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E. Recami and G. Ziino[x]: EXPLAINING $\Delta I = \frac{1}{2}$ RULE AND THE EXISTENCE OF K_{oS}, K_{oL} : A NEW FOUR-QUARK SCHEME.

ABSTRACT :

A new four-quark scheme is suggested, in which the strange and charm quarks λ, c are considered as the members of the 1/2 representation (with I = 0) of a new "strange isospin" $\overline{S}$. The new scheme is shown to be already contained in SU(4), and allows a different internal classification of hadron supermultiplets. Moreover, from the conservation law of the total isospin $\underset{\approx}{T} = \underset{\approx}{I} + \underset{\approx}{\overline{S}}$, we succeed in particular in explaining: (i) the $\Delta I = 1/2$ rule for strangeness violating weak interactions; (ii) the existence of K_{oS}, K_{oL}.

1. - Let us observe that all the good features of the four-quark model[1-3] can be more straightforwardly derived from considering the third and fourth[4] quarks λ, c as the members of the 1/2 representation of a new "strange isospin" $\overline{S}$.

In fact, let us consider the four quarks (with their electric charge) as in the charm model[1, 2]:

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$$\begin{cases} n\,(q = -1/3)\,; \\ p\,(q = +2/3)\,; \end{cases} \qquad \begin{cases} \lambda\,(q = -1/3)\,; \\ c\,(q = +2/3)\,, \end{cases} \tag{1}$$

where of course n, p, λ constitute the fundamental representation of SU(3).

As well-known, the quarks n, p are the members of the ordinary isospin I, with strangeness $S = 0$ and charm $C = 0$:

$$n\,(I_3 = -1/2)\,; \qquad p\,(I_3 = +1/2)\,. \tag{2}$$

Analogously, we can well consider the quarks λ, c as the members of the 1/2 representation (with $I = 0$) of a new isospin $\bar{S}$, that we shall call "strange isospin" (or "strange spin"):

$$\lambda\,(\bar{S}_3 = -1/2)\,; \qquad c\,(\bar{S}_3 = +1/2)\,. \tag{3}$$

In this way, as the "strange isospin" doublet λ, c is an ordinary--isospin singlet, so it is natural to consider the ordinary-isospin doublet n, p as a strange-isospin singlet.

Of course, in the present scheme the two quantum numbers strangeness, $\bar{S}$, and charm, C, are very simply substituted in the case of our four quarks by the degrees of freedom $\bar{S}_3 = -1/2$ and $\bar{S}_3 = +1/2$, respectively, of the strange-isospin $\bar{S}$:

$$2\bar{S}_3 = \begin{cases} -1 = S(\lambda)\,; \\ +1 = C(c)\,, \end{cases}$$

or better:

$$\frac{S(\lambda)}{2} = \bar{S}_3(\lambda) = -\frac{1}{2}\;; \tag{4a}$$

$$\frac{C(c)}{2} = \bar{S}_3(c) = +\frac{1}{2}\;. \tag{4b}$$

The generalized Gell-Mann and Nishijima formula of the charm model:

$$Q = I_3 + \frac{S+C}{2} + \frac{B}{2}\;,$$

in our scheme reads in the equivalent form:

$$Q = I_3 + \bar{S}_3 + \frac{B}{2} \quad . \tag{5}$$

2.- We immediately want to forward three important consequences of our present, symmetric scheme.

Firstly, our quark model yields a straightforward explanation of the rule

$$\Delta I = \frac{1}{2} \; , \tag{6}$$

for the (strangeness violating) weak interactions, as a consequence of the conservation of the vector $\underset{\sim}{I}+\underset{\sim}{\bar{S}}$, that we shall call "total isospin" $\underset{\sim}{T}$:

$$\underset{\sim}{T} = \underset{\sim}{I} + \underset{\sim}{\bar{S}} \, . \tag{7}$$

In the other words, if we postulate(5) the "total isospin" conservation law to hold for both all weak and the strong interactions, then we get immediately the $\Delta I = 1/2$ rule for all $\Delta S = 1$ weak interactions (or better for all $\Delta \bar{S} = 1/2$ weak interactions).

