THE REGGEON CALCULUS AS A CRITICAL PHENOMENON; CALCULATIONS OF THE CRITICAL INDICES

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A typical problem in the Reggeon calculus ([1]) is the computation of multipomeron exchanges between two particle lines:

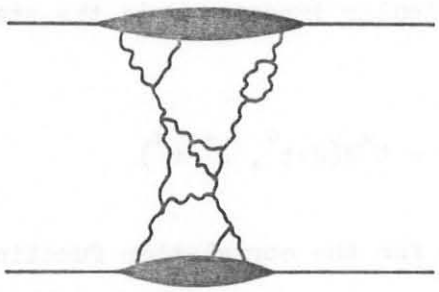


Fig. 1

This graph represents a contribution to the scattering amplitude. Note that we have multipomeron effects in the propagator, like radiative corrections, and we have multipomeron couplings to the external particle lines.

Besides the two body scattering amplitude, we hope to calculate other observables: for instance the particle distributions, the multiplicities, the correlations, etc. Each Pomeron line can be cut: a cut corresponds to a contribution to the production cross section.

The problem is in the intrinsic non-perturbative nature of the calculation for $\alpha_p \approx 1$. Even for small Pomeron coupling, complicated graphs give large effects for $\ell ns \to \infty$ (s = squared C.M. energy). The cuts in the J plane are all at the same place J=1 and the limit $\ell ns \to \infty$ corresponds to $E \equiv 1-J \to 0$. Therefore the problem is an infrared one (E is like an energy, being conserved at the interactions) and the limit $\alpha_p \to 1$ is the analogue of the limit for the temperature $T \to T_c$ in the theory of the critical phenomena ([2,3]).

Formally the problem is equivalent to the evaluation of the partition function

$$Z(J) = \int d\psi d\psi^+ \; e^{i(\mathcal{A}+J^+\psi+J\psi^+)} \tag{1}$$

where

$$\mathcal{A} = \int d^D r d\tau \left\{ \frac{i}{2}\psi^+ \overleftrightarrow{\partial}_\tau \psi - \alpha' |\nabla_2 \psi|^2 + h|\psi|^2 + \frac{i\lambda}{2}\left(\psi^+\psi^2 + \psi^{+2}\psi\right)\right\} \tag{2}$$

and the identification is $\tau = i\ \ell n\ s$.

The parameter $h \equiv \alpha_p - 1$ is something like the temperature difference $t \equiv T - T_c$. Note that for generality we take the transverse space to have dimensions = D.

This picture leads us to parallel the features of the critical phenomena ([4]). Of particular importance is the static scaling hypothesis for $W \equiv \ell n\ Z$

$$W(t,J,J^+) = t^{\times} W(J\cdot t^{\alpha},\ J^+\cdot t^{\alpha}) \tag{3}$$

and the scaling law for the correlation function

$$\langle T(\psi(\vec{r},\tau)\psi^+(0,0)\rangle \equiv G(\vec{r},\tau;t) = t^A g\left(\frac{|r|}{t^B},\ \frac{\tau}{t^C}\right) \tag{4}$$

Physics take place at $t \simeq 0$ and one does not explain why the temperature is at the critical value. We can rewrite the scaling law for $t = 0$ as

$$G \sim \tau^{A/C}\ f\left(\frac{|r|}{\tau^{B/C}}\right) \tag{5}$$

A, B, C, x, α are called the critical indices. In general the critical indices are related by linear relations and in our case, as long as we consider Pomeron Green functions, there are three independent indices, which for $t = 0$ become two. They are specified by the expression of the scattering amplitude for large $\ell n\ s$:

$$A(S,k^2) \sim is(\ell n\ s)^{\eta}\cdot\tilde{g}(K^2\cdot(\ell n\ s)^{\nu}) \tag{6}$$

For instance, $\eta = \frac{A+DB}{C}$, $\nu = 2\ B/C$.

The critical indices do not depend on the actual form of the interaction, e.g. on the value of λ in eq. (2), and more generally on the actual features of the problem, e.g. the continue or discrete nature of the space $(\vec{r},\tau)$, as long as the dimensionality and the symmetry properties stay the same.

This is the fascinating aspect of this theory, the relevant observables of the high energy strong interactions being in principle computable without

input.

