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G. Ziino[x] and E. Recami: PHYSICAL MEANING OF THE CABIBBO ANGLE AND THEORETICAL EXPLANATION OF THE NONLEPTONIC $\Delta I = 1/2$ RULE.

Within SU(4) symmetry[1], the fundamental four-quark representation (p, n, c, s) may correctly be splitted into two quite symmetrical doublets

$$\binom{p}{n} \; , \quad \binom{c}{s} \tag{1}$$

the first one defined as the 1/2 (quark) representation of ordinary isospin $\vec{I}$, the second one as the 1/2 (quark) representation of a new isospin $\vec{R}$, or "strange isospin", and both assumed to be singlets under the groups $SU_R(2)$, $SU_I(2)$ respectively: $\vec{R}$ conservation (as well as $\vec{I}$ conservation) will automatically follow from SU(4) invariance. According to this scheme, the opposite charm and strangeness quantum numbers, carried by c and s respectively, are properly replaced by the corresponding two degrees of freedom for the strange-isospin third component R_3,

$$R_3|c\rangle = (C/2)|c\rangle = 1/2 \, , \quad R_3|s\rangle = (S/2)|s\rangle = -1/2, \tag{2}$$

so that the generalized Gell-Mann/Nishijima formula will read in the equivalent form

$$Q = I_3 + R_3 + B/2 \, . \tag{3}$$

On this ground, let us now assume that weak interaction, although violating both $SU_I(2)$ and $SU_R(2)$ symmetries, are invariant under the group

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$$SU_T(2) = SU_I(2) \otimes SU_R(2) \tag{4}$$

generated by the "total isospin"

$$\vec{T} = \vec{I} + \vec{R}\,. \tag{5}$$

The $T = 1/2$ (quark) representation will be two-fold degenerate according to (1). Then, if $SU_T(2)$ symmetry were not broken by any other interaction, weak processes would be invariant under the substitution

$$\binom{p}{n} \rightleftarrows \binom{c}{s}$$

which means, more generally, that any state $|I=1/2,\ R=0\rangle$ (or $|I=0,\ R=1/2\rangle$) would have a fifty-fifty probability to develop (weakly) either into a final state such as $|I=1/2,\ R=0\rangle_{fin}$ or into the symmetrical one $|I=0,\ R=1/2\rangle_{fin}$. This (bare) property can conveniently be expressed by choosing the new degenerate basis $(|T=1/2\rangle_1, |T=1/2\rangle_2)$ obtained from $(|I=1/2,\ R=0\rangle, |I=0,\ R=1/2\rangle)$ by a $\pi/4$ rotation :

$$\begin{cases} |T=1/2\rangle_1 \xrightarrow{\text{weakly}} |I=1/2,\ R=0\rangle_{fin}\,, \\ |T=1/2\rangle_2 \xrightarrow{\text{weakly}} |I=0,\ R=1/2\rangle_{fin}\,. \end{cases} \tag{6}$$

Actually, $SU_T(2)$ symmetry is broken at least by medium-strong interactions, which are invariant under $SU_I(2)$ alone. So, the (bare)weak Hamiltonian $H_W^{(o)}$ is to be replaced by

$$H_W = H_W^{(o)} + H_S \tag{7}$$

where the strong $SU_T(2)$-violating term H_S is anyhow assumed to be vanishing within the weak-interaction range[2].

Applying degenerate perturbation theory to zeroth order (any higher order is automatically ruled out by the above assumption on H_S), we shall thus find the unperturbed basis $(|T=1/2\rangle_1, |T=1/2\rangle_2)$ rotated by a given mixing angle θ_c', dependent only on the particular structure of H_S. Hence, (6) will correctly read as

$$\begin{cases} |T=1/2\rangle_{1'} \xrightarrow{\text{weakly}} |I=1/2,\ R=0\rangle_{fin}\,, \\ |T=1/2\rangle_{2'} \xrightarrow{\text{weakly}} |I=0,\ R=1/2\rangle_{fin}\,, \end{cases} \tag{8}$$

where (apart from an arbitrary phase factor)

$$\begin{cases} |T=1/2\rangle_{1'} = \cos\theta_c |I=1/2,\ R=0\rangle + \mathrm{sen}\,\theta_c |I=0,\ R=1/2\rangle \\ |T=1/2\rangle_{2'} = -\mathrm{sen}\,\theta_c |I=1/2, R=0\rangle + \cos\theta_c |I=0, R=1/2\rangle \end{cases} \tag{9}$$

and $\theta_c = \theta'_c + \pi/4$.

