bradyons) also of tachyonic quarks (see Refs.(87)); and tachyons have been shown to suffer a repulsion when usual particles feel an attraction[87]. If we choose this way out, incidentally, then the sign of the cosmological (hadronic and cosmical) constants might be accordingly changed for the inside cases.

9. 2. - WHY BIG-BANGS?

Lastly, let us put forth a possible hint for understanding the big-bang "explosion" within the present theory. After any cosmic expansion and recontraction[88], we are probably left with a big, collapsing ensemble of 10^{80} neutrons.

If neutrons are strong black-holes, we can imagine that the "Second law of black-hole termodynamics"[89] holds even for them: i. e. that, when a couple of neutrons coalesce, they form a new "strong-black-hole" whose horizon-surface is larger than the sum of the two initial horizon-surfaces (=neutron-surfaces).

It is easy to calculate, then, that the 10^{80} neutrons constituting our cosmos, when coalescing all together, will form a new, Super "strong black-hole" with horizon-surface:

$$S > 10^{80} 4\pi r^2 \gtrsim 10^{51} \text{m}^2 \tag{142}$$

where $r \equiv r(N) \gtrsim 10^{-13}$ cm.

It means that, due to neutron "melting" during the final period of cosmos-contraction, we have a process that builds up a new cosmos which (at the end of such a process) must have a radius

$$\boxed{R > 10^{25} \text{m}} \tag{143}$$

as follows from eq. (142). The previous consideration may constitute a starting point for investigating the big-bang "explosion" that supposedly create a new cosmos.

The fact that we pass from a gravitational "black-hole" (old cosmos) to a "strong (Super) black-hole" should not be misunderstood, since - due to our theory of the hierarchy of "universes" - inside each new "cosmos" the (internal) observer would just feel what we usually call "gravitational" interactions.

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APPENDIX A

Einstein's equations with cosmological term can be written

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\varrho}_{\varrho} - \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}; \quad \left[\Lambda > 0\right] \tag{A1}$$

or in the equivalent form:

$$R_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\varrho}_{\varrho}). \tag{A2}$$

Let us linearize eqs. (A2) with respect to De Sitter (non-static) space. To this aim let us put

$$g_{\mu\nu} = e^{2\sigma} (\delta_{\mu\nu} + h_{\mu\nu}), \qquad \left[\delta_{\mu\nu} \equiv \eta_{\mu\nu}\right] \tag{A3}$$

where $e^{2\sigma} \delta_{\mu\nu}$ is De Sitter-space metric tensor[90] with $e^{-\sigma} = \Lambda + \Lambda x_\mu x^\mu / 12$, and where the quantities $h_{\mu\nu}$ are first-order corrections to the components $\delta_{\mu\nu}$ = diag(1, -1, -1, -1). In other words, we require that

$$\left| h_{\mu\nu} \right| \ll 1. \tag{A4}$$

By inserting eq. (A3) into eq. (A2), under the conditions (A4), we obtain, following the procedure in ref. (90), the linearized Einstein equations (with respect to the De Sitter background):

$$(\partial_\mu \partial^\mu + 2\Lambda) \psi \approx \frac{16\pi G}{c^4} T^{\varrho}_{\varrho} \tag{A5a}$$

$$(\partial_\mu \partial^\mu + \frac{2}{3}\Lambda) \psi_{\alpha\beta} \approx \frac{16\pi G}{c^4} (T_{\alpha\beta} - \frac{1}{4} \delta_{\alpha\beta} T^{\varrho}_{\varrho}), \tag{A5b}$$

where

$$\begin{cases} \psi = \delta^{\mu\nu} h_{\mu\nu} \equiv h; \\ \psi_{\alpha\beta} = e^{2\sigma} (h_{\alpha\beta} - \frac{1}{4} \delta_{\alpha\beta} h). \end{cases} \tag{A6}$$

Eqs. (A5a) are relativistically covariant massive equations for scalar and tensorial fields, respectively, with[90]:

$$2\Lambda \equiv \left(\frac{m_o c}{\hbar}\right)^2, \tag{A7a}$$

$$\frac{2}{3}\Lambda \equiv \left(\frac{m_2 c}{\hbar}\right)^2, \tag{A7b}$$

where m_o is the scalar-field mass and m_2 the tensor-field mass.

