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G. Alberi and F. Baldracchini

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EXOTIC FINAL STATE INTERACTION
IN DEUTERON BREAK-UP AND
Δ N THRESHOLD PARAMETERS

by

G. Alberi*, Istituto di Fisica Teorica, Miramare, Trieste

and

F. Baldracchini, Advanced School of Physics, Miramare, Trieste

ABSTRACT

We formulate a simple model for the final state interaction with an intermediate Δ in deuteron break-up induced by medium energy hadrons and we test its prediction. The result is in very poor agreement with data if the intermediate Δ has its real width and it becomes comparable with data, for small width $\Gamma \sim 10$ MeV. This is explained by the strong coupling of the decay meson to the spectator nucleon, which provides the Born term for our transition amplitude. Although data and theory are in a rather primitive stage to determine the ΔN scattering length, we see that the effect is sensitive to this fundamental quantity.

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1. - INTRODUCTION

It is a well known experimental fact that the high momentum part of the spectator distribution in deuteron break-up induced by high energy hadrons, cannot be described by the spectator model. The disagreement seems to derive from a strong final state interaction of the two nucleon, which form a virtual ΔN state. This idea was first suggested by the Dubna-Warsaw Collaboration [1] (Aladashvili et al. 1975), who examined the high momentum tail of the spectator for the reaction $pD \rightarrow pnp$ in the charge preserving and charge-exchange sector; the large difference between the two sectors led them to the above hypothesis. We [2] have recently studied the channel dependence of the effect, for other 10 reactions and we found that various ordinary multiple scattering diagrams were not able to reproduce the data: assuming the presence of a ΔN virtual state in the nucleon-nucleon final state interaction, we could reproduce qualitatively the channel dependence of the effect. Here we develop further this idea, constructing a simple model: this model is based on the calculation of the diagram of Fig. 1. The diagrammatic technique to study rescattering processes was proposed long time ago and it was applied (Shapiro) with great success by his collaborators. As often stressed in the literature it boils down to Watson multiple scattering theory, if the spectator nucleons are taken on the mass shell; but it has the advantage to be originally relativistic, and it can describe easily exotic process, like the one we study here.

In Sect. 2 we calculate explicitly the diagram of Fig. 1, neglecting the dependence on the internal variables of all vertices, but for the deuteron, on the account that this dependence is the most dramatic one. We calculate also the $\Delta N \rightarrow NN$ inelastic amplitude, using a 2-channel K-matrix approach and the $\pi N \Delta$ vertex, using the Rarita-Schwinger spinor for the Δ.

In Sect. 3 we calculate the 3 body phase space in terms of the various kinematic variables we are interested.

Finally in Sect. 4 we discuss the detailed predictions of the model for various experimental data.

We conclude in Sect. 5 discussing other possible approaches.

2. - FORMULATION OF THE MODEL

The diagram of Fig. 1 can be written down using Feynman rules (Bjorken and Drell)

$$\hat{T} = \frac{i}{(2\pi)^4} \int d^4\xi \, \frac{G(\xi)\hat{T}_1((M-\xi_o)^2,(E-\xi_o)^2|\vec{\Delta},\Delta_o)\hat{T}_2((E-\xi_o)^2,E,\cos\theta)}{(\xi_o^2-\xi^2-m^2+i\varepsilon)((M-\xi_o)^2-\xi^2-m^2+i\varepsilon)((E-\xi_o)^2-(\vec{\Delta}-\vec{\xi})^2-m_\Delta^2+i\varepsilon)} \tag{2.1}$$

where M is the deuteron mass and m, m_Δ are the nucleon and the delta mass: E is the total energy of the final two nucleons and θ their C.M. angle: $(\vec{\Delta},\Delta_o)$ is the momentum transfer and G is the deuteron vertex function. Expressing G through the non relativistic deuteron wave function (Bertocchi and Capella, Alberi et al. 1972) and reducing the lower internal line to the mass shell we get

$$\hat{T} = \frac{\pi}{(2\pi)^4} (16\pi^3 M)^{\frac{1}{2}} \int \frac{d^3\xi}{\xi_o} \psi(\xi) \frac{\hat{T}_1 \hat{T}_2}{(E-\xi_o)^2-(\vec{\Delta}-\vec{\xi})^2-m_\Delta^2+i\varepsilon} \tag{2.2}$$

Neglecting the off-mass shell dependence of T_1 and T_2 we can perform the integration, provided we choose for $\psi(\xi)$ a form of the Hulthen type (Hulthen and Sugawara); we can also take in account that the Δ is an unstable particle and substitute to m_Δ, $m_\Delta - i\Gamma/2$ where Γ is its measured width.

Taking a pole wave function, for the deuteron, and neglecting $\Gamma^2/4$ compared to m_Δ^2, we get

$$\hat{T} = \frac{\pi}{(2\pi)^4} (16\pi^3 M)^{\frac{1}{2}} \sqrt{N}\, \hat{T}_1(\vec{\Delta},\Delta_o)\, \hat{T}_2(E,\cos\theta) \frac{i\pi^2}{m\,\Delta} Y \tag{2.3}$$

where $Y(\chi) = \frac{\Delta}{i\pi^2} \int \frac{d^3\xi}{(\xi^2+\chi^2)} \frac{1}{(E-\xi_o)^2-(\vec{\Delta}-\vec{\xi})^2-m_\Delta^2+im_\Delta\Gamma}$

a. The triangular graph.

Using the non relativistic approximation for the propagator (Shapiro) one gets

$$Y(\chi) = \frac{2\Delta}{i\pi} \int \frac{\xi^2 d\xi \, d\cos\theta'}{\xi^2 + \chi^2} \quad \frac{1}{(2m_\Delta(E-\xi_o-m_\Delta)-(\vec{\Delta}-\vec{\xi})^2 + i\Gamma m_\Delta)}$$

The angular integration is easily done, taking as z axis the direction of $\vec{\Delta}$

$$Y(\chi) = \frac{1}{i\pi} \int_o^\infty \frac{\xi \, d \, \xi}{\xi^2+\chi^2} \ln \frac{(\xi- \Delta/2)^2-\varepsilon'^2-i\Gamma m_{\Delta/2}}{(\xi+\Delta/2)^2-\varepsilon'^2-i \, \Gamma m_{\Delta/2}}$$

where

$$\varepsilon'^2 = (1 - \frac{m_\Delta}{m}) \; (m_\Delta m-(2\chi^2+\Delta^2)/4) \;\; + \frac{m_\Delta}{m} \varepsilon^2$$

and ε is the C.M. momentum of the two final nucleons. The use of standard complex plane integration method leads to the closed form

$$Y(\chi) = \ln \frac{i\chi -\Delta/2 + \sqrt{\varepsilon'^2+i\Gamma m_{\Delta/2}}}{i\chi +\Delta/2 + \sqrt{\varepsilon'^2+i\Gamma m_{\Delta/2}}} \tag{2.4}$$

b. K-matrix formalism.

Being the energy of the two final nucleons, reasonably low, because of the cut off in $t = -\Delta^2+\Delta_o^2$, we expect that only the ΔN channel is open: furthermore since we are close to the ΔN threshold, we expect the S wave to be dominant in that channel. Being the isospin 1 and the parity even, the nucleons have to be in a spin singlet state because of Pauli principle, therefore their only possible orbital angular momentum is 2. This is an ideal situation for applying the multichannel K-matrix formalism (Ross and Shaw). In their approach the T-matrix is expanded in spherical harmonics

$$T_{fi} = 8\pi \sqrt{s}\ 4\pi \sum_{JM\ell_f\ell_i} Y_{\ell_f}^{M-\nu_f}(\hat{K}_f)\ Y_{ei}^{M-\nu_i}(\hat{K}_i)^{\dagger}$$

$$\langle JM \mid \ell_f, M-\nu_f; S_f, \nu_f\rangle \ \langle JM \mid \ell_i, M-\nu_i; S_i \nu_i\rangle \times \tag{2.5}$$

$$\times \langle f \mid q^{\ell} \frac{1}{M - iq^{2\ell+1}} q^{\ell} \mid i\rangle$$

($S_{i,f}$ is the spin of the channel)
where q^{ℓ} is a diagonal operator, where matrix elements have exactly that form with q the C.M. momentum of the channel and ℓ its orbital angular momentum. The M operator is closely related to the K-matrix (Pilkuhn 1967)

$$M = q^{\ell}\ K^{-1}\ q^{\ell} \tag{2.6}$$

In our case there are only 2 channels; channel (1) is NN in the state $\ell_1 = 2$, $S_1 = 0$ and channel (2) is ΛN for $\ell_2 = 0$, $S_2 = 2$. The transition matrix element is

$$T_{12} = 8\pi \sqrt{s}\ \sqrt{4\pi}\ Y_2^{M'}(\Omega_f) \frac{(-)\ q_1^2\ M_{12}}{(M_{11} - iq_1^5)(M_{22} - iq_2) - M_{12}M_{21}} \tag{2.7}$$

where

$$M_{ij} = \frac{1}{A_{ij}} + \frac{1}{2} B_{ij}\ q_i^2\ \delta_{ij} \tag{2.8}$$

in the effective range approximation. While for the channel (2) this approximation is justified, being $q_2 \sim 0$, for the channel (1) it is meant, only as 2° order polinomial approximation for the function $q_1^5 \cot \delta_2$ in the region of interest.

c. Spin analysis.

