## DIRECT DETERIMINATION OF ETA-ZERO BRANCHING RATIOS ${ }^{(+)}$.-

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SUMMMARY. -
This work reports a determination of the decay branching ra tios of the eta-zero meson into $2 \gamma, 3 \pi^{0}, \pi^{0} \gamma \gamma, \pi^{+} \pi^{-} \pi^{0}$ and $\pi^{+} \pi^{-} \gamma$ modes.

The etas were produced at BNL Cosmotron in the reaction $\pi^{-} \mathrm{p} \rightarrow \mathrm{n} \eta_{o}$ at $752.5 \mathrm{MeV} / \mathrm{c}$ pion momentum, close to the threshold value of the eta zero production.

The experiment was designed to minimize the systematic errors due to separate evaluations of single channel rates by different expe

[^0]riments. The trigger has been obtained from neutron time of flight and all the decay modes have been detected simultaneously in double spark chamber array. The size of the available sample has been $\underline{i}$ mited by the Cosmotron final shut down.

The results indicate an $\eta^{0} \rightarrow \pi^{0} \gamma \gamma$ contribution consistent with zero. The branching ratios for the other channels are $\eta^{\circ} \rightarrow$ $\rightarrow 2 \gamma: 38.6 \pm 5.9 \% ; \eta^{0} \rightarrow 3 \pi^{0}: 28.1 \pm 5.7 \% \quad \eta^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}: 25.8 \pm$ $\pm 7.9 \% ; \eta^{\circ} \rightarrow \pi^{+} \pi^{-} \gamma: 7.4 \pm 8.5 \%$.

## 1. - INTRODUCTION. -

Eta-zero meson produced in the reaction:

$$
\pi^{-} \mathrm{p} \rightarrow \eta^{\circ} \mathrm{n}
$$

can be detected measuring the emission angle and the momentum of the associated neutron. An unbiasses extimate of the $\eta^{\circ}$ decay-branching ratios can be obtained from an $\eta^{\circ}$ sample selected with this technique.

The cross section for $\eta^{\circ}$ production in $\pi-p$ interaction as a function of the incoming pion momentum is shown in Fig. 1. The cross section reaches its maximum at approximately $760 \mathrm{MeV} / \mathrm{c}$, close enough to the production threshold to constrain the associated neutron within a narrow forward cone. This fact allows the measurement of the neu tron angle and time of flight with a detector of acceptable size. Fig. 2 shows the correlation between neutron emission angle and its time of flight for $\eta^{\circ}$ production, at different incoming momenta.

The $\eta^{0}$ sample used in the present experiment has been obtai ned at $752.5 \mathrm{MeV} / \mathrm{c}$, as best compromize between production cross section and maximum neutron angle.

At this momentum the $\eta^{\circ}$ production cross section is about 2.6 mbarns. Other reactions with a neutron in the final state are listed in Table I together with their relative cross sections.

It can be seen that the $\eta^{\circ}$ signal at $752.5 \mathrm{MeV} / \mathrm{c}$ is approxima tely $1 / 7$ of the total background contributions.

The neutron detector can be selective for $\eta^{\circ}$ productions, when it subtends a narrow angle near the $\eta^{\circ}$ kinematic limit ( $\sim 22^{\circ}$ ) as shown in Fig. 3 where the angular distributions for the neutron in the $\pi^{-} p \rightarrow n \eta^{0}$ reaction and for $\pi^{-} p \rightarrow n \pi^{0} \pi^{0}$ phase space produc tion are plotted.


FIG. 1


FIG. 2
4.

TABLE I



FIG. 3

It can be seen that $\sim 52 \%$ of the neutrons from $\eta^{\circ}$ will appear in 50 interval covering the kinematic peack, while only $\sim 16 \%$ of the background neutron will go in the same interval. With the neutron de tector covering the angle interval $17.4 \div 22.6^{\circ}$, the signal to background ratio will be enhanced from $1 \div 7$ to about $1 \div 2$.

