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THEORETICAL APPROACHES TO $K\ell_3$ DECAY *

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SUMMARY. A discussion on Kl_3 decay is given, in the framework of $SU(3) \times SU(3)$ equal-time and light-cone commutation relations.

Kl_3 decay is undoubtedly one of the most interesting and popular topics in the field of weak interactions. The reason, essentially, is that not only the experimental information on Kl_3 decay allows precise checks of some basic assumptions of the theory of weak interactions, but, also, it represents a sensible test of the theoretical approaches which are available. My talk will be devoted to a short survey on Kl_3 decay, looking at the problem from the point of view of current algebra, light-cone algebra and related techniques.

Let me start by briefly introducing the Kl_3 process. A very gratifying point is that the experimental data strongly recommend the simple Cabibbo description of semileptonic decays, based on V - A hadronic currents with octet behaviour under SU(3). Accordingly, the object of interest for the decay $K \to \pi \ell \bar{\nu}_\ell$ is the matrix element

$$\langle \pi^0(q) | V_\mu^{(K^+)} | K^-(p) \rangle = \frac{1}{\sqrt{2}} \left\{ f_+(k^2)(p+q)_\mu + \right.$$
$$\left. + f_-(k^2)(p-q)_\mu \right\} \quad , \quad k = p - q \tag{1}$$

where $V_\mu^{(K^\pm)} = V_\mu^{(4 \pm i5)}$ is the vector part of the $\Delta S = \Delta Q$

hadron current, and $m_\ell^2 \leq k^2 \leq (m_K - m_\pi)^2$ in the physical decay region.

A third form factor is usually introduced :

$$f(k^2) = f_+(k^2) + \frac{k^2}{m_K^2 - m_\pi^2} f_-(k^2) \tag{2}$$

which is induced by the SU(3) breaking :

$$i\langle\pi|\partial_\mu V_\mu|K\rangle = (m_K^2 - m_\pi^2) f(k^2) \tag{3}$$

Rather than in terms of f_+, f_-, the process is better described by the form factors f_+, f, with which most of the theoretical predictions are concerned. It seems reasonable to assume a linear variation for all Kl_3 form factors :

$$\begin{aligned} f_\pm(k^2) &= f_\pm(0)\left(1 + \frac{\lambda_\pm}{m_\pi^2} k^2\right), \\ f(k^2) &= f(0)\left(1 + \frac{\lambda_0}{m_\pi^2} k^2\right), \\ \lambda_0 &= \lambda_+ + \xi(0)\frac{m_\pi^2}{m_K^2 - m_\pi^2}, \quad \xi(0) \equiv \frac{f_-(0)}{f_+(0)} \end{aligned} \tag{4}$$

Actually, the assumption of linear dependence for both f_+, f amounts to implicitly neglect λ_-, so that, within our parametrization (4), only $\xi(0)$ and λ_+ (or λ_0 and λ_+) are significant.

How looks our experimental knowledge of $\xi(0)$ and λ_+ ? The least we can say, to be optimistic, is that it is controversial. The best established case is $K^+_{\mu 3}$, where an overall fit gives [1]

$$\xi(0) = -0.85 \pm 0.20 \; ; \; \lambda_+ = 0.045 \pm 0.012 \tag{5}$$

Ke_3 data, on the other hand, seem to suggest smaller values of λ_+ (typically, 0.029 $\pm$ 0.006) so that, by combining $K\mu_3$ and Ke_3 experiments we find [2]

$$\xi(0) = -0.65 \pm 0.20 \; ; \; \lambda_+ = 0.034 \pm 0.006 \tag{6}$$

Finally, the result $\xi(0) = 0.00 \pm 0.18$ can be found in the literature, which has been derived from the branching ratio $\Gamma(K^0_{\mu 3})/\Gamma(K^0_{e3})$ assuming $\lambda_+ \approx 0.03$.[1] Anyway, most of the data point towards a negative and sizeable value of $\xi(0)$; since $\xi \equiv 0$ in the SU(3) limit, this result does not seem, at first sight, easy to fit in the current picture of SU(3) as an approximate symmetry of the hadrons.

Now, the thoretical aim is to evaluate the form factors, or, at least, to get predictions on them.

A first possibility is represented by simple theoretical models, like e.g. K^* dominance of f_+, which gives $\lambda_+ = \frac{m_\pi^2}{m_{K^*}^2} \approx 0.024$, in rough agreement with most of the Ke_3 data.