To calculate the cross section, we have to average on the spin state of the deuteron and sum on the orbital angular momentum states of the final nucleons

$$\frac{d\,\sigma}{d\,\cos\theta} \sim \frac{1}{3} \sum_{M=-1}^{+1} \sum_{M'=-2}^{+2} \tag{2.9}$$

$$\left| V_{s'_z S_z} \langle 1M | \tfrac{1}{2}\, s'_z\ \tfrac{1}{2}\, s_z \rangle \langle 3/2\ S_z\ \tfrac{1}{2}\, s_z |\ 2M' \rangle\ Y_2^{M'}(\cos\theta) \right|^2$$

where $V_{s'_z S_z}$ is the spin dependent part of T_1

$$V_{s'_z S_z} = \bar{U}_\mu(S_z)\, P_\mu \frac{G^*}{m_\pi}\, u(s_z) \tag{2.10}$$

where U and μ are Rarita-Schwinger and Dirac spinor and P_μ is the nucleon momentum; θ is the C.M. angle of the nucleon in the final state: the spin notation is given in Fig. 2. As stressed before, this term depends slowly on the nucleon interval momentum and can be calculated for the value $P_\mu \equiv (\vec{0},m)$. This is close to the most probable value, according to the deuteron wave function. For this configuration the momentum of the intermediate Δ is fixed and equal to the 3-momentum transfer $\vec{\Delta}$. Being $\vec{\Delta}$ along the z axis, the form of the Rarita-Schwinger spinors is very simple

$$U_\mu\ (\pm 3/2) = u\ (\pm\tfrac{1}{2})\ \varepsilon_\mu(\pm 1)$$

$$U_\mu\ (\pm\tfrac{1}{2}) = \frac{1}{\sqrt{3}}\, u(\mp\tfrac{1}{2})\ \varepsilon_\mu(\pm 1) + \frac{\sqrt{2}}{\sqrt{3}}\, u\ (\pm\tfrac{1}{2})\ \varepsilon_\mu(o)$$

because

$$\varepsilon_\mu\ (\pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ i \\ 0 \\ 0 \end{pmatrix} \qquad \varepsilon_\mu(o) = \frac{1}{m_\Delta} \begin{pmatrix} 0 \\ 0 \\ \Delta_o \\ \Delta \end{pmatrix}$$

Therefore, only the matrix elements $V_{\pm\frac{1}{2},\pm\frac{1}{2}}$ survive, because only $\varepsilon_\mu(o)P_\mu(o)$ is different from zero. It is easy to calculate from (2.8) the angular distribution of the nucleons

$$\frac{d\sigma}{d\cos\theta'} \sim \frac{1}{3}\, |V_{\frac{1}{2}\frac{1}{2}}|^2 \left(\frac{3}{2}\, |Y_2^1(\cos\theta)|^2 + |Y_2^0(\cos\theta)^2\right) \tag{2.11}$$

Integrating on $\cos\theta$, one gets

$$\int d\Omega \frac{d\sigma}{d\cos\theta} \sim \frac{5}{6}\, |V_{\frac{1}{2}\frac{1}{2}}|^2 \tag{2.12}$$

which establishes the relation with the differential cross section for Δ production on nucleon

$$\left.\frac{d\sigma}{dt}\right|_P \sim |V_{\frac{1}{2}\frac{1}{2}}|^2 \tag{2.13}$$

The index P means pseudoscalar.

This relation becomes more complicated if $\hat{T}_1$ is the sum of a pseudoscalar and a vector meson exchange ($\pi+\rho$). The $\rho N\Delta$ vertex is, according to Jackson and Pilkuhm in the notation of Bjorken and Drell

$$W_{s_z' S_z} = \bar{U}_\mu(S_z) \left[G_1\, g_{\mu\nu} - \frac{G_2}{m+m_\Delta}\, \Delta_\mu \gamma_\nu\right] \gamma_5 u(s_z') \tag{2.14}$$

Doing explicitly the matrix elements, we realize that $W_{(1/2,\pm 3/2)}=0$ then the differential cross section for vector meson exchange is simply given by

$$\left.\frac{d\sigma}{dt}\right|_V = |W_{\frac{1}{2}\frac{1}{2}}|^2 + |W_{\frac{1}{2}-\frac{1}{2}}|^2 \tag{2.15}$$

on the other side the explicit calculation of (2.9) and the angular integration gives

$$\int d\Omega \frac{d\sigma}{d\cos\theta} = \frac{7}{12}\, |W_{\frac{1}{2}-\frac{1}{2}}|^2 + \frac{1}{2}\, |W_{\frac{1}{2}\frac{1}{2}}| + \frac{5}{6}\, |V_{\frac{1}{2}\frac{1}{2}}|^2 \tag{2.16}$$

$$\sim \frac{1}{2} \left.\frac{d\sigma}{dt}\right|_{P+V} + \frac{1}{3} \left.\frac{d\sigma}{dt}\right|_P$$

With another type of $\rho N\Delta$ coupling, recently used by Joynson for studying the reaction $pp \to n\Delta^{++}$,

$$W_{s_z' S_z} = G\, \bar{U}_\mu(S_z)\, \varepsilon_{\lambda\nu\kappa\mu}\, (P_\nu+\Delta_\nu)(P_\kappa-\Delta_\kappa)\, u\,(s_z) \tag{2.17}$$

one obtains

$$\int d\Omega \frac{d\sigma}{d\cos\theta} = \frac{5}{6} |V_{\frac{1}{2}\frac{1}{2}}|^2 \qquad (2.18)$$

as done in the Appendix A.

d. Isospin.

Analogously to the spin case, we fix the notation in Fig. 3 and we write down the isospin sum

$$T(\tau_z, T_z) \sim \sum_{t'_z T'_z} \langle 00|\tfrac{1}{2}\, t_z, \tfrac{1}{2} t'_z\rangle \langle \tfrac{1}{2} t'_z 1 \tau_z | 3/2\, T'_z\rangle \times$$

$$\times \langle 3/2\, T'_z\, \tfrac{1}{2}\, t_z |\, 1, T_z\rangle \langle \tau_a, \tau_{az}; 1, \tau_z | \tau_c\, \tau_{cz}\rangle\, T$$

which becomes for the case of charge exchange

$$T(1,1) \sim \sqrt{\frac{2}{3}}\, T \langle \tau_a\, \tau_{az}\, 1\, \tau_z |\, \tau_c\, \tau_{cz}\rangle$$

This expression can be easily connected when squared and spin averaged, to the differential cross section for the process $ap \to c\Delta^{++}$

$$|T(1,1)|^2 \sim \frac{2}{3} \frac{d\sigma}{dt}\bigg|_{ap \to c\Delta^{++}} \qquad (2.19)$$

3. - REDUCTION OF THE PHASE SPACE

The use of relativistic invariance leads to a simple (Byckling and Kajantie) reduction of the 3-body phase space for a $2 \to 3$ process (Fig. 4)

$$d\sigma = \frac{\Sigma|T|^2}{\Phi} \frac{d^3p_c d^3p_1 d^3p_2}{8(2\pi)^5} \delta^4(p_a+p_b-p_1-p_2-p_c)$$

$$= \frac{\Sigma\,|T|^2}{\Phi 2 2^6 \pi^4}\; q\; dm_{12} d\Omega\; dt \tag{3.1}$$

where $\Phi = 2\sqrt{\lambda(s\ m_a^2, m_b^2)}$ and λ is the triangular function; Ω is the solid angle of one of the two nucleons in their center of mass. For the other symbols, see Fig. 4.

From energy-momentum conservation

$$(p_a - p_c)^2 = (p_b - p_1 - p_2)^2$$

which in the C.M. of the two nucleons becomes

$$t = M^2 + m_{12}^2 - 2\,E\,m_{12} \tag{3.2}$$

that is a relation between the energy E of the deuteron in the C.M. of the two nucleons and t.

From relativistic invariance we get

$$(p_b-p_2)^2 = M^2+m^2-2Eq_o+2Pq\cos\theta = M^2+m^2-2Ms_o \tag{3.3}$$

being $p_s = (\vec{p}_s, s_o)$ and q the momentum of the nucleon in the rest system of the deuteron and in the C.M. of the nucleon pair respectively. $\vec{P}$ is the momentum of the deuteron in the latter system.