Experimentally, θ_c is just to be identified with the Cabibbo angle, which (except for a shift by $\pi/4$) becomes then a measure of the weak-$SU_T(2)$-degeneracy breaking due to strong interactions: more explicitly, the presence of H_S causes an actual decreasing of the bare (Cabibbo) value $\theta_c^{(o)} = \pi/4$ so as to inhibit those (weak) processes involving a nonconservation of ordinary isospin $\vec{I}$ (or strange isospin $\vec{R}$).

Applying (9) to quark states, we actually find the two degenerate doublets

$$\begin{cases} \binom{p'}{n'} = \cos\theta_c \binom{p}{n} + \mathrm{sen}\,\theta_c \binom{c}{s}, \\ \binom{c'}{s'} = -\mathrm{sen}\,\theta_c \binom{p}{n} + \cos\theta_c \binom{c}{s}, \end{cases} \tag{10}$$

where $\binom{p'}{n'}$ may be coupled only to $(\bar{p}\,\bar{n})$, and $\binom{c'}{s'}$ only to $(\bar{c}\,\bar{s})$. Taking also into account the leptonic T-doublet $\binom{\nu_\ell}{\ell}$, $(\ell = \mu, e)$, we are thus led to the following general isocurrent:

$$\vec{J}^{\lambda} = (\bar{p}\,\bar{n})\gamma^{\lambda}\vec{T}\binom{p'}{n'} + (\bar{c}\,\bar{s})\gamma^{\lambda}\vec{T}\binom{c'}{s'} + (\bar{\nu}_\ell\,\bar{\ell})\gamma^{\lambda}\vec{T}\binom{\nu_\ell}{\ell} \tag{11}$$

where for simplicity the factors $(1+\gamma^5)$ have been omitted.

Besides yielding a theoretical interpretation of the Cabibbo angle, this symmetry model is immediately seen to predict[3]:

a) The nonleptonic $\Delta I = 1/2$ rule, extended to first-order[4] R-violating (weak) processes, as merely due to the conservation of total isospin $\vec{T}$: its strict validity is in particular to be expected not only for $s \rightleftarrows p$, but even for $c \rightleftarrows n$.

b) The $I = 1/2$ hadron-current rule for $n \rightleftarrows p$ (plus leptons), and an analogous $R = 1/2$ rule for $s \rightleftarrows c$ (plus leptons).

c) A quite identical dynamical behaviour for the corresponding (leptonic or nonleptonic) processes $n \rightleftarrows p$, $s \rightleftarrows c$, and for the analogous ones $s \rightleftarrows p$, $c \rightleftarrows n$.

If moreover we take into account the new "revised" formulation of the Dirac fermion-antifermion theory built up by one of the present authors(5), the same model can be easily shown to involve the further predictions :

d) Actual electromagnetic-like form $\bar{\psi}\gamma^{\lambda}\psi$ for neutral currents conserving both charm and strangeness (according to (11), the corresponding ratio between neutral and charged processes cross-sections will take on the theoretical value $r = 1/4$, in good agreement with experiments(6)).

e) Suppression to all orders of charm- and strangeness-violating neutral processes.

For an explicit check, let us denote by $\bar{\psi}_a$, $\psi_{\bar{a}}$ the respective fermion and antifermion (Dirac) fields, and by χ_a, $\chi_{\bar{a}}$ the corresponding "chiral" fields $\frac{1}{\sqrt{2}}(1+\gamma^5)\psi_a$, $\frac{1}{\sqrt{2}}(1-\gamma^5)\psi_{\bar{a}}$. Using the new, correct charge-conjugation definition(5)

$$C^{-1}\psi_a C = \psi_{\bar{a}} = \gamma^5\psi_a \tag{12}$$

it is not difficult to get the general result(5)