If we restrict ourselves to the case of stationary sources, then the only nonvanishing components of $T_{\alpha\beta}$ is $T_{oo} = \gamma c^2$ where γ is the mass density; so that $T^{\varrho}_{\varrho} = T_{oo}$. In the case of a spherically symmetric mass-distribution we get from eqs. (A5a), (A5b) the following solutions (holding outside the spherically symmetric mass M):

$$\begin{cases} \psi = \dfrac{4GM}{c^2 r} \exp\left[-\sqrt{2\Lambda}\, r\right] ; \\ \psi_{oo} = -\dfrac{3GM}{c^2 r} \exp\left[-\sqrt{\dfrac{2\Lambda}{3}}\, r\right] \end{cases} \tag{A8}$$

In order to find the correspondence between ψ, ψ_{oo} and the gravitational potential $V = \phi/m$, let us calculate the acceleration $\vec{a}$ in the case of stationary field:

$$\frac{dx^r}{ds} = -\Gamma^s_{\mu\nu} u^\mu u^\nu = -\frac{1}{2} \delta^{rs} g_{oo,s} \; ; \qquad \begin{bmatrix} r,s = 1,2,3 \\ \mu,\nu = 0,1,2,3 \end{bmatrix},$$

or equivalently $\vec{a} = -(c^2/2)\vec{\Delta} g_{oo}$. On the other hand, from eq. (A3) we obtain

$$g_{oo} = e^{2\sigma} + e^{2\sigma} h_{oo} \equiv 1 + \frac{2V}{c^2} .$$

Now let us split $V = V_o + V_1$ and let us identify

$$\frac{2V_1}{c^2} \equiv e^{2\sigma} h_{oo} \tag{A9}$$

$$\frac{2V_o}{c^4} \equiv e^{2\sigma} - 1. \tag{A10}$$

Further, from eq. (A6) we have

$$e^{2\sigma} h_{oo} = \psi_{oo} + \frac{1}{4} e^{2\sigma} \psi ;$$

therefore eq. (A9) becomes:

$$\frac{2V_1}{c^2} = -\frac{3GM}{c^2 r} \exp\left[-\sqrt{\frac{2\Lambda}{3}}\, r\right] + e^{2\sigma} \frac{GM}{c^2 r} \exp\left[-\sqrt{2\Lambda}\, r\right] \tag{A9'}$$

where we have taken into account the solutions (A8). In the special case $\Lambda = 0$, eqs. (A9') takes the form of the usual Newtonian gravitational potential

$$\phi \equiv mV = -\frac{mGM}{r} . \tag{A11}$$

When $\sqrt{2\Lambda}\, r \longrightarrow 0$, at the moment $t = 0$, eq. (A9') tends to the same form (A11).

To study the general case, let us rewrite eq. (A9') into the form

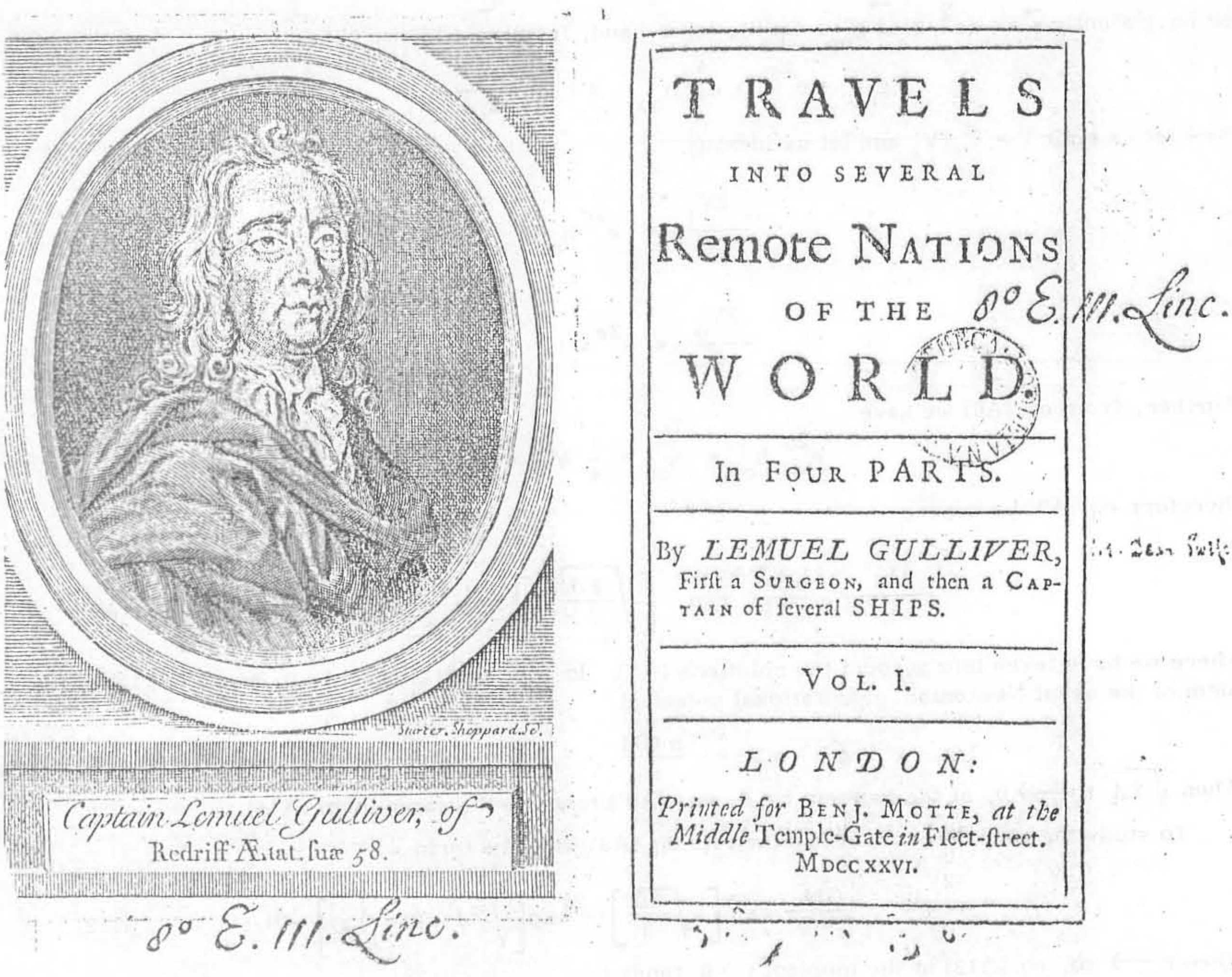
$$\frac{2V_1}{c^2} = \frac{GM}{c^2 r} \exp\left[-\sqrt{\frac{2\Lambda}{3}}\right] (e^{2\sigma} \exp\left[-\sqrt{2\Lambda}\, r(1-\frac{1}{\sqrt{3}})\right] - 3). \tag{A12}$$

