EXPLOITING A SINGLE MEASUREMENT OF THE σ_L/σ_T RATIO IN THRESHOLD ELECTROPRODUCTION

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Summary.

We discuss the possibility of exploiting a single measurement of the ratio $[\sigma_L/\sigma_T]_{(th.)}$ at a low value of K^2 to draw a curve of this ratio in a certain K^2- range.

The relevance to different theoretical frameworks and models of a separate knowledge of the longitudinal and transverse pion electroproduction cross sections[1] $\sigma_{L,T}$ at low energies has often been emphasized in the literature[2]. In the charged pion case, for instance, it is known that the axial vector form factor of the nucleon $G_A(t)$ plays the main role in the current algebra and PCAC treatment of σ_T[3]; on the other hand, the value of the pion form factor $F_\pi(k^2)$ is essential to the dispersive evaluation of σ_L[4]. Although at the moment only the sum of the two cross sections has been measured near threshold at a limited number of values of the virtual photon mass k^2[5], a measurement of the ratio σ_L/σ_T at threshold at k^2 approximately -4 Fermi^{-2} is being planned in the very near future. The aim of this note is to discuss how, from knowledge of this single experimental result, one might try to parametrize this ratio in that whole range of k^2 values, $0 \leq |k^2| \lesssim 10$ Fermi^{-2}, where the total cross section is experimentally known[5].

We start by defining the quantity (the (+) refers to the charge of the produced pion):

$$S^{(+)}(x) = (x - x_\pi)^2 \, R^{(+)}(x) . \qquad (1)$$

where

$$R^{(+)}(x) = \left[\sigma_L/\sigma_T \right]^{(\pi^+)}_{(th.)} = (-k^2) \left[\frac{|\mathcal{L}_{0+}/k_0|^2}{|E_{0+}|^2} \right]^{(\pi^+)}_{(th)} \qquad (2)$$

$x = k^2/m_\pi^2$, and x_π is the value of x corresponding to the pion pole at $t = m_\pi^2$ which contributes to $\mathcal{L}_{0^+}(th)$, i.e.

$$x_\pi = 2 + m_\pi/M . \tag{3}$$

(we remind that at the physical threshold the variables t and K^2 are related, namely $t(th) = M/(M+m_\pi) \cdot (K^2 - m_\pi^2)$.

In the physical region, $x \leq 0$, $S^{(+)}(x)$ will be an unknown function of x with the property, which is a mere consequence of its definition, that it must be vanishing at $x = 0$. We will assume that $S^{(+)}(x)$ is a regular function in the physical region, at least for those small x values, $0 \leq |x| \lesssim 20$, where it appears on experimental grounds that the sum of the two cross sections is in fact regular. The properties of $S^{(+)}(x)$ in the unphysical region, $x > 0$, will be in general more involved. For our purposes, however, it will be sufficient to know that

a) $S^{(+)}(1)$ is fixed by the requirement that E_{0^+} and $\mathcal{L}_{0^+}$ coincide[6] at that point, which corresponds to the configuration $|\vec{k}| = 0$ at threshold. This gives

$$S^{(+)}(1) = -(x_\pi - 1)^2 . \tag{4}$$

b) $S^{(+)}(x_\pi)$ is fixed by the definition of residue at the pole, which gives in a straightforward way

$$S^{(+)}(x_\pi) = B \left[\frac{F_\pi(K_\pi^2)}{E^{(\pi^+)}_{0^+(th)}(K_\pi^2)} \right]^2 . \tag{5}$$

$$B = (-)\, g_{\pi N}^2\, x_\pi \left[\frac{m_\pi}{2M(2M+m_\pi)} \right]^2 \frac{M+m_\pi}{M} \left[\left(\frac{2M+m_\pi}{m_\pi} \right)^2 - x_\pi \right] .$$

where $F_\pi(k^2)$ is the electromagnetic form factor of the pion. In practice, the ratio which appears in the bracket on the r.h.s. of eq. (5) is not measurable; however, we know it at the near point $k^2 = 0$ from photoproduction experiments[7], and we shall write therefore

$$S^{(+)}(x_\pi) = B\left[\frac{1}{E_{0^+(+k)}^{(\pi^+)}(0)}\right]^2 \left(1 + \Delta^{(+)}\right). \tag{6}$$

