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G. Cabras and E. Flaminio: RESOLUTION OF KINEMATICAL AMBIGUITIES IN BUBBLE CHAMBER EVENTS USING THE F.S.D. IONIZATION MEASUREMENTS. -

SUMMARY. -

We have used the ionization measurements performed on an F.S.D. in order to solve kinematical ambiguities in a low momentum $\bar{p}p$ bubble chamber exposure.

We present here details of the method used, and discuss the problems we have met because of the many low-momentum tracks present in our events.

1. - INFORMATION PROVIDED BY THE F.S.D. -

The image of a track on a bubble chamber picture, consists of a sequence of dark, rather fuzzy, spots where groupings of dark grain represent bubbles or bubble clusters.

The bubbles, of varying size and shape, are generally distributed along the trajectory of the ionizing particle; they are moreover shifted by small, random amounts, with respect to the most likely path of the particle.

In the image of a track one can often see clusters of bubbles which, due to coalescence or imperfect optical resolution, remain unresolved.

It is customary to call "blobs" the projections of unresolved bubble clusters along the path of a particle, and "gaps" the projections of intervals between consecutive blobs.

The linear structure of a track is determined by its bubble density; if we consider a bubble as the smallest blob, then the track is made up of a succession of blobs separated by an equal number of gaps; we can therefore consider a track like a sequential repetition of a basic cell which consists of a gap followed by a blob.

On the other hand, the F.S.D., by consecutively scanning a given frame, provides at each intersection with a given track the binary information hit-miss, which constitutes a different representation of the same linear structure.

In this way a group of bubbles is recorded as a string of one or more hits, and a gap as a string of one or more misses.

Each "segment" along a track appears therefore as a succession of T binary elements, consisting of H hits and M misses ($T = H + M$).

This basic information is translated in the filter program into a measurement of the bubble density of the track.

We will call:

- b : the bubble density on film
- a/2 : the maximum separation between the center of a bubble image and the center of a scan line for recording a hit. (a = effective spot size)
- $\overline{\cos\alpha}$: the average value of the cosinus of the angle between the track direction and the direction perpendicular to the scan line (average over all slices).

From a knowledge of the values of T and M_i for each of the N slices into which a track can be divided, and from the value $\cos\alpha_i$, one can calculate the number of misses M_i^x corrected for the angle of the track (M_i^x is the number of misses which would have been found if the track had been straight and pointing in a direction forming an angle $\overline{\cos\alpha}$ with the normal to the scan lines).

If we now define:

$$\overline{M}^x = \frac{1}{N} \sum_{i=1}^{N} M_i^x \quad \text{(N = total number of slices)}$$

we get for the bubble density times effective spot size the expression:

$$(ba) = \overline{\cos\alpha}\ \ln\left(\frac{T}{\overline{M}^x}\right) \qquad (1)$$

From (1), b could be computed, if we knew the effective spot size a.

However a, although roughly constant for a given event on a given view, can vary considerably during the measurement and is to be considered, in general, unknown.

We will come back to this point later.

The standard deviations on (ba) can be computed from those on $\overline{M}^x$, using the following formula:

$$\sigma\,(ba) = \overline{\cos\alpha}\;\frac{\sigma\,(\overline{M}^x)}{\overline{M}^x}$$

where $\sigma\,(\overline{M}^x)$ may either be the external or the internal error on $\overline{M}^x$.

However both errors on the quantity (ba) turn out to be non-symmetric, and one computes therefore the upper and lower errors:

$$\sigma^-(ba) = \overline{\cos\alpha}\left[\ln\left(\frac{T}{\overline{M}^x}\right) - \ln\left(\frac{T}{\overline{M}^x + \sigma\,(\overline{M}^x)}\right)\right]$$

$$\sigma^+(ba) = \overline{\cos\alpha}\left[\ln\left(\frac{T}{\overline{M}^x - \sigma\,(\overline{M}^x)}\right) - \ln\left(\frac{T}{\overline{M}^x}\right)\right]$$

where we take $\sigma\,(\overline{M}^x)$ to be either the internal or the external errors on $\overline{M}^x$.