Further enchancement of the $\eta^{\circ}$ signal can be obtained from con venient cuts in the invariant mass of the system of particles accom panying the neutron in the final state. Fig. 4 shows the correlation between the neutron emission angle and neutron time of flight for different values of the invariant mass of the other particles in the final state. The correlation has been evaluated at $752.5 \mathrm{MeV} / \mathrm{c}$ incoming pion momentum. Fig. 5 shows the invariant mass distribution for two pions produced according to phase space with the neutron within the selected angular region. The dotted line is the MC prediction for the invariant mass from neutrons produced with $₹^{{ }^{1}}$ 's, taking account the experimental errors in the time of flight and angle determination.

With a proper choice for an invariant mass cut, it is possible to eliminate a large fraction of the background, still mantaining the signal almost unreduced.

The final analysis has been performed with an invariant mass cut at 540 MeV corresponding approximately to $1.85: 1$ signal+background/background ratio.

## 2. - EXPERIMENTAL APPARATUS. -

The experimental apparatus consisted of a small hydrogen tar get surrounded by a spark chamber array: four large steel chambers to detected electron showers and four small thin foils chambers to detect charged decay products. Three meters distant from the target, at $20^{\circ}$ with respect to the beam, was a neutron hodoscope, to select the neutron from the eta production reaction by measuring emission angle and time of flight. It subtended approximatly 5.2 degrees of polar angle $1 / 13$ of the total azimuth.

## 2.1.- Beam transport.-

The circulating beam of the Cosmotron was extracted and then focussed at $3 \mathrm{GeV} / \mathrm{c}$ upon a copper target placed between the po lefaces of bending magnet $\mathrm{B}_{1}$ (Fig. 6). The negative particles, focussed by the quadrupoles $Q_{1}-Q_{2}$, were momentum selected by the bending magnet $B_{2}$.

Pitching magnets $P_{1}-P_{2}$ were placed to raise the beam 38 inches above the plane of the internal proton beam of the Cosmotron.
6.


FIG. 4

Invarlant mass distributions from simulation.

From $\pi^{\circ} p \rightarrow n n^{0} n^{\circ} \quad$ (Phase Space)
-- $-\infty-\infty$ From $\eta^{0}$ production at $752,5 \mathrm{MeV} / \mathrm{C}$
-.-.-. Lower limit on Invariant Mass from


FIG. 5

In this way the chance of spurious interactions in our apparatus has been reduced.

The beam momentum has been determined measuring the cen tral value of the magnetic field in the momentum selecting magnet with a calibrated Hall probe. The momentum loss introduced by counters and thin foil spark chambers preceding the target resulted to be 10.5 $\mathrm{MeV} / \mathrm{c}$.

The momentum values have been checked through the CM ope ning angle distributions of the two gamma decay of the $\eta^{\circ}$ and of the charge exchange $\pi^{\circ}$.


FIG. 6

## 2.2.- Target.-

The liquid hydrogen target consisted of a spherical Mylar ball, 4 cm in diameter and 0.002 inches wall thickness. The target was sur rounded by a Mylar vacuum jacket and mounted on a aluminium frame to which the small thin-foil spark chamber were attached.

## 2.3.-Spark-chambers.-

The main spark chambers surrounding the target (Fig. 7) were made of fifty one steel plates, $75 \mathrm{~cm} \times 75 \mathrm{~cm} \times 2 \mathrm{~mm}$ with 3 mm gaps between the plates. Thịs array provides a total of 5.5 radiation lenght of steel in the direction perpendicular to the plates, ensuring that vir tually all photons passing through the chambers were converted to

## 8.

electron-positron showers. The low energy cut off for $\gamma$-ray detect dion in the system has been evaluated at 35 MeV . To detect low enter gy charged particles and to determine accurately their origins, four small chambers were arrayed close to the target. These chambers were made of five. $001^{\prime \prime}$ aluminium foils stretched and glued over $12^{\prime \prime} \times 7^{\prime \prime} \times 1 / 16^{\prime \prime}$ aluminium frames.


