ON THE EXTRACTION OF THE PARTICLE-NEUTRON AMPLITUDE FROM PARTICLE-DEUTERON SCATTERING WITH BREAK UP (II)

G. Alberi

Istituto di Fisica Teorica dell'Università, Miramare, Trieste

Istituto Nazionale di Fisica Nucleare, Sezione di Trieste

6 Marzo 1973

ABSTRACT

In this second report we consider the internal motion of nucleons in the deuteron and its effect on the single scattering expression of the break-up cross section. In the outset we show that there is effect of the Lorentz transformation only on the interference terms of the cross-section. Then we point out the difficulties of the on shell extrapolation of the deuterium data and we propone, a practical, albeit approximate method of performing an "unfolded" phase shift analysis, that is taking in account the Fermi motion.

1. Introduction.

In liquid or gaseous deuterium experiment the neutron target is not only moving together with the deuteron, but it is moving in the deuteron C.M. and this movement is extremely faster than the first one. This movement is called in the literature as "Fermi motion". This effect was first considered by Ericson e Ericson (ER 66) for incident pions as a contribution to the velocity dependent part of π-nuclei effective interaction: the effect, on the optical potential is the adding of one term of the order of $(m_\pi/m_N)^2$. Successively Fäldt and Ericson (FAE 68) described with success π- deuteron total cross-section data using a multiple scattering expansion truncated to the double scattering term. Their theoretical assumption consists in neglecting the binding effect of the nucleus but to take in account their kinetic ebergy. While in the single scattering term the full effect of this hypothesis is calculated, the double scattering term is approximated " a la Glauber". This simplification is abandoned by J. Wallace (WA) who considered the effect of the Fermi motion on the whole angular distribution of πD elastic scattering at 766 MeV (BRA 68), studying therefore the effect even on the shoulder for $|t| \geq .4\ (GeV/c)^2$ where the double scattering is known to dominate (ALB 69); the effect is rather small. He is taking all the π N amplitudes on the mass shell, trusting in a slow dependence of such amplitudes on the off shell parameter. This assumption is very appealing, but it was not possible till now to test it in a clear cut case [+].
The so called "off-mass shell dependence", was evaluated numerically by Landau (LAN 71) again for the case of πD elastic

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[+] The best kinematical region to test the off-mass shell dependence is the backward region of πD scttering for energies around the 33 resonance (WIL 72), where the single scattering is known to dominate.

scattering at 766 MeV for high momentum transfer. His ansatz is to assume that the amplitudes depends only on the energy squared s; the off-shell effect is calculated simply by taking for s, its off shell value.

The elastic scattering is a very complicated case because even if you stick to the on shell prescription, there are two possibilities fro the definition of s, the initial and the final s for the scattering which are different, because the energy is not conserved in the vertex,whereas for the quasi elastic case, the final energy for the scattering corresponds to the off shell energy, while the on shell energy is given by the initial energy of the internal nucleon on the mass-shell. It is important to notice that Landau is choosing a parametrization for the πN amplitude, which consists of the sum of a resonant and of diffractive part, which could influence considerably the importance of the effect. With the same model for the K^+N amplitude he calculated the Fermi motion effect on the defect of the total K^+D cross section finding a very small defference with Glauber theory result.

The above type of work is concerned with the calculation of the particle deuteron total or elastic differential cross section, once the elementary particle neutron and particle-proton amplitude are known. The practical situation is usually just the reverse: knowing experimentally the particle deuteron total or differential elastic, differential anelastic cross section, we want to determine the particle neutron amplitude. This problem was solved already by Wilkin et al. (CO 70) for the extraction of the K^+N total cross section from the K^+D total cross section data. The procedure is to solve by iterative method the following integral equation

$$\sigma(s) = \langle\sigma\rangle_s + \int[\sigma(s) - \sigma(s')]\,\psi^2(p)\,d^3p \qquad (1.1)$$

where

$$\langle\sigma\rangle = \sigma_D - \langle\sigma_p\rangle + \delta\sigma$$

That is at the beginning, they insert in the right hand side of the equation the folded total cross section and determine the first approximation for the unfolded one. Then this result of the first interation is inserted in the right hand side and so on. The method breaks down after the third interation, because of the appearance of unexpected oscillations. The same method was used by G. Lynch (LYN 70) with the modification of fitting the result of any interaction, with a smooth curve. The mathematical problem of the unfolding is studied in detail by Bundaru and Stamatescu (BUN 72).

We will propose here a method for unfolding the phase shifts and elasticities for a separate channel. The procedure makes use of the derivative formula for the folded cross section.

In section 2 we show that only on the interference term there is record left of the Lorentz transformation from the deuteron Lab. system, to the particle nucleon C.M. system (even if we take in account the internal motion of the nucleons).

In section 3 we define the off-mass shell dependence and we clarify the meaning of the hypothesis which is done in the literature.

In section 4 the derivative formula is expressed through characteristic deuteron quantities ($\langle p^2 \rangle$, $\langle p^4 \rangle$) and the unfolding procedure is outlined.

In Appendix C we review some of the deuteron wave functions, used in the literature, giving some new parametrizations.

In Appendix D the expression of the CEX and CP scattering with the inclusion of the D wave.

In Appendix C the case of half phase space is considered in the charge preserving process.

2. Relativistic invariant treatment.

In the previous paper w have assumed in the calculation of the single scattering amplitude, represented by the diagrams of Fig. I 2, that the incident particle is so fast that we can consider the bound nucleons as fixed in the laboratory system. If the same notation of I is mantained we have for the single scattering operator

$$\hat{T} = (16\pi^3 m_1)^{1/2} m \Big[\Big\{ (m^* + m_1 - E_5)^{1/2} (m + E_4)^{1/2} (A + \tfrac{B}{2}(E_2+E_3))$$

$$- \frac{1}{(m^* + m_1 - E_5)^{1/2} (m + E_4)^{1/2}} \Big[(A - B/_2 (E_2+E_3))(P^2 - \Delta^2/_4) +$$

$$+ B(m + m^* + E_4 + m_1 - E_5)\vec{P}.\vec{Q} + B/_2 (m_1 - E_5 - E_4)\vec{Q}.\vec{\Delta}$$

$$+ i(A - B/_2(E_2+E_3))\vec{\sigma}(\vec{\Delta}\times\vec{P}) + iB(E_4+E_5-m_1)\vec{\sigma}.(\vec{Q}\times\vec{P})$$

$$+ i B/_2 (m + m^* + E_4 + m_1 - E_5)\vec{\sigma}.(\vec{\Delta}\times\vec{Q}) \Big] \Big\} \Psi(-\vec{p}_5) +$$

$$\Big\{ (m^{*\prime} + m_1 - E_4)^{1/2} (m + E_5)^{1/2} (A + B/_2 (E_2+E_3)) -$$

$$\frac{1}{(m^{*\prime} + m_1 - E_4)^{1/2} (m + E_5)^{1/2}} \Big[(A - B/_2 (E_2+E_3))(P'^2 - \Delta^2/_4) +$$

$$B(m + m^{*\prime} + E_4 + m_1 - E_5)\vec{P}'.\vec{Q} + i(A - B/_2 (E_2+E_3))\vec{\sigma}.(\vec{\Delta}\times\vec{P}')$$

$$+ i\frac{B}{2}(E_5+E_4-m_1)\,\vec{\sigma}\cdot(\vec{Q}\times\vec{P}') +$$

$$+ i\frac{B}{2}(m+m^{*\prime}+E_4+m_1-E_5)\,\vec{\sigma}\cdot(\vec{\Delta}\times\vec{Q})\Big]\Big\}\psi(-\vec{p}_4)\Big] \qquad (2.1)$$

where

$$\vec{P} = \frac{(\vec{p}_4-\vec{p}_5)}{2} = -\vec{P}'$$

$$m^{*2} = (m_1-E_5)^2 - p_5^2$$

$$m^{*\prime 2} = (m_1-E_4)^2 - p_4^2$$

In a more compact notation

$$\hat{T} = (16\pi^3 m_1)^{1/2}\Big[(C_4 + iD_4\vec{\sigma}_4\cdot\hat{i} + iF_4\vec{\sigma}_4\cdot\hat{j} + iG_4\vec{\sigma}_4\cdot\hat{k})\,\psi(-\vec{p}_5)$$

$$+ (C_5 + iD_5\vec{\sigma}_5\cdot\hat{i} + iF_5\vec{\sigma}_5\cdot\hat{j} + iG_5\vec{\sigma}_5\cdot\hat{k})\,\psi(-\vec{p}_4)\Big] \qquad (2.2)$$

where

$$C_4 = m\Big\{(m^*+m_1-E_5)^{1/2}(m+E_4)^{1/2}\big(A+B/2\,(E_2+E_3)\big) -$$

$$\frac{1}{(m^*+m_1-E_5)^{1/2}(m+E_4)^{1/2}}\Big[\big(A-B/2\,(E_2+E_3)\big)(P^2-\Delta^2/4) +$$

$$B(m+m^*+E_4+m_1-E_5)\,\vec{P}\cdot\vec{Q} + B/2\,(m_1-E_5-E_4)\,\vec{Q}\cdot\vec{\Delta}\Big]\Big\}$$

$$D_4 = \frac{m}{(m^*+m_1-E_5)^{1/2}(m+E_4)^{1/2}}\Big[n\big(A-\frac{B}{2}(E_2+E_3)\big) + \ell B(E_4+E_5-m_1)\Big]$$

$$F_4 = \frac{m}{(m^*+m_1-E_5)^{1/2}(m+E_4)^{1/2}}\left[v\left(A-\frac{B}{2}(E_2+E_3)\right)+jB\,(E_4+E_5-m_1)\right]$$

$$G_4 = \frac{m}{(m^*+m_1-E_5)^{1/2}(m+E_4)^{1/2}}\left[n\left(A-\frac{B}{2}(E_2+E_3\right)+n'\frac{B}{2}(m+m^*+E_4+m_1-E_5)\right]$$

where

$$u = p_{4y}p_{5z} - p_{4z}p_{5y}$$

$$v = -p_{5x}p_{4z} + p_{4x}p_{5z}$$

$$w = p_{4x}p_{5y} - p_{4y}p_{5x}$$

$$l = -\frac{1}{2}p_2(p_{4z}-p_{5z}) - u/2$$

$$j = -v/2$$

$$n = \frac{1}{2}p_2(p_{4x}-p_{5x}) - w/2 \qquad n' = p_2 p_{3x}$$

A, B may be still operators in the isospin space.

