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ABSTRACT -

The scaling properties of the inclusive process $p + p \rightarrow \pi$ + anything are studied and shown to account satisfactorily for the observed forward peak and breaks in the cross section if the Pomeron dominates for small values of the momentum transfer $Q^2 < 0.3\ GeV^2$ while ordinary Regge exchanges dominate for $Q^2 > 3\ GeV^2$ giving rise to scaling in the sense of Bjorken.

In the past few years inclusive hadronic processes such as

$$(1) \qquad p + p \longrightarrow \pi + \text{anything}$$

have been extensively studied experimentally(1) and are now receiving increasing attention from theorists(2). One reason for theoretical interest is that these processes are expected to show some form of scaling asymptotically(2, 3). Following Feynman(2) the cross section for (1) in the scaling limit is of the form

$$(2) \qquad d^2\sigma = \frac{\pi}{E} f(k_T^2, x = k_L/\sqrt{s})\, d\, k_T^2\, dk_L$$

where $k_{T,L}$ are the transverse and longitudinal momentum in CM of the detected pion with energy E and $f(k_T^2, x)$ is a function of k_T^2 and $x = k_L/\sqrt{s}$ only, such that $f(k_T^2, x = 0) \neq 0$. If the x-dependence of $f(k_T^2, x)$ is weak then from eq (2) the average number of pions is given by

$$n_\pi = C_\pi \ln(s) + \text{const.}$$

(3)

$$C_\pi = \frac{\pi}{\sigma_{inel}(p-p)} \int_0^\infty dk_T^2 f(k_T^2, 0)$$

where $\sigma_{inel}(p - p) \simeq 30.5$ mb is the total inelastic p - p cross section. Recently Bali et al.[4] have used eq. (3) as a test of the Feynman scaling hypothesis by comparing the coefficient of the logarithmic term, calculated for various CM energies $\sqrt{s}$, with data from cosmic rays, using the following form of $f(k_T^2, x)$

(4) $$f(k_T^2, x) = G_o \exp(-2.44 (k_T + k_T^2/M)) \exp(-4a x^2)$$

G_o and a are parameters slowly dependent on the energy $\sqrt{s}$ and M is the proton mass.

This note reports a further study of the scaling properties of the process (1) with the following findings:

(i) scaling cannot be tested using eq. (3); this conclusion is not based on the uncertainties of the cosmic ray data with which theory is compared although it is argued that the said data have over-estimated the coefficient C_π, but on the fact that C_π is actually saturated by contributions from the non-scaling sector of kinematics while the scaling part makes a small contribution to it

(ii) for large values of the momentum transfer $Q^2 > 3$ GeV^2 there is scaling in the Bjorken sense determined via duality by ordinary Regge exchanges and not by the Pomeron. This result is in disagreement with previous indications[5] that the Pomeron plays a dominant role in scaling asymptotics but is partially supported by a recent suggestion of Bloom and Gilman[6] that a substantial part of the observed scaling of inelastic e - p scattering is non-diffractive.

(iii) hitherto unexplained features (kinks and a forward peak)[1] in the experimental cross section of (1) can be understood by assuming that there are two dominant production mechanisms represented by a peripheral and a double Regge exchange graph as shown in figs 1a and 1b respectively. The peripheral graph dominates for large values of