When $r \longrightarrow \infty$, eq. (A12) at the moment $t = 0$ tends to

$$\phi = -\frac{3GmM}{2r} \exp\left[-\sqrt{\frac{2\Lambda}{3}}\, r\right] .$$

APPENDIX B

For briefness' sake, we do not include hare this Appendix B. It will appear in the printed version of this report, and will deal with an introduction to "Projective Relativity" and to "Conformal Relativity".



Sturter. Sheppard Sc.

Captain Lemuel Gulliver, of Redriff Ætat. suæ 58.

8° E. III. Linc.

TRAVELS INTO SEVERAL Remote NATIONS OF THE WORLD.

In FOUR PARTS.

By LEMUEL GULLIVER, First a SURGEON, and then a CAPTAIN of several SHIPS.

VOL. I.

LONDON:

Printed for BENJ. MOTTE, at the Middle Temple-Gate in Fleet-street. MDCCXXVI.

FIG. 1- The title page of the original edition of the famous book by J. Swift.

REFERENCES (AND FOOTNOTES)

1) See e. g. T. Fulton, F. Rohlish and L. Witten: Rev. Mod. Phys. 34, 442 (1962): A. O. Barut and R. B. Haugen: Ann. of Phys. 71, 519 (1972); H. A. Kastrup: Ann. Phys. (Lpz.) 7, 388 (1962); J. Wess: Nuovo Cimento 18, 1086 (1960); J. Nierderle and J. Tolar: Chech. Journ. Phys. B23, 871 (1973); G. Mack: Nuclear Phys. B5, 499 (1968). See also E. Cunningham: Proc. Lond. Math. Soc. 8, 77 (1910); H. Bateman: Proc. Lond. Soc. 8, 223 (1910).

2) See e. g. A. O. Barut and R. B. Haugen: ref. (1); F. Hoyle and J. V. Narlikar: Nature 233, 41 (1977); Ann. of Phys. 62, 44 (1971); "Action at a distance in physics and cosmology" (Freeman Pub. S. Francisco, Cal.; 1973); F. Hoyle: quoted by P. C. W. Davies in Nature 255, 191 (1975); E. A. Lord: Nuovo Cimento 11B, 185 (1972); S. J. Aldersley: Phys. Rev. 15D, 370 (1977).

3) In terms of unchanged measure-units.

4) With respect to the fixed frame. Of course, different physical system a priori can be differently dilatated: cf. refs. (5).

5) M. Pavšič: Int. Journ. Theor. Phys. 14, 299 (1975); Nuovo Cimento B41, 397 (1977).