The internal and external errors on $\overline{M}^x$ are in turn given by the following expressions:

$$\sigma_{int}(\overline{M}^x) = \sqrt{\frac{\sum_{i=1}^{N}(\overline{M}^x - M_i^x)^2}{N(N+1)}}$$

$$\sigma_{ext}(\overline{M}^x) = \sqrt{\frac{\overline{M}^x(T-\overline{M}^x)}{T\cdot N} + k\,(T-\overline{M}^x)^2}$$

where the constant k represents the square of the average loss of hits/hit.

The value of (ba) given by (1) has still to be corrected to take into account the following effects:

(a) Demagnification
(b) Track orientation in space.

A correction factor g is calculated which takes both these effects into account. It has the effect of bringing the measurements onto a plane parallel to the film plane and of making them directly comparable track by track.

The corrected bubble density will then be given by the expression:

$$b_{corr} = b_{meas.} \cdot g$$

2. - USE OF THE IONIZATION MEASUREMENTS. -

Measurements have been performed on one of the F.S.D.[s] of C.N.A.F. (Centro Nazionale Analisi Fotogrammi) in Bologna[1].

The film was taken exposing the C.E.R.N. 81 cm H.B.C. to a beam of antiprotons having a momentum of ~ 1100 MeV/c.

The measurements of ~ 50 k pictures have been used to the purpose of the present test.

Only events having two charged prongs and one or more visible k^o decays have been used.

The final states we have to separate are the following:

$k^+ \pi^- \bar{k}^o$	(4-c fit)
$k^- \pi^+ k^o$	(4-c fit)
$k^+ \pi^- \bar{k}^o \pi^o$	(1-c fit)
$k^+ \pi^- \bar{k}^o z^o$	(no fit)
$\pi^+ \pi^- k^o (\bar{k}^o)$	(1-c fit)
$\pi^+ \pi^- k^o z^o$	(no fit)
$k^- \pi^+ k^o \pi^o$	(1-c fit)
$k^- \pi^+ k^o z^o$	(no fit)

The problem we have to face involves the resolution of π/k ambiguity.

The possibility of resolving such ambiguity on the basis of ionization on a given track depends very much upon the momentum.

In Fig. 1 we show the predicted $1/\beta^2$ dependence of bubble density as a function of momentum for pions and kaons.