FIG. 7 - Main counter and spark chamber array.
A $90 \%$ Neon, $10 \%$ helium gas mixture was constantly flowing through the chambers with the addition of a $5 \%$ of alcohol vapor. Sparious shaking was reduced by external dampers ${ }^{(1)}$.

A beam hole 9 cm in diameter was cut in the upstream chamber. The hole was covered with $0.001^{\prime \prime}$ aluminium foils in the first and last five gaps. A rectangular hole $4^{\prime \prime} \times 3.75^{\prime \prime}$ was cut in the downstream chamber to remove the steel from the:solid angle subtended by the neutron detector. This hole was also covered with aluminium foils to detect charged particles.

## 2.4. -Neutron detector.-

The neutron detector shown in Fig. 8 consisted of 21 scintilladion counters ( $26 \mathrm{~cm} \times 45 \mathrm{~cm} \times 2.5 \mathrm{~cm}$ ) alternated with two gaps $0.001^{\prime \prime}$ aluminium foil spark chambers. The charged particles coming from neutron interactions were detected requiring coincidences between any pair of adjacent counters and were observed in the corresponding spark chambers. In order to improve the time resolution of the spark chambers, electrolumiscent panels were attached at the location of each scintillator. Panels corresponding to counters giving pulses in coincidence with the trigger signal were illuminated, to show in the pictures which counters have been triggered by the charged prongs.

The neutron detectors was shilded by an array of anticoincidence counters.


FIG. 8 - Neutron detector array.
2.5.- Counter logic.-

The counter layout for the trigger logic is shown in Fig. 7. Counters 1, 2, 3', 3, de fine the beam. Counter $3^{\prime}$, with a beam defining hole 4 cm in diameter, was used in anticoin cidence to eliminate off axis beam particles and charged pro ducts from upstream interactions. Two downstream anti--counters 4 and $4^{\prime}$, were used as high efficiency veto for noninteracting beam particles.

The neutron detector logic requires a coincidence between at least one pair of adjacent counters in the neutron detector in anticoincidence with counters 5-5'.

A good event trigger was generated by a coincidence between one pulse from the telescope $123^{\prime} 344^{\prime}$, and one pulse from the neutron detector logic; the telescope pulse was stretched to 48 nsec corresponding to the desired range of the neutron tin:e of flight.

The output of the time to pulse-height converter was digitized in 8 binary-bits and the resulting number, displayed on indicator lights, appeared in the correspondent spark chamber picture.

## 3. - DATA ACQUISITION. -

The sample has been obtained from 70.000 pictures at 752.5 $\mathrm{MeV} / \mathrm{c}$. In addition 8.000 pictures at $721.5 \mathrm{MeV} / \mathrm{c}, 3.000$ pictures at $729.5 \mathrm{MeV} / \mathrm{c}$ and 27.000 pictures at $737.5 \mathrm{MeV} / \mathrm{c}$ have been used to obtain information on the background since at these momenta the kine matic limit of the neutron angle from $\eta^{\circ}$ production is lower than the minimum angle subtended by the neutron detector.

Several cuts were made in the scanning table to eliminate background events and speed up the scanning procedure. All photograps with neutron time of flight corresponding to $\beta>0.6$ were by-passed. In addition during preliminary analysis of the data (confirmed by a Monte Carlo study of the recoil proton in the meutron detector) ${ }^{(2)}$, it
was found that approximately $90 \%$ of the good $\eta^{\circ}$ events would trigger only one or two consecutive counter pair in the neutron detector. All photographs with more than four consecutive lights on in the neutron detector were therefore disregarded. All events remaining after the se two cuts were scanned in detail for charged particles and electron showers in the main detector. Events were measured when a good neutron recoil trach was present in the neutron detector, in the region identified by the luminescent strip and the beam reack was clear ly visible in the upstreams large chamber and at least in the innermost gap of the upstream small chamber.