Alternatively, one can exploit general statements on the weak currents, like current algebra (and light-cone algebra), concerning their transformation and partial conservation properties.

SU(3), of course, is the first candidate, and the information which comes from the algebra of SU(3) charges alone is the well-known relation [3]

$$f(0) \equiv f_+(0) = 1 + O(\varepsilon_8^2) \tag{7}$$

which states that the departure of $f_+(0)$ from its SU(3) symmetric value is of second order in the breaking. This result, although of extreme importance, has not proved enough, so far, to produce a quantitative prediction on Kl_3 decay, since a reliable evaluation of the corrections $O(\varepsilon_8^2)$ is not at hand [4]; the only immediate indication which we can derive, assuming octet dominance, is the limitation $f_+(0) < 1$. Moreover, a relation of the kind in Eq.(7) is not easy to test, since only the product $f_+(0) \times \sin\theta_c$ is determined by experiment, θ_c being the Cabibbo angle.

We are lead therefore to extend our considerations from SU(3) to the SU(3) x SU(3) algebra generated by the vector and axial vector currents, supplemented by the PCAC hypothesis, namely, to soft-pion theory. Soft-pion theorems are, among the predictions of current algebra, the easiest to test experimentally and, as well known, they work in many cases so well that we are inclined to consider the smallness of the pion mass, which is the essence of the PCAC assumption, not as a dynamical accident but, more seriously, as the manifestation of an approximate chiral SU(2) x SU(2) symmetry of the hadron world, exactly realized in the limit $m_\pi = 0$. In our case, the soft-pion result is represented by the Callan-Treiman relation, which states that, in the limit of vanishing mass and momentum of the emitted pion :

$$f_+(m_K^2) + f_-(m_K^2) = \frac{f_K}{f_\pi} \tag{8}$$

or, in terms of the form factor f :

$$f(m_K^2) = \frac{f_K}{f_\pi} \tag{9}$$

Now, it happens that if we assume, according to PCAC, the Callan-Treiman result to hold without sensible variations on the pion mass-shell too, its simple linear extrapolation to the physical region turns out to be inconsistent with the experimental indication, since we find, from Eq.(4), inserting the experimental values of λ_+ :

$$\xi(0) \simeq -0.32 \quad \text{and} \quad \xi(0) \approx -0.21,$$

in contrast with the figures (5) and (7).

This is not very pleasant, of course, if we believe that the soft-pion picture has some adherence to the physical world. Then, unless we want to invent some extra effects to explain this failure, the corrections to the PCAC result, due to the finite size of the pion mass, must be taken into account before trying a comparison with the experiment. The prescription to extrapolate soft-pion theorems onto the mass shell is not unique and depends, so to say, on the author's taste. I choose to review three among the most recent examples.

A very simple approach is represented by the rest-frame saturation of equal-time commutation relations of SU(3) x SU(3) charges and divergences, involving the non strange axial charges $\overline{Q}^{(\pi)}$ to which, essentially, the pion is associated [5], and such that, in the spirit of PCAC, $\dot{\overline{Q}}^{(\pi)}=\int d\vec{x}\,\partial_\mu A_\mu^{(\pi)}=O(m_\pi^2)$. The result is [6]

$$\begin{aligned}\left(1+\frac{m_\pi}{m_K}\right)f[(m_K-m_\pi)^2] &= \frac{f_K}{f_\pi}+\\ &+\frac{2}{m_K m_\pi f_\pi}\langle 0|[\dot{\overline{Q}}^{(\pi^0)},Q^{(K^+)}]|K^-(\vec{p}=0)\rangle-\\ &-\frac{\sqrt{2}\,m_\pi}{m_K}\frac{1}{\pi}\int\frac{dq_0\,\rho(q_0,\vec{q}=0,\vec{p}=0)}{q_0(q_0-m_\pi)(q_0-m_K)},\end{aligned}\tag{10}$$

where

$$\begin{aligned}\rho(q,p)&=\sum_n\delta(q-p_n)\langle 0|\chi^{(\pi^0)}|n\rangle\langle n|\partial_\mu V_\mu^{(K^+)}|K^-(p)\rangle-\text{c.t.}=\\ &=\int d^4x\,e^{iqx}\langle 0|[\chi^{(\pi^0)}(x),\partial_\mu V_\mu^{(K^+)}(0)]|K^-\rangle\equiv\rho(q^2,k^2),\\ k&=p-q\;;\quad(\Box+m_\pi^2)\,\partial_\mu A_\mu^{(\pi)}(x)=m_\pi^2 f_\pi\,\chi^{(\pi)}(x)\end{aligned}\tag{11}$$