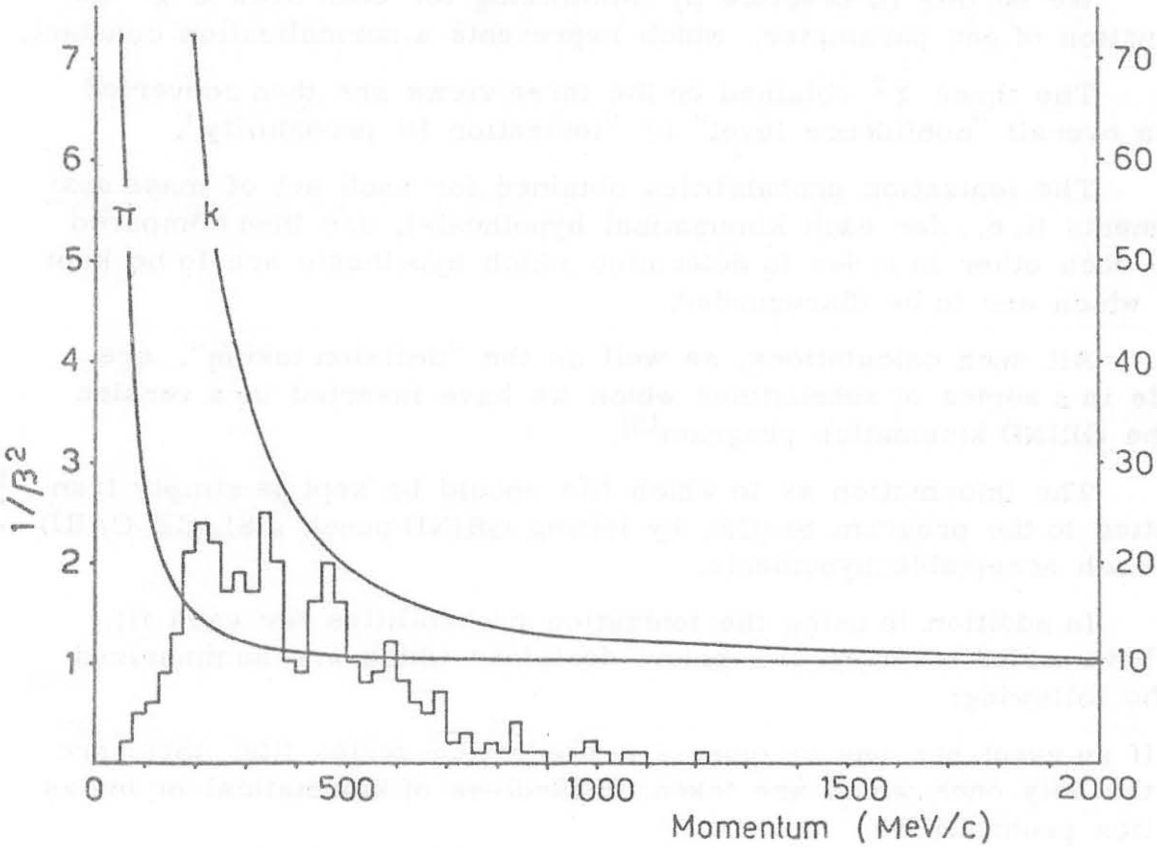

FIG. 1

It can be seen that the separation becomes critical for momenta greater than $\sim$ 800 MeV/c.

It should also be noted that for momenta smaller than $\sim$250 MeV/c for kaons and $\sim$100 MeV/c for pions the bubble density becomes so high that the measurements are made difficult due to a well know "saturation effect"(2).

In Fig. 1 we have also shown the measured momentum distribution for a sample of events of the above topology in our experiment.

It can be seen from these data that a sizable fraction of the events have tracks with momenta in the "saturation" region. We must therefore be aware of this problem.

Since the measurements which the F.S.D. provides are only proportional to the bubble density, the proportionality constant being about the same for all tracks on a given view, what we can do is only a test of the consistency of ratios of bubble density among different tracks, for a given set of mass assignements.

We do this in practice by minimizing for each view a χ^2 as a function of one parameter, which represents a normalization constant.

The three χ^2 obtained on the three views are then converted in an overall "confidence level" or "ionization fit probability".

The ionization probabilities obtained for each set of mass assignements (i. e., for each kinematical hypothesis), are then compared with each other in order to determine which hypothesis are to be kept and which are to be disregarded.

All such calculations, as well as the "decision taking", are made in a series of subroutines which we have inserted in a version of the GRIND kinematics program[3].

The information as to which fits should be kept is simply transmitted to the program SLICE, by letting GRIND punch a SLICE CARD for each acceptable hypothesis.

In addition to using the ionization probabilities for each fit, we have added a series of a-priori decisions which are summarized in the following:

(a) If an event has one or more 4-c production-vertex fits, these are the only ones which are taken, regardless of kinematical or ionization probabilities.
(b) All 1-c fits and all missing-mass hypothesis are let to compete exactly on the same footing, regardless of kinematical probabilities for 1-c fits. Ionization probability only will be used in the choice.
(c) Presence or absence of a multivertex fit will not imply acceptance or rejection of the corresponding hypothesis.

To the purpose of ionization fit selection among several competing hypothesis, we have used an "ionization fit probability", which has been computed for each kinematical hypothesis by multiplying the confidence levels obtained on each of the three views of the event.

An interpretation will then be rejected if its "ionization fit probability" is less than 1/8 of the interpretation which has the best probability.

3. - DETAILS OF THE METHOD. -

The method we have used follows closely that described in reference (2).

The measurements of (ba) for each track on each view, have been used to calculate a χ^2 defined as:

$$\chi_i^2 = \sum_{j=1}^{NTR} \left[\left(\left(\frac{M}{T}\right)_{meas_{ij}} - \left(\frac{M}{T}\right)_{pred_{ij}} \right) / \left(1.5\, \delta \left(\frac{M}{T}\right)_{meas_{ij}} \right) \right]^2$$

where NTR is the total number of useful tracks in the event and i is the index of the view. The factor of 1.5 in the denominator has been introduced in order to get a correct stretch function distribution.