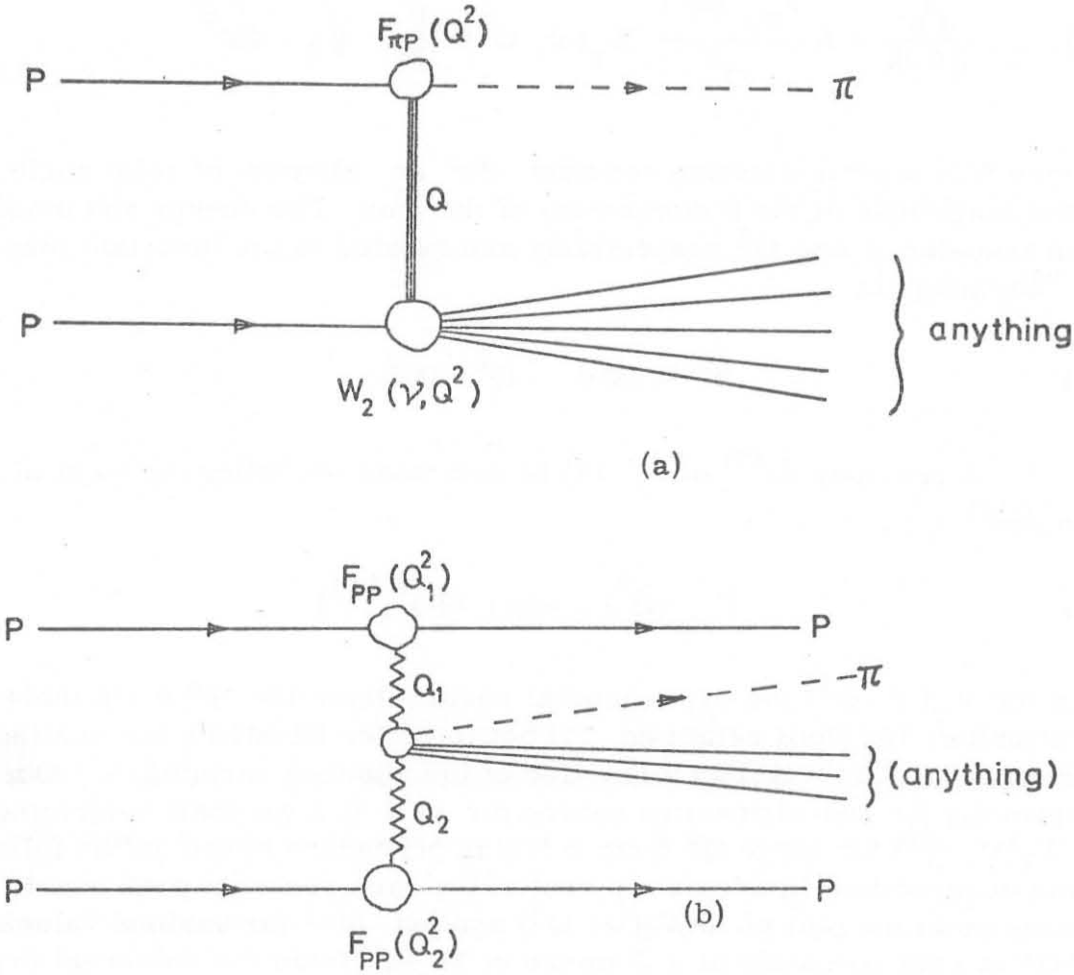

FIG. 1 - a) The dominant peripheral graph in the scaling limit; the internal line does not necessarily correspond to a one particle state.
b) A simple multi-Regge graph, with two echanged Pomerons, which dominates the production process as $\omega \rightarrow 0$.

$Q^2 > 3\ GeV^2$ and determines scaling while the multi-Regge graph with Pomeron exchange dominates for small values of $Q^2 < 0.3\ GeV^2$ giving rise to the forward peak. The intermediate region $0.3 \leq Q^2 \leq 3\ GeV^2$ is not covered by the present model; however the existence of three kinematical regions dominated by different production mechanisms accounts naturally for the observed breaks in the cross section.

Considering the peripheral graph of fig. 1a) and invoking a generalized Wu-Yang hypothesis the cross section for (1) can be written, for large Q^2, in terms of the elastic π-p form factor $F_{\pi p}(Q^2)$ and the inelastic structure function $W_2(\nu, Q^2)$ of deep inelastic e - p scattering. One finds in CM

(5) $$\frac{d^2\sigma}{d\Omega dk} = A \frac{F^2_{\pi p}(Q^2)}{Q^4} W_2(\nu, Q^2) \frac{\sqrt{s}}{M} \sqrt{s - 4M^2}$$

where A is a normalization constant, $d\Omega$ an element of solid angle and k the magnitude of the 3 momentum of the pion. The energy and momentum transfer ν and Q^2 respectively are related to the invariant mass W^2 of "anything" by