For further development we write the formal expression of (2.1)

$$\hat{T} = (16\pi^3 m_1)^{1/2}\left[\hat{T}_4\,\psi(-\vec{p}_5) + \hat{T}_5\,\psi(-\vec{p}_4)\right] \tag{2.3}$$

with self understanding meaning of T_4 and T_5.

In the assumption that there is no $\ell = 2$ component of the wave function, the sum of the matrix elements squared is

$$\sum_{fi} |\langle f|\hat{T}|i\rangle|^2 = \Big(\sum_{fi}|\langle f|\hat{T}_4|i\rangle|^2\Big)\psi^2(p_5) \quad +$$

$$\Big(\sum_{fi}|\langle f|\hat{T}_5|i\rangle|^2\Big)\psi^2(p_4) + 2\sum_{fi}\mathrm{Re}\big[\langle f|\hat{T}_4|i\rangle^*\langle f|\hat{T}_5|i\rangle\big]\psi(p_4)\psi(p_5) \tag{2.4}$$

where $|i\rangle$ and $|f\rangle$ are the initial and final spin states of the nucleon pair. The initial states cover the triplet subspace, and the final states run over the whole Hilbert space. It is possible to show, using group theoretical techniques (GOL 64, p.862 n.18), that

$$\frac{1}{3}\sum_{fi}|\langle f|\hat{T}_4|i\rangle|^2 = \frac{1}{2}\sum_{mm'}|\langle m|\hat{T}_4|m'\rangle|^2 \tag{2.5}$$

where m,m' are the final and initial spin projector of the nucleon+. Therefore the first two terms in expression (2.4) are related to the differential cross-section which are invariant quantities, while the last term has to be expressed through the amplitudes C_i D_i F_i G_i , which are the spin amplitudes for a general reference frame. The sum of the matrix elements squared

$$\frac{1}{3}\sum_{fi}|\langle f|\hat{T}|i\rangle|^2 = \frac{d\Omega^*}{dV^{(2)}}\phi^{(2)}\big[U_4\psi^2(p_5) + U_5\psi^2(p_4)\big]$$
$$+ \frac{2}{3}\mathrm{Re}\big[3C_4^*C_5 + G_4^*G_5\big]\psi(p_4)\psi(p_5) \tag{2.6}$$

where

$$U_i = \frac{1}{2}\frac{dV^{(2)}}{d\Omega^*}\frac{1}{\phi^{(2)}}\sum_{m,m'}|\langle m|T_i|m'\rangle|^2$$

therefore only in the interference term there is trace left of the difference between the particle deuteron and the particle

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+ In this particular case this relation can be shown directly calculating both sides nucleon reference frame.

3. Folded differential cross section.

As seen in the previous paper (I) and in the section before the single scattering approximation is expressed through the following form.

$$\hat{T} = (16\pi^3 m_\Delta)^{1/2} \left[\hat{T}_4 (s_{34}, t_{23}, m^*) \psi(-\vec{p}_5) + \right. \tag{3.1}$$
$$\left. \hat{T}_5 (s_{35}, t_{23}, m^{*\prime}) \psi(-\vec{p}_4) \right]$$

The dependence on the virtual mass $m^*, m^{*\prime}$ is present, since the scattering process is only quasi-real. The relation between the Mandelstam variables is still satisfied, provided the virtual mass is taken for the internal nucleon

$$s_{34} + t_{23} + t_{24} = m_3^2 + m_2^2 + m_4^2 + m^{*2} \tag{3.2}$$

which compared with the relations of type (B.1), gives the equivalence

$$m^{*2} = t_{15} \tag{3.3}$$

The off mass shell dependence is not known and it is expressed in (3.1) as a double dependence on m^* of the amplitude

$$s = m_2^2 + m^{*2} + 2E_2\sqrt{m^{*2} + p_5^2} + 2\vec{p}_2 \cdot \vec{p}_5 \tag{3.4}$$

One dependence through s and the other one direct.One get the first one plotting s as function of m . The direct dependence of the amplitude on m^* is not known, but what ever it is, the variation of m^* affects the dependence of T_4 on s : for instance let us take a separable dependence

$$\hat{T}(s,t,m^*) = \Phi(s,t)\,\chi(m^*) \tag{3.5}$$

this will modify the normalization of the s dependence. Let us now take a general dependence and expand it in Taylor series around the point $m^* = m$

$$\hat{T}(s,t,m^*) = \hat{T}(s,t,m) + \left.\frac{\partial \hat{T}}{\partial m^*}\right|_{m^*=m} (m^*-m) \tag{3.6}$$

It is not unreasonable to assume the

$$\left.\frac{\partial \hat{T}}{\partial m^*}\right|_{m^*=m} \cong 0 \tag{3.7}$$

and therefore for m^* near to m

$$\hat{T}(s,t,m^*) \cong \hat{T}(s,t,m) \tag{3.8}$$

This is a function which in principle can be calculated because it corresponds to the on mass shell dependence of T and s. In s there is still the dependence on the virtual mass and on the momentum of the particle. In the above assumption (3.7), this is an approximate way to calculate both the effect of virtuality of the process and the effect of Fermi motion.

The variable t_{23} does not depend on the virtual mass of the internal nucleon, but if we use a partial wave expansion for $\hat{T}$ through th C.M. amplitudes, we need to determine the corresponding C.M. angle +. In this determination, we have to take in account that the internal nucleon has unphysical mass: this is done considering for every value of $\vec{p}_5$, the internal nucleon is thought as a physical particle with a light mass. Therefore for the scattering on nucleon 4

$$\cos\theta^* = \frac{t_{23} - m_2^2 - m_3^2 + 2E_2^* E_3^*}{2q_2^* q_3^*}$$

where

$$E_i^* = \sqrt{m_i^2 + q_i^{*2}}$$

$$q_3^* = t\,(s_{34}, m_3, m)$$

$$q_2^* = t\,(s_{34}, m_2, \sqrt{(m_4 - E_5)^2 - p_5^2}\,)$$

where
$$t\,(s, m_1, m_2) = \frac{[s - (m_1 + m_2)^2]\,[s - (m_1 - m_2)^2]}{4s}$$

and analogously for the other scattering.

+This dependence is easily seen, looking at formula (2.5) where U_i, with above assumption, become simply

$$\left.\frac{d\sigma}{d\Omega^*}\right|_i = |f_i|^2 + |g_i|^2$$

The two C.M. angles obtained in that way are different in general, unless $\vec{p}_4$ and $\vec{p}_5$ are identical. Therefore if we want to parametrize the data for the deuteron cross section as a function of $\cos\theta^*$, which in the most suitable choice for the phase shift analysis, we have to take some care in the integration: i.e. we have to consider 2 different angular regions

1) the region of large angles $-1 \leq \cos\theta^* \leq 1 + t(\Delta_0)/2q^2$ that we call the "spectator" region
2) the region of small angles $1 + t(\Delta_0)/2q^2 \leq \cos\theta^* \leq 1$ that we call the "overlapping" region

q is the C.M. momentum of the elementary particle nucleon scattering, in the assumption that the internal nucleon is at rest in the laboratory system and

$$t(\Delta_0) = m_2^2 + m_3^2 - 2E_2\sqrt{m_3^2 + p_2^2 + \Delta_0^2 + 2p_2\Delta_{0\parallel}} + 2p_2^2 + 2p_2\Delta_{0\parallel}$$

and $\Delta_{0\parallel} \sim \dfrac{1}{2p_2}\left[\dfrac{\Delta_0^2}{4m^2} - (1 + E_2/m)\right]\Delta_0^2 \quad , \quad \Delta_0^2$

In the first region the amplitudes sum incoherently in the folded differential cross section: in other words the kinematical region where 4 spectator, is not overlapping with the region where 5 is spectator, as it shown on fig. 3 of I.

In this case we fix the angle θ^* and we make the convention that this angle is referred to the particle 3 - 4 in the region where $p_5 < p_4$ and to the particles 3 - 5 in the region where $p_4 < p_5$. These two regions are not distinguishable if the two nucleons are identical as in charge exchange: in this case experimentally the slower nucleon is assumed as "spectator" and the angle is referred to the faster nucleon and the scattered particle.

In this region the folded deuterium cross section are

$$F_{CEX}(\theta^*) = \int U_{CEX}(s_{34}, \theta^*) \frac{2\phi^{(2)}}{\phi} |\psi(p_5)|^2 d^3p_5$$

$$F_{QE}(\theta^*) = \int \left[U_N(s_{34}, \theta^*) + U_p(s_{34}, \theta^*)\right] \frac{2\phi^{(2)}}{\phi} |\psi(p_5)|^2 d^3p_5$$

if

$$F(\theta^*) = \left.\frac{d\sigma}{d\Omega^*}\right|_D \tag{3.10}$$

is the <u>folded</u> deuterium differential cross section and

$$U(\theta^*) = \left.\frac{d\sigma}{d\Omega^*}\right|_N = |f_N|^2 + |g_N|^2 \tag{3.11}$$

is the unfolded nucleon cross section.

In the "overlapping" region the amplitudes sum coherently, that is, the two regions overlap to a certain extent and therefore we have to change our attitude.

Since <u>none</u> of the nucleon can be assumed as a "spectator" we have to choose one and refer our scattering angle to the process with that nucleon as "spectator".

The choice is rather arbitrary, but we mantain the above convention when the two nucleon are identical and we choose the proton for the quasi-elastic scattering.