The quantity $(M/T)_{meas_{ij}}$ is computed from the given values of $(ba)_{ij}$ and $\overline{\cos\alpha}_{ij}$ by inverting the expression (1) of the previous paragraph:

$$\left(\frac{M}{T}\right)_{meas_{ij}} = e^{-(ba)_{ij}/\overline{\cos\alpha}_{ij}}$$

The error on this quantity is computed as follows:

$$\delta \left(\frac{M}{T}\right)_{ij} = \left| e^{-((ba)_{ij} - \sigma (ba)_{ij})/\overline{\cos\alpha}_{ij}} - e^{-(ba)_{ij}/\overline{\cos\alpha}_{ij}} \right|$$

In calculating the χ^2 we consider as "useful" all tracks present in the event, having the bubble density measured, including the beam track and tracks from k^0 decays but excluding those having too small a geometrical correction factor g_{ij} (g_{ij} must be ≥ 0.3).

The predicted value of the ionization for a given mass interpretation of a given track is given by

$$D_{pred_{ij}} = \frac{\gamma_i}{\beta_j^2 g_{ij}}$$

where j is the index of the track, $\beta_j^2 = 1 + M^2/p_j^2$ and γ_i is the normalization factor to be determined by the fit, separately for each view. This amounts to say that we know the ionization of each track for the event in the given view, up to an arbitrary normalization factor.

The value of $(M/T)_{pred_{ij}}$ can be obtained from $D_{pred_{ij}}$ as follows:

$$\left(\frac{M}{T}\right)_{pred_{ij}} = e^{-\frac{\gamma_i}{\beta_j^2 g_{ij} \sqrt{\cos \alpha_{ij}}}}$$

As we have already pointed out, the F.S.D. seems to be unable to cope with the measurements of bubble density for tracks which are heavily ionizing, because of the saturation effect. Even if no gaps are present in a track segment, for reasons which are not well understood, hits may be lost.

To correct for this effect we have (like it has already been done in other experiments(2)) introduced a correction factor in the expression of $(M/T)_{pred_{ij}}$:

$$\left(\frac{M}{T}\right)_{pred_{ij}} = \left(\frac{M}{T}\right)_{pred_{ij}} + v_i \left(\frac{H}{T}\right)_{pred_{ij}}$$

where v_i is a parameter to be determined.

Using this expression, the χ^2 will now be written in the following form:

$$\chi^2(\gamma_i, v_i) = \sum_{j=1}^{NTR} \left[\left(\left(\frac{M}{T}\right)_{meas_{ij}} - \left(\frac{M}{T}\right)_{pred_{ij}}(\gamma_i) - v_i\left(\frac{H}{T}\right)_{pred_{ij}}(\gamma_i)\right)/\left(1.5\ \delta\left(\frac{M}{T}\right)_{ij}\right)\right]^2$$

The minimization of the χ^2 is performed first in an iterative way with respect to the normalization parameter γ_i. Once a minimum has been reached, the minimization with respect to the second parameter v_i is performed in an analytical way, be solving the equation:

$$\frac{\partial \chi^2}{\partial v_i} = 0$$

The solution of such equation is:

$$v_i = \frac{\sum_{j=1}^{NTR} \left\{ \left[\left(\frac{M}{T}\right)_{meas_{ij}} - \left(\frac{M}{T}\right)_{pred_{ij}} \right] \left(\frac{H}{T}\right)_{pred_{ij}} \right\} / \left(1.5\, \delta \left(\frac{M}{T}\right)_{ij}\right)^2}{\sum_{j=1}^{NTR} \left[\left(\frac{M}{T}\right)_{pred_{ij}} \right]^2 / \left[1.5\, \delta \left(\frac{M}{T}\right)_{ij} \right]^2}$$

The whole process is repeated over and over again until a minimum with respect to both parameters has been reached.

During the iterations the parameter v_i is allowed to vary between 0 and 0.05.