Using the notation

$$x = \frac{P}{M} \qquad y = \frac{E}{M} \qquad \text{and} \quad y^2-x^2= 1$$

we get from the above relation $\cos\theta$ as function of the nucleon momentum

$$\cos\theta = \frac{y\,q_o-s_o}{xq} \tag{3.4}$$

Imposing that

$$\cos^2\theta \;=\; \frac{(y\,q_o - s_o)^2}{x^2\;q^2} < 1$$

we get the following inequality

$$y^2m^2 - 2y\;q_o\;s_o + s_o^2 + q^2 \;<\; 0$$

which gives the kinematical limits on y and x

$$y_{\pm} \;=\; \frac{q_o s_o \,\pm\, q p_s}{m^2}$$

$$\sqrt{y_-^2-1} \;<\; x \;<\; \sqrt{y_+^2-1} \qquad (3.5)$$

Using equation (3.2) and (3.4) we write down the Jacobians

$$\left|\frac{d\,\cos\theta}{d\;p_s}\right| = \frac{p_s}{s_o}\;\frac{1}{xq} \qquad \left|\frac{dt}{dx}\right| = \frac{2m_{12}Mx}{y} \qquad \left|\frac{dm_{12}}{dq}\right| = \frac{4\;q}{m_{12}} \qquad (3.6)$$

$$\frac{d\sigma}{dp_s} \;=\; \frac{p_s}{s_o}\;\frac{M}{\Phi^2 2^3\pi^3}\int dq^2 \int_{x_-}^{x_+}\frac{dx}{y}\;\Sigma\;|T|^2$$

where

$$x_+ \;=\; \min\,(\sqrt{y_+^2-1},\; x(t_+))$$

$$x_- \;=\; \max\,(\sqrt{y_-^2-1},\; x(t_-))$$

and

$$x(t) = \frac{1}{2m_{12}M}\;\sqrt{(M^2-m_{12}^2)^2+t^2-2t(M^2+m_{12}^2)}$$

$t_{\pm}$ are limits of the Chew-Low plot (Byckling and Kajantie)

$$t_{\pm} \;=\; m_a^2 + m_c^2 - \Big[(s_{ab}+m_a^2-M^2)(s_{ab}-m_{12}^2+m_c^2) \pm\sqrt{\lambda(s_{ab},m_a^2,M^2)\,\lambda(s_{ab},m_{12}^2,m_c^2)}\Big] \Big/ (2s_{ab})$$

The allowed region for q^2 is given by the condition $x_+ > x_-$.

4. - COMPARISON WITH EXPERIMENTAL DATA

In this section we consider the prediction of the model for the two charge exchange reactions $K^+D \to K^{*o}pp$ and $pD \to npp$ and compare them with existing experimental data. If we assume that the corresponding reactions $K^+p \to K^{o*}\Delta^{++}$ and $pp \to n\Delta^{++}$ occur through peripheral meson exchange, the π-meson and vector meson exchanges are weighted in a different way in the two reactions. Therefore, one can in principle find useful information about the coupling of vector mesons to the deuteron (+).

For the angular distribution of the protons in their center of mass a pure π-meson exchange gives a definite prediction, coming from (2.11)

$$\frac{d\sigma}{d\cos\theta} \sim (1+3\cos^2\theta)\ \tfrac{1}{2}\ |V_{\frac{1}{2}\frac{1}{2}}|^2 \tag{4.1}$$

This prediction is independent from the specific form of the matrix element, which depends only on m_{12} and t. The data of Poster et al. for the process $K^+D \to K^{o*}$ pp at 2 GeV/c are showing reasonable agreement with this simple prediction (Fig. 5). From the angular distribution of the decay products of K^{o*}, one finds experimentally that the π-exchange is dominant (Buchner et al. and Giacomelli et al.) and this feature is present also at high momenta of the spectator (++). Therefore we neglect for this reaction the contribution of vector meson exchanges.

The t-distribution for the same process, can be studied inserting expression (2.3) in (3.1), and the result is

$$\begin{aligned}\frac{d\sigma}{dt} &= \frac{2M\ q_1^2\ s_1}{\lambda(s,m_k^2,M^2)}\ \frac{H^2}{\pi}\ \frac{2}{3}\ \frac{5}{6}\ \frac{d\sigma}{dt}\Big|_{K^+p\to K^{o*}\Delta^{++}} \times \\ &\times \frac{1}{m^2}\int_{2.13}^{2.19} s_{12}\left|F_{12}\right|^2 \frac{1}{\Delta^2}\left|Y(\alpha)-Y(\beta)\right|^2 q\ d\sqrt{s_{12}} + B\end{aligned} \tag{4.2}$$

(+) It is clear however that the best reaction for this purpose, would be $\pi^+D \to \pi^o pp$.

(++) Private communication of G. Giacomelli.

where s_1 refers to the reaction $K^+p \to K^{o*}\Delta^{++}$, with the proton of the deuteron at rest, and $\left.\frac{d\sigma}{dt}\right|_{K^+p\to K^{o*}\Delta^{++}} = 14.56\ e^{5.71t}$ mb/(GeV/c)^2 (Flaminio, UCRL K^+N); H is the normalization factor for the Hulthen wave function. F_{12} is the partial wave transition amplitude for the $NN\to\Delta N$ in the $I=1$ channel ($T_{12} = 8\pi\sqrt{s}\ \sqrt{4\pi}\ F_{12}\ Y_2^M\ (\Omega)$).
The background B includes the impulse approximation and the ordinary final state interaction considered in Alberi et al. (1976). The interferences are neglected here because the D-wave spin singlet part of these terms, which can interfere, is only a small part of B.

The background (dashed line) and the result of (4.2), are given in Figure 6, where the parameters of the ΔN interaction at low energy are

$$A_{22} = 8\ F \qquad B_{22} = 1.32\ F \qquad A_{12} = (.438\ F)^3$$

The other parameters $A_{11} = -1.80224\ (\mathrm{GeV/c})^{-5}$ $B_{11} = 2.76819\ (\mathrm{GeV/c})^3$ are determined using the N-N phase shift analysis for $l = 2$ and $S = 0$. The width of the Δ is assumed to be $\Gamma = 10$ MeV. Trials with the experimental width of the Δ , have shown that it is impossible to get results, comparable with experiment using reasonable values for the ΔN scattering length. This could be understood, representing the propagation of the Δ in coordinate space $e^{i\eta r-\gamma r}$ where $\gamma\sim\Gamma$: the finite width of the Δ means that before interacting the Δ could decay and result in a $N\pi N$ state. However, if the π is virtually absorbed by the other nucleon, this does not amount to an absorption from the final channel NN. Since this probability is quite high in that region, as shown by Riska et al.(1976) for a strictly related process, it is plausible that also the effective width in the treatment is very small.

The invariant mass distribution of the two protons in the final state for the reaction $pD\to npp$ (Aladashvili et al. 1976) shows a peak around the threshold for the ΔN channel. This was interpreted in our previous work (Alberi and Baldracchini, 1976), with the presence of a virtual ΔN state in S-wave but no serious attempt was done to fit the data both in form and in absolute value. Here we do that, using the formulas (2.3), (2.7) and (2.12) to get:

$$\frac{d\sigma}{d\,m_{12}} = \frac{M}{m^2}\,\frac{H^2}{2\pi}\,\frac{m_{12}^2 q\, s_1(s_1-4m^2)}{\lambda(s_{ab},m_a^2,M^2)}\,\frac{5}{6}\,\frac{2}{3}\,\left|F_{12}\right|^2 \times$$

$$\times \int_{t-1}^{.15} d|t|\frac{1}{4}\int_{-1}^{z_{max}}(1+3\cos^2\theta)\,d\cos\theta\,\left.\frac{d\sigma}{dt}\right|_{pp\to n\Delta^{++}}|Y(\alpha)-Y(\beta)|^2\,\frac{1}{\Delta^2} + B' \tag{4.2}$$

the upper limit for the $\cos\theta$ integration is determined by the lower limit for p_s, and it is

$$z_{max} = (y(t)q_o - s_o(.3))/(x(t)q_o)$$

being $p_s > .3$ GeV/c. We extrapolate the value of $\left.\frac{d\sigma}{dt}\right|_{pp\to n\Delta^{++}}$ from 2.8 GeV/c (NN and ND Interactions) down to 1.65 GeV/c, using the scaling law $q_1^2 s\ \frac{d\sigma}{dt}$ = const valid for any particle exchange; this law was checked in the region above 2.8 GeV/c. The result is

$$\left.\frac{d\sigma}{dt}\right|_{pp\to n\Delta^{++}} = 272\ e^{11.8t}\,\mathrm{mb}/(\mathrm{GeV/c})^2$$

The result of (4.3) are compared with experiment in Fig. 7 a). Both π and ρ exchange are used to describe the reaction $pp\to n\Delta^{++}$, with the Pilkuhn ansatz for the $\rho N\Delta$ coupling (2.14) being $\frac{g^2_{\pi N\Delta}}{4\pi} = .35$ (Riska et al.) and being the form factor F(t) in (B.3) equal to e^{6t}; this corresponds to substitute (2.16) to $5/6\left.\frac{d\sigma}{dt}\right|_{pp\to n\Delta^{++}}$, in (4.3). A reasonable guess for the background B' is done, consistently with experimental data: a real calculation of B' would not improve qualitatively the situation. The ΔN parameters are the same as used for the reaction $K^+D\to K^{o*}pp$. The dashed line is obtained using formula (4.3) as it is, which corresponds to assume that the reaction $pp\to n\Delta^{++}$ occurs via π-meson exchange only.