All spark structures accepted as electron showers had to sa tisfy minimum requirements concerning their direction in space and number of sparks per unit lenght. These requirements introduced an effective low energy cut off of 35 MeV .

During the analysis of the data, it was noticed that a large fraction of events with a charged particle in or close to the hole cut in the steel plates of the downstream chamber, or with a gamma ray which converted close to the hole, were associated with a short time of flight $(\beta>0.6)$. With an error of $\sim 2$ nanoseconds on the neutron time-of-flight, $99.8 \%$ of all neutron associated with eta are expected to appear with beta in the range $0.3 \div 0.6$. The enhancement of high beta events seems be due to negative pions and $\gamma$-rays produced on the edge of the solid angle subtended by the neutron detector. These particles may interact in front of the neutron detector anticounters and produce fast neutrons or soft gammas.

The effect can be reduced removing from the sample all events with charged particles and/or showers visible in the 2 cm region sur rounding the neutron hole.

In addition, to reduce the effect of efficiency variations in the thin foil-chambers, charged particles were required to produce at least one spark in the first gap of large chamber in addition to one or more sparks in the small chambers. These criteria introduced an effective low energy cut-off of $\sim 45 \mathrm{MeV}$ on the charged particles from $\eta^{\circ}$-decay.

Coordinate measurements were done with a precision of about $10 \mu \mathrm{~m}$ on the film. The overall error, due mainly to the recon struction parameters, is about 2.5 mm in space. The neutron produc tion angle has been determined with an error smaller than 0.5 degrees.

## 4. - MONTE CARLO EVENTS SIMULATION. -

The general purpose simulation program NVERTEX ${ }^{(3)}$ was used to evaluate the detection efficiency for different $\eta$-decay modes
and to take into account the measuring and reconstruction errors.
Detection efficiency is less than unity since the spark chambers covered only two third of the solid angle surrounding the target. In addition final state particles can be lost as a consequence of the low energy cut-off on gammas and charged particles. Different final states are not expected to appear with equal detection efficiency because of possible triggering of anticounters by gammas. or charged particles, while spurious triggers can derive from $\gamma$-ray interactions in the N. D.

For the event simulation the angular distribution of the $\eta^{0}$ was assumed to be isotropic in the production center of mass. The Monte Carlo events were also thrown with the assumption of constant matrix elements for the decays. Within statistics, other matrix elements gave indistinguishible results. In order to determine the probability that a given decay mode of the eta zero meson will appear with a topology of i-gammas and j-charged particles in the fiducial volume, the cuts extimated for the experimental sample were applied also to the simulated data.

The probabilities evaluated at $752.5 \mathrm{MeV} / \mathrm{c}$, for an invariant mass cut of 540 MeV , are shown in Table II, together with the detection efficiencies for $N \pi^{0} \pi^{0}$ background reaction generated according to phase space.

## 5. - NEUTRON TIME OF FLIGHT. -

In order to evaluate the neutron time of flight it was necessary to determine which counters pair in the N.D. sent the first pulse into the time-to-pulse height converter.

TABLE II
Monte Carlo detection - Probabilities at $752.5 \mathrm{MeV} / \mathrm{c}$