where $\Delta^{(+)}$ is an unknown parameter. Since both F_π and E_{0^+} are expected to be smooth increasing functions of k^2 for $k^2 \leq 0$, we would guess $\Delta^{(+)}$ to be a small fraction of unity, $|\Delta^{(+)}| \ll 1$. If we had at our disposal a number of experimental values of the function $S^{(+)}(x)$ in the physical region , we would use eq. (4) as a constraint which every fit to $S^{(+)}(x)$ should satisfy. In fact, eq. (4) has been fruitfully utilized in a previous paper[8] to select between different polynomial fits to the total cross section. In our case, one experimental value at most will be available, but we can still use the constraints eq. (4) and (although in a more limited sense) eq. (6) to check if simple parametrizations of $S^{(+)}(x)$ are, in principle, acceptable. Suppose for instance that we start from the simplest fit to $S^{(+)}(x)$ vanishing at $x = 0$:

$$S^{(+)}(x) = \alpha x. \tag{7}$$

The constraint eq. (4) fixes then the value of the parameter α :

$$\alpha = -(x_\pi - 1)^2. \tag{8}$$

In terms of $R^{(+)}(x)$, eqs. (7) and (8) correspond to the curve drawn in fig. 1. In particular, one has at the point $x = -8$:

$$\mathcal{R}^{(+)}_{(-8)} \simeq 0{,}10 . \tag{9}$$

while the value of $\Delta^{(+)}$ is

$$\Delta^{(+)} \simeq 0{,}10 . \tag{10}$$

which is in agreement with our expectation that $|\Delta^{(+)}| \ll 1$. A value of $\mathcal{R}^{(+)}_{(-8)}$ as in eq. (9) would be in disagreement with estimates based on current algebra and PCAC[3], whilst computations based on dispersion theory[4] would be compatible, provided that the pion form factor were equal to the proton charge form factor. Clearly, if the result of a measurement of $\mathcal{R}^{(+)}_{(-8)}$ were significantly different from eq. (9), one of the conclusions should be that a linear fit to $S^{(+)}_{(x)}$ does not appear to be adequate. Let us suppose that this is in fact the case and investigate the features of a quadratic fit to $S^{(+)}_{(x)}$:

$$S^{(+)}_{(x)} = x\,(a + bx) . \tag{11}$$

From eq. (4) one has

$$a = -\left[b + (x_\pi - 1)^2 \right] . \tag{12}$$

and the parameter b has now to be fixed by the measured value $\mathcal{R}^{(+)}_{(-8)}$, which we shall call π from now on. In fig. 1 we have also drawn, for different values of π, different curves of $\mathcal{R}^{(+)}_{(x)}$; we remember that $\Delta^{(+)}$ is connected to π in the parametrization (11) by the relation

$$\Delta^{(+)} \simeq 0{,}26 - 1{,}40\, r . \qquad (13)$$

If we believe that the quadratic fit eq. (11) is adequate (which should be a reasonable assumption in the small x range we are exploring) we see that small values of r , $r \ll 1$, correspond to a small $\Delta^{(+)}$; this would lead us to predict such values on the basis of very simple arguments. In any case, we can say that a value of r between, say, 0,20 and 0,40 would be quite compatible with the simple quadratic parametrization eq. (11); as fig. 1 shows, this would imply that the ratio σ_L / σ_T should remain remarkably constant in the range $5 \lesssim |x| \lesssim 20$. Higher values of r would lead us to conjecture the need of higher terms in a fit to $S^{(+)}(x)$. This would make things less simple, and we probably should be obliged to wait for further measurements of $R^{(+)}(x)$ before being able to draw separate curves of σ_L and σ_T.

In conclusion, we have seen how even a single measurement r of $R^{(+)}(x)$ might enable us to check if simple parametrizations of $R^{(+)}(x)$ in the low-x range are, or not, acceptable. To make this analysis a little more predictive, we have assigned three arbitrary values 0,20, 0,30 and 0,40 to r . Using the quadratic parametrization eq. (11) and the best fit to the total cross section given in ref. (8) (of course, other data might be used and/or a different curve of the total cross section might be obtained) we have drawn in fig. 2 the three different resulting curves of σ_T in the low-x range. These curves, and the corresponding ones for σ_L , should then be analyzed in terms of different models. This, we hope, will lead to an improved knowledge of the nucleon axial vector and pion electromagnetic form factors.