In particular the critical indices should be the same if we add to the action in eq. (2) more complicated forms of Pomeron interactions. Indeed we have written only the relevant structure, any other coupling or modification of the Pomeron trajectory being irrelevant, in the critical phenomena jargon, in the limit $E \to 0$ ([5]). The actual computation is of course a formidable task. A method is the renormalization group with the ϵ-expansion ([6]), which consists in integrating over the high E degrees of freedom. Putting

$$\psi(E) = \begin{cases} \psi_1(E) \text{ for } |E| < \Lambda \\ \psi_2(E) \text{ for } |E| > \Lambda \end{cases}$$

we define a new action through

$$e^{i\mathcal{A}'(\psi_1)} = \int d\,\psi_2\; e^{i\mathcal{A}(\psi_1+\psi_2)}$$

and after rescaling the variable E we look for the fixed point $\mathcal{A} = \mathcal{A}' = \mathcal{A}^*$. If $D = 4$, $\mathcal{A}^*$ = free theory, therefore for small ϵ, $D = 4-\epsilon$, we can take a perturbative expansion. The Pomeron Green functions are determined by the anomalous dimensions of the field ψ

$$< \psi\,(E)\; \psi^+(o) > \sim E^{-1-\eta}$$

and by the anomalous dimensions of the slope α' in terms of E; this amounts to fixing the ratio K^2/E^ν in the limit $E \to 0$.

The ϵ-expansion gives for the two critical indices ([2,3,7,8])

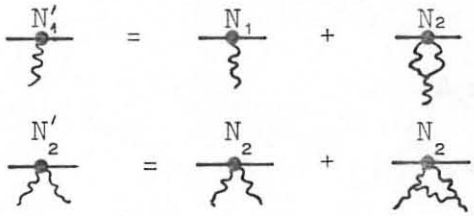
$$\begin{aligned} &\eta = .17 \text{ for } 0(\epsilon), = 0.38 \text{ for } 0(\epsilon^2) \\ &\nu = 1.08 \text{ for } 0(\epsilon), = 1.18 \text{ for } 0(\epsilon^2) \end{aligned} \qquad (7)$$

(for $\epsilon = 2$ everywhere, the one loop approximation gives $\eta = .12$ and $\nu = 1.06$).

As long as the Pomeron Green functions are not the only ingredients of the theory, η and ν are not the only independent critical indices.

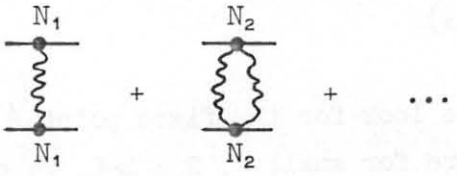
To begin with, we want to compute scattering amplitudes, and therefore to couple the Pomerons to the particle lines. Those couplings are "external operators" from the point of view of our field theory and they carry anomalous dimensions too ([5]).

For instance if we add to $\mathcal{A}$ a term $N_1\psi+N_2\psi^2$ representing the couplings  and  we will find in $\mathcal{A}'$ the values N_1' and N_2' which are determined by integrating over the high degrees of freedom of the loops appearing in the development:



coupled equations.

When we look to the contribution to the total cross section given by



we have to take into account the behaviour of N_1 and N_2. In particular the effective coupling N_1 goes to a constant for $E \to 0$, from above. This is related to the fact that the J-plane cut due to the triple Pomeron graph

has a minus sign.

A numerical analysis of this effect in the one loop approximation gives a warning. It takes, in this model, a very long time (high value of $\ell n\, s$) before the asymptotic regime $\sigma_T \sim (\ell n\, s)^{\eta}$ sets in ([9]).

External operators and the accompanying anomalous dimensions also appear in the computation of the multiple diffractive production of low masses - there typically one has vertices like Pomeron - Pomeron particle as external operators ([2,10]).