| Topology decay mode | $\begin{aligned} & \mathrm{or} \\ & \mathrm{OC} \end{aligned}$ | $\begin{aligned} & 18 \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 2 \gamma \\ & \mathrm{OC} \end{aligned}$ | $\begin{aligned} & 3 \gamma \\ & \mathrm{OC} \end{aligned}$ | $\begin{aligned} & 42 \\ & \mathrm{OC} \end{aligned}$ | $\begin{aligned} & 5 \gamma \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 68 \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 0 \gamma \\ & 1 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 15 \\ & 1 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 2 \gamma \\ & 1 \mathrm{C} \end{aligned}$ | $\begin{aligned} & \text { or } \\ & 2 \mathrm{C} \end{aligned}$ | 12 2 C | $\begin{aligned} & 2 \gamma \\ & \text { 2 } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\eta \rightarrow 2 \gamma$ | $\begin{array}{r} 0.0087 \\ +0.005 \end{array}$ | $\begin{array}{r} .362 \\ \pm .012 \end{array}$ | $\begin{array}{r} .462 \\ \pm .014 \end{array}$ | -- | -- | -- | -- | -- | -- | -- |  |  | -- |
| $\eta \rightarrow \pi^{\circ} \gamma \gamma$ | $\begin{array}{r} 0.007 \\ \pm . .001 \end{array}$ | $\begin{array}{r} .0 .101 \\ \pm .003 \end{array}$ | $\begin{array}{r} .287 \\ \pm .006 \end{array}$ | $\begin{array}{r} .340 \\ \pm .006 \end{array}$ | $\begin{array}{r} .141 \\ \pm .004 \end{array}$ | -- | -- | -- | -- | -- | -- | -- | -- |
| $\eta \rightarrow 3 \pi^{\circ}$ | $\begin{array}{r} .0009 \\ +.0002 \end{array}$ | $\begin{array}{r} .0015 \\ \pm .0011 \end{array}$ | $\begin{array}{r} .0923 \\ \pm .0031 \end{array}$ | $\begin{array}{r} .260 \\ \pm .006 \end{array}$ | $\begin{array}{r} .278 \\ \pm .006 \end{array}$ | $\begin{array}{r} .165 \\ \pm .004 \end{array}$ | $\begin{array}{r} .0361 \\ +.0018 \end{array}$ | -- |  | -- |  | -- | -- |
| $3 \rightarrow \pi^{+} \pi^{-} \pi^{\circ}$ | $\begin{array}{r} .0006 \\ \pm .0003 \end{array}$ | $\begin{array}{r} .008 \\ +.002 \end{array}$ | $\begin{array}{r} .010 \\ \pm .002 \end{array}$ |  | -- | -- | -- | $\begin{array}{r} .024 \\ \pm .002 \end{array}$ | $\begin{aligned} & .0162 \\ & \pm .006 \end{aligned}$ | $\begin{array}{r} .108 \\ \pm .005 \end{array}$ | $\begin{array}{r} .063 \\ \pm .004 \end{array}$ | $\begin{array}{r} .243 \\ \pm .008 \end{array}$ | $\begin{array}{r} .162 \\ \pm .006 \end{array}$ |
| $3 \rightarrow \pi^{+} \pi^{-} \gamma$ | $\begin{array}{r} .004 \\ \pm .001 \end{array}$ | $\begin{array}{r} .018 \\ \pm .002 \end{array}$ |  | -- | -- | -- | -- | $\begin{array}{r} .111 \\ \pm .005 \end{array}$ | $\begin{array}{r} .194 \\ \pm .007 \end{array}$ | -- | $\begin{array}{r} .182 \\ +.007 \end{array}$ | $\begin{array}{r} .305 \\ \pm .005 \end{array}$ | -- |
| $\pi^{\circ} \pi^{0}$ | $\begin{array}{r} .002 \\ \pm .001 \end{array}$ | $\begin{array}{r} .020 \\ \pm .004 \end{array}$ | $\begin{array}{r} .051 \\ \pm .002 \end{array}$ | $\begin{array}{r} .056 \\ \pm .008 \end{array}$ | $\begin{array}{r} .020 \\ +.004 \end{array}$ | -- | -- | -- | -- | -- | -- | -- | -- |