The scattering angle $\theta^{*\prime}$ relative to the second amplitude in (2.1) is related to θ^* through t_{23}

$$\cos\theta^{*'} = \frac{t_{23} - m_2^2 - m_3^2 + 2E_2^{*'}E_3^{*'}}{2q_2^{*'}q_3^{*'}}$$

$$q_3^{*'} = t\,(s_{35}, m_3, m_5)$$

$$q_2^{*'} = t\,(s_{35}, m_2, \sqrt{(m_1 - E_4)^2 - p_4^2}\,) \qquad (3.11)$$

In this region the folded differential cross section (neglecting the relativistic effect for the interference term)

$$F_{CEX}(\theta^*) = \int U_{CEX}(\theta^*, s_{34})\,|\psi(p_5)|^2\,\frac{2\phi^{(2)}}{\phi}\,d^3p_5 -$$

$$\frac{k^2}{\phi}\int \mathrm{Re}\left\{\frac{1}{3}\,g^*_{CEX}(\theta^*, s_{34})\,g_{CEX}(\theta^{*'}, s_{35}) + f^*_{CEX}(\theta^*, s_{34})\,f_{CEX}(\theta^{*'}, s_{35})\right\}\times$$

$$\times\sqrt{\left.\frac{d\Omega^*}{dV^{(2)}}\phi^{(2)}\right|_{s_{34}}\left.\frac{d\Omega^{*'}}{dV^{(2)}}\phi^{(2)}\right|_{s_{35}}}\cdot\psi(p_4)\,\psi(p_5)\,\frac{dV^{(3)}}{d\Omega^*} \qquad (3.12)$$

$$F_{QE}(\theta^*) = \int\left[U_N(\theta^*, s_{34}) + U_P(\theta^*, s_{34})\right]\psi^2(p_5)\,\frac{2\phi^{(2)}}{\phi}\,d^3p_5$$

$$+\,2\,\frac{k^2}{\phi}\int \mathrm{Re}\left\{\frac{1}{3}\,g^*_P(\theta^*, s_{34})\,g_N(\theta^{*'}, s_{35}) + f^*_P(\theta^*, s_{34})\,f_N(\theta^{*'}, s_{35})\right\}$$

$$\times \sqrt{ \left. \frac{d\Omega^*}{dV^{(2)}} \phi^{(2)} \right|_{s_{34}} \left. \frac{d\Omega^{*'}}{dV^{(2)}} \phi^{(2)} \right|_{s_{35}} } \cdot \psi(p_4)\psi(p_5) \frac{dV^{(3)}}{d\Omega^*} \qquad (3.13)$$

Till now, the analysis of neutron amplitude was done using the formalism given in I. We will call it "folded" analysis: this can be improved using the formules (3.8), (3.9), (3.12), (3.13) The method, we suggest here, is to parametrize the amplitudes as function of s and ϑ^* and determine the parameters with a best fit of the folded cross section, in a large region of energies and the whole range of angles, with expression (2.12) or (2.13). The parameters of the "folded" analysis can be the starting values of the "unfolding" procedure.

4. Qualitative consideration on Fermi motion folding effects.

In order to gain intuitive insight of the folding effects due to the Fermi motion we consider only the incoherent part of the cross section, neglecting in this way the interference: this is a good approximation even in the "overlapping" region because the interference is always a minor part of the differential cross section (in the worst case of the order 10%)+. With this simplification we can apply the simple Taylor expression of differential cross section inside the integral proposed by Faeldt and Ericson (FAE 68) and then revived by G. West (WES 71)

$$U(s) = U(s_0) + \left.\frac{dU}{ds}\right|_{s=s_0}(s-s_0) + \frac{1}{2}\left.\frac{d^2U}{ds^2}\right|_{s=s_0}(s-s_0)^2 + \ldots \quad (4.1)$$

where

$$(s-s_0) = -2(E_2+\omega_1)T - (2E_2+\omega_1)\epsilon + 2p_2 p_{s\parallel} \quad (4.2)$$

where ϵ is of the order of the binding energy of the deuteron and T is the kinetic energy of the spectator nucleon. Neglecting the contribution of the binding energy

$$s - s_0 \sim -2(E_2+\omega_1)T + 2p_2 p_{s\parallel} \quad (4.3)$$

The flux factor which according to (3.9) has to be folded together with the differential cross section, can be approximated in the following manner (HIR 70). Starting from (I.3.25) and

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+ This is not true for an experiment with good angular resolution. In the forward direction the interference is counting for 100%.

neglecting the off shell effect

$$\frac{2\phi^{(2)}}{\phi} \sim \frac{\sqrt{(p_2 p)^2 - (m_2 m)^2}}{m p_2} = \frac{\sqrt{p_2^2 m^2 + p_2^2 p_5^2 + 2 p_2 m E_2 p_{5\parallel}}}{m p_2} \tag{4.4}$$

$$\sim \sqrt{1 + 2\frac{E_2}{p_2 m} p_5 \cos\theta_5} \sim 1 + \frac{E_2}{p_2 m} p_5 \cos\theta_5$$

Using the Taylor expansion for the cross section (4.1) and the above expression for the flux factor, we find

$$\begin{aligned} F(s) = {} & U(s_0) \int d^3p \left(1 + \frac{p\cos\theta}{\beta_2 m}\right)\psi^2(p) + \\ & + \left.\frac{dU}{ds}\right|_{s=s_0} \times \int d^3p\,(s-s_0)\left(1 + \frac{p\cos\theta}{\beta_2 m}\right)\psi^2(p) + \\ & + \frac{1}{2}\left.\frac{d^2U}{ds^2}\right|_{s=s_0} \times \int d^3p\,(s-s_0)^2\left(1 + \frac{p\cos\theta}{\beta_2 m}\right)\psi^2(p) + \dots \end{aligned} \tag{4.5}$$

where

$$\int d^3p\,(s-s_0)\left(1 + \frac{p\cos\theta}{\beta_2 m}\right)\psi^2(p) = -\left(\frac{1}{3}E_2 + m_1\right)\frac{\langle p^2\rangle}{m} \tag{4.6}$$

$$\int d^3p\,(s-s_0)^2\left(1 + \frac{p\cos\theta}{\beta_2 m}\right)\psi^2(p) =$$

$$= \frac{1}{3}\frac{1}{m^2}\left(-E_2^2 + E_2 m_1 + 3m_1^2\right)\langle p^4\rangle + \frac{4}{3}p_2^2\langle p^2\rangle \tag{4.7}$$

Note that we have assumed the non relativistic approximation for the kinetic energy of the spectator. We notice that in the first order term (4.6) only the term in $\cos^2\theta$ is surviving, that is in absence of the flux factor the term in cos θ does not contribute at all in the folding with Fermi motion, this statement is correct only to the first order, but in the case of cross section rapidly rising with energy as for instance the differential cross section $K^+N \to K^+N$ at fixed angle (around 700 MeV/c and cos $\theta^* = .75$), is almost exact: in other words the behaviour with energy of the cross section can be approximated by a straight line and for straight line the linear contribution to the folding with Fermi motion is exactly zero. The above derivative formula, is accurate inside few per cent (see Figs. 1,2,3), for a cross section which can be fitted locally be a second order polinomial. The unfolding procedure is therefore consisting of the following steps

1) Parametrize the phase shifts and elasticities, obtained in the folded analysis.
2) Minimize the χ^2, fitting the experimental cross section with the derivative formula (derivatives can be calculated numerically or even analitically).
3) The new parameters, found, in this minimization program, give the unfolded phase shifts and elasticities.

We notice that the first term of the derivative formula is giving the form of the cross section used for the "folded" phase shifts analysis. Therefore the unfolding means just adding two terms to the first one.

ACKNOWLEDGEMENTS

I am deeply indebted to L.Bertocchi for encouragement and suggestions, to G.Bialkowski for an interesting discussion on the off-mass shell problem and to E.Castelli for handing me over the data of Fig.8 before publication. I' m grateful also to D.Zanello,M.Sessa and P.Poropat for numerous and alive discussions.

APPENDIX C

First of all we give some definitions, used in the literature concerning the wave function of the deuteron in coordinate and in momentum space and the so called form factors. The representation of the deuteron wave function in the momentum space and the spin space of the deuteron is

$$\psi(q) = \psi_0(q) - \frac{1}{\sqrt{2}}\left[3(\vec{J}\cdot\hat{q})^2 - 2\right]\psi_2(q) \tag{C.1}$$

where ψ_0, ψ_2 are respectively the S and D wave function, and $\vec{J}$ is the spin of the deuteron. The operator which multiplies the D wave is called the tensor operator and can be more traditionally expressed through the spin of the individual nucleons

$$\hat{O} = \frac{1}{\sqrt{2}}\left[3(\vec{J}\cdot\hat{q})^2 - 2\right] = \frac{1}{\sqrt{8}}\left[3(\vec{\sigma}_4\cdot\hat{q})(\vec{\sigma}_5\cdot\hat{q}) - \vec{\sigma}_4\vec{\sigma}_5\right] \tag{C.2}$$

The matrix elements of this operator on the various triplet states are linear superposition of spherical harmonics with $\ell = 2$.

It is easy to show in either rapresentation that

$$\mathrm{tr}\,\hat{O} = 0 \qquad \mathrm{tr}\,\hat{O}^2 = 3 \tag{C.3}$$

Therefore the norm of the wave function is

$$\langle\psi|\psi\rangle = \int \psi_0^2(q)\, d^3q + \frac{1}{3}\,\mathrm{tr}\int \hat{O}^2 \psi_2^2(q)\, d^3q =$$

$$= \int \psi_0^2(q)\, d^3q + \int \psi_2^2(q)\, d^3q = P_S + P_D = 1 \tag{C.4}$$

The same can be done in the coordinate space

$$\varphi(r) = \varphi_0(r) + \frac{1}{\sqrt{8}}\left[3(\vec{\sigma}_4\cdot\hat{r})(\vec{\sigma}_5\cdot\hat{r}) - \vec{\sigma}_4\vec{\sigma}_5\right]\varphi_2(r) \tag{C.5}$$

Conventionally

$$\varphi_0(r) = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r}$$
$$\varphi_2(r) = \frac{1}{\sqrt{4\pi}} \frac{w(r)}{r} \qquad (C.6)$$

The relation of the momentum wave functions with u and w are

$$\psi_0(q) = \frac{1}{\pi\sqrt{2}} \int_0^\infty r\,dr\, u(r)\, j_0(qr) \qquad (C.7)$$

$$\psi_2(q) = \sqrt{\frac{2}{\pi}} \int_0^\infty r\,dr\, w(r)\, j_2(qr)$$

In standard single scattering approximation for elastic scattering the operator $e^{i\vec{\Delta}.\vec{r}/2}$, has to be averaged on the ground state of the deuteron (where $\vec{\Delta}$ is the three-momentum transfer)

$$\langle\psi| e^{i\vec{\Delta}.\vec{r}/2} |\psi\rangle = S_0(\Delta/2) - \hat{O}(\Delta/2)\, S_2(\Delta/2) \qquad (C.8)$$

since $\hat{O}^2 = -\frac{1}{\sqrt{2}}\hat{O} + 1$ on triplet states

where

$$S_0(k) = \int_0^\infty dr\, j_0(kr) \left[u^2(r) + w^2(r) \right]$$
$$\qquad (C.9)$$
$$S_2(k) = 2\int_0^\infty dr\, j_2(kr) \left[u(r)\,w(r) - \frac{w^2(r)}{\sqrt{8}} \right]$$

The first is called "charge" form factor and second "quadrupole" form factor. The connection to the charge and the quadrupole moment of the deuteron is given by the following relations

$$e\, S_0(0) = e \qquad (C.10)$$

$$e \lim_{k \to 0} \frac{S_2(k)}{k^2} = \frac{\sqrt{2}}{3} Q$$

We list hereafter some deuteron wave function and compare afterwards their features.