6) H. A. Kastrup: ref. (1); A. O. Barut and R. B. Haugen: ref. (1); M. Pavšič: refs. (5).

7) Even if, in conformal relativity, other transformations - as the "special conformal" ones - are considered, which here we do not need. Incidentally, let us notice that in the following we shall use for simplicity the same symbol for objects and for their charges (in this case, gravitational charges: i. e. masses).

8) The conformally-invariant chronotopical-coordinates(4) are defined differently from eq. (6) since - in the c=G=1 units - it is $[T] = [L]$, but $[M] = [L^{-1}]$.

9) E. Recami and P. Castorina: Lett. Nuovo Cimento 15, 347 (1976); R. Mignani: Lett. Nuovo Cimento 16, 6 (1976).

10) P. Caldirola and E. Recami: "Proceeding of the 1976 Conference on the Concept of Time" (S. Margherita L.), to appear; Report INFN/AE-77/6 (1977).

11) Even one of the present authors used different numbers in ref. (9).

12) According to the international conventions we shall use I. S. units, as far as convenient.

13) Following ref. (9), we might ask ourselves which will be the physical dimensions of the universal constants N_S, N_W of the nuclear fields. Being interested in the physical dimensions, we can go on assuming numerically $N_S = N_W = 1$ (in terms of their measure-units). Then, we might try to be guided by the coincidence that from eq. (31) one gets a value of g_W not far from $1,6 \times 10^{-19}$. [If N_W was a pure number, then $g_W^2 \approx (1,6 \times 10^{-19})^2$ Joule x m, so that numerically $g_W \approx e$]. We might therefore assume $g_W = e$ even dimensionally. From that assumption, N_W results then to have the physical dimensions of the inverse of a dielectric constant ε_o:

where we used Maxwell's symbols. $$\left[N_W\right] = \left[\varepsilon_o^{-1}\right]$$

What we are more interested in is the following. For the reasons we are going to see in this paper, let us make the (second) assumption that - dimensionally -

$$\left[N_S\right] = \left[G\right] \tag{34}$$

It follows that:

$$\left[g\right] = \left[M\right]. \tag{35}$$

We shall in the following deal with the problem of the value of the ratio g/M for hadrons. Notice since now that the starting positions either $N_S = G = 1$ or $N_S \neq G$ lead to two possible, different, alternative procedures.

14) Except for the fact that eqs. (30), (32) hold only in the $pp\pi$ case, and should be changed in other case.

15) P. F. Browne: Inter. Journ. Theor. Phys. 15, 73 (1976); R. v. B. Rucker: "Geometry, Relativity and the fourth dimension" (Dover; New York, 1977).

16) As already mentioned, we are skipping the "weak" case (which could be associated only to the "electromagnetic" case, as well-known). For the "weak universe" (see the following) we would nevertheless get a characteristic radius of the order of 1 cm.

17) Cf. e. g. H. Weyl: Ann. Phys. 59, 129 (1919); A. Eddington: "The expanding universe"(Univ. of Mich. Press; Ann Arbor, 1958); P. A. M. Dirac: Proc. Roy Soc. A165, 199 (1938); E. R. Harrison: Physics Today (Dec. 1972), p. 30; Ya. B. Zeldovich: Soviet Physiscs-Uspekhi 11, 381 (1968); R. Hagedorn: in "Cargès lectures in physics", vol. 6, (E. Schatzman Ed.), Gordon and Breach, (New York, 1973), p. 643; I. W. Roxburgh: Nature 268, 504 (1977); F. Hoyle and J. V. Narlikar: ref. (2).

18) We shall use the metric with signature (+---).

19) See e. g. S. Weinberg: "Gravitation and Cosmology" (J. Wiley Pub. ; New York, N. Y. , 1972).

20) M. Pavšič : unpublished (1976); C. Sivaram and K. P. Sinha: Phys. Letters 60B, 181 (1976) and references therein; Progr. Theor. Phys. 55, 1288 (1976); E. A. Lord, K. P. Sinha and C. Sivaram: Prog. Theor. Phys. 52, 161 (1974). See also B. M. Tinsley: Phys. Today (June 1977); p. 32; P. G. O. Freund, A. Maheshwari and E. Schonberg: Astrophys. Journ. 157, 857 (1969); N. Dadhich: Journ. Phys. A10, 1111 (1977); K. Lake and R. C. Roeder: Phys. Rev. D15, 3513 (1977).

21) See e. g. J. L. Anderson: "Principles of Relativity Physisc" (Academic Press; New York, N.Y. 1976).