Other critical indices determine the behaviour of the multiplicity

distribution, in particular a critical exponent γ gives the behaviour of the moments ([2],[11],[12])

$$< n^p > \sim (\ell n\ s)^{p\cdot(1+\gamma)}$$

The correlations coefficients are given by other independent critical indices ([12])

$$\frac{<n^p>}{<n>^p} = c_p^*$$

The fact that the coefficients c_p^* constitute a set of (observable) critical indices is clear once one realizes that their formal expressions in our Pomeron field theory have zero dimensions, in units of the variable E. They are therefore independent of any input parameter. In this case the analogue in the ferromagnetism is the response to a variation of the temperature around T_c. Indeed to calculate the multiplicity distribution one has to cut Pomeron lines, weight each produced particle by a parameter Z and sum. This amounts to change the intercept of the cut lines to be h(Z): the multiplicity distribution is then given by derivatives with respect to Z at Z = 1, which corresponds to $T = T_c$. With the cutting rules of the Gribov calculus one founds

$$\gamma = \eta \qquad \text{to all orders in } \epsilon.$$

The indices C_2 and C_3 have been evaluated at $O(\epsilon)$ ([12]) :

$$C_2 = 1 + \frac{\epsilon}{8} = 1.25, \qquad C = 1 + \frac{7\epsilon}{8} = 1.8$$

in surprisingly good agreement with experiments.

One can make a more drastic use of the universality property, and change the nature of the space time by going for instance from the continuum to a lattice formulation. In this version ([13]) of the theory

$$\psi(\vec{r},\tau) \to \psi_j, \quad \text{where } j\text{: lattice site.}$$

Decomposing $\psi = \Phi + i\chi$, the action becomes

$$\mathcal{A} = \sum_j \Big\{ E(\Phi_j \chi_{j+\tau} - \chi_j \Phi_{j+\tau}) + \\ + K\left[(\Phi_{j+r_x} + \Phi_{j+r_y})\Phi_j + (\chi_{j+r_x} + \chi_{j+r_y})\chi_j\right] + \\ + i\, G\, \Phi_j (\Phi_j^2 + \chi_j^2) \Big\} . \tag{8}$$

Here we have already fixed the dimensions $D = 2$. Actually to avoid complications with the continuum limit and to have more freedom (anyhow that should not matter!) one adds two extra interactions

$$L(\Phi_j \Phi_{j+\tau} + \chi_j \chi_{j+\tau}) + M(\Phi_j^2 + \chi_j^2)$$

The field variables are taken to be continuous and bounded by $-1<\Phi,\chi<1$. What to do with this version of the problem? The authors choose to compute the "high temperature expansion" according to which if

$$\langle u \rangle = \frac{\int u\, e^{A/T}}{\int e^{A/T}} \simeq \left(\frac{T-T_c}{T_c}\right)^{-p}$$

then expanding in powers of $1/T$, $\langle u \rangle = \sum_\ell a_\ell\, T^{-\ell}$, one has

$$\frac{a_\ell}{a_{\ell-1}} = T_c \cdot \left(1 + \frac{p-1}{\ell}\right) \tag{9}$$

where $a_\ell = \frac{1}{\ell!} \int u \cdot (\mathcal{A})^\ell$. This gives in principle T_c and the critical index p. In the actual computation the expansions of $G(0,0,t)$, of $\int d^2 r G(r,0,t)$ and of similar quantities obtained by integrating G or deriving W (see eqs (3) and (4)) are calculated up to $O(T^{-3})$. One then looks for the values of the parametrs in the action of eq. (8) which make the various T_c determine by eq.(9) as near each other as it is possible. Of course there is some freedom, for instance some of the parameters can be put equal to zero from the beginning. The relations among non independent critical indices are used to obtain an idea on the quality of the approximation. The results are that

$$\eta \approx 0.5 \div 1 \qquad \text{and} \qquad \nu \approx 1.5 \div 2 .$$

to be compared with eq. (7).

Various considerations indicate that the value for η is overestimated. The results are not without ambiguity. for instance the three Pomeron coupling turns out to be real in the solution (instead that immaginary).