In shorthand notation

$$\left(1+\frac{m_\pi}{m_K}\right)f[(m_K-m_\pi)^2]=\frac{f_K}{f_\pi}+\delta(m_\pi)\tag{12}$$

It goes without saying that, for $m_\pi \doteq 0$, the Callan-Treiman relation is reproduced. It is easily realized that the extrapolation is performed along a parabola in the (q^2,k^2) plane, represented in Fig. 1, connecting the soft-pion point $q^2=0,\ k^2=m_K^2$ to the nearest point of the physical region, $q^2=m_\pi^2,\ k^2=(m_K-m_\pi)^2$: indeed, Eq. (10) is a dispersion relation where both the "mass" q^2 and the momentum transfer k^2 are varying along this line, whose intersections with the "lines of singularities" represent the various contributions to the dispersive sum.

Let us inspect very quickly the corrections to the Callan-Treiman relation, which appear in Eq. (12). The higher commutator $[\dot{Q},Q]$ is essential in order to get rid of the pion crossed mass singularity at $q^2 = m_\pi^2$, $k^2 = (m_K + m_\pi)^2$, proportional to $f[(m_K + m_\pi)^2]$, whose appearance would prevent us the possibility to extrapolate the Callan-Treiman relation to a single physical point. Such a commutator is, of course, outside the current algebra frame; so, this is the ideal case where to test models of chiral SU(3) x SU(3) symmetry breaking. We have, moreover, kinematical corrections (which are seen to work in the right direction but to be not enough) and a dispersive continuum which might be, as a matter of fact, an $O(m_\pi \ln m_\pi)$, since the integration threshold is $O(m_\pi)$. A quantitative discussion of the corrections is not easy: opinions are diverging, and so far, a definite conclusion has not been reached. For example, on the basis of simple, model dependent estimates, Banerjee(7) has claimed that the Callan-Treiman relation (namely, approximate chiral SU(2) x SU(2) symmetry) and the experimental data can be compatible: a dip of $f(k^2)$ somewhere around $k^2 = (m_K - m_\pi)^2$ is required. This conclusion, which has received support from several other authors, is tackled by the opponent party, trying to interpret data rather in terms of near SU(3). Anyway, a good experimental information on $f(k^2)$ near the edge of the physical region would be of much help in clarifying things. The situation concerning $f(k^2)$ is sketched, qualitatively, in Fig. 2.

The second method which I would like to mention starts from the SU(3) x SU(3) light-cone algebra, and has been developed by G. Furlan and myself(8). Light-cone physics has represented one of the most exciting topics of these recent times and shows nice features of simplicity. Being not willing to enter details, I limit to remind that it seems reasonable to abstract the light-cone current commutation relations from the free quark model (9). One of the arguments in favour of this assumption is that this model has been already successful in suggesting the SU(3) x SU(3) equal-time algebra of charges (and, perhaps, of divergences).

As a result, introducing the operators $\left(x^{\pm}=\frac{1}{\sqrt{2}}(x^0 \pm x^3),\ \vec{x}_{\perp}\equiv(x^1,x^2)\right)$:

$$\begin{aligned} Q_a(q,x^+) &= \int dx^- d\vec{x}_{\perp}\, e^{i(q^+x^- - \vec{q}_{\perp}\vec{x}_{\perp})} V_a^+(x) \\ \overline{Q}_a(q,x^+) &= \int dx^- d\vec{x}_{\perp}\, e^{i(q^+x^- - \vec{q}_{\perp}\vec{x}_{\perp})} A_a^+(x) \end{aligned} \tag{13}$$

we have at disposal the SU(3) x SU(3) light-cone algebra

$$\begin{aligned} \left[Q_a(q,x^+), Q_b(k,x^+)\right] &= i f_{abc} Q_c(q+k,x^+) \\ \left[Q_a(q,x^+), \overline{Q}_b(k,x^+)\right] &= i f_{abc} \overline{Q}_c(q+k,x^+) \\ \left[\overline{Q}_a(q,x^+), \overline{Q}_b(k,x^+)\right] &= i f_{abc} Q_c(q+k,x^+) \end{aligned} \tag{14}$$

The importance of defining the operators $Q_a, \overline{Q}_a$ in Eq. (13) stems from the fact that at $q^+ = \vec{q}_{\perp} = 0$ they coincide, for conserved currents, with the familiar SU(3) x SU(3) charges $Q = \int d\vec{x}\, J^0(x)$ and therefore, while being appropriate to light-cone kinematics, they are of use in dealing with partial conservation properties when the symmetry is broken.