(6) $$W^2 = 2M\nu - Q^2 + M^2$$

A previous fit(7) of eq. (5) to data used the following form of $F_{\pi p}(Q^2)$

(7) $$F_{\pi p}(Q^2) = \exp(-6(Q^2)^{1/4})$$

and for $W_2(\nu, Q^2)$ the experimental results from the 10^o e - p inelastic scattering. We shall retain eq. (7) but to better illustrate the scaling forms of $\nu W_2(\nu, Q^2)$ as a function of the Bjorken variable $\omega = Q^2/2M$ especially its non-diffractive nature for $\omega \gtrsim 0.2$ we shall determine $\nu W_2(\nu, Q^2)$ for large Q^2 from a fitting procedure based on the following observation: in deep e - p scattering each resonant peak which shows up in the plot of $\nu W_2(\nu, Q^2)$ against ω^{-1} for various values of Q^2 is approximately at a distance of M^2/Q^2 from the universal branch of the νW_2 curve. Therefore if for each resonant peak one performs the translation

(8) $$\omega^{-1} \longrightarrow \omega^{-1} + M^2/Q^2$$

the new positions of the resonances and their corresponding values of νW_2 approximate in the average the scaling branch of νW_2. In other words the resonant contributions to νW_2 scale in the resonant masses since from eqs. (6) and (8)

(8') $$\omega^{-1} + M^2/Q^2 = 1 + W^2/Q^2$$

Hence from duality we conclude that ordinary Regge exchanges dominate νW_2 in the scaling limit at least that part of it from $\omega = 1$ to its maximum around $\omega = 0.2$. To determine νW_2 from these considerations we set for large Q^2

$$(9) \qquad \nu W_2(\nu, Q^2) = \nu W_2(\omega) = \omega^{\gamma} \sum_{n=0}^{\infty} C_n \omega^n$$

subject to the following conditions:

a) as $\omega \rightarrow 1$ νW_2 should be smaller than some preassigned value typically of the order of 10^{-3}. We choose to normalise νW_2 this way because we are not interested, in the present circumstances, with its threshold behaviour which is related to the asymptotic Q^2 - dependence of elastic e - p form factors as $Q^2 \longrightarrow \infty$. From eq (9) this normalisation is easily satisfied if the C_n are exponential coefficients.

$$(10) \qquad C_n = B \frac{(-C)^n}{n!}$$

with constants C and B; B is fixed up by the value of νW_2 at the maximum.

b) νW_2 should have a maximum for ω around 0.2; from this condition we find a relation between the two constants γ and C

$$(11) \qquad \gamma / C = 0.2$$

There is therefore effectively one free parameter C; for C between 5 and 6 eqs. (9), (10) and (11) give indeed a smooth average to the resonant peaks as shown in fig. 2 for C = 5.7 where νW_2 is plotted against ω^{-1}. The experimental points are values of νW_2 at the resonant peaks only, in the 10.0, 13.5 and 16.0 GeV 6^o e - p scattering for $Q^2 = 1.0$, 1.7 and 2.4 GeV^2 respectively[6, 8]. The non resonant experimental points which cluster around the smooth curve have been omitted for clarity. Fig. 2 clearly confirms that for large Q^2 ordinary Regge exchanges determine the behaviour of νW_2 in agreement with our previous conclusion

The double differential cross section in eq. (5) for π^- production is plotted in fig. 3 (dashed curve) against $(Q^2)^{1/4}$ making use of eqs. (7), (9), (10) and (11) for incident proton LAB energy of 30 GeV[1]. The full curve is obtained using the 10^o e - p data for νW_2[8]. For comparison the cross section from eqs. (2) and (4) (dashed-dotted curve) is also shown. It follows from this figure that in the whole range of Q^2 shown eq. (4) is nowhere near being even an indicative behaviour of the data. On the other hand for $Q^2 > 3$ GeV^2 corresponding to $k_T > 1$ GeV the data are in agreement with scaling the Bjorken way this scaling being accounted for entirely in terms of dominance of ordinary Regge exchanges.