22) See e. g. L. Landau and Lifshitz: "Théorie du Champ" (MIR Pub. , Moscow, 1966).

23) If M represents the whole-cosmos mass and g the whole-hadron strong-charge, then - strictly speaking - m_G should be the "graviton" mass outside the cosmos, i. e. the mass of "gravitons" exchanged between two cosmoses. See the following.

24) Remember that attributing a gravitational charge to gravitons is consistent only with an actual non-linearity of the field equations.

25) D. H. Weinstein and J. Keenney: Nature 247, 140 (1974); cf. also W. Yourgrau and J. F. Woodward: Acta Phys. Hung. 30, 323 (1971).

26) G. Arcidiacono: "Relatività e Cosmologia" (Veschi Pub. ; Roma, 1973), and references therein . See also A. G. Agnese and P. Calvini: Phys. Rev. D12, 3804 (1977); A. Agnese, M. La Camera and A. Wataghin: Nuovo Cimento 66B, 202 (1970).

27) See e. g. I. M. Freeman: Am. Journ. Phys. 43, 644 (1975) and refs. therein. See also refs. (9, 10) and G. W. Gibbons and H. Hawking: Phys. Rev. D15, 2738 (1977).

28) See e. g. F . Hoyle: American Scientist 64, 197 (1976). Let us notice that it seems somewhat unjustified (both philosophically and physically) to assume - as usually done - that our own cosmos coincides with the "totality" of all what exists. Actually, we often use the word "universe" in that sense. But we perhaps behave as the past scientists who believed that the Earth (and, later the Solar system) constitute the whole universe, when we assume today the universe to be not much larger than what we can presently "see". It is probably more reasonable to consider a priori our cosmos as a part of a larger entity (or "big universe").

29) Since we found - at a certain extent - a series of "Chinese boxes" (or "Russian dolls") in the direction of the micro, nothing a priori forbids us to assume a similar structure even in the direction of the macro (i. e. in the opposite direction in scale).

30) Cf. e. g. A. O. Barut: Phys. Rev. D3, 1747 (1971); A. O. Barut and J. Nagel: Phys. Letters 55B, 147 (1975); T. Sawada: Nuclear Phys. B71, 82 (1974). See also ref. (9).

31) See e. g. A. Salam: in "Developments in High-Energy Physics (Proceedings of Varenna LIV Course)" page 415 folls. (Academic Press; New York, N. Y. ; 1972); C. Sivaram and K. P. Sinha: Lett. Nuovo Cimento 8, 324 (1973); 9, 740 (1974); 10, 227 (1974); Phys. Letters 60B, 181 (1976); E. A. Lord, K. P. Sinha and C. Sivaram: Progr. Theor. Phys. 52, 161 (1974); Curr. Sci. 44, 143 (1975); A. Salam and J. Strathdee: Preprint IC/76/125 (ICTP, Trieste; 1976) . See also E. Recami and P. Castorina: ref. (9); M. N. Mahanta: Lett. Nuovo Cimento 16, 242 (1976); S. Blaha: Lett. Nuovo Cimento 18, 60 (1977); A. Inomata: Lett. Nuovo Cimento 18, 73 (1977); see also ref. (42bis).

32) P. Caldirola: Nuovo Cimento 17, 68 (1940); 19, 25 (1942); M. Pavšič: Lett. Nuovo Cimento 17, 44 (1976); Nuovo Cimento B41, 397 (1977); P. Caldirola: Lett. Nuovo Cimento 18, 465 (1977). See also E. Recami and G. Ziino: Nuovo Cimento 33A, 205 (1976) and references therein.

33) S. Weinberg: ref. (19), page 392.

34) The same holds for eq (36) if we consider the external interactions, instead of the internal ones.

35) J. Swift: "Travels into several remote nations of the world by Lemuel Gulliver" (Benj. Motte Pub. London, 1726); see also F. Rabelais: "Les Ouvres, cotenans cinq livres de la vie, faicts et dicts heroiques de Gargantua et de son fils Pantagruel. . . ", Lyon (1565). See Fig. 1.

36) In our language, the usual Mack's principle essentially asserts that: "the inertia of a body is a result of the interaction between that body and all the other bodies in the universe"; see E. Mach: "The Science of Mechanics", Chapt. 1 (La Salle, Ill.; 1942); B. Berkeley: "The Principles of Human knowledge" (London, 1937); A. Einstein: "The meaning of Relativity" (Princeton, N. Y.; 1955), page 100; D. J. Raine: Montly Notice RAS 171, 507 (1975).

37) The internal observer would call such interaction "gravitational", but we call it "strong".

38) It this process is allowed.