In Fig. 7b) the Joynson couplig for $\rho N\Delta$ is tried. The dashed line is the result for A_{22} = 16 F, which shows the sensitivity to the ΔN scattering length. *

From the comparison between Figures 7a) and 7b) we realize that the result is quite sensitive to the form of the $\rho N\Delta$ coupling, but it is also sensitive to the value of the ΔN scattering length. We can conclude also that to take in account ρ-exchange, gives a suppression in the effect,

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* The results are insensitive to A_{12}, B_{22}. For big changes of A_{12}, they become very small.

smaller in 7a) and larger in 7b). This suppression could explain the systematic overevaluation of the effect by the simple probabilistic model of Alberi et al. (1976) (Fig. 17) in the charge exchange reactions like $\pi^+ D \to \pi^o pp$, where the vector meson exchange is dominant.

It is worth mentioning here that we cannot describe the whole mass spectrum with our model, because the effective range expansion is valid only around the ΔN threshold.

Let us consider now the spectator distribution where the anomaly was first discovered (Alberi et al. 1976). This distribution is very important from the practical point of view, because it provides and easy way to check the validity of the spectator model: to avoid the complication of the high momentum tail, the neutron cross section was extracted, using only the events with $p_S \le .25$ GeV/c. In this procedure it was implicitly assumed that the dominant phenomenon at high momenta, gives a negligible contribution for $0 \le p_s \le .25$ GeV/c.

The kinematics of the spectator distribution is discussed in Sect. 3. The complete formula for the reaction $pD \to npp$ is

$$\frac{d\sigma}{dp_s} = \frac{M}{\phi^2 2^3 \pi^3} \frac{p_s}{\sqrt{m^2+p_s^2}} \int dq^2 \int_{\max(x_-, x(t_{min}))}^{\min x_+, x(t=-.1)} \frac{dx}{y} \Sigma|T|^2$$

where $\phi = 2\sqrt{\lambda(s_{ab}, m_a^2, M^2)}$

$$\Sigma|T|^2 = (16\pi^3 M) 2^6 \pi q_1^2 s_1 \Bigg[\frac{2}{3} \frac{d\sigma}{dt}\Big|_{an\to cp} \psi^2(s) - \frac{4}{3}\sqrt{\pi}\frac{\sqrt{s_{12}}}{2m\,\Delta}\sqrt{\frac{2}{3}}$$

$$\sqrt{\frac{d\sigma}{dt}}\Big|_{an\to cp}\sqrt{\frac{d\sigma}{dt}}\Big|_{ap\to c\Delta^{++}} \frac{H}{\pi\sqrt{2}}(s) \mathrm{Re}\,\{i(Y(\alpha)-Y(\beta))T_{12}\} \times$$

$$\times Y_2^o(\cos\theta) + \frac{1}{3}\,\frac{2}{3}\,\frac{s_{12}}{4m^2\Delta^2}\,\frac{d\sigma}{dt}\Big|_{ap\to c\Delta^{++}} \frac{H^2}{2\pi^2} |Y(\alpha)-Y(\beta)|^2 \times |F_{12}|^2$$

$$\times \frac{5}{4}(1+3\cos^2\theta) \Bigg] \qquad (4.4)$$

The numerical outcome of this expression is compared in Fig. 8 with the data of Aladashvili et al. (1977) for the spectator distribution with $|t| \leq .1(\text{GeV/c})^2$ Our theoretical curve is normalized to the experimental points at p_s = .05 GeV/c; the modification of the spectator model predictions due to normal final state interactions were studied in the above quoted paper and amounts at most to 10% corrections. The effect of the peak in the invariant mass of the two protons, is resulting in enhancement at high momenta, because of kinematical reasons high momenta correspond to high invariant masses of the two protons, as shown in the Dalitz plot of Fig. 9.

The dominance of this phenomenon at high momenta of the spectator, seems true also for other 10 reactions studied by Alberi et al. (1976): indeed the relative importance of the effect was explained by the relative strength of the Δ production on nucleon.

5. - GENERAL DISCUSSION

The general idea of this paper is to interpret the bump in the invariant mass of the two protons in the charge exchange reactions on deuteron, as final state interaction of the two nucleons with an intermediate Δ. This interpretation would be consistent with the analysis (Satoh et al.) of the analogous phenomenon for the Λp channel in the deuteron break up induced by medium energy K^-.

This approach is not inconsistent with the approach of Poster et al. where in the hypothesis of pure π-exchange, one uses for the coupling of the π to the deuteron, the data for the reaction $\pi^+ D \to pp$. This idea was exploited in Aladashvili et al.(1977)giving reasonable agreement with experimental data, for the reaction $pD \to n\, pp$. In this treatment one implicitly assumes that the ρ coupling to the deuteron is 0, as it would be for our model in the Joynson coupling. On the other side this conclusion is difficult to accept, on the basis that the high momentum tail is present also for reactions like $\pi^+ D \to \pi^o pp$ where the ρ exchange is very important (Alberi et al. 1976). Actually these reactions could be useful to extract, using the Poster method, the ρ-coupling to the deuteron.

The current theoretical treatment for $\sigma(\pi^+ D \to pp)$ (Green and Niskanen, Riska et al. 1976) using the meson exchange approach for weak and electromagnetic interactions, is a possible alternative to our approach. This approach is much more complicated, but it has the advantage of being able to calculate more carefully the effect in the low mass region. Indeed there is a relativistic effect, Seagull term, which is not taken in account in a non-relativistic scattering theory and is important at low masses (Riska and Brown). The global effect was calculated already by Mosconi and Ricci for electron disintegration of the deuteron: it turns out to be additive to the spectator part and of the order of 2% at small momentum transfers. The calculation for the hadron charge exchange case, is in preparation and will be published in the next future.

With our approach, however, we have gained physical understanding of an exotic final scattering state occurring at 300 MeV C.M. kinetic energy of the two proton and we have related it to the high momentum tail of the spectator distribution. The "energy" of this virtual state can be determined by the comparison with experimental data. The parameters are clearly useful to study the same state in the ΔN channel: this could be done for the state seen in the reaction $\gamma^4He \to \Delta N(NN)$ (Argan et al.); here an $I = 2$ part is not excluded, but since we know how much is in $I = 1$ state, one could use our approach to extract information on the $I = 2$ sector.

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FIGURE CAPTIONS

Fig. 1 - Diagram for nucleon-nucleon final state interaction with Δ-N intermediate state.

Fig. 2 - Spin notation.

Fig. 3 - Isospin notation.

Fig. 4 - Kinematical variables in deuteron break-up process.

Fig. 5 - Angular distribution of one recoil proton in the 2 proton C.M., for the process $K^{+}D \to K^{o*}pp$ at 2 GeV/c. The data are taken from the preprint of Poster et al. The theoretical curve is the simple expression $(1+3\cos^2\theta)$ form (4.1), normalized on the data.

Fig. 6 - The t distribution for $2.13 \lesssim M_{pp} \leq 2.19$ GeV for the process $K^{+}D \to K^{o*}pp$ at 2 GeV/c. The dashed line is the background.

Fig. 7 - The mass distribution of the two protons in the process $pD \to n\,pp$ at 1.65 GeV/c. The data are from Aladashvili et al. (1976). The curves are obtained using (4.3). The dashed line in (a) corresponds to pure (4.3), the solid line to a $\pi+\rho$ exchange using Pilkhun ansatz for the vector coupling. In (b) the Joynson ansatz is used and the dashed line corresponds to a larger value of the ΔN scattering length.

Fig. 8 - The spectator distribution for $pD \to n\,pp$ at 1.65 GeV/c. The data are from Aladashvili et al. (1976). The solid curve is the complete theory (4.4). The dashed curve is the spectator model.

Fig. 9 - The Dalitz plot p_s-M_{pp} for $pD \to n\,pp$ at 1.65 GeV/c. In the smoked area the cross section is suppressed for kinematical reasons ($x_+ - x_-$ is small).

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APPENDIX A

Expression (2.2) can be computed more carefully than in the main text, taking in account the angular dependence of the lower vertex of T_1

$$T_1 = A \frac{1}{t-m_e^2} B \qquad (A.1)$$

(m_e is the mass of the exchanged particle) B depends on the spin projections s'_z, S_z (See Fig. 2). The expression of the triangular diagram is therefore

$$\hat{T} = -\frac{\pi(16\pi^3 M)\sqrt{N}}{(2\pi)^4} A \frac{1}{t-m_e^2} I(S_z,s'_z)\, T_2 \qquad (A.2)$$

where

$$I(S_z,s'_z) = \int \frac{d^3\eta}{m} \frac{B(S_z,s'_z)}{(\Delta^2+\eta^2-2\Delta\eta\cos\theta'+\chi^2)(2m_\Delta(E-m-m_\Delta)-\eta^2-\frac{m}{m_\Delta}(\vec{\Delta}-\vec{\eta})^2+i\varepsilon)}$$

In the definitions of the Rarita-Schwinger spinor, the polarization vector is the rotated version of the definition, of Sect. 2c

$$\varepsilon_\mu(\pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm\sin\phi'+i\cos\phi'\cos\theta' \\ \mp\cos\phi'+i\sin\phi'\cos\theta' \\ -i\sin\theta' \\ 0 \end{pmatrix} \qquad \varepsilon_\mu(0) = \frac{1}{m_\Delta} \begin{pmatrix} \eta_o\cos\phi' \quad \sin\theta' \\ \eta_o\sin\phi' \quad \sin\theta' \\ \eta_o\cos\theta' \\ \eta \end{pmatrix}$$

being ϕ', θ' the polar angles of $\vec{\eta}$, in a system with $\vec{\Delta}$ as z axis.