The procedure, now, very similar to the previous one, consists in saturating the simple algebraic structure (14). The result which is derived for Kl_3 decay is the modified version of Eq. (12):

$$\begin{aligned} \frac{1}{2}\left(1+\frac{m_\pi}{m_K}\right) f\left[(m_K - m_\pi)^2\right] + \tilde{\delta}(m_\pi) &= \\ = \frac{f_K}{f_\pi} - \frac{f_K}{2 f_\pi(0)} \end{aligned} \tag{15}$$

where $f_\pi(0) = f_\pi(m_\pi = 0)$ is the value of f_π given by the Goldberger-Treiman relation, exactly valid for massless pions, and $\tilde{\delta}(m_\pi)$, whose explicit expression this time I omit, represents, again, a dispersive continuum of order m_π. Having started from a light-cône commutator, the dispersive line is, in this case, the straight line, represented in Fig. 1, of equation

$$q^2 = \frac{m_\pi}{m_\pi - m_K} k^2 + m_\pi m_K \tag{16}$$

Actually, the straight line can be obtained from the parabola, characteristic of the equal-time method, as a degenerate case, by taking an appropriate limit which requires, for consistency, the convergence of form factors at infinity. The straight line, moreover, does not touch the soft-pion point $q^2 = 0,\ k^2 = m_K^2$, which means that in the present method we get the separation of the result into a finite part plus $\tilde{\delta}(m_\pi)$ corrections only after that the soft-pion limit has been imposed, in a sense, like a constraint; this is why $f_\pi(0)$ appears in Eq. (15). Apart from this difficulty in handling the soft-pion limit $m_\pi = 0$, the fact of working along a straight line is really an advantage, since the lines of singularities are met only once (and not twice), and this results in a remarkable simplification in the set of graphs contributing to $\tilde{\delta}(m_\pi)$. Looking at Fig. 1 we see that crossed mass singularities are excluded; as a consequence, the pion crossed contribution, which I mentioned before, is disposed of from the beginning, and we directly get a prediction at a single physical point without introducing higher, model dependent commutators. Furthermore, the q^2-channel (lines $q^2 = M_n^2$) should be strongly depressed, since form factors are evaluated at large (timelike) momentum transfer, of order $1/m_\pi$, while simplifications are possible in the k^2-channel (lines $k^2 = M_m^2$), where momentum transfers are $O(m_\pi)$. Now, to be quantitative, we could try a comparison of Eq. (17) with experimental data, neglecting, as a first approximation, the $\tilde{\delta}(m_\pi)$ effects and exploiting only kinematical coefficients, corrections to the Goldberger-Treiman relation included. The result is that the figures (5) and (6) are pretty well reproduced since we find, according to the alternative values of λ_+ :

$$\xi(0) \approx -0.75 \qquad \lambda_+ = 0.045$$
$$\xi(0) \approx -0.65 \qquad \lambda_+ = 0.034 \qquad (17)$$

This is not very conclusive, of course, and the indication coming from a simple-minded estimate is that the $\tilde{\delta}(m_\pi)$ effects might be sizeable, although not dramatic. So, let us consider the figures (17), at least, as an encouraging indication.

An alternative and interesting light-cone approach to Kl_3 decay has been proposed by Brandt and Preparata [10], who find, among other things

$$f_+(0) \approx 0.94 \; ; \; \xi(0) \approx -0.73 \qquad (18)$$

In their method light-cone commutators are used in connection with appropriate finite-energy sum rules in the "mass" variable q^2 and play, essentially, a role analogous to that of Regge poles in conventional dispersion relations. The dispersive lines are, in this case, referring to Fig. 1, the vertical lines $k^2 = m_\pi^2$ and $k^2 = m_K^2$. It is very difficult to establish connections (if any) with our approach. We think that our method is more economical and involves, in practice, very little of the light-cone approach, since the simple "charge-charge" algebra (14) is taken as the starting point. Brandt and Preparata, on the other hand, use higher, model dependent commutators, which they abstract from the simple model of Gell-Mann, Oakes and Renner [11]; in their treatment, therefore, the bilocal operators appear to play the fundamental role, and more results are obtained, as a test of more assumptions.