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References.

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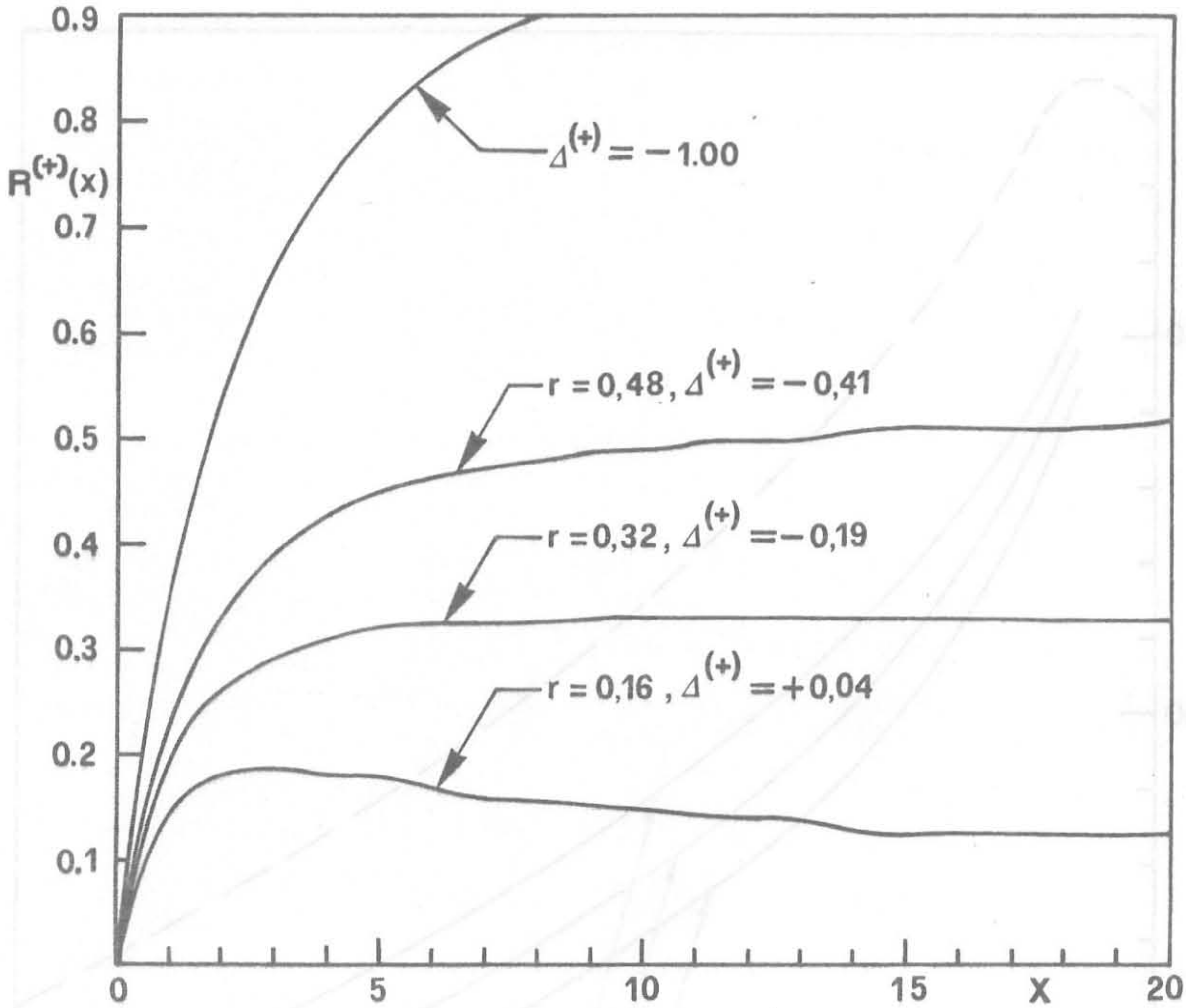
Figure captions.

Fig. 1.

The ratio σ_L / σ_T at threshold, assuming the parametrization eq. (11), for different values or π.

Fig. 2.

$K_L/|\vec{q}|\ \sigma_T\,(\mathrm{th.})\ (\mu b)$ for different values of π (dotted curves). The full line is $K_L/|\vec{q}|\left[\sigma_T + \varepsilon \sigma_L\right]$ of ref.(8).