1. Asymptotic wave function.

The simplest deuteron wave function one can think of, is the so called "asymptotic or pole" wave function; this wave function, only for the S state, contains only one parameter, the binding energy of the deuteron $-\frac{\alpha^2}{m}$, and is very useful for simple model calculation

$$\psi_0(q) = \frac{\sqrt{\alpha}}{\pi}\frac{1}{q^2+\alpha^2} \qquad \varphi_0(r) = \sqrt{\frac{\alpha}{2\pi}}\,\frac{e^{-\alpha r}}{r} \tag{C.11}$$

The wave function in coordinate space has wrong behaviour at small distances.

2. Hulthen wave function (HUL 57)

This wave function is going to finite value for small distances

$$\psi_0(q) = \frac{H}{\pi\sqrt{2}}\left(\frac{1}{q^2+\alpha^2} - \frac{1}{\beta^2+q^2}\right) \tag{C.12}$$

$$\varphi_0(r) = \frac{H}{\sqrt{4\pi}}\left(e^{-\alpha r} - e^{-\beta r}\right)\frac{1}{r}$$

where

$$H = \left(\frac{1}{2\alpha} + \frac{1}{2\beta} - \frac{2}{\alpha+\beta}\right)^{-1/2}$$

This wave function is already realistic enough to attempt with it rough calculations. However one has to bear in mind that in a χ^2 analysis of elastic electron scattering it can be rejected with a high level of confidence (ELI 69). This result can be understood looking at the comparison between the charge form factors for the Hulthen w.f. and the Bressel and Kerman w.f. (fig. 6), which gives a reasonable fit to the electron scattering data.

3. Moravcsic-Gartenhaus wave function.

The Gartenhaus wave function was parametrized by Moravcsic, with simple exponentials (MOR 58).

$$\psi(q) = \frac{M}{\pi\sqrt{2}} \sum_{i=1}^{8} (-1)^i \frac{1}{\alpha_i^2 + q^2}$$

$$\varphi(r) = \frac{M}{\sqrt{4\pi}} \frac{1}{r} \sum_{i=1}^{8} (-1)^i e^{-\alpha_i r}$$

where (C.13)

$$M = \left(\sum_{i,j=1}^{8} (-1)^{i+j} \frac{1}{\alpha_i + \alpha_j} \right)^{-1/2} +$$

Although this wave function was not considered in the analysis of electron scattering, it is easy to realize that it is very similar to the Bressel-Kerman wave function from the comparison of the form factors (fig. 6). The D part of the wave function was parametrized in three different region of the coordinate space and therefore it is not too helpful in actual calculations.

4. Bressel-Kerman wave function.

The previous wave function u(r) is presenting an r^3 behaviour at r = 0, which is simulating the so called core, coming from the repulsive part of the potential (showing evidently in the $^1S_0$ nucleon nucleon phase shift). This wave function presents the same characteristic at small distance (so called "soft core"). The S wave and D wave part can be parametrized as exponentials (BAX 68)

$$u(r) = r^2 \sum_{i=1}^{6} B_i e^{-\beta_i r}$$

$$w(r) = r^2 \sum_{i=1}^{13} C_i e^{-\gamma_i r} + \qquad \text{(C.14)}$$

This analytic form is particularly useful for the calculation of the electric and quadrupole form factors.

5. Gaussian wave function.

Sometimes for rough calculations, it is useful to represent the wave function in momentum space as a single gaussian

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+ The parameters are reported on table I

$$\psi_0(q) = \left(\frac{2\delta}{\pi}\right)^{3/4} \exp[-\delta q^2]$$

$$\delta = 66 \ (\text{GeV/c})^{-2} \tag{C.15}$$

The behaviour of such model wave functions is compared with the Gartenhaus Moravcsic in Fig.7.

6 Multigaussian wave function.

It is well known that any function can be represented as superposition of gaussians of different slopes. To mantain the analytic form of most of the result of deuteron physics, and in the same time to stick to realistic wave function, we can parametrize it in term of gaussians

$$\psi_0(q) = \sum_{i=1}^{5} F_i \exp[-\eta_i q^2]$$

$$\psi_2(q) = q^2 \sum_{j=1}^{5} G_j \exp[-\xi_j q^2] \tag{C.16}$$

The fit of the Gartenhaus-Moravcsic wave function in the momentum space is extremely good for q $\lesssim$ .4 while for q $>$ .4 GeV/c the two curves are slightly departing as shown on fig. C.3. The numbers for the amplitude and the slopes for a P_S = .933 and P_D = .067 are given on Table I. This parametrization was used all over the previous (I) and the present paper.

On fig. 7 the behaviour of the Hulthen wave funtion (only S wave) is shown. This comparison shows very little difference between the two wave functions for q $\lesssim$ .25 GeV/c, but further on there is very poor resemblance. The difference in this last region is the cause of the discrepancy between the correspondent charge form factors (Fig. 6)

In the figure 8 we show the behaviour of the probability distribution for nucleon having momentum q

$$\frac{d\mathbb{P}}{dq} = \left[\psi_0^2(q) + \frac{1}{3}\,\mathrm{tr}(\hat{\delta}^2)\,\psi_2^2(q)\right] q^2$$

$$= \left[\psi_0^2(q) + \psi_2^2(q)\right] q^2 \qquad (C.17)$$

This is arbitrarily normalized in such a way that

$$\int_0^{.25} \frac{d\mathbb{P}}{dq}\, dq = 1 \qquad (C.18)$$

which corresponds to assume weight 1 in the extraction of the neutron cross section. It is easy to see that there is a great difference in the large momentum region between Gartenhaus-Moravcsic with only S- wave and the same with both waves. The Hulthen distribution with only S wave stays in the middle.

APPENDIX D

The matrix element for the break up single scattering amplitude, which were listed in (I) in (3.4, 5, 6, 7) become with the inclusion of the D wave in the deuteron ground state:

a) for the singlet spin state of the nucleon pair in the final state

$$\langle 00;00|T|00;1\nu\rangle = i a_g k\,[\,\delta_{\nu 0}\psi_0(p_5) - \langle 0|\hat{O}_5|\nu\rangle\psi_2(p_5) - \delta_{\nu 0}\psi_0(p_4) + \langle 0|\hat{O}_4|\nu\rangle\psi_2(p_4)\,] \tag{D.1}$$

$$\langle 1T_z;00|T|00;1\nu\rangle = i b_g k \langle NN|\vec{\tau}\cdot\vec{\tau}_4|D\rangle \times [\,\delta_{\nu 0}\psi_0(p_5) - \langle 0|\hat{O}_5|\nu\rangle\psi_2(p_5) + \delta_{\nu 0}\psi_0(p_4) - \langle 0|\hat{O}_4|\nu\rangle\psi_2(p_4)\,] \tag{D.2}$$

b) for the triplet spin state

$$\langle 00;1\nu'|T|00;1\nu\rangle = \langle \nu'|a_f + i\sigma_{4z}a_g|\nu'\rangle k \times [\,\delta_{\nu'\nu}\psi_0(p_5) - \langle \nu'|\hat{O}_5|\nu\rangle\psi_2(p_5) + \delta_{\nu'\nu}\psi_0(p_4) - \langle \nu'|\hat{O}_4|\nu\rangle\psi_2(p_4)\,] \tag{D.3}$$

$$\langle 1 T_z ; 1 \nu' | T | 00 ; 1 \nu \rangle = \langle NN | \vec{\tau} \cdot \vec{\tau}_4 | D \rangle^*$$

$$\times \langle \nu' | b_f + i \sigma_{4z} b_g | \nu' \rangle^* K^* \left[\delta_{\nu' \nu} \psi_0(p_5) \right.$$

$$\left. - \langle \nu' | \hat{O}_5 | \nu \rangle \psi_2(p_5) - \delta_{\nu' \nu} \psi_0(p_4) + \langle \nu' | \hat{O}_4 | \nu \rangle \psi_2(p_4) \right] \quad (D.4)$$

where

$$\hat{O}_i = \frac{1}{\sqrt{2}} \left(3 (\vec{J} \cdot \hat{p}_i)^2 - 2 \right)$$

We walk pleasantly along the same road of I and find the result for the cross sections

$$\left. \frac{d\sigma}{d\Omega^*} \right|_{CEX} = \frac{|g^{CEX}|^2}{3} {}^1W^+ + \frac{2}{3} \left(|f^{CEX}|^2 + |g^{CEX}|^2 \right) {}^3W_1^- + \frac{1}{3} |f^{CEX}|^2 \, {}^3W_0^- \quad (D.5)$$

$$\left. \frac{d\sigma}{d\Omega^*} \right|_{QE} = \frac{|g^+|^2}{3} {}^1W^- + \frac{2}{3} \left(|f^+|^2 + |g^+|^2 \right) {}^3W_1^+ + \frac{|f^+|^2}{3} {}^3W_0^+$$

$$+ \frac{|g^-|^2}{3} {}^1W^+ + \frac{2}{3} \left(|f^-|^2 + |g^-|^2 \right) {}^3W_1^- + \frac{|f^-|^2}{3} {}^3W_0^- \quad (D.6)$$

where the new weight factors are defined according to (3.24) of I, that is choosing the Jew (Ref. 6 of I) convention for the phase space.

$${}^1W^{\pm} = \frac{1}{p_2} \int \frac{d^3 p_5}{E_5} \left\{ \left[\psi_0(p_5) - \langle 0 | \hat{O}_5 | 0 \rangle \psi_2(p_5) \right]^2 + \right. \quad (D.7)$$

$$2 \left| \langle 0 | \hat{O}_5 | 1 \rangle \right|^2 \psi_2^2(p_5) \pm \left(\psi_0(p_5) - \langle 0 | \hat{O}_5 | 0 \rangle \psi_2(p_5) \right) \times \left(\psi_0(p_4) - \right.$$

$$\left. \left. \langle 0 | \hat{O}_4 | 0 \rangle \psi_2(p_4) \right) \pm 2 \, \mathrm{Re} \left[\langle 0 | \hat{O}_5 | 1 \rangle^* \langle 0 | \hat{O}_4 | 1 \rangle \psi_2(p_4) \psi_2(p_5) \right] \right\} q \sqrt{s_{34}}$$

$$^3W_1^{\pm} = \frac{1}{p_2}\int \frac{d^3p_5}{E_5}\Big\{ [\psi_0(p_5) - \langle 1|\hat{O}_5|1\rangle \psi_2(p_5)]^2 + |\langle 1|\hat{O}_5|0\rangle|^2 \psi_2^2(p_5)$$

$$+ |\langle 1|\hat{O}_5|-1\rangle|^2 \psi_2^2(p_5) \pm (\psi_0(p_5) - \langle 1|\hat{O}_5|1\rangle\psi_2(p_5))(\psi_0(p_4)$$

$$- \langle 1|\hat{O}_4|1\rangle\psi_2(p_4)) \pm \mathrm{Re}\,[\langle 1|\hat{O}_5|0\rangle^* \langle 1|\hat{O}_4|0\rangle + \langle 1|\hat{O}_5|-1\rangle^* \langle 1|\hat{O}_4|-1\rangle]\times$$

$$\times \psi_2(p_4)\psi_2(p_5)\Big\} \, 9\sqrt{s_{34}} \qquad \text{(D.8)}$$

$$^3W_0^{\pm} = {}^1W^{\pm}$$

This last form factor and the first corresponds to the spin of the nucleon pair, directed along the orthogonal to the quantization axis, while the second corresponds to the direction of the spin along the quantization axis.