For a pseudoscalar exchange the lower vertex is (Jackson and Pilkuhn)

$$B_p(S_z,s_z) = \frac{G^*}{\mu} U_\mu (S_z) P_\mu u(s'_z) \qquad (A.3)$$

where P is the nucleon momentum. The matrix elements are, neglecting terms of the order η^2, D^2, $\eta\Delta$, small compared to square masses

$$B_p(\pm 3/2, \pm 1/2) \cong \frac{G^*}{\mu} (-i\Delta) \sin\theta' \sqrt{\frac{C}{2}}$$

$$B_p(\pm 3/2, \mp 1/2) = 0$$

$$B_p(\pm 1/2, \pm 1/2) \cong \frac{G^*}{\mu} \frac{1}{m_\Delta} \sqrt{\frac{2C}{3}} (2\eta\eta_o + \eta\Delta_o - \eta_o \cos\theta')$$

$$B_p(\pm 1/2, \mp 1/2) \cong \frac{G^*}{\mu} (-i) \Delta \sin\theta' \sqrt{\frac{C}{6}} \qquad (A.4)$$

where

$$C = (E+m)(E_\Delta + m_\Delta)$$

For a vector meson exchange

$$B_v(S_z, s'_z) = \bar{U}_\mu(S_z) \left[G_1 g_{\mu\nu} - \frac{G_2}{m+m_\Delta} \Delta_\mu \gamma_\nu \right] \gamma_5 u(s'_z) \qquad (A.5)$$

where the ν index is saturated with the polarization vector of the exchanged vector meson. In the same approximation as above

$$B_\nu(\pm 3/2, \pm 1/2) \cong \pm \frac{i \sin\theta'}{\sqrt{2C}} \left\{ G_1 g_{3\nu} (B\eta \cos\theta' - D\Delta) + \frac{G_2}{m+m} (-\Delta C \, \delta_{\nu 3}) \right\}$$

$$B_\nu(\pm 3/2, \mp 1/2) \cong \frac{\sin\theta'}{\sqrt{2C}} \left\{ G_1 \frac{B\eta}{2} (1+\cos\theta')(-ig_{1\nu} \mp g_{2\nu}) + \frac{G_2}{m+m_\Delta} (\delta_{1\nu} \pm i\delta_{2\nu})(-i\Delta C) \right\}$$

$$B_\nu(\pm 1/2, \pm 1/2) \cong \frac{\sin\theta'}{\sqrt{6C}} \left\{ G_1 \frac{B\eta}{2} (1-\cos\theta')(ig_{2\nu} \mp g_{2\nu}) + \frac{G_2}{m+m_\Delta} (\delta_{1\nu} \pm i\delta_{2\nu})(-i\Delta C) \right\}$$
$$\pm \frac{1}{m_\Delta} \sqrt{\frac{2}{3C}} \left\{ G_1 \eta_o g_{3\nu} \cos\theta' (B\eta\cos\theta' - D\Delta) + \frac{G_2}{m+m_\Delta} (-\eta\Delta_o - \eta_o \Delta\cos\theta') C \, \delta_{3\nu} \right\}$$

$$B_\nu(\pm 1/2,\mp 1/2) \cong \pm i\,\frac{\sin\theta'}{\sqrt{6C}}\left\{-G_1 g_{3\nu}(B\eta\,\cos\theta'-D\Delta)+\frac{G_2}{m+m_\Delta}\,\delta_{3\nu}\quad C\Delta\right\}$$
$$+\frac{1}{m_\Delta}\sqrt{\frac{2}{3C}}\,\left\{G_1\;\eta\eta_o\,\frac{B}{2}\,\sin^2\theta'(g_{1\nu}\mp i g_{2\nu})\right.$$
$$\left.+\frac{G_2}{m+m_\Delta}\,(\delta_{1\nu}\mp i\delta_{2\nu})(-\eta\Delta_o-\eta_o\,\cos\theta')\;C\right\}$$

Both in expressions (A.4) and (A.6), also terms in $\sin\phi'$, $\cos\phi'$, $\sin\phi'\cos\phi'$ are neglected, because they vanish in the ϕ' integration. The terms $\sin^2\phi'$ $\cos^2\phi'$ are replaced with 1/2, being their integral π. So these terms can be just inserted in (A.2) for integration.

If the nucleon momentum is equal to 0,$(\vec{\eta},\eta_o)$ becomes $(\vec{\Delta},\Delta_o)$ and looking at expressions (A.4),(A.6), one realizes that only few matrix elements survive as indicated in (2.11) and (2.15). For the coupling (2.16)

$$B'_\nu(S_z,s'_z) = -2\;G\;\bar{U}_\mu(S_z)\;\varepsilon_{\lambda\nu k\mu}\;\Delta_\nu\eta_k\qquad u\;(s'_z)$$

Most of the matrix elements contain a factor $e^{i\phi'}$, which cancels in the integration. Therefore we are left with

$$B'_\nu\;(\mp 1/2,\pm 1/2) = \pm\,\frac{2G}{\sqrt{3C}}\;i\;\eta\;\frac{\Delta^2\sin^2\theta'}{m_\Delta}\;(\eta_o^2-\eta^2) \tag{A.8}$$

In the limit P = 0, also this matrix element is 0 and one obtains (2.17).

APPENDIX B

The π-meson exchange amplitude for the reaction $N_a N_b \to N_c \Delta$ is

$$T_p = GG^*\;\bar{u}(c)\gamma_5\;u(a)\;u_\lambda(\Delta)b_\lambda\;u(b)F(t) \tag{B.1}$$

where b_λ is the four momentum of the nucleon and F(t) is the form factor which takes in account of the absorption effect in the final state. Calculating the average on the initial and sum on final spin states, we obtain

$$\Sigma \, |T_p|^2 = \mathrm{Tr}\left[(\not{a}-m)(\not{c}+m)\right] \mathrm{Tr}\left\{(\not{b}+m)(\not{\Delta}+m_D)\right.$$

$$\left.\frac{1}{3} b_\lambda b_\mu \left[\frac{2}{m_\Delta^2} \Delta_\mu \Delta_\lambda - g_{\mu\lambda} - \gamma_\mu \gamma_\lambda + \frac{1}{m_\Lambda} (\gamma_\lambda \Delta_\mu - \gamma_\mu \Delta_\lambda)\right]\right\}$$

$$= \frac{32}{3} (a \cdot c - m^2)(b \, \Delta + m m_\Delta) \left(\frac{(b \, \Delta)^2}{m_\Delta^2} - b^2 \right) \qquad \text{(B.2)}$$

The π-meson exchange contribution to the differential cross section is

$$\left.\frac{d\sigma}{dt}\right|_\rho = \frac{\frac{1}{4} \Sigma |T_p|^2}{64 \pi s q^2} = \frac{|F(t)|^2}{(\mu^2 - t)^2} \frac{G^2}{4\pi} \frac{G^{*2}}{4\pi} \times \frac{b_\Delta^2 \pi}{6\mu^2 s q^2} \qquad \text{(B.3)}$$

$$\times \; (-t)((m+m_\Delta)^2 - t)$$

where b_Δ is the 3-momentum of the nucleon b in the rest system of the Δ.

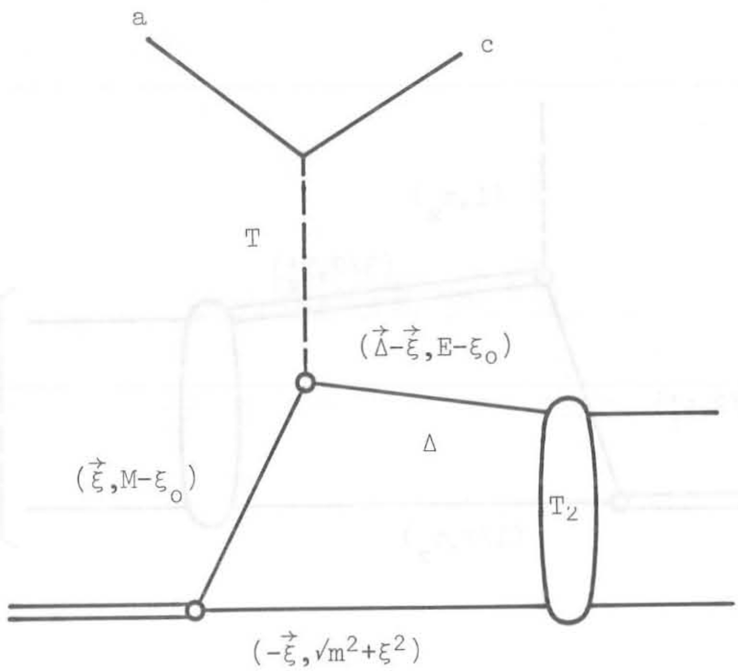

a
c
T
$(\vec{\Delta}-\vec{\xi}, E-\xi_o)$
Δ
$(\vec{\xi}, M-\xi_o)$
T_2
$(-\vec{\xi}, \sqrt{m^2+\xi^2})$