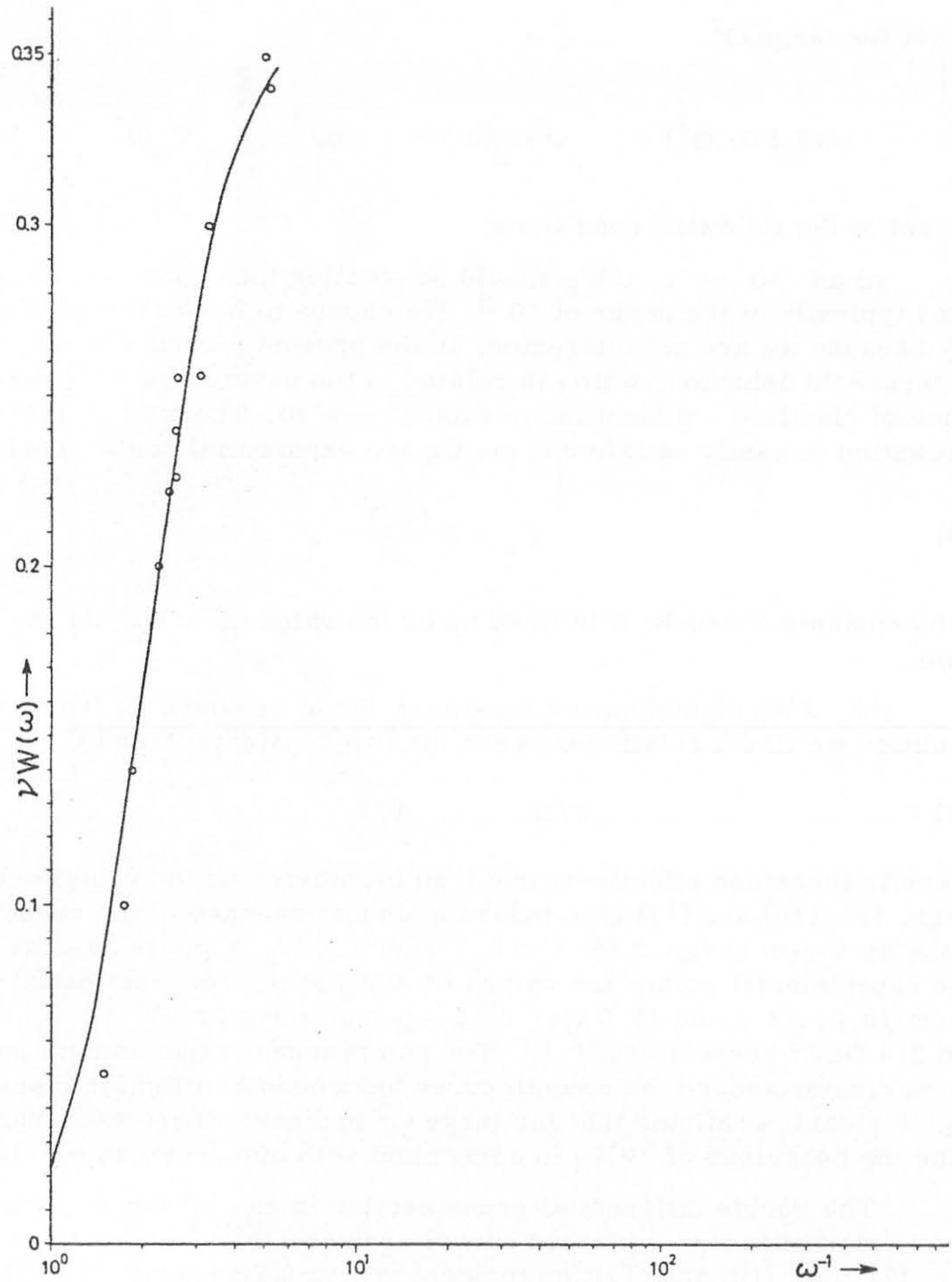

FIG. 2 - Plot of νW_2 given by eqs. (9), (10) and (11) for $C = 5.7$ against ω^{-1}. The experimental points are the resonant contributions to νW_2 in the 10.0, 13.5, 16.0 GeV 6^o e - p data for $Q^2 = 1.0$, 1.7 and 2 - 4 GeV^2 respectively when the corresponding values of ω^{-1} undergo the translation $\omega^{-1} \rightarrow \omega^{-1} + M^2/Q^2$.