$$\bar{\psi}_a\gamma^{\lambda}\psi_b = \frac{1}{2}(\bar{\chi}_a\gamma^{\lambda}\chi_b + \bar{\chi}_{\bar{a}}\gamma^{\lambda}\chi_{\bar{b}}) = \bar{\psi}_b\gamma^{\lambda}\psi_a \tag{13}$$

where $\bar{\chi}_a\gamma^{\lambda}\chi_b = \bar{\chi}_{\bar{b}}\gamma^{\lambda}\chi_{\bar{a}}$ and $\bar{\chi}_{\bar{a}}\gamma^{\lambda}\chi_{\bar{b}} = \bar{\chi}_b\gamma^{\lambda}\chi_a$ are just equivalent to the V-A and V+A currents respectively. Therefore, setting $b = a$, we automatically have :

$$\bar{\chi}_a\gamma^{\lambda}\chi_a = \frac{1}{2}(\bar{\chi}_a\gamma^{\lambda}\chi_a + \bar{\chi}_{\bar{a}}\gamma^{\lambda}\chi_{\bar{a}}) = \bar{\psi}_a\gamma^{\lambda}\psi_a \tag{14}$$

and point d) is fully verified. Within $SU_T(2)$ symmetry, eq. (14) will in particular be valid even for charm- and strangeness-violating neutral currents ; so that, taking into account (11), it must be

$$(\bar{\chi}_p\gamma^{\lambda}\chi_c - \bar{\chi}_n\gamma^{\lambda}\chi_s - \bar{\chi}_c\gamma^{\lambda}\chi_p + \bar{\chi}_s\gamma^{\lambda}\chi_n)\,\text{sen}\,\theta_c = 0 \tag{15}$$

which leads to an automatical first-order suppression of charm- and strangeness-violating neutral processes. Indeed, according to (14), the same processes might equivalently occur through either higher-order charged-current or charged-anticurrent diagrams. Yet, owing to the resulting opposite couplings for the single reaction pairs $c \rightleftarrows n$, $\bar{c} \rightleftarrows \bar{n}$ and $s \rightleftarrows p$, $\bar{s} \rightleftarrows \bar{p}$, the respective amplitudes $A^{(+)}$, $A^{(-)}$ of the former and the latter diagrams would necessarily have opposite signs. And so we must have to any order

$$A^{(+)} = \frac{1}{2}(A^{(+)} + A^{(-)}) = - A^{(-)} = 0 \qquad (16)$$

which fully verifies point e).

Finally, it is worth spending a few words on the physical meaning of the new group $SU_T(2)$. Within $SU_T(2)$ symmetry, eq. (3) goes over into

$$Q = T_3 + B/2 \qquad (3')$$

which is the same as saying that, when weak interactions are "turned on", the strong-symmetry group SU(4) "collapses" to $SU_T(2)$, thus giving rise to a rougher classification of hadrons, in terms i. e. of less conserved quantum numbers. An interesting consequence is that weak intermediate bosons (now members of the T-triplet) cannot be expected to be produced as single real particles (although, in the dynamics of weak interactions, they would behave as single virtual particles having a unique $SU_T(2)$-symmetric mass): in this sense, if truly existing, such bosons may have already been found out (at least in the lowest part of their mass spectrum) since, within $SU_T(2)$ symmetry, most of the known spin-1 mesons are indeed to be classified as degenerate members of the T-triplet.

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Notes and References. -

(1) - See, e.g., B. J. Bjørken and S. L. Glashow, Phys. Letters 11, 255 (1964); S. L. Glashow, J. Iliopulos and L. Maiani, Phys. Rev. 2D, 1285 (1970).

(2) - This assumption, which allows $SU_T(2)$ symmetry not to be broken in the dynamics of weak interactions, is strongly supported by the experimental fact that the weak-force range is much smaller than the pion Compton-wavelength.