FIG. 1

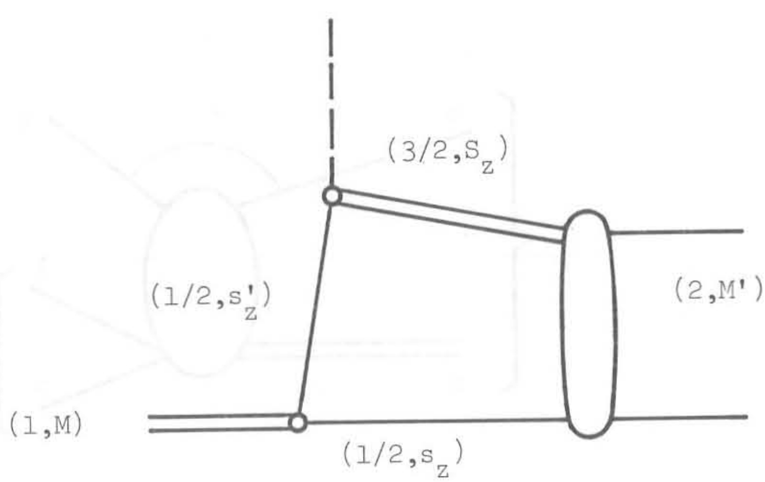

$(3/2, S_z)$
$(1/2, s'_z)$
$(2, M')$
$(1, M)$
$(1/2, s_z)$

FIG. 2

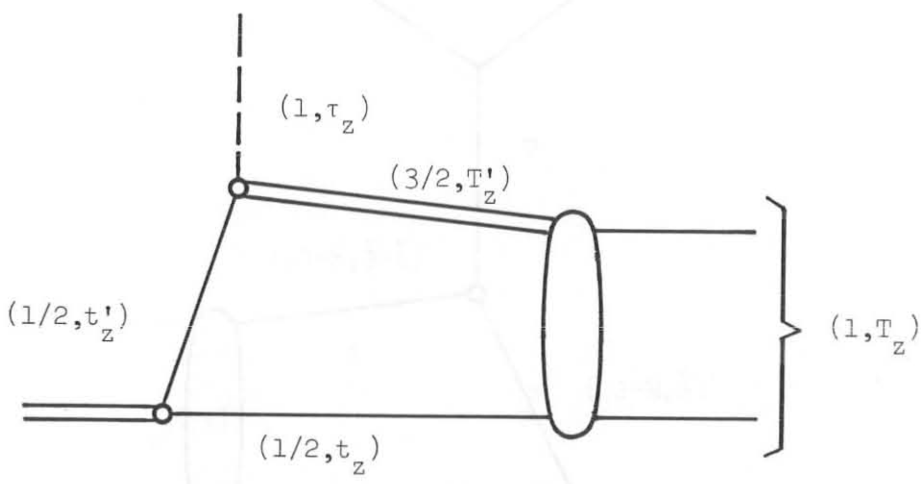


FIG. 3

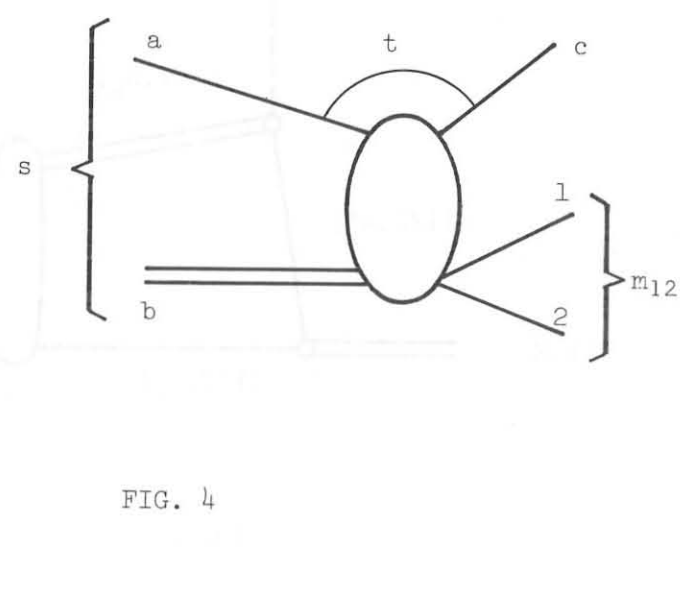


FIG. 4

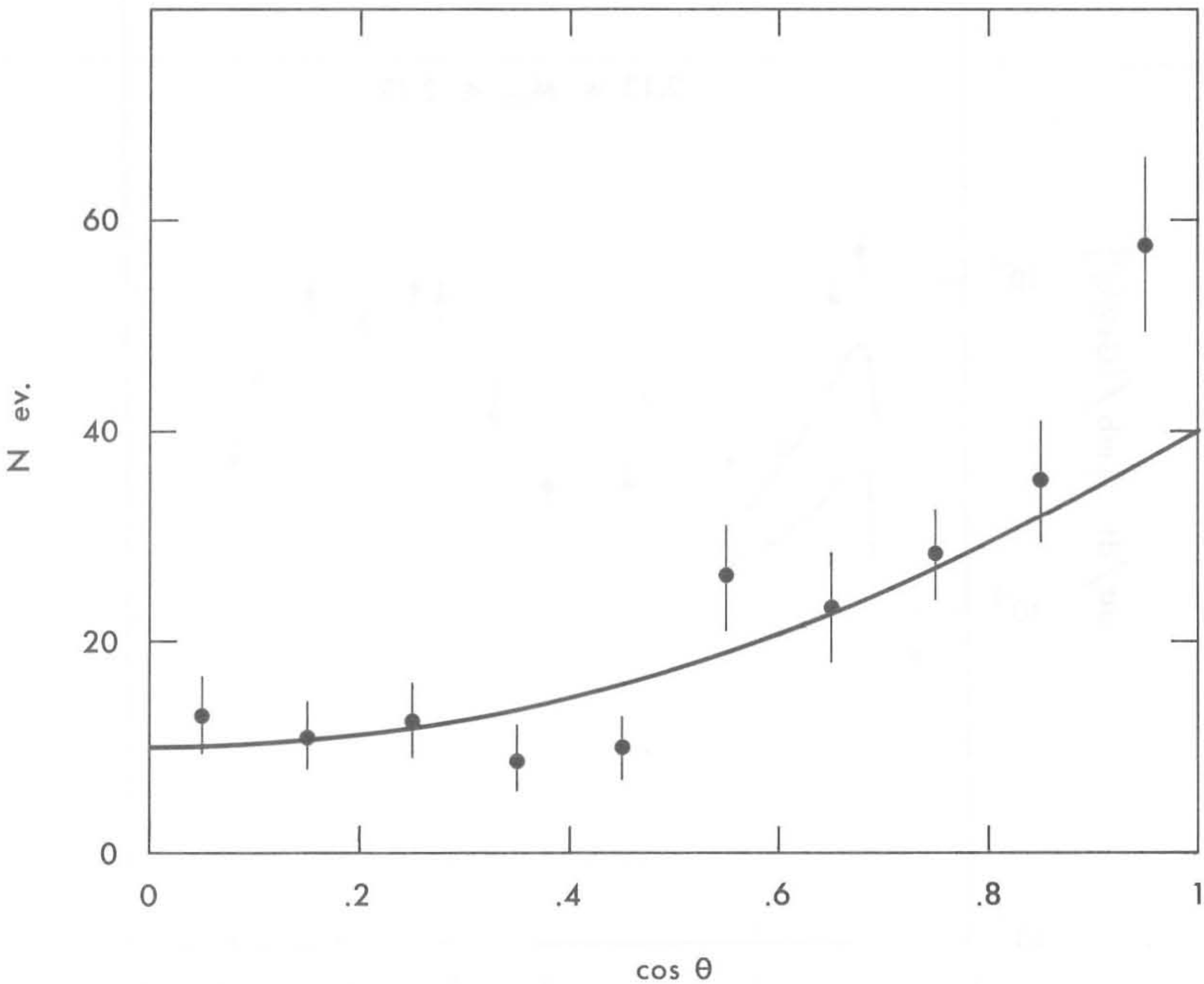
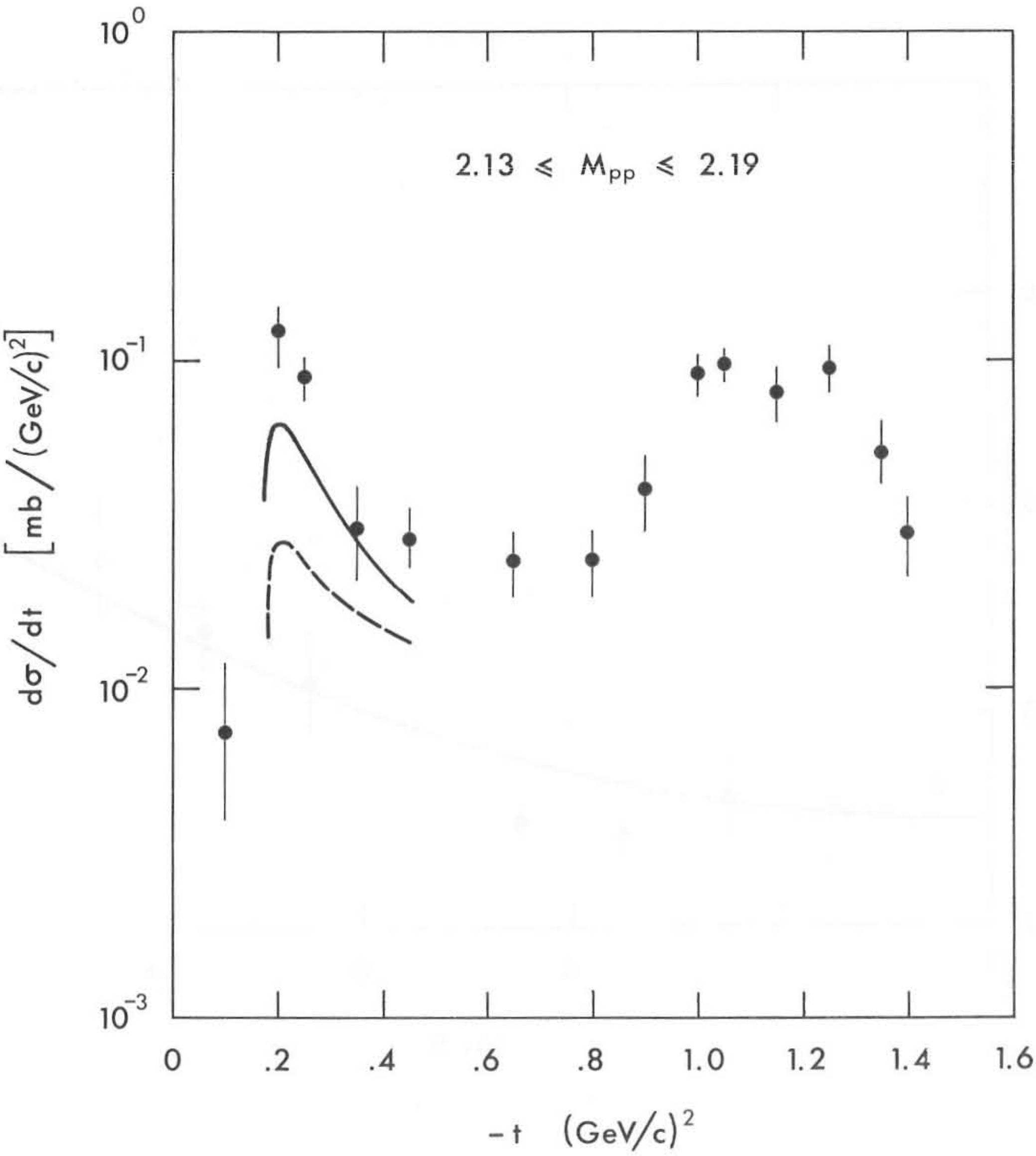