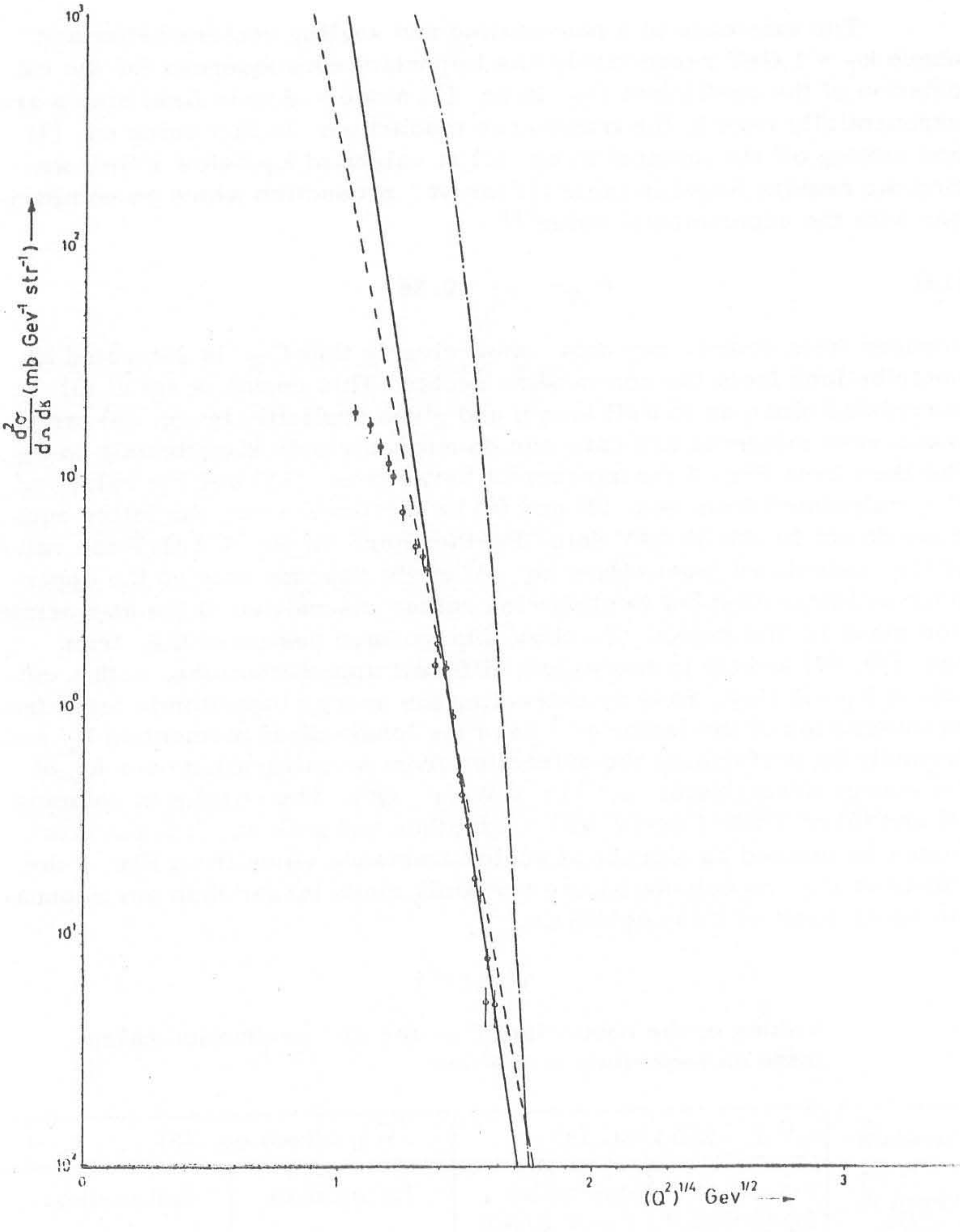

FIG. 3 - Comparison of the peripheral process cross section for π^- production with data of 30 GeV p - p inelastic scattering. The cross section is plotted against $(Q^2)^{1/4}$; the full curve corresponds to νW_2 given by the experimental data of the 10^o e - p inelastic scattering, the dashed curve corresponds to νW_2 given by eqs. (9), (10) and (11) and the dash-dotted curve to the cross section given by the parametrisation of Bali et al.[(4)].