(3) - Another very simple consequence of $\vec{T}$-conservation in weak-interactions, with regard to neutral kaons and anti-kaons, is that only a K^o_S and a K^o_L will be actually "seen" through weak interactions (as far as we neglect CP violation, at least). In fact, let us consider the usual K^o, $\bar{K}^o$ meson states, which in our formalism read

$$\begin{cases} |K^o\rangle = |\bar{n}s\rangle = |R=\frac{1}{2}, I=\frac{1}{2}; R_3=+\frac{1}{2}, I_3=-\frac{1}{2}\rangle ; \\ |\bar{K}^o\rangle = |n\bar{s}\rangle = |R=\frac{1}{2}, I=\frac{1}{2}; R_3=-\frac{1}{2}, I_3=+\frac{1}{2}\rangle . \end{cases}$$

From the definition of T, we get:

$$\begin{cases} |K^o\rangle = \frac{1}{\sqrt{2}}|T=1; T_3=0\rangle + \frac{1}{\sqrt{2}}|T=0; T_3=0\rangle ; \\ |\bar{K}^o\rangle = \frac{1}{\sqrt{2}}|T=1; T_3=0\rangle - \frac{1}{\sqrt{2}}|T=0; T_3=0\rangle , \end{cases}$$

where of course it must be:

$$|K^o_S\rangle \equiv |T=1; T_3=0\rangle ; \qquad |\bar{K}^o_L\rangle \equiv |T=0; T_3=0\rangle ,$$

so that, by weak interactions, we shall "see" only the eigenstates $|K^o_S\rangle, |K^o_L\rangle$ of T, T_3.

(4) - From now on, by saying "n-order" we will implicity mean "n--order in the Fermi coupling constant G".

(5) - See G. Ziino, Lett. Nuovo Cimento 15, 449 (1976), and G. Ziino, Why an electromagnetic-like form $\bar{\psi}\gamma^\lambda\psi$ for neutral weak currents, to appear. In the first paper, that author shows that the two fermion and antifermion (Dirac) fields ψ_a, $\psi_{\bar{a}}$ can really have opposite intrinsic parities only if they are solutions of the symmetrical equations

$$i\gamma^\lambda \frac{\partial}{\partial x^\lambda}\psi_a = \mu_o \psi_a , \qquad i\gamma^\lambda \frac{\partial}{\partial x^\lambda}\psi_{\bar{a}} = -\mu_o \psi_{\bar{a}}$$

which involve, besides the new relation $\psi_{\bar{a}} = \gamma^5 \psi_a$, the following basis transformation between ψ_a, $\psi_{\bar{a}}$ and the two (fermion and antifermion) "chiral" fields

$$\chi_a = \frac{1}{\sqrt{2}}(1+\gamma^5)\psi_a \,, \qquad \chi_{\bar{a}} = \frac{1}{\sqrt{2}}(1-\gamma^5)\psi_a \,:$$

$$\psi_a = \frac{1}{\sqrt{2}}(\chi_a - \chi_{\bar{a}}) \,, \qquad \psi_{\bar{a}} = \frac{1}{\sqrt{2}}(\chi_a + \chi_{\bar{a}}) \,.$$

In the second paper, moreover, the problem of vector currents is dealt with, and the conclusion is achieved that fermion and antifermion charged currents may only have the respective "chiral" forms $\bar{\chi}_a\gamma^\lambda\chi_b$, $\bar{\chi}_{\bar{a}}\gamma^\lambda\chi_{\bar{b}}$, while neutral ones the electromagnetic-like form $\bar{\psi}\gamma^\lambda\psi$. It is worthwile - at this point - to stress that the total isospin $\vec{T}$ has nothing to do with the "weak isospin" introduced in the first quotation of this ref. (5). Quantity $\vec{T}$ is defined so as to act directly on the Dirac fields, whereas the "weak isospin" was defined so as to act directly on the (corresponding) chiral fields (and only in this sense it was shown to generate the only possible isospin-like models for weak interactions). A more complete work, that takes into account also invariance under the SU(2) group generated by the "weak isospin", is indeed the subject of work in progress. Cf. also E. Recami and G. Ziino, Nuovo Cimento 33A, 205 (1976).

(6) - See, e.g., F. J. Hasert et al., Phys. Letters 46B, 138 (1973); and Nuclear Phys. B73, 1 (1974); B. Aubert et al., Phys. Rev. Letters 32, 1454 and 1457 (1974); A. Benvenuti et al., Phys. Rev. Letters 32, 800 (1974); S. Barish et al., Phys. Rev. Letters 33, 448 (1974).