39) See e. g. Y. Nambu: Scient. Amer. 235, (5), 48 (Nov. 1976); W. K. Jentschke: Cern Courier (2), 4 (Feb. 1976).

40) In order to comply with the standard conventions of general relativity, we can for instance assume $G=N_S=c=1$, wherefrom it would follow e. g. $\underline{m}_p=\underline{m}(N)\approx 1.2\times10^{-52}$cm; $\underline{e}\approx 1.3\times10^{-34}$cm; $\hbar\approx 2.6\times10^{-66}$ cm^2; Planck mass $\equiv (\hbar c/G)^{1/2}\approx 2.2\times10^{-5}$g $\approx 1.6\times10^{-33}$cm; and so on.

41) Notice, incidentally, that $\lambda\approx 10^{-33}$cm is considered to be a "fundamental lenght" in general relativity; cf. e. g. D. I. Blokhintsev: Nuovo Cimento 16, 382 (1960); E. A. Raucher: Lett. Nuovo Cimento 7, 361 (1973); M. Toller: Nuovo Cimento B40, 27 (1977); J. A. Wheeler: in "Battelle Rencontres", C. M. De Witt and J. A. Wheeler Eds. (Benjamin Pub; New York, 1968), p. 242; V. Canuto and J. L. Lodenquai: Astrophys. Journ. 211, 342 (1977); Ya. B. Zeldovic and I. D. Novikov: "Stars and Relativity" (Chicago, Ill.; 1971), pages 64÷66 and so on. Notice, furthermore, that $\underline{g}\approx 50\,\underline{e}$, quantity $\underline{e}$ being the electron or proton electric charge, in agreement with eqs. (28), (30).

42) See e. g.; M. A. Markov: Zh. Eksp. Teor. Fiz. 51, 878 (1966); E. R. Harrson: ref. (17); W. K. Jentschke: ref. (39); Ya B. Zeldovich and I. D. Novikov: ref. (41), page 73.

42a) After the completion of the present work, we received the interesting preprint A. Salam: Preprint IC/77/6 (ICTP., Trieste; 1977), where similar considerations are presented. See also preprint IC/74/55 (ICTP, Trieste, 1974); C. J. Isham and D. Storey: Preprint IC/77/9 (ICTP, Trieste, 1977); and E. W. Mielke: Phys. Rev. Letters 39, 530 (1977).

43) See e. g. R. H. Dicke: Nature 192, 440 (1961); B Carter: (unpublished); C. B. Collins and S. W. Hawking: Astrophys. Journ. 180, 317 (1973). As before, we never dilatate their distance r, even if the interactions under exam are functions of r (that must run in suitable ranges).

44) For an example of unification of gravitational and electromagnetic interactions, within conformal relativity, see M. Pavšič: Preprint PP/500 (Catania Università, 1976), to appear in Nuovo Cimento B. See also G. Arcidiacono: ref. (26); Rendic. Acc. Lincei 18, 515 (1955).

45) Essentially, our model is an Einstein model, with non-zero cosmological constant but non-static. Cf. B. M. Tinsley: ref. (20); D. D. Ivanenko: (to appear).

46) See e. g. Fig. 27. 2, page 719 of ref. (47). See also, e.g. J. D. Barrow: "Modern Cosmological Models", preprint (Dept. Astrophys., Univer. Oxford), 1977.

47) C. W. Minsner, K. S. Thorne and J. A. Wheeler: "Gravitation" (W. H. Freeman Pub.; San Francisco, Cal.; 1973); H. A. Lorentz: "Problems of Modern Physics" (New York, 1967: reprint of the 1927 Ed).

48) By packing together about 10^{80} neutrons, we'ld get a sphere with radius $r\approx 5\times10^{11}$ m $\approx$ 25 light-minutes, corresponding (in four dimensions) to $R_0\approx 3\times10^{11}$m $\approx$ 15 light-minutes.

49) See e. g. G. Arcidiacono, ref. (26).

50) Actually, the gravitational force does not act through the abstract space, but propagates only along the "cosmos", i. e. the balloon hyper-surface. The effect is analogous to the one produced by surface tension in a soap bubble, as already noticed. Eq. (81) can just be derived by considering the effects of the "surface-tension" on a hyper surface embedded in a four-dimensional space.

51) Notice that relation (82) does not mean that G grows with time, since $\bar{t}$ and $\bar{R}$ are (maximal-expansion) constants. The same held, a priori, in eq. (73).

52) See D. W. Sciama: Monthly Notices RAS 113, 34 (1953). See also, e. g. D. M. Eardeley: Phys. Rev. Letters 33, 442 (1974); C. W. Misner, K. S. Thorne and J. A. Wheeler: ref. (47); H. Bondi: Monthly Notices RAS 142, 333 (1969).