The existence of a non-scaling and scaling sectors below and above k_T = 1 GeV respectively has important consequences for the calculation of the coefficient C_π in eq. (3) since hadronic final states are exponentially rare in the transverse momentum In fact using eq. (4) and cutting off the integral in eq (3) at values of k_T below 1 GeV we find the results listed in table (1) for π^- production which on comparison with the experimental value[4]

(12) $$C_{\pi^-} = 0.36$$

deduced from cosmic ray data, show clearly that C_π is saturated by contributions from the non-scaling sector. This result is not at all surprising since as is well known and given explicitly by eq (4) large transverse momenta are rare and do not contribute significantly to C_π. But then from Fig. 3 the agreement between eq (12) and the values of C_π calculated from eqs. (3) and (4) is fortitous since the latter equations do not fit the 30 GeV data. Furthermore for $k_T < 1$ GeV the value of C_π calculated from either eq. (4) or (5) depends only on the upper limit of integration but is otherwise rather insensitive to the approximation made in this region To show this we have computed C_π from eqs. (5), (7) and (9) in two widely different approximations, with a cut--off at k_T = 1 GeV, first by extracting the energy logarithmic term from an integration of the factor E^{-1} over the longitudinal momentum k_L and secondly by performing the extraction from an integration over k_L of the energy denominator ν^{-2} in $\nu W_2(\nu, Q^2)$. The results in columns (4) and (5) of Table I agree with each other and with eq. (12) but this cannot be clained as a proof of scale invariance since from Fig 3 the values of C_π so calculated are certainly much larger than any resonable upper limit of this coefficient.

TABLE I

Values of the coefficient C_π for π^- production calculated as described in the text.

Incident LAB Energy in MeV	C_π from eq. (4)		C_π from eq. (5)	
	cut-off at k_T=0.5 GeV	cut-off at k_T=0.8 GeV	1st method	2nd method
12.5	0.18	0.23	0.22	0.24
19.2	0.23	0.30	0.30	0.30
30.0	0.27	0.35	0.34	0.34

We thus conclude that not only does eq. (3) not constitute a quantitative test of the scaling hypothesis but that the experimental value of the coefficient C_{π} deduced from cosmic ray data assuming a kaon to pion ratio $K/\pi = 0.2$ is at best a very liberal upper limit.

In both the purely hadronic process (1) and in e - p scattering[8] we find that experimentally scaling sets in when the transverse momentum of the pion and electron respectively is greater than about 1 GeV. The experimental situation is shown in Table II. This fact does not contradict the small transverse momentum hypothesis needed to obtain scaling in the field theoretical model of Drell, Levy and Yan[9] because the transverse momentum referred to there is the transverse momentum of the component particles (partons) in "anything". The transverse momenta of the individual partons can be small while their sum is large.

TABLE II

Values of the transverse momentum of detected electron and pion in the processes e + p → e + anything and p+p → → π + anything respectively at the point where scaling sets in.

Process	Incident LAB Energy in GeV	LAB Scattering Angle (or Other Specification)	Transverse Momentum of Detected Particle, at Scaling "Threshold", in GeV
e+p → e+anything	13.5	6°	0.89
	16.0	6°	1.11
	10.99	10°	0 975
	13.5	10°	1.06
	15.2	10°	0.985
p+p → π+anything	12.5	$(k_L)=0.6$GeV	1.18
	30.0	9.2°	0.95

The strong correlation between the transverse momentum and scaling manifestations can also be seen in fig. 4 where the cross section in eq. (5) for π^- production is plotted against k_T^2 for incident proton LAB energy of 12.5 GeV[1] The new fact which emerges from this plot is that the hitherto unexplained break at $k_T^2 = 1.39$ GeV^2 is associated with the passage from the non-scaling to the scaling sector. Fig 4 and Table II together suggest that it is the proton mass and not the average transverse momentum of secondaries[4] which sets the energy scale in scaling asymptotics. This fact was already used in eq. (8).