Fig. 5



10^0
10^-1
10^-2
10^-3
2.13 ≤ M_pp ≤ 2.19
dσ/dt [mb/(GeV/c)^2]
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
-t (GeV/c)^2

Fig. 6

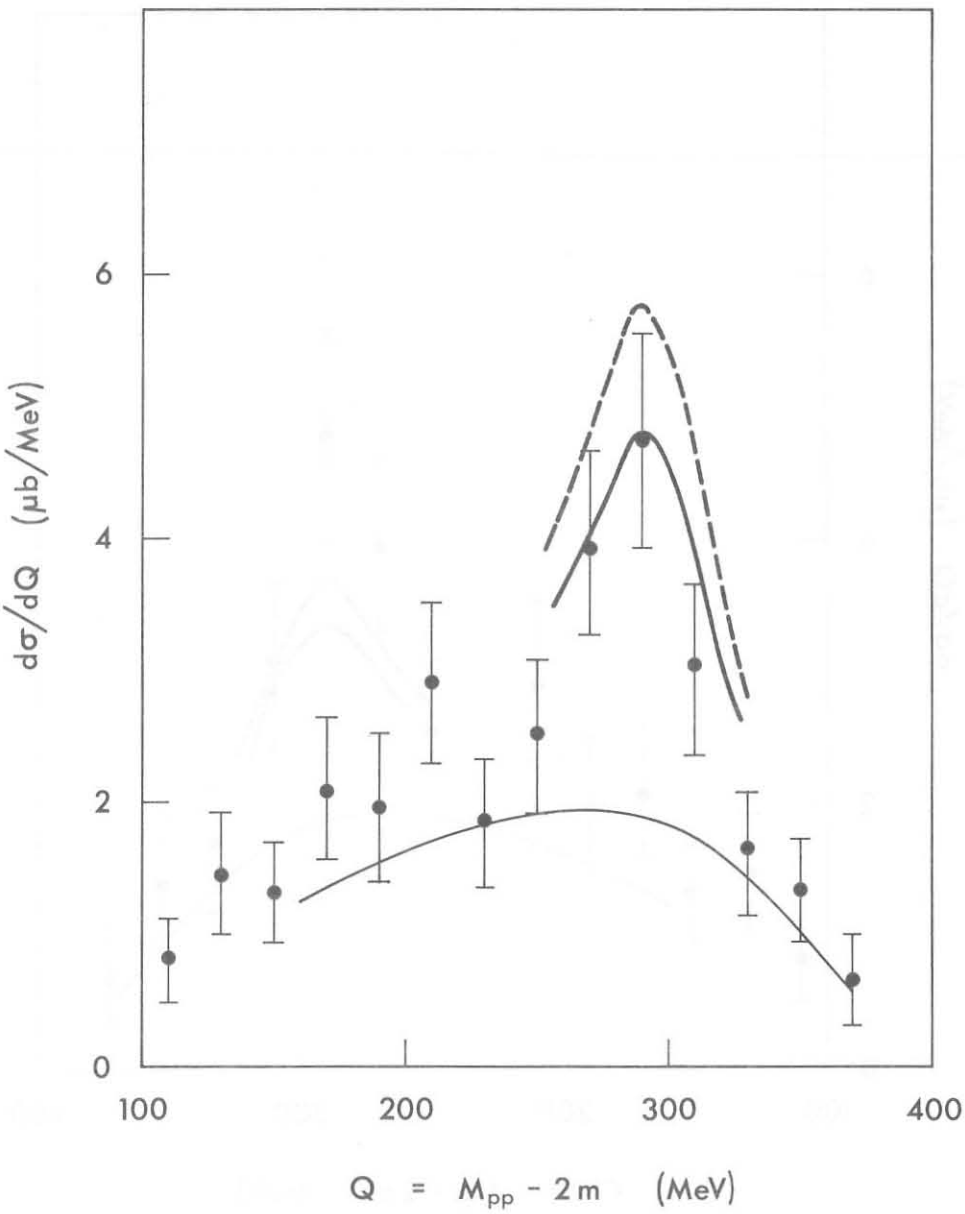


Fig. 7a