Acknowledgements

I would like to thank Prof. G. Furlan for helpful discussions.

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Figure captions

Fig. 1 — · — · — equal-time parabola

———— light-cone path Eq. (16)

- - - - lines of singularities $q^2 = M_n^2$, $k^2 = M_m^2$.

—··—·· — lines of Ref. (10)

Fig. 2 The divergence form factor $f(k^2)$.

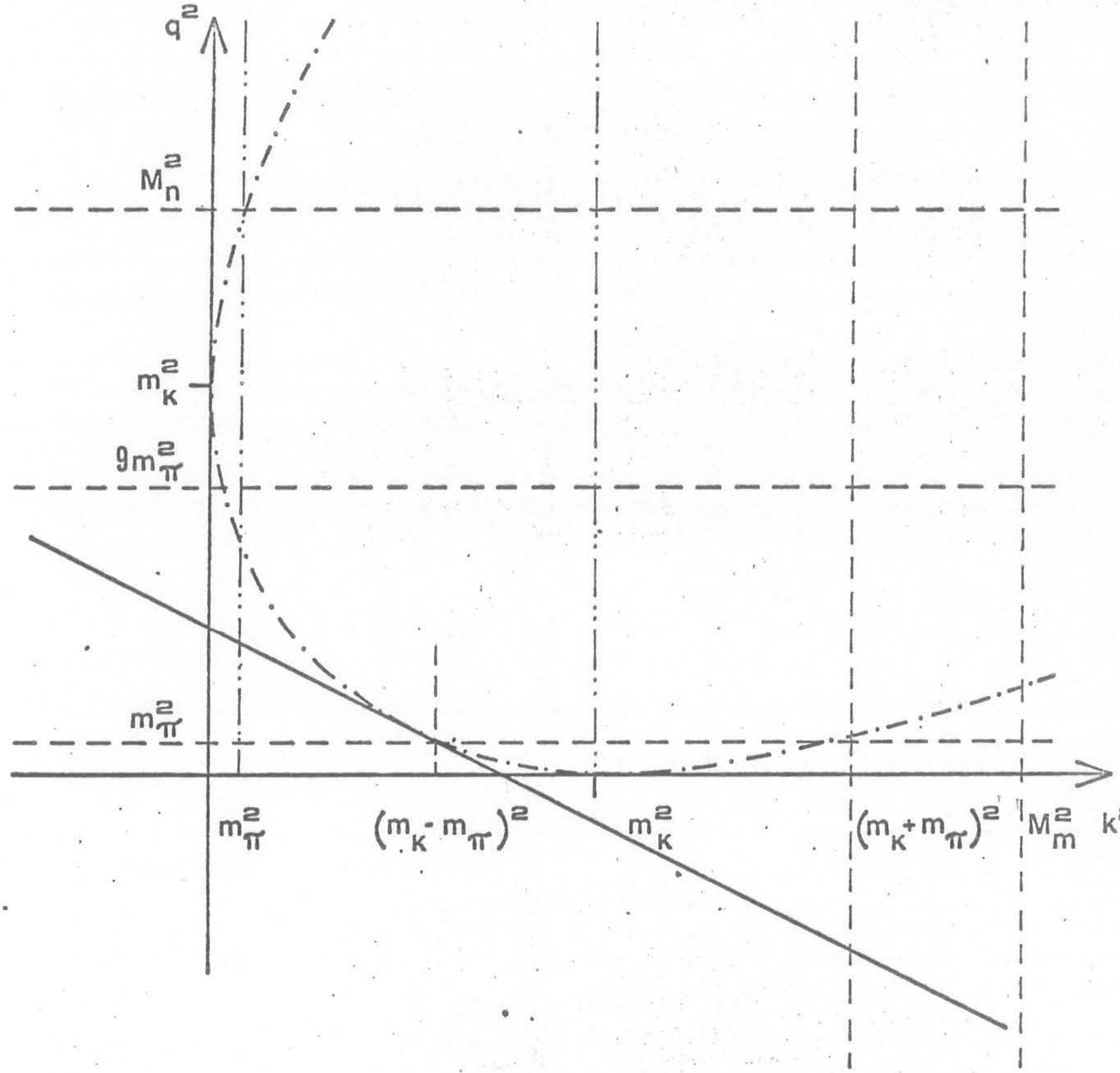

Fig. 1

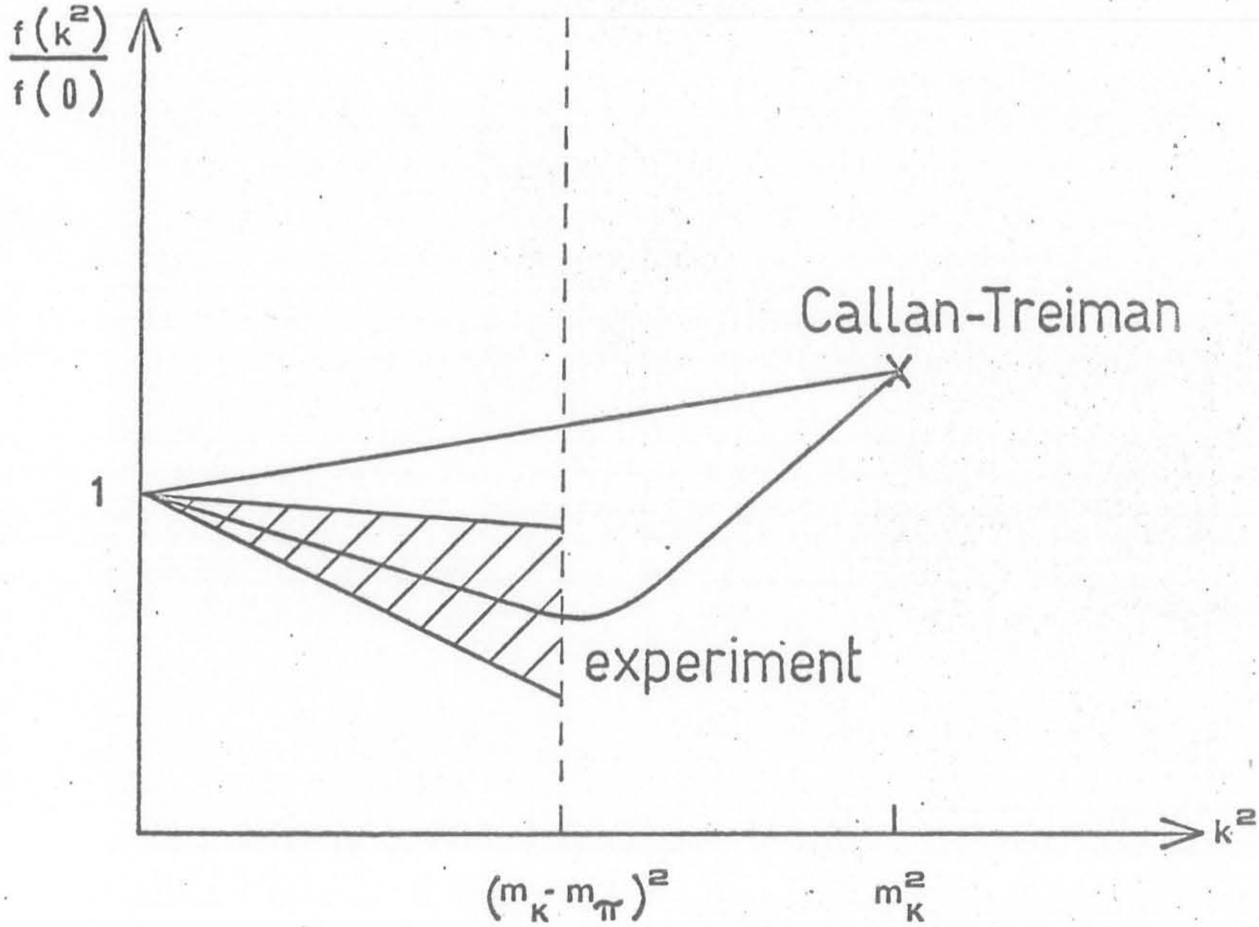

Fig. 2