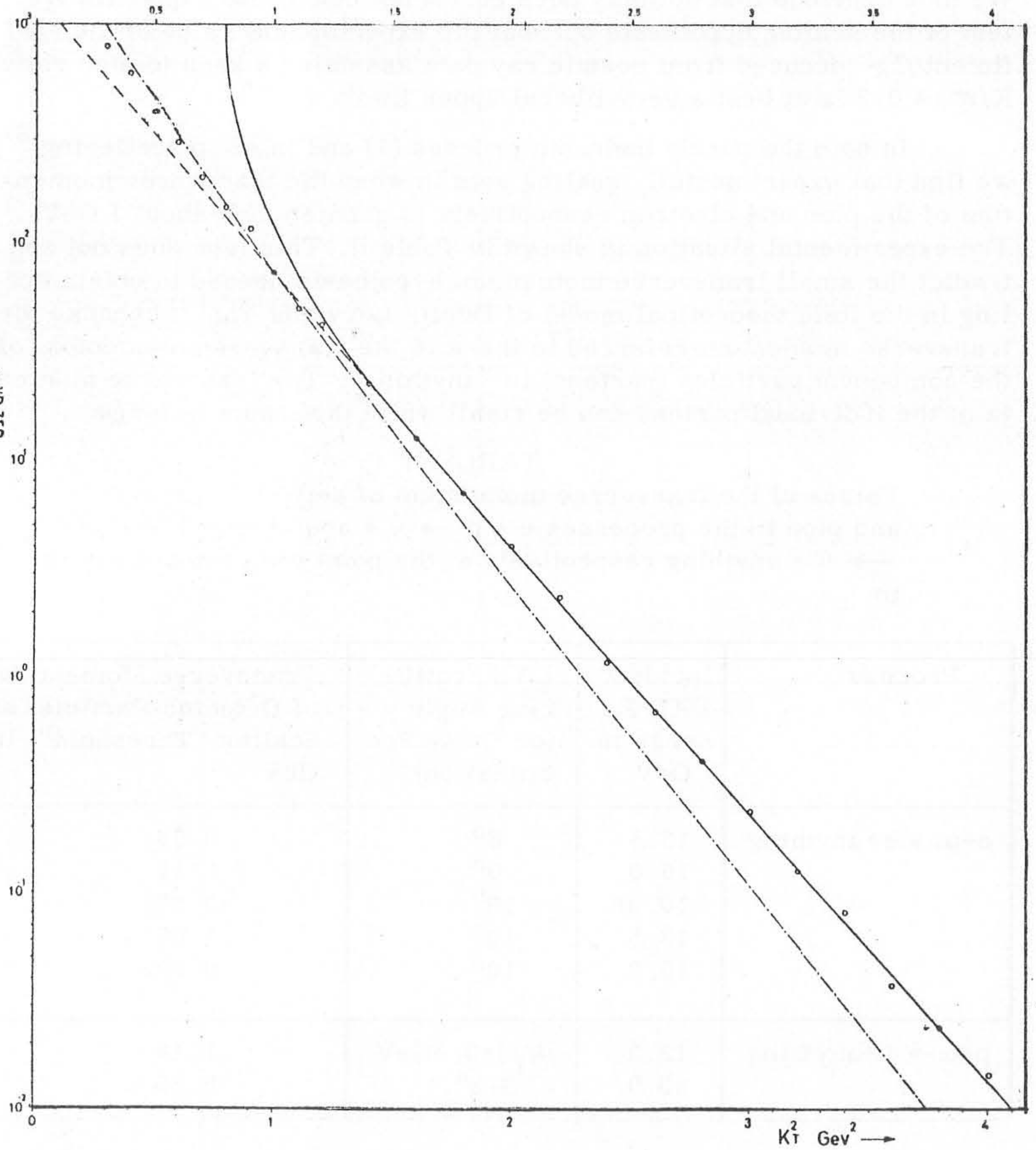

FIG. 4 - Comparison of the peripheral process cross section for π^- production with data of 12. 5 GeV p - p inelastic scattering. The cross section is plotted against k_T^2 ; the full curve corresponds to νW_2 given by eqs. (9), (10) and (11) and the dash-dotted curve to the parametrization of Bali et al.[4] the dashed curve is a continuation of the full curve after the break at k_T^2 = 1. 39 into the region where the peripheral process no longer dominates.