Secondly, let us consider the K_o and $\bar{K}_o$ meson states, according to our (more symmetric) scheme :

$$|K_o\rangle = |n\bar{\lambda}\rangle = |\bar{S} = \tfrac{1}{2}; \quad I = \tfrac{1}{2}; \quad \bar{S}_3 = +\tfrac{1}{2}; \quad I_3 = -\tfrac{1}{2}\rangle ;$$

$$|\bar{K}_o\rangle = |\bar{n}\lambda\rangle = |\bar{S} = \tfrac{1}{2}; \quad I = \tfrac{1}{2}; \quad \bar{S}_3 = -\tfrac{1}{2}; \quad I_3 = +\tfrac{1}{2}\rangle .$$

From the above definition (7), we get :

$$|K_o\rangle = \frac{1}{\sqrt{2}}|T=1 \; ; \; T_3 = 0\rangle + \frac{1}{\sqrt{2}}|T=0 \; ; \; T_3 = 0\rangle \; ; \tag{8a}$$

$$|\bar{K}_o\rangle = \frac{1}{\sqrt{2}}|T=1 \; ; \; T_3 = 0\rangle - \frac{1}{\sqrt{2}}|T=0 \; ; \; T_3 = 0\rangle \, , \tag{8b}$$

where of course it must be(6) :

$$\left\{ \begin{aligned} |K_{o_S}\rangle &= |T = 1 ; \quad T_3 = 0\rangle ; \\ |K_{o_L}\rangle &= |T = 0 ; \quad T_3 = 0\rangle . \end{aligned} \right. \tag{9}$$

Since, in our assumptions, the total isospin $\underset{\sim}{T}$ is conserved in weak interactions, it follows immediately that only the eigenstates, $|K_{o_S}\rangle$, $|K_{o_L}\rangle$, of T, T_3 can be actually "seen" through weak interactions.

Thirdly, still from conservation of $\underset{\sim}{T}$, one can directly derive the CVC-hypothesis, in the case of $\Delta\bar{S}=0$ currents. Besides, even in the case of $\Delta\bar{S}\neq 0$ isocurrents, one can define a generalized current (carrying T) and then derive the conservation of the generalized vector isocurrent, thus extending the original CVC-hypothesis to the total-isospin case.

3. - Let us now come to the question of the algebraic structure underlying our four-quark model. Following Avilez-Valdez[7], from our required symmetries $SU(2)_I$, $SU(2)_{\bar{S}}$, SU(3) we are automatically led to the sympletic group Sp(4) in four dimensions[8], which however is contained[8] in SU(4): $Sp(4) \subset SU(4)$. Therefore we stress that the usual charm model[1,2] itself must in particular include $SU(2)_{\bar{S}}$ as well as $SU(2)_I$.

If we adopt SU(4) symmetry, then by our scheme we shall merely forward a different internal classification of supermultiplets.

For instance, the spin 1/2 baryon 20-plet of SU(4) becomes:

$$
\begin{array}{llllll}
\bar{S}=0; & & ; I=\frac{1}{2}; & I_3=+\frac{1}{2} & \longrightarrow & npp \equiv P^+ \equiv P \\
 & & & & \longrightarrow & c\lambda p \equiv P^+(c\lambda) \\
 & & & I_3=-\frac{1}{2} & \longrightarrow & npp \equiv N^o \equiv N \\
 & & & & \longrightarrow & c\lambda n \equiv N^o(c\lambda) \\
\bar{S}=\frac{1}{2}; & \bar{S}_3=+\frac{1}{2}; & I=0 & & \longrightarrow & cc\lambda \equiv C^+(c\lambda) \\
 & & & & \longrightarrow & cpn \equiv C^+ \equiv C \\
 & & I=1; & I_3=+1 & \longrightarrow & cpp \equiv \Sigma^{++}(c) \\
 & & & I_3=0 & \longrightarrow & cpn \equiv \Sigma^+(c) \\
 & & & I_3=-1 & \longrightarrow & cnn \equiv \Sigma^o(c) \\
 & \bar{S}_3=-\frac{1}{2}; & I=0 & & \longrightarrow & c\lambda\lambda \equiv \Lambda^o(\lambda c) \\
 & & & & \longrightarrow & \lambda pn \equiv \Lambda^o \equiv \Lambda \\
 & & I=1; & I_3=+1 & \longrightarrow & \lambda pp \equiv \Sigma^+ \\
 & & & I_3=0 & \longrightarrow & \lambda pn \equiv \Sigma^o \\
 & & & I_3=-1 & \longrightarrow & \lambda nn \equiv \Sigma^-
\end{array}
$$