To spare to the reader the effort of calculating the matrix elements of the tensor operator, we list them hereafter

$$p^2\langle 1|\hat{O}|1\rangle = \frac{1}{2}(3p_z^2 - p^2)\frac{1}{\sqrt{2}}$$

$$p^2\langle 0|\hat{O}|0\rangle = (p^2 - 3p_z^2)\frac{1}{\sqrt{2}}$$

$$p^2\langle 0|\hat{O}|1\rangle = \frac{3}{\sqrt{2}}(p_z p_x + i p_z p_y)\frac{1}{\sqrt{2}} \qquad \text{(D.10)}$$

$$p^2\langle 1|\hat{O}|-1\rangle = \frac{3}{2}(p_x^2 - p_y^2 - 2ip_xp_y)\frac{1}{\sqrt{2}}$$

the other are calculated using the hermiticicity and time reversal invariance of the operator.

APPENDIX E

We consider in this appendix the case of charge preserving scattering: in this wase the two nucleon in the final state are distinguishable and for convenience we can call the neutron 4 and the proton 5. The iso-spin state of the two nucleons in the final state is in this case

$$|pn\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |0\rangle\right) \tag{E.1}$$

the singlet spin matrix element is therefore derived from I.3.4 and 5

$$\begin{aligned} {}^1T &= \frac{1}{\sqrt{2}}\left(\langle 00;00|T|00;1\nu\rangle + \langle 10;00|T|00;1\nu\rangle\right) \\ &= \frac{i}{\sqrt{2}}\left[(-a_g+b_g)\psi(-p_4) + (a_g+b_g)\psi(-p_5)\right] \end{aligned} \tag{E.2}$$

and the triplet from I.3.6 and 7

$$\begin{aligned} {}^3T^{\nu,\nu'} &= \frac{1}{\sqrt{2}}\left(\langle 00;1\nu'|T|00;1\nu\rangle + \langle 10;1\nu'|T|00;1\nu\rangle\right. \\ &= \frac{1}{\sqrt{2}}\langle\nu'|\{[a_f-b_f+i\sigma_{4z}(a_g-b_g)]\psi(p_4) \\ &\quad + [a_f+b_f+i\sigma_{5z}(a_g+b_g)]\psi(p_5)\}|\nu\rangle \end{aligned} \tag{E.3}$$

Squaring the matrix elements, averaging on initial spin states and summing over final states we get

$$\begin{aligned} \frac{1}{3}\sum_{f,i}|T_{fi}|^2 &= \frac{1}{2}\left\{\frac{|a_g|^2}{3}E^- + \left(|a_f|^2 + \frac{2}{3}|a_g|^2\right)E^+\right. \\ &\quad + \frac{|b_g|^2}{3}E^+ + \left(|b_f|^2 + \frac{2}{3}|b_g|^2\right)E^- + \\ &\quad \left. + 2\,\mathrm{Re}\left[a_g^* b_g + a_f^* b_f\right]\left(\psi^2(p_5) - \psi^2(p_4)\right)\right\} \end{aligned} \tag{E.4}$$

where

$$E^{\pm} = \left[\psi(p_5) \pm \psi(p_4)\right]^2$$

If we assume in an ideal experiment to kill all the events with neutron spectator ($\psi(p_4) \equiv 0$), we get, integrating on the phase space, the differential cross section for the elastic process K^+N : doing the same for the other nucleon, we obtain the differential cross section for the elastic process K^+P

$$\left.\frac{d\sigma}{d\Omega^*}\right|_{K^+N} = \frac{1}{2}\left\{|a_f|^2 + |b_f|^2 + 2\,\mathrm{Re}\,[a_f^* b_f] + |a_g|^2 + |b_g|^2 + 2\,\mathrm{Re}\,[a_g^* b_g]\right\}\frac{dV^{(2)}}{d\Omega^*}\,\phi^{(2)}$$

$$\left.\frac{d\sigma}{d\Omega^*}\right|_{K^+P} = \frac{1}{2}\left\{|a_f|^2 + |b_f|^2 - 2\,\mathrm{Re}\,[a_f^* b_f] + |a_g|^2 + |b_g|^2 - 2\,\mathrm{Re}\,[a_g^* b_g]\right\}\frac{dV^{(2)}}{d\Omega^*}\,\phi^{(2)} . \tag{E.5}$$

That is, identifying it with proper combination of isospin amplitudes, we obtain

$$a_f = \frac{\sqrt{2}}{4}\,(f_0 + 3f_1)$$

$$b_f = \frac{\sqrt{2}}{4}\,(f_0 - f_1) \tag{E.6}$$

and the same for a_g and b_g .

Integrating on the whole phase space the interference terms in (E.4) vanishes and we get (I.3.18)

$$\left.\frac{d\sigma}{d\Omega^*}\right|_{QE} = \frac{|g^+|^2}{3}\,w^- + \left(|f^+|^2 + \frac{2}{3}|g^+|^2\right) w^+$$

$$+ \frac{|g^-|^2}{3}\,w^+ + \left(|f^-|^2 + \frac{2}{3}|g^-|^2\right) w^- \tag{E.7}$$

If we integrate only on half space defined by the kinematical condition $p_5 < p_4$

$$\left.\frac{d\sigma}{d\Omega^*}\right|_D = \left.\frac{d\sigma}{d\Omega^*}\right|_{K^+N} \times \frac{W^+ + W^-}{2} + \left(|f^+|^2 - |f^-|^2 \right. \tag{E.8}$$

$$\left. + \frac{|g^+|^2 - |g^-|^2}{3} \right) \times \frac{W^+ - W^-}{2} - 2\,\mathrm{Re}\left(f^{+*} f^- + g^{+*} g^-\right) \int_{p_5 < p_4} \psi^2(p_4)\, \frac{2\phi^{(2)}}{\phi}\, d^3 p_5$$

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TABLE I

α_i (GeV/c)	B_i $(GeV/c)^{-5/2}$	β_i (GeV/c)	C_i $(GeV/c)^{-5/2}$	γ_i (GeV/c)
.5382	8.827×10^{-4}	8.019×10^{-2}	1.344×10^{-4}	9.357×10^{-2}
.8514	1.17×10^{-2}	1.971×10^{-1}	6.12×10^{-3}	2.347×10^{-1}
.3589	2.089×10^{-2}	4.437×10^{-1}	3.432×10^{-2}	6.254×10^{-1}
.8668	9.694×10^{-2}	5.388×10^{-1}	-5.042×10^{-1}	7.421×10^{-1}
.3743	3.120×10^{-2}	1.0139	1.5400	8.541×10^{-1}
.6875	3.155×10^{-1}	1.5920	9.6698	1.3273
1.18003			-9.2825	1.2064
.04570			-2.4939	2.4081
			-2.612×10^{-1}	1.9934
			-2.342×10^{-1}	2.5158
			5.9456	6.2232
			3.1971	4.3509
			-1.026×10^{-3}	7.1188

TABLE II

F_i $(GeV/c)^{-3/2}$	η_i $(GeV/c)^{-2}$	G_i $(GeV/c)^{-7/2}$	ξ_i $(GeV/c)^{-2}$
4.9965	2034.129	2.9742	7.925
24.3705	408.220	21.4327	28.461
9.4082	87.934	10.6916	111.632
2.1399	21.495	75.5395	99.161
-.06246	1.702	188.3457	388.459

TABLE III

T = 0	A	B	C	D	E
$S_{1/2}$	4.32E-2	3.25E-2	-1.21E-3	-2.04	1.71E1
$P_{1/2}$	-1.04E-2	1.16E-4	6.43E-5	3.51E1	-2.88E2
$P_{3/2}$	1.96E-2	-1.75E-3	4.16E-5	6.88	-1.11E1
$D_{3/2}$	-7.34E-4	9.26E-5	-2.50E-6	-5.09	-1.32
$D_{5/2}$	-1.00E-3	7.62E-5	-1.53E-6	4.02E-1	-2.23

TABLE IV

T = 1	A	B	C	D	E
$S_{1/2}$	-2.20E-1	3.76E-3	-5.68E-5	0.	0.
$P_{1/2}$	-2.17E-2	1.56E-3	-3.34E-5	0.	0.
$P_{3/2}$	9.84E-3	-3.08E-4	-1.97E-6	1.04E+ 1	-1.31E 1
$D_{3/2}$	-6.97E-6	-9.42E-6	3.33E-7	2.17E 2	8.70E-1
$D_{5/2}$	-6.39E-4	6.10E-5	-1.44E-6	8.67	-1.83E 1

FIGURE CAPTIONS

Fig. 1 The excitation function for $K^+D \to K^+NP$ for $\cos\theta^* = .75$ in the single scattering approximation using Glasgow solution A_{III} of the energy dependent search for the KN phase shifts. The dot-dashed line is the fixed center approximation (no Fermi motion). The continuous line is the result of folding the K^+N differential cross section, with a numerical integration (only the incoherent part). The dashed line is the result of the derivative formula up to the 2° order. The experimental data are from Ref. GIA 73. (The parameter of Lea, Martin and Oades parametrization (LEA 68) for the solution A_{III} are given on table III, IV).