Lastly we point out that it is not possible to fit all the data of Figs. 3 and 4 for $0 \leq k_T \leq 1$ GeV if for small Q^2, $\nu W_2(\nu, Q^2)$ is proportional to Q^2

$$\nu W_2(\nu, Q^2) \propto Q^2 \tag{13}$$

as suggested by models with Pomeron dominance[10]. This is also true even when eq. (13) is combined with the replacement

$$\exp(6-(Q^2)^{1/4})/Q^4 \longrightarrow \exp(-b\,Q^2)/Q^2 \tag{14}$$

where b is a constant. However for $0 \leq k_T^2 < 0.3$ GeV^2 and b between 9 and 10 GeV^{-2} eqs (13) and (14) yield results not incompatible with the observed forward peak of the 12.5 GeV data[1]. Unfortunately the experimental points determining this peak are so closely strung one after the other that an unambiguous graphical comparison is difficult Instead we compare in Table III the values of the exponential $\exp(-bQ^2)$ for b = 10 GeV^{-2} with those of the experimental forward peak proportional to $\exp(-15\,k_T^2)$ for three values of Q^2 corresponding to k_T^2 in the range $0 \leq k_T^2 \leq 0$ 3 GeV^2; as can be seen there is reasonable agreement.

TABLE III

Comparison of the theoretical diffraction formula (eq. (14) in the text) proportional to $\exp(-b\,Q^2)$ for b = 10 GeV^{-2} with the experimental forward peak $\exp(-15\,k_T^2)$.

k_T^2	Q^2	$\mathrm{Exp}(-15k_T^2)$	$\mathrm{Exp}(-10\,Q^2)$
0.114	0.1	1.71	1.0
0.146	0.2	2.19	2 0
0 180	0.3	2.7	3.0

The cross section given by eqs (13) and (14), in particular the value $b \simeq 10$ GeV^{-2}, can be understood on the basis of Fig. 1b in which a Pomeron is exchanged at each of the two proton vertices[11]. According to eqs. (13) and (14) the cross section for this graph has the form

$$\frac{d^2\sigma}{dQ^2\,dW} \propto \exp(-\chi(W^2)\,Q^2) \tag{15}$$

where in the limit $W^2 \rightarrow \infty$ (small Q^2 and large ν)

$$\lim_{W^2 \to \infty} \chi(W^2) = b \tag{16}$$

Now from Fig. 1b the exponential on the right hand side of eq. (15) is bounded by the fourth power of the elastic p - p form factor[12]

$$F^4_{pp}(Q^2) \simeq \exp(-10\,\beta^2 Q^2) \tag{17}$$

where β is the velocity of CM relative to LAB. At 12.5 and 30 GeV incident LAB energies $\beta^2 = 0.86$ and 0.94 respectively; hence from eq. (17) one expects b to be about 10.0 GeV^{-2}. Experimentally b = 15 GeV^{-2}[12]; thus Pomeron dominance of the structure function $\nu W_2(\nu, Q^2)$ in the limit $\omega = Q^2/2M\nu \to 0$ accounts satisfactorily for the observed diffraction peak and predicts a value of the width close to the experimental.

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(10) - This proportionality also follows from the relation of νW_2 to the total photon absorption cross section $\sigma_T + \sigma_L$ on protons.

(11) - Fig. 1b is a simple generalization of the familiar diffraction dissociation graph; see H. Satz, CERN report TH. 1175 (1970). It also generalizes the Brodsky - Kinoshita - Terazawa mechanism for generating large hadron production cross sections in ADONE (Frascati) see Phys. Rev Letters 25, 972 (1970).

(12) - In the range of Q^2 considered here $\overline{Q^2} \simeq k_T^2$; to relate the inelastic forward peak with the elastic one we use the fit of Krisch; A.D. Krisch, Phys. Rev. Letters 19, 1149 (1967).