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$R^{(+)}(x)$
$\Delta^{(+)} = -1.00$
$r = 0{,}48$, $\Delta^{(+)} = -0{,}41$
$r = 0{,}32$, $\Delta^{(+)} = -0{,}19$
$r = 0{,}16$, $\Delta^{(+)} = +0{,}04$
0
5
10
15
X
20

Fig. 1

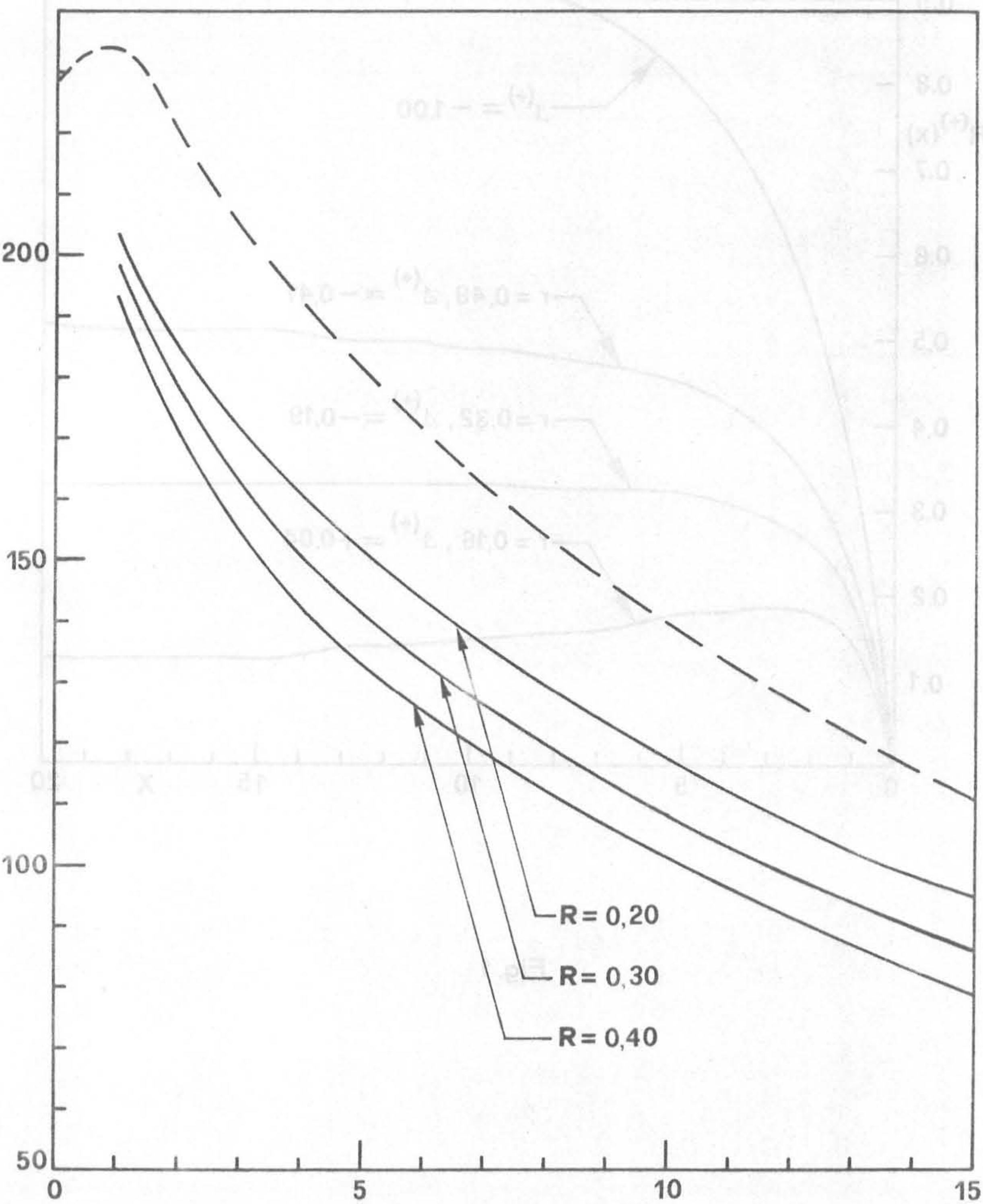


Fig.2