Fig. 2 The same for $\cos\theta^* = .85$

Fig. 3 The interference effect for $\cos\theta^* = .75$. The dashed line is without interference. The continuous line is with interference

Fig. 4 The interference effect for $\cos\theta^* = .85$

Fig. 5 The same as fig. 1 but the derivative formula is calculated up to the 3rd order.

Fig. 6 The square of charge form factor $S^2(t/4)$ for different deuteron wave functions

1) dot - dashed line, asymptotic w.f.

2) dashed line: Hulthen w.f.

3) dotted line : Bressel and Kerman

4) continuous line : Gartenhaus Moravcsik

Fig. 7 Comparison of different deuteron wave functions in the momentum space

1) continuous line :Gartenhaus Moravcsik (S wave and D wave)

2) dashed line : multigaussian

3) dotted line : Hulthen (only S wave)

4) dott - dashed : 1 gaussian (only S wave)

Fig. 8 Probability distribution of the slow proton at $p_2 = .98$ GeV/c

1) dashed line : G.M. (only S wave)

2) continuous line: G.M. (S wave and D wave)

3) dotted line : Hulthen (only S wave)

The experimental data are from BGRT collaboration.

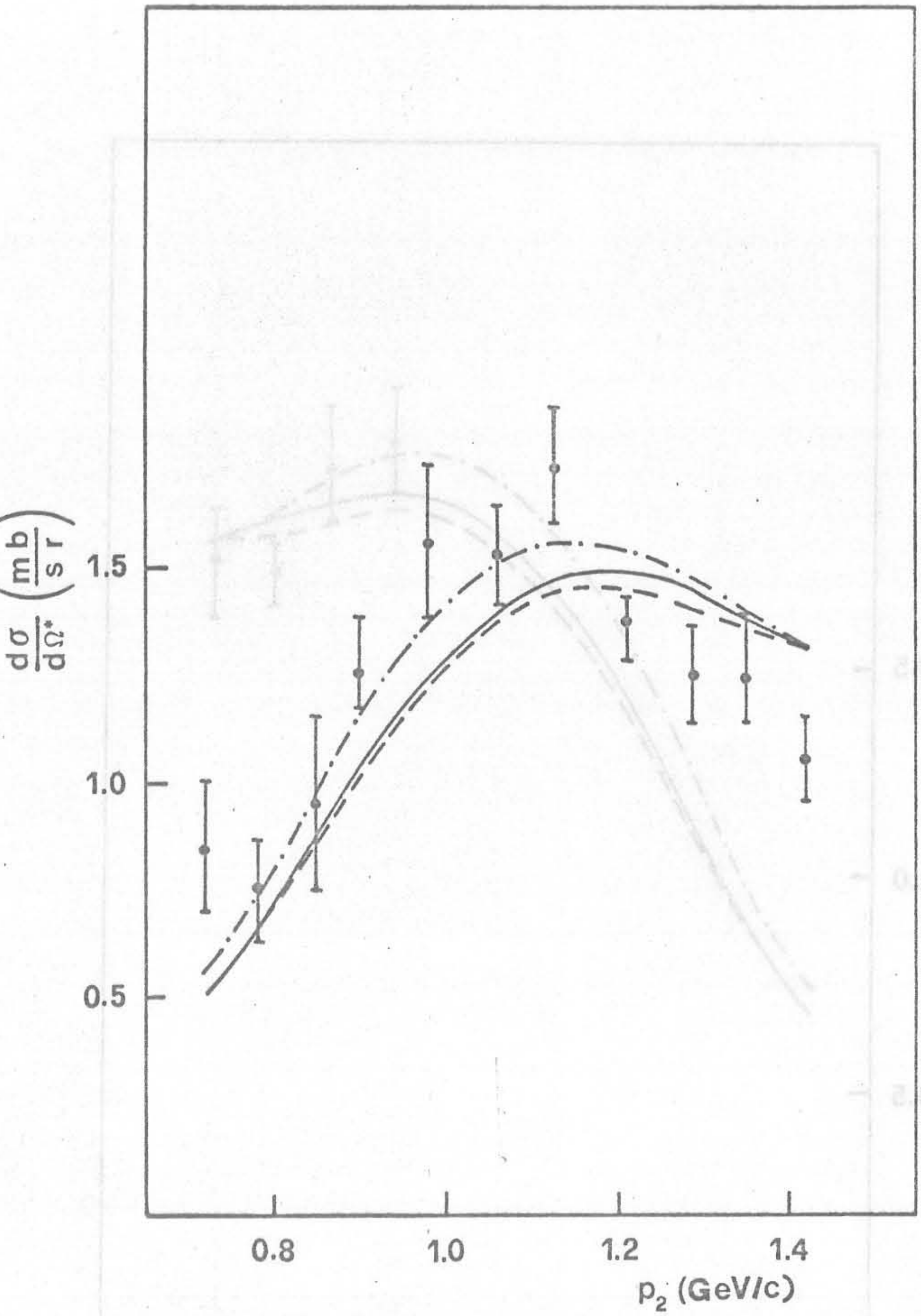
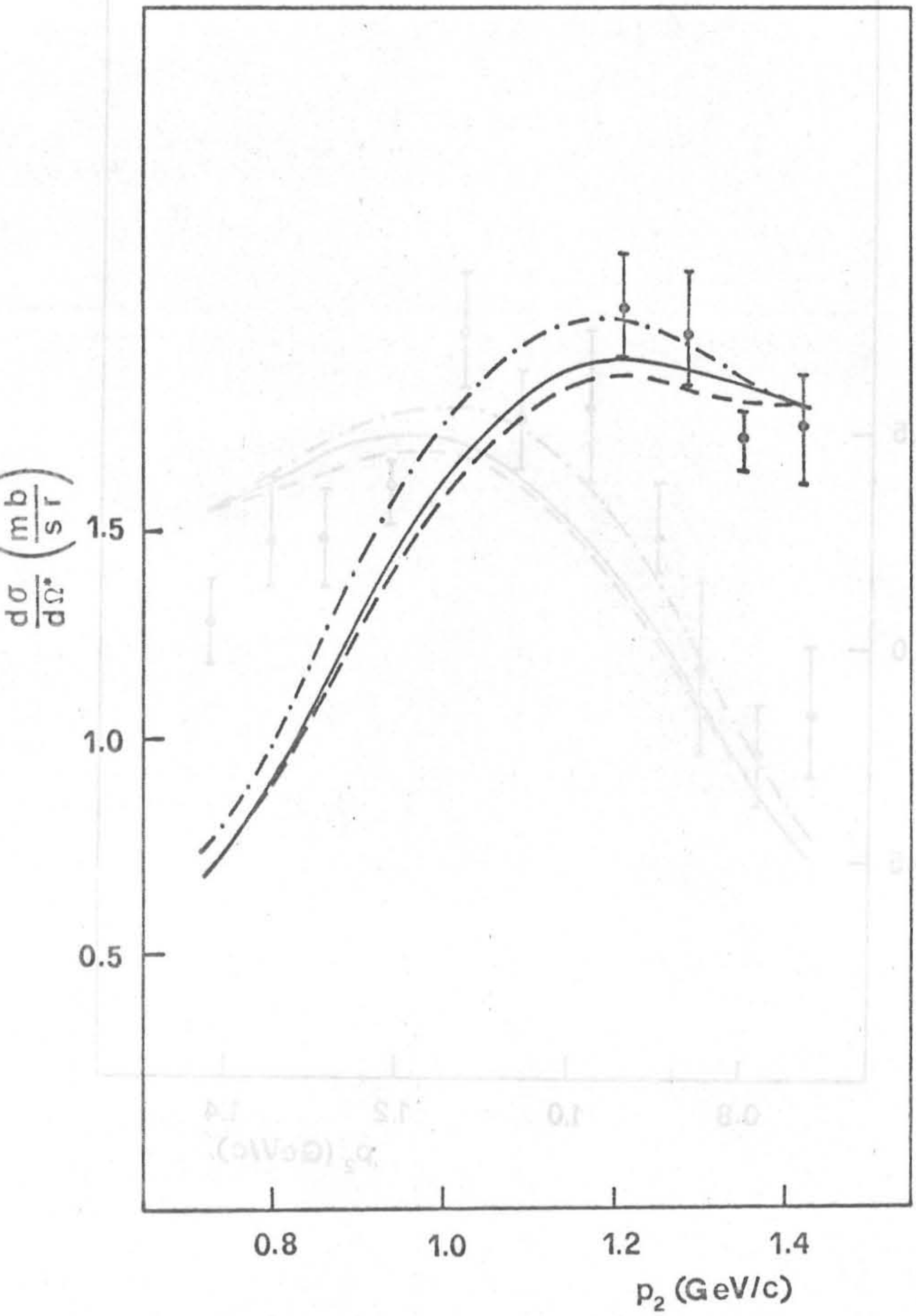

$\frac{d\sigma}{d\Omega^*}\left(\frac{mb}{sr}\right)$
1.5
1.0
0.5
0.8
1.0
1.2
1.4
p_2 (GeV/c)

Fig. 1



$\frac{d\sigma}{d\Omega^*}\left(\frac{mb}{sr}\right)$
1.5
1.0
0.5
0.8
1.0
1.2
1.4
p_2 (GeV/c)