4. - COMPARISON WITH THE DECISIONS TAKEN AT THE SCANNING TABLE. -

As a first check of the performance of the program and of the goodness of the measurements, we have compared the decisions taken by the program on the basis of such measurements with the decisions one would have taken by direct inspection of the events on the scanning table.

The criteria one uses in deciding among different hypothesis on the scanning table are not, in fact very different from those used in the program, since in both cases they amount to a check of relative ionizations of different tracks of the same event on each given view.

In performing the comparison we have disregarded those events for which the program had chosen a 4-c fit, since this choice is made independently of any ionization information, and hence is not very representative of the reliability of the ionization measurements.

The results of the comparison based on visual inspection of ~ 200 events, are shown in Fig. 2.

Here one can see on the left an histogram of the difference between the number of fits selected by the program an those selected by us.

It can be seen that in all cases but one the difference is positive, meaning that the program is somewhat more "generous" in keeping fits than we have been. It can moreover be seen that in about 60% of the cases there was complete agreement between the two decisions.

In about 40% of the cases the resolution of the ambiguities was complete and there was agreement.

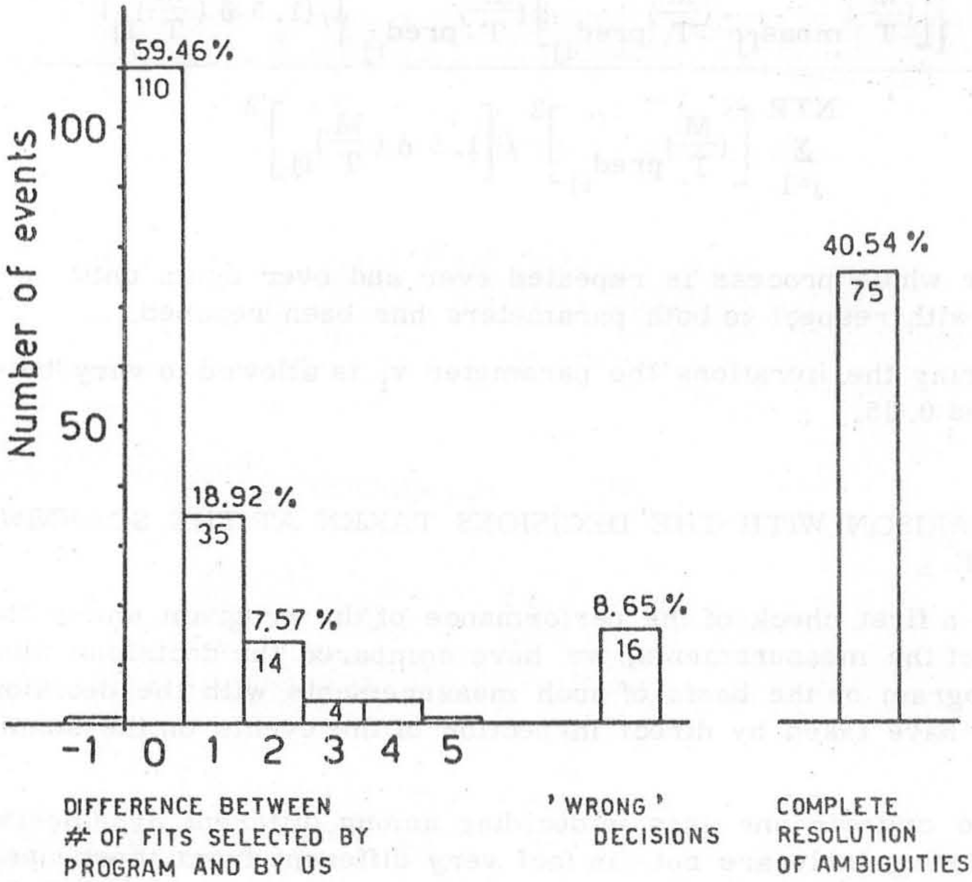

FIG. 2

In about 9% of the cases the program picked up an apparently "wrong" fit.

In order to investigate the reason behind the $\sim$ 9% of "wrong" choices, we have looked at these events in more detail.

There can be three possible reasons for the apparent disagreement between our decisions and those made by the program:

a) the F.S.D. measurements underestimate the true ionization, because of saturation
b) the measurements are O.K., but our use of them is incorrect
c) our decisions may in some cases be the wrong ones.