53) R. J. Cook: Nuovo Cimento B35, 25 (1976). See also J. A. Wheeler: in "General Relativity and Gravitation (GR7)", G. Shaviv and J. Rosen Eds. (J. Wiley Pub. New York; 1975), p. 321.

54) Eq. (82), as we noticed, reveals that in our four-dimensional model the cosmos maximal radius is equal to its "Schwarzschild radius" (as calculated, in a 3-dimensional space) $R_S = 2GM/c^2$. Cf. eq. (85) and the end of Sect. 4. 2. Therefore, the cosmos expansion/contraction theory (which indeed seems to require even photons to go back to the same space singularity, after that expansion is finished) suggests for the cosmos a particular motion "inside the horizon"(55), where the expansion ("white-hole" phase) turns into collapse ("black-hole" phase) as soon as the maximal radius $\bar{R} = R_S$ is reached(52). Even if many problems are of course left unsolved on this respect, nevertheless we can now recall that, inside the "horizon"(55), the radial coordinate seems actually to play the rôle of a time: in other words, during the expansion phase the cosmos might behave as the interior of a white-hole, and during contraction as a black-holes's interior(52). See end of Sect. 4. 2.
Therefore, our impossibility to stop our motion along the time axis would become equivalent to the impossibility of stopping the motion along the radial coordinate inside a horizon.

55) The meaning of the expression "inside the horizon" is given e.g. in the paper V. De Sabbata, M. Pavši and E. Recami: Lett. Nuovo Cimento 19, 441 (1977); see also C. T. Cunningam: Preprint DAP-395 (Calt Pasadena, Cal.; 1975).

55b) After the completion of this work , we knew about a similar result by J. B. Barbour and B. Bertotti: Nuovo Cimento B38, 1 (1977). See also V. Canuto, S. H. Hsieh and P. J. Adams; Phys. Rev. Letters 39, 429 (1977); E. Elizalde: J. Math. Phys. 19, 526 (1978).

56) See e. g. E. Recami and R. Mignani: Rivista Nuovo Cimento 4, 271 (1974). See also C. A. Lopez: (unpublished); W. Pauli: "Theory of Relativity" (New York, N. Y., 1958), p. 4; and references in M. Ruderfer: Lett. Nuovo Cimento 13 , 9 (1975).

57) Let v and v_{obs} be the light-speed (in a given medium) relative to the medium itself and to the observer, respectively. Then, if $n_o \equiv c/v$, we can define $n_{obs}(\theta) \equiv c/v_{obs}(\theta)$. From the velocity composition law, it follows that the "observed refraction-index" is $n_{obs}(\theta) = (cn_o + u\cos\theta)/(c + un_o)$, where u is the medium-speed. Now, let us consider the abstract "space" as a medim; such a "medium" (with respect to the hyper-balloon) "flows" orthogonally to our 3-dimensional space at any point. In other words, since at any point $\vec{u}$ is parallel to $\vec{R}$, then it is always: $\vec{u} \perp \vec{v}$ and $\cos\theta = 0$. Now, if photons are objects at rest in the abstract space ($n_o = 0$), then we get $n_{obs}(90^o) = 1$, so as it actually happens for the "vacuum" in our cosmos. What precedes, by the way, seems to suggest that Michelson-Morley experiments are consistent also with an "ether" moving with the light-speed orthogonally to our 3-dimensional space at any point.

58) See e. g. S. Weinberg: ref. (19); Ya. B. Zeldovich and I. D. Novikov: ref. (41). Our conventions follow essentially J. L. Anderson: ref. (21). See also E. Finlay-Freundlich: "Cosmology", in vol. 1, (No. 8), Intern. Encycl. Unified Sc. (Chicago, 1951).

59) Such a "confinement" (even if possibly partial, due to quantum-effects(60)) is particularly meaningful for us in the case(9) of strong black-holes.

60) See e. g. S. W. Hawking: Comm. Math. Phys. 25, 152 (1972); 43, 199 (1975); Scient. Am. 236, 34 (1977, Jan.); "GR8" lectures (Watrloo, Ont.; 1977).

61) Such a solution can be interpreted (cf. Sect. 7) as yielding the potential GM/r superposed to a geometric background ($\sim 1 + \frac{\Lambda r^2}{3}$) of De Sitter type(19), in full accord with our cosmological Einstein-De Sitter model. All the same will hold, mutatis mutandis, in the strong case. See also ref. (42').