Another possibility is to try to evaluate approximately the functional integration, by some saddle point method ([14]). Coming back to the physical variable $y \equiv \ell n\, s$, the problem is the evaluation of

$$Z = \int d\varphi\, d\chi\, e^{-\int_{-\infty}^{+\infty} dy\, d^2r\, \mathcal{L}}$$

where

$$\mathcal{L} = 2i\varphi\, \partial_y\chi + \alpha' \left[(\nabla_2\varphi)^2 + \nabla_2\chi)^2 \right] - V(\varphi,\chi) \qquad (10)$$

and

$$V(\varphi,\chi) = (h+i\lambda\varphi)\cdot(\varphi^2+\chi^2) \ .$$

Let us now look for the stationary points of the potential V, i.e. the roots of $\frac{\partial V}{\partial \varphi} = \frac{\partial V}{\partial \chi} = 0$. They are

$$(\varphi = \chi = 0), \qquad \left(\varphi = \frac{2ih}{3\lambda}\ ,\ \chi = 0\right), \qquad \left(\varphi = \frac{ih}{\lambda}\ ,\ \chi = \pm\frac{h}{\lambda}\right) .$$

From an analysis based on perturbation theory it is apparent that the critical temperature corresponds to a non zero positive value of the bare intercept h, which is related to λ (this relations gives in fact a critical line): $h \sim \lambda^2$. Moreover the bare coupling constant λ is expected to diverge when the renormalized one λ_R approaches its fixed point value. Therefore what is relevant to our problem is the case in which h and λ are large.

It is easy to see that in such a case the first two stationary points are outside the domain of convergence of the functional integral, whereas expanding around the other two

$$\varphi = \frac{ih}{\lambda} + x \qquad\qquad \chi = \pm\frac{h}{\lambda} + Z$$

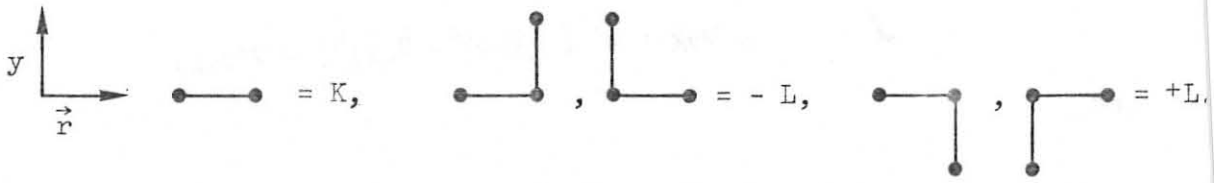
one has a gaussian integral

$$\int dx\, dz\, \ell^{-h\left\{2\left(x \mp \frac{iz}{2}\right)^2 + \frac{z^2}{2}\right\}}$$

The calculation is pursued by representing the continuum in y and $\vec{r}$ with a lattice and the φ, χ integration is performed at each lattice site around the two stationary points, which are indicated by a dichotomic variable $S_j = \pm 1$. In this cay one obtains for Z

$$Z = \sum_{\{S_j\}} e^{-\sum_{ij} K(ij) S_i S_j + \sum_{ijk} L(ijk) S_i S_j S_k}$$

In the space $(y,\vec{r})$ the interactions are non zero only among nearest sites, with the specifications:



The important point is that the symmetry of the interactions with respect t P (space reversal), T ("time", or y-reversal), S (spin reversal, or $\chi \to -\chi$ are the same as in the original Lagrangian.

The problem is now very similar to a standard Ising model problem. The authors of this development do not (for the moment) attempt to evaluate the critical indices. They are satisfied to argue that a critical point should exists, that is should be repulsive in L and attractive in K. They further argue that since the correlation function is an expectation value of the product of two spins, it should be bounded. This, toghether with the attractiveness of the fixed point in K, gives a (Froissart) bound

$$\eta \leq 2 , \qquad \eta \leq \nu \leq 2 .$$

It is amusing to note that the stationary points in φ, χ used for the saddle point integration have, as expected, a "classical" interpretation. Indeed i is possible to solve completely the classical equations of motion obtained by varying the Lagrangian of eq. (10), in the extreme approximation $\alpha' = 0$ (It turns out that the system evolves in closed orbits in the (φ,χ) plane, w periodical motion (the "time" is the rapidity y. Actually in the classical solution φ is pure immaginary). The particular orbit which passes through t stationary points has an infinite period. In the limit of large ℓn S the

favoured solution (in the sense of minimal action) tends to this orbit: here the system spends a lot of time near a stationary point, then with a sudden transition flips to the other.

This "classical solutions", which corresponds to the sum of the tree diagrams, i.e. no loops, in Fig. 1, gives for the scattering amplitude the geometrical picture of an expanding gray disk.

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