As a test of these possibilities, we have made independent measurements of the ionization on these, as well as on other events and we have moreover looked for any peculiar characteristics which these events might have.

We have in fact to remember that we are dealing with low-momentum tracks, and that the F.S.D. measurements may suffer from this.

We have found that in most of the 16 events which appeared to be wrongly assigned, there was at least one outgoing track with a predicted ionization for the k interpretation greater than 6 times minimum.

This means that our correction for heavy tracks is inadeguate in some extreme cases. As a further check on the quality of the measurements we have made microscope measurements on some of these events.

5. - COMPARISON WITH MICROSCOPE MEASUREMENTS. -

Measurements of the mean-gap-length have been performed an 7 events which had been wrongly assigned and on 2 more events which had been correctly classified.

The measurements have been performed in view 2 only, and the resulting histograms of m. g. l. distribution have been used to determine the bubble density, by means of the formula suggested by Willis[4]:

$$b = \frac{N}{\sum_{1}^{N} l_i - N l_o} \quad \left(\pm \frac{b}{\sqrt{N}}\right)$$

where N is the total number of gaps with length greater than the cutoff length l_o; and l_i is the gap length of the i^{th} gap longer than l_o.

The results of these measurements were than normalized to those performed on the beam track.

In Table I these results are compared with the F.S.D. results on the same events, normalized in the same way.

Events 1 and 2 had been correctly classified. For them the results of the two measurements are in good agreement.

Events 3, 4 and 5 are also in reasonable agreement, although for these the program appeared to select the "wrong" hypothesis.

It might well be possible that our "eyeball" selection is incorrect, or that the measurements disagree on the remaining two views.

Events 6-9 present a definite disagreement between the two sets of measurements.

It should be noted that the microscope measurements do not suffer from the saturation effect to the same extent as the F.S.D.

TABLE I

	MICROSCOPE			F.S.D.		
N^O TR	NORM. B.D.	ERROR	%	NORM. B.D.	ERROR	%
1) A3+	0.8582	0.1135	13.22	0.8315	0.1585	19.07
A2-	1.1530	0.1327	11.51	0.9987	0.1723	17.25
2) A2-	0.7303	0.0745	10.21	0.8354	0.1639	19.62
A3+	1.2222	0.1304	10.67	1.3892	0.2890	20.80
3) A3+	1.1685	0.1444	12.35	1.3349	0.2722	20.39
A2-	1.3399	0.2650	19.78	1.4633	0.2877	19.66
4) A2-	1.4987	0.1527	10.19	1.4170	0.2481	17.51
A3+	1.0870	0.1036	9.53	1.2077	0.2244	18.58
5) A2+	0.9661	0.1123	11.62	0.9305	0.1932	20.76
A3-	1.3788	0.1654	12.00	1.2337	0.2441	19.78
6) A3-	0.9785	0.1131	11.56	1.6767	0.4553	27.16
A2+	1.4425	0.2108	14.62	3.1874	0.9348	29.33
7) A2-	1.4329	0.1769	12.35	1.8650	0.3829	20.53
A3+	1.0108	0.0937	9.27	1.1652	0.2418	20.75
8) A2+	1.0415	0.1974	18.95	1.2391	0.4263	34.40
A3-	0.8277	0.1435	17.33	1.1820	0.3862	32.67
9) A3+	1.1762	0.1239	10.54	1.3017	0.2385	18.33
A2-	1.7064	0.2074	12.15	1.1310	0.3659	32.35

We believe that the correction we are using for very low-mo mentum and/or very steep tracks is inadeguate in these cases, which however represent a very small percentage of all events.

The method seems to work in an overall fairly satisfactory way, and to help in resolving a rather large fraction of ambiguous events; this in spite of the rather bad working conditions of the chamber optics during the run, and of the very low-momentum tracks we had to deal with.

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We are also grateful to Drs. P. Duinker, A. Ghiselli and A. Gu glieri for several useful discussions, and to the staff of the Bologna F.S.D. where the measurements were performed.

FOOTNOTES AND REFERENCES. -

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