Fig. 2

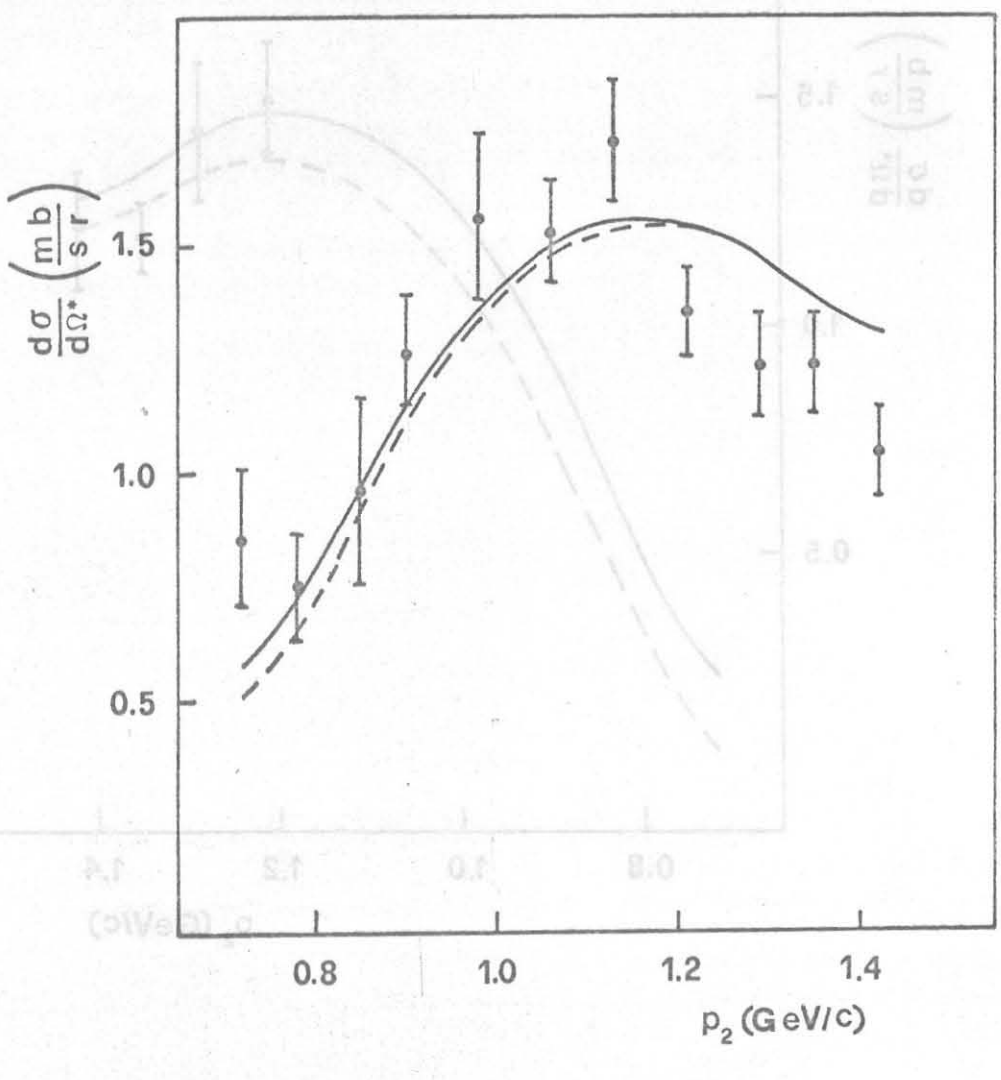
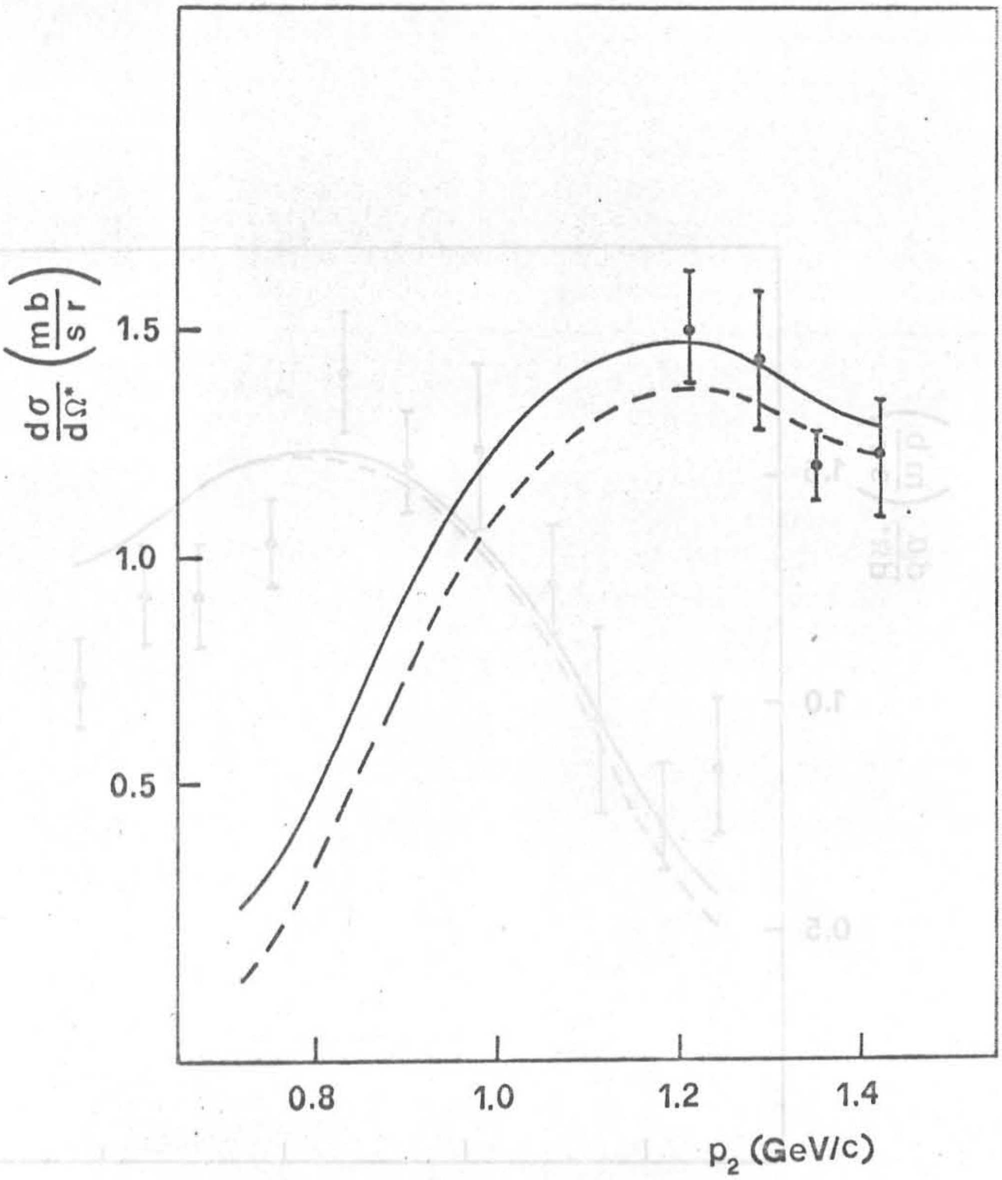

$\frac{d\sigma}{d\Omega^*}\left(\frac{mb}{sr}\right)$
1.5
1.0
0.5
0.8
1.0
1.2
1.4
p_2 (GeV/c)

Fig. 3



$\frac{d\sigma}{d\Omega^*}\left(\frac{mb}{sr}\right)$
1.5
1.0
0.5
0.8
1.0
1.2
1.4
p_2 (GeV/c)