62) Our present philosophy, based on the assumption of the dilatation-covariance of physical laws, is in the direction of answering the problem posed by Einstein in his last writing(63) (i. e. the problem why field equations admit also solutions of the type $g_{\mu\nu}(x^\sigma/a)$ whenever they admit a solution $g_{\mu\nu}(x^\sigma)$).

63) A. Einstein: in "Cinquant'anni di Relatività", M. Pantaleo Editor (Firenze, 1955), page XX. See quotation at the very top of this article.

64) See e. g. J. Weber: "General Relativity and Gravitational waves" (Interscience Pub.; New York, N. Y.; 1961).

65) Remember also that - to compare the strenghts of the initial and final fields - we do not scale r.

66) See e. g. E. Recami and C. Spitaleri: Scientia 110, 197 (1975) and references therein. See also J. Silk: Nature 265, 710 (1977). and references therein; S. J. Aldersley: ref. (2).

67) Compare also note(13) and eq. (34).

68) A. Chodos, R. J. Jaffe, K. Johnson, C. B. Thorn and V. Weisskopf: Phys. Rev. D9, 3471 (1974).
69) See e. g. G. Cavallo: Nature 245, 313 (1973) and references therein. See also C. B. Collins and S. W. Hawking: Monthly Notice RAS 162, 307 (1973); A. M. Wolfe: Astrophys. Journ. (Letters) 159, L 61 (1970); A. J. Fennelly: Astrophy. Journ. 207, 693 (1973).
68) A. Chodos, R. J. Jaffe, K. Johnson, C. B. Thorn and V. Weisskopf: Phys. Rev. D9, 3471 (1974).
69) See e. g. G. Cavallo: Nature 245, 313 (1973) and references therein.
70) See ref. (60) and S. W. Hawking: Nature 248, 30 (1974); in "Black-holes", C. and B. S. De Witt Editors (Gordon and Breach; New York, N. Y.; 1973); in "Quantum Gravity", C. J. Isham, R. Penrose and S. W. Sciama Editors (Claredon Press, Oxford, U. K.; 1974); J. B. Hartle and S. W. Hawking: Phys. Rev. D13, 2188 (1976); G. W. Gibbons and S. W. Hawking: Preprint (Univ. of Cambrige, U. K.; Sept. 1976); T. Damour and R. Ruffini: Phys. Rev. D14, 332 (1976).
71) Or other solitonic solutions of Einstein-type equations. For the connections between solitons and black-holes, see e. g. E. Recami and P.Castorina : ref. (9); J. Pati and A. Salam: Phys. Letters 61B, 375 (1976); A. Salam and J. Strathdee: Preprint IC/76/107 (ICTP, Trieste; 1976); B. Hu: Lett. Nuovo Cimento 18, 267 (1977); S. Coleman: Phys. Rev. D11, 2088 (1975); G. Bowtell and A. E. G. Stuart: Phys. Rev. D15, 3580 (1977). For connections between Einstein and Yang-Mills theories, see also e.g.R. Tabenski: Journ. Math. Phys. 17, 1 (1976); P. Urban and R. Burghardt: Acta Phys. Austr. 43, 293 (1975); M. Camenzind: "GR8" (Waterloo, Ont.; 1977).
72) See e. g. J. D. Bekenstein: Ann. of Phys. 91, 75 (1975), especially Sect. 5; D.W.Sciama: (priv. comm.).
73) V. De Sabbata, M. Pavšič and E. Recami: ref. (55); E. Recami and R. Mignani: ref. (56): V. De Sabbata, M. Pavšič, E. Recami and K. T. Shah: (in preparation).
74) a) G. Parisi: Phys.Rev. D11, 970 (1975); Y. Nambu: Phys. Rev. D10, 4262 (1974); H. B. Nielsen and P. Olesen: Nucl. Phys. B61, 45 (1973); b) T. W. Kibble: Preprint ICTP/75/5 (Trieste, Feb. 1976).
75) In $c=G=\varepsilon_0=1$ units, the proton mass in $m_p=1.2\times10^{-52}$cm, and its electric charge is $e=1.3\times10^{-34}$ cm $\approx \approx 10^{18} m_p$. See also T. Piran: Preprint (Dept. Astrophys., Univ. Oxford), 1977.
76) See J. Sarfatti: Found. of Phys. 5, 301 (1975), or rather L. P. Hughston: "A particle classification scheme based on the theory of twistors" (an Oxford Univ. Thesis, the Mathem. Inst., Oxford, U. K., 1976), unpublished.
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88) See e. g. the end of Sect. 4. 2.

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91) Thanks are due to G. Schiffrer for calling our attention to this verse in Goethe's "Faust".