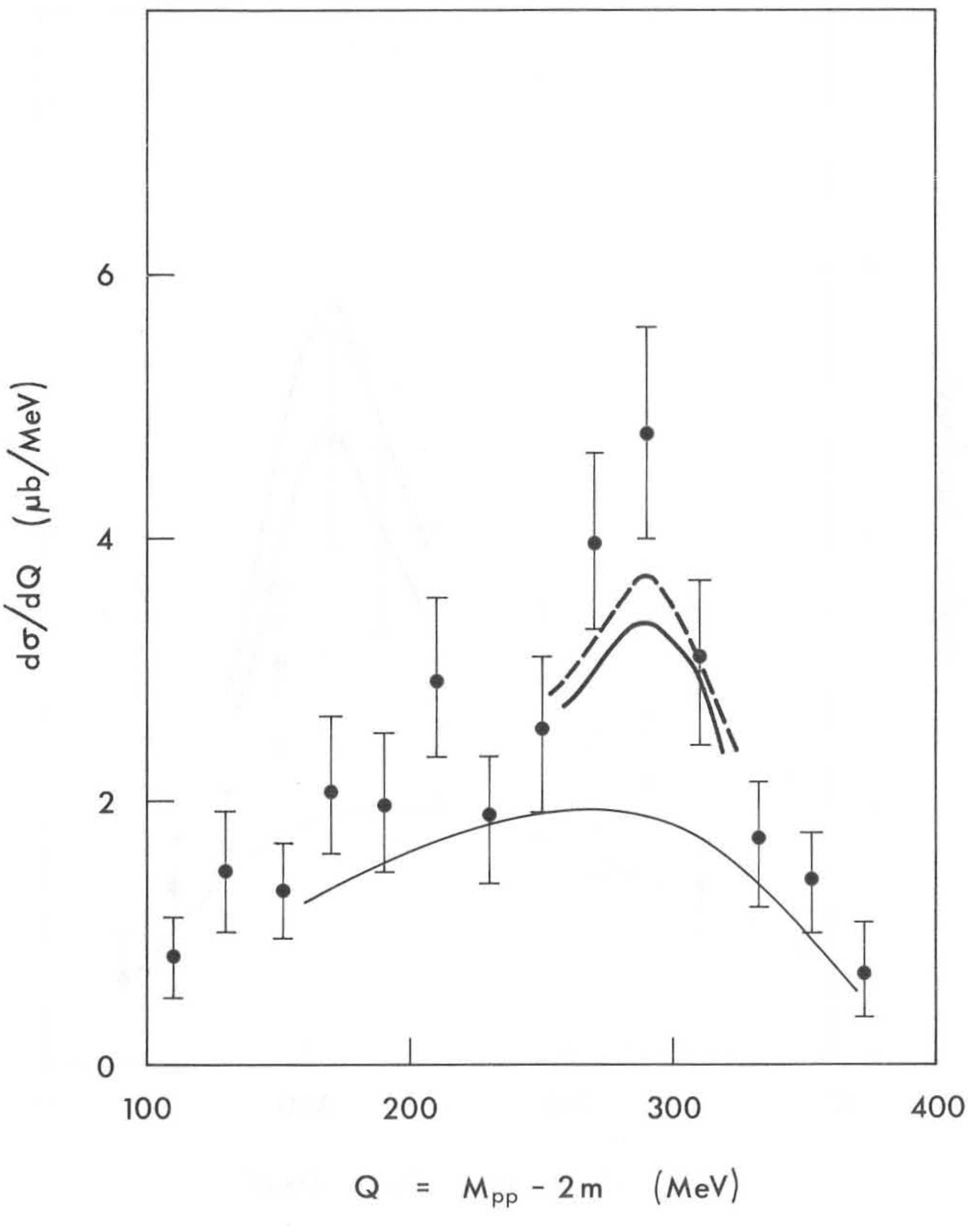


Fig. 7b

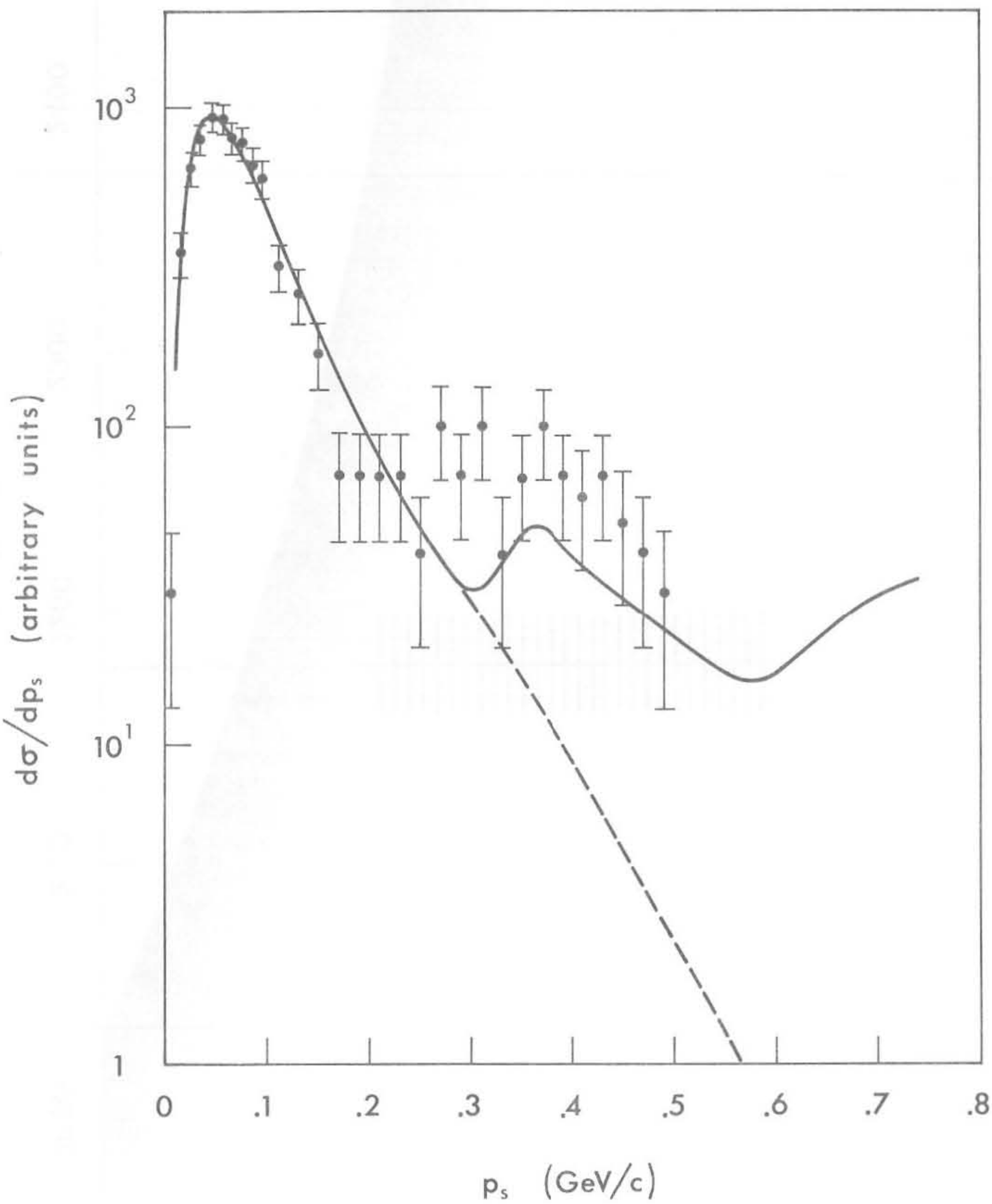
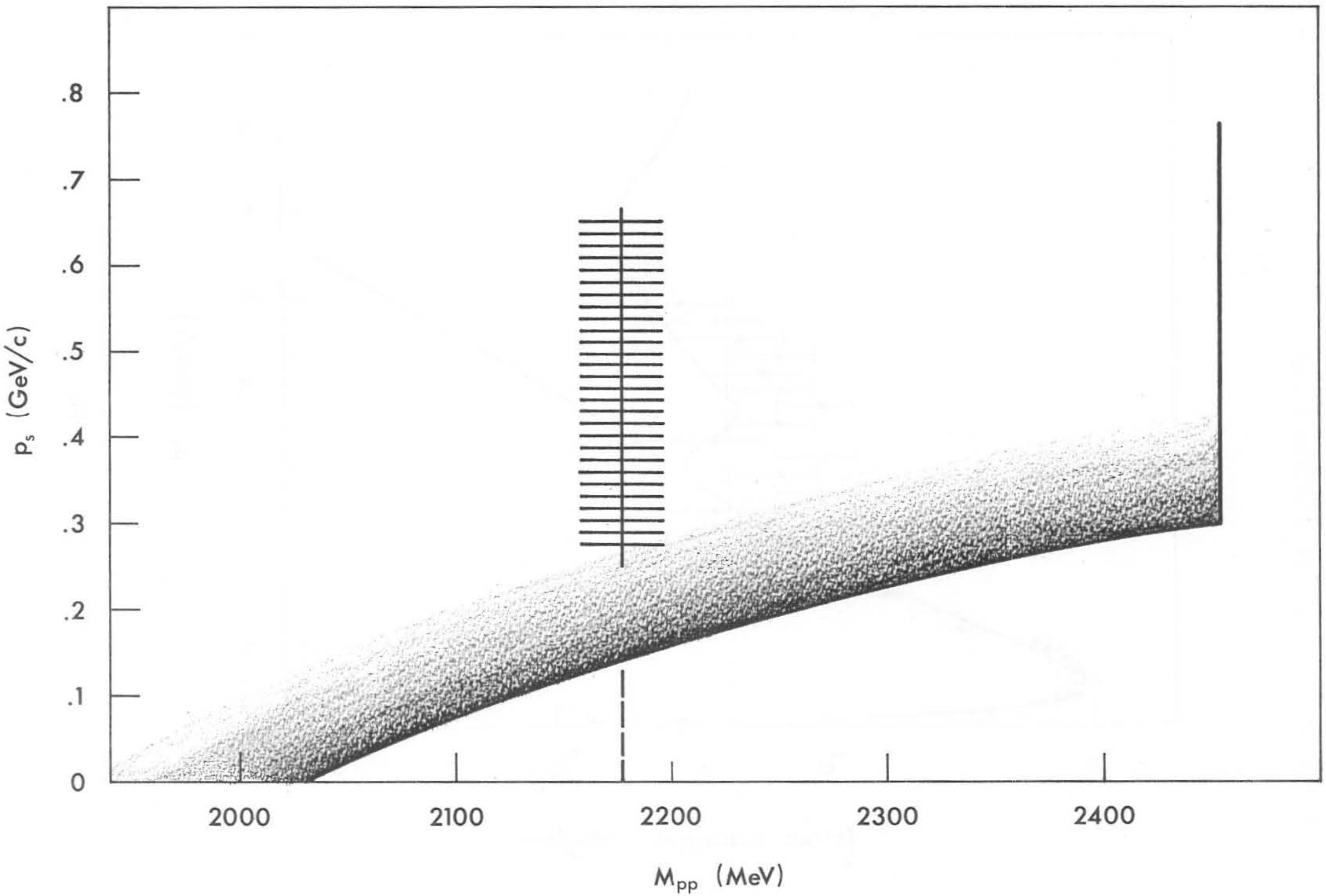


Fig. 8



p_s (GeV/c)
.8
.7
.6
.5
.4
.3
.2
.1
0
2000
2100
2200
2300
2400
M_{pp} (MeV)