Fig. 4

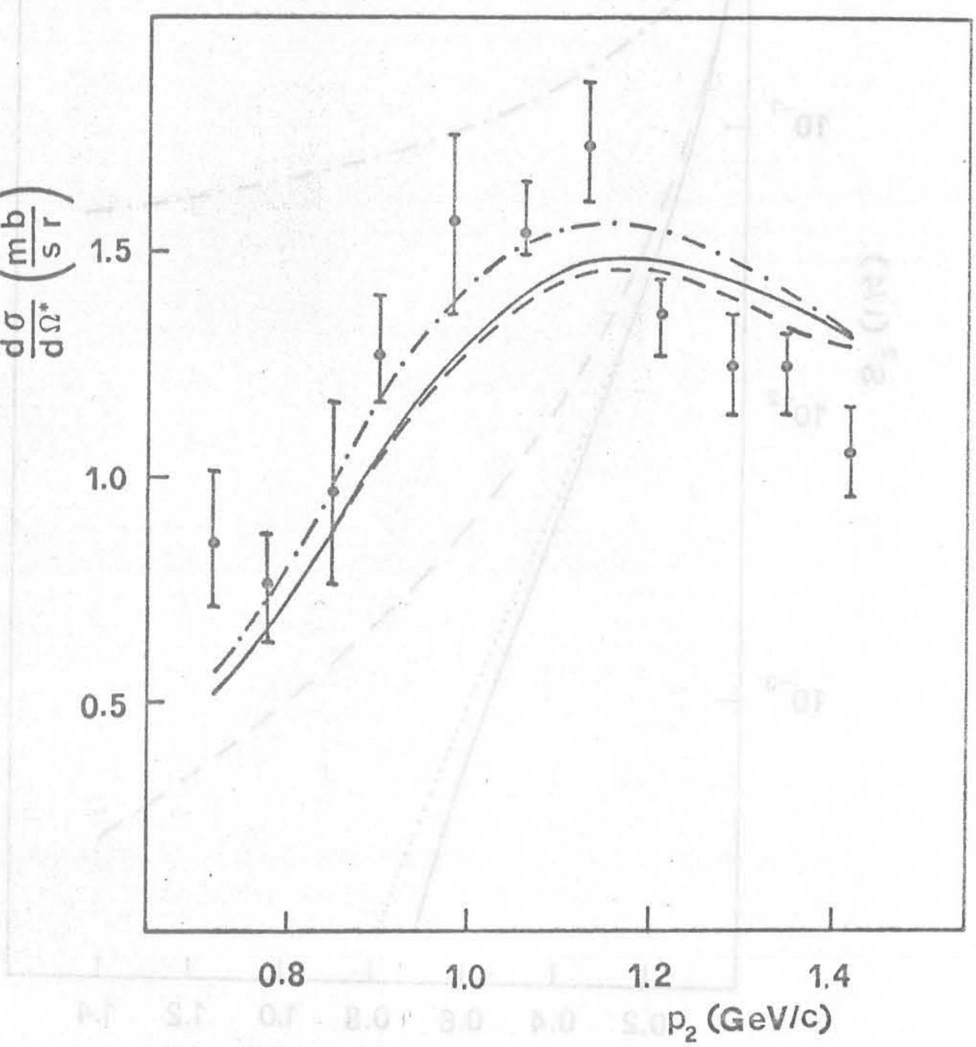


Fig. 5

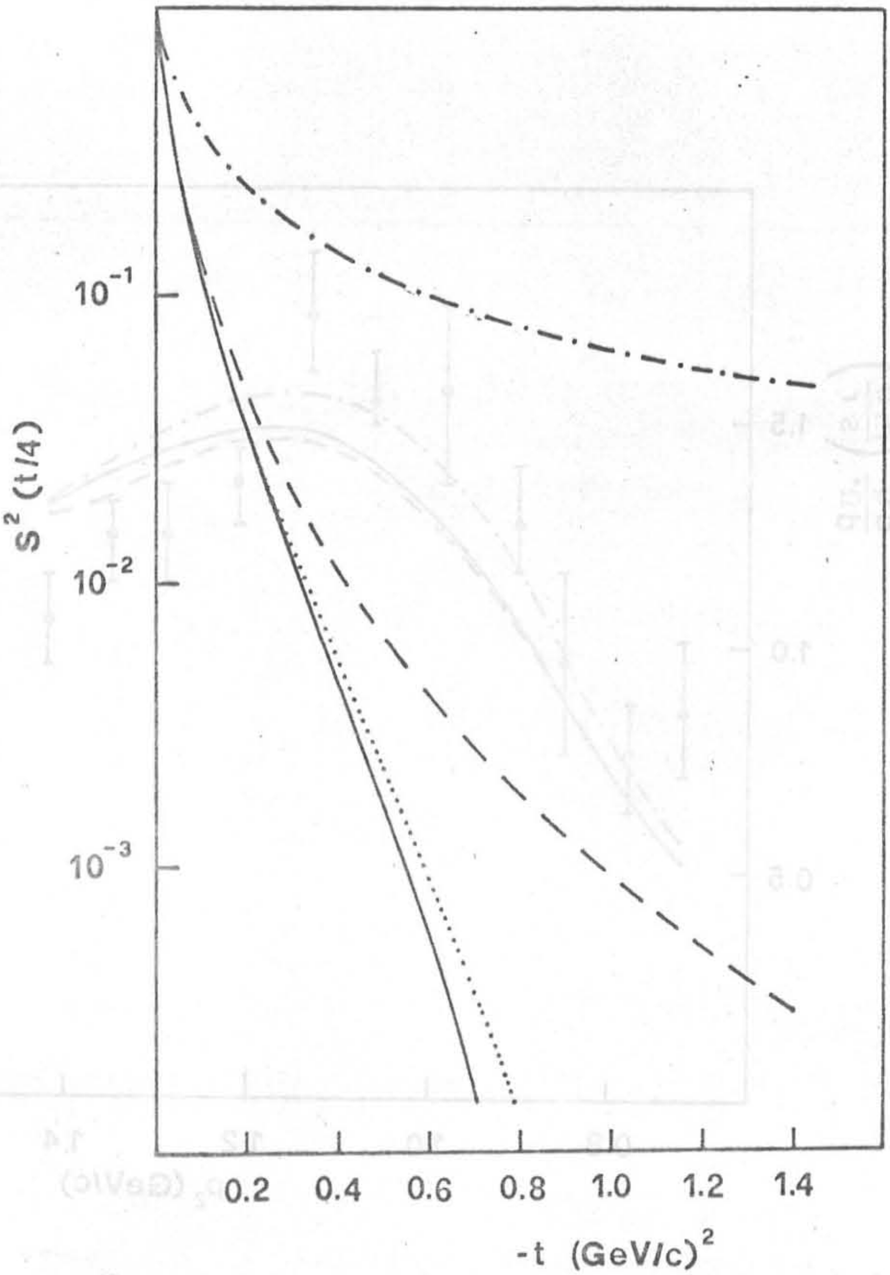


Fig. 6

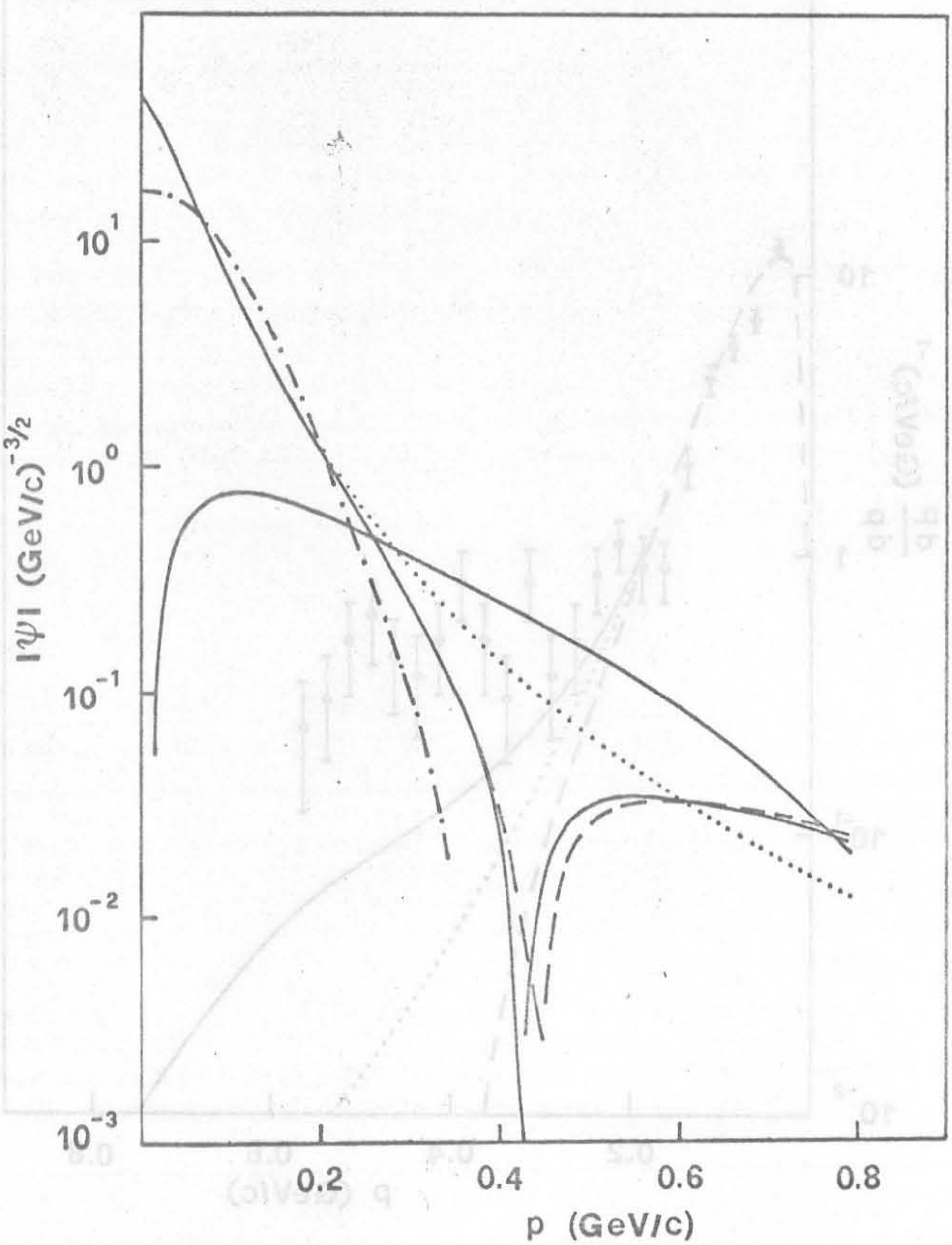


Fig. 7

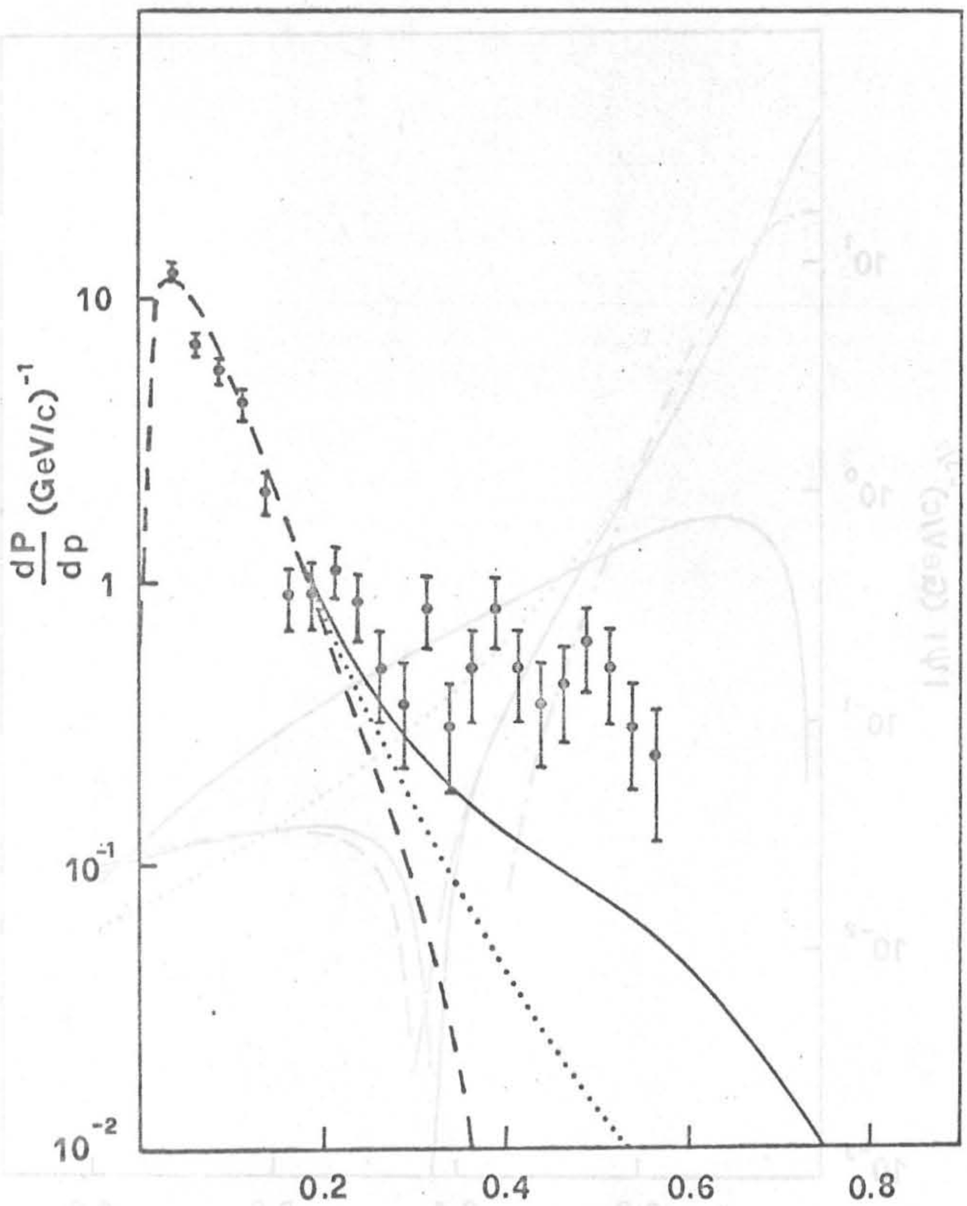


Fig. 8