$$\begin{array}{lllcl}
\overline{S} = 1\,;\ \overline{S}_3 = +1\,; & I = \frac{1}{2}; & I_3 = +\frac{1}{2} & \longrightarrow & ccp \equiv \Xi^{++}(cc) \\
 & & I_3 = -\frac{1}{2} & \longrightarrow & ccn \equiv \Xi^{+}(cc) \\
\overline{S}_3 = 0\,; & & I_3 = +\frac{1}{2} & \longrightarrow & c\lambda p \equiv \Xi^{+}(c\ ) \\
 & & I_3 = -\frac{1}{2} & \longrightarrow & c\lambda n \equiv \Xi^{o}(c\ ) \\
\overline{S}_3 = -1\,; & & I_3 = +\frac{1}{2} & \longrightarrow & \lambda\lambda p \equiv \Xi^{o} \\
 & & I_3 = -\frac{1}{2} & \longrightarrow & \lambda\lambda n \equiv \Xi^{-}
\end{array}$$

where $N^o \equiv N$ is the neutron, $P^+ \equiv P$ the proton, $\Lambda^o \equiv \Lambda$ the Lambda--baryon and $C^+ \equiv C$ the newly discovered "charmed proton"[(9)]. The baryon names are preceded by their empirical "chemical formula" in terms of quarks.

It is noticeable e.g. the large mass-splitting in the $\overline{S}_3 = \pm 1/2$ doublet of baryons Λ^o, C^+; but it can be due to the so-called medium-strong interactions, which in fact are known to be invariant not under group SU(3), but only under the group SU(2) generated by ordinary isospin. More generally, the mass-splitting due to medium-strong interactions is expected to depend on both $\overline{S}$ and $\overline{S}_3$, so that the "medium-strong" Hamiltonian is not expected to be invariant under the group SU(2) generated by $\overline{S}$.

Since we assumed strong interactions to be invariant also under $SU(2)_{\overline{S}}$, then the SU(3) symmetry among n, p, λ becomes - by a mere $SU(2)_{\overline{S}}$-rotation - the SU(3) among n, p, c. Therefore, by applying the Gell-Mann and Okubo formula to the alternative SU(3)-fundamental-representation n, p, c, besides the usual mass relation[(10)]

$$\frac{3}{4}\Lambda + \frac{1}{4}\Sigma = \frac{1}{2}N + \frac{1}{2}\Xi\ ,$$

we can symmetrically write at the first order:

$$\frac{3}{4}C^+ + \frac{1}{4}\Sigma(c) = \frac{1}{2}N + \frac{1}{2}\Xi(cc)\ , \tag{10}$$

where $\Sigma(c)$ is the mass of the triplet $\Sigma^o(c)$, $\Sigma^+(c)$, $\Sigma^{++}(c)$, and $\Xi(cc)$ of the doublet $\Xi^+(cc)$, $\Xi^{++}(cc)$. From the last two relations, one may also get:

$$\frac{3}{4}C^+ + \frac{1}{4}\Sigma(c) - \frac{1}{2}\Xi(cc) = \frac{3}{4}\Lambda^o + \frac{1}{4}\Sigma - \frac{1}{2}\Xi\ . \tag{10'}$$

The previous considerations will be derived elsewhere also from more general interaction-symmetry considerations[11].

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(3) - We are neglecting the recent evidence in favour of heavy leptons, and therefore of possible existence of further quarks.

(4) - Of course, each quark (n, p, λ, c) might exist with the three apparent signs (red, blue, yallow) of the "strong charge".

(5) - Notice that, in the formalism put forth by Ziino, the conservation of the total-isospin vector is a consequence of the charge independence of weak interactions. See G. Ziino, Lett. Nuovo Cimento 13, 95 (1975); ibidem (to appear); and (in preparation).

(6) - S. S. Schweber, An introduction to relativistic Q. F. T. (Evanston, Ill., 1961), p. 288.

(7) - C. Avilez-Valdez, Internal Report DESY-T-75/4 (Nov. 1975). After the completion of the present work in preprint form (Catania Univ. Preprint, Feb. 1976), we were made aware of that Internal Report, where similar ideas are discussed.(to be published in Proceedings of the Smalenice Conference).

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(9) - G. Blietszhau et al., Phys. Letters (to appear).

(10) - E. Lifchitz and L. Pitayevski, Theorie quantique relativiste (MIR Pub., Moscow, 1973), 2^{me} partie, p. 209.

(11) - A larger group containing our requested symmetries and suitable for more than four quarks looks to be $SU(4) \otimes SU(2)_V$, where V is an auxiliary quantum number; see M. Noga and C. Cronström, Phys. Rev. D1, 2414 (1970), and (private communication).