The time-of-flight of a neutron from the target to its interaction point is related to the $A / D$ time by the following equation:

$$
T_{A / D}=T_{N}+T_{L}^{(i)}+T_{o}^{(i)}+T_{R}^{(i)}=T_{N}+T(i)
$$

where $T_{A / D}=$ time shown in the $A / D$ lights;
$\mathrm{T}_{\mathrm{N}}=$ time of flight of the neutron;
$\mathrm{T}_{\mathrm{L}}^{(\mathrm{i})}=$ the time taken by the light to travel from the point of emission in the $i^{\text {th }}$ scintillator to the photomultiplier. (The effective index of refraction has been measured and its value is $\mathrm{n}=2.4)^{(4)}$;
$T_{o}^{(i)}=$ a correction term characteristic of counter pair i, i+1, to compensate intrinsic delay of photomultipliers, circuits and cables;
$\mathrm{T}_{\mathrm{R}}^{(\mathrm{i})}=$ time of flight of the recoil proton to reach the $i^{\text {th }}$ counter, as calculated from the residual range of the proton in the scintillator.
'To resolve the ambiguity as to which counter pair gave timing signal, we have to find the earliest signal from the different counter pair i triggered by the recoil charged particle. This will be given from the i pair for which $T(i)$ is minimum.

The time constant $\mathrm{T}_{\mathrm{o}}^{(\mathrm{i})}$ were determined by calibration with elastic scattering events: 5.000 pictures have been taken triggering the neutron detector, pair by pair, on elastic scattering. These mea surements gave a time spread of about 2 nanoseconds.

## 6. - BACKGROUND EVALUATION. -

The $\eta^{\circ}$ sample has been obtained at $752.5 \mathrm{MeV} / \mathrm{c} \pi^{-}$incident momentum. At this energy, relevant contributions can derive from the following background reactions

$$
\begin{aligned}
\pi^{-} \mathrm{p} \rightarrow \pi^{0} \mathrm{n} ; \quad & \rightarrow \mathrm{n} \pi^{+} \pi^{-} \\
\pi^{-} \mathrm{p} & \rightarrow \mathrm{n} \pi^{0} \pi^{0} \pi^{0} \\
& \rightarrow \mathrm{n} \pi^{0} \pi^{0 ;} \quad \pi^{-} \mathrm{p} \\
& \rightarrow \mathrm{n} \pi^{+} \pi^{-} \pi^{0}
\end{aligned}
$$

Charge exchange contributions can easily be eliminated with the invariant mass cut.

The remaining background is excepted to derive mainly from two pions final states, since the phase space available for three pions final states is very limited.

Background contributions to each topology have been evaluated on samples produced at lower beam momenta, having the neutron de tector outside the kinematic limit for the neutron associated to the $\eta^{\circ}$. For this reason data have been taken at $721.5,729.5$ and 737.5 $\mathrm{MeV} / \mathrm{c}$. At this last momentum, $\mathcal{Z}^{\circ}$ events can send the neutron in the inner half of the neutron detector $\left(\theta_{\mathrm{N}}<19^{\circ}\right)$. Therefore, for this sample, only events with $20^{\circ}<\theta_{\mathrm{N}}<22.6^{\circ}$ have been considered for the background evaluation.

Table III shows the number of events observed with different topologies for the background and the signal sample together with the relative number of incident pions. It can be noticed that the limi ted variations in the beam momentum do not appear to affect, within statistics, the partition of background events among the different chan nels. The background contribution at $752.5 \mathrm{MeV} / \mathrm{c}$ can therefore be obtained normalizing properly the data at lower momenta.

TABELLA III

| Topologies | $\begin{aligned} & \therefore \text { Invariant Mass cut }=110 \mathrm{MeV} / \mathrm{c}^{2} \\ & \qquad{ }_{\mathrm{N}}<22^{\circ} \end{aligned}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}^{\circ}$ of pions | $0.502 \times 10^{9}$ | $0.0086 \times 10^{9}$ | $0.550 \times 10^{9}$ | $3.124 \times 10^{9}$ |
| $\begin{aligned} & \text { Momentum } \\ & \mathrm{MeV} / \mathrm{c} \end{aligned}$ | 721.5 | 727. 5 | 737. 5 | 752.5 |
| $0 \gamma 0 \mathrm{C}$ | 7 | 2 | 8 | 85 |
| $1 \gamma_{0 C}$ | 22 | 7 | 29 | 287 |
| $2 \gamma 0 \mathrm{C}$ | 44 | 15 | 58 | 527 , |
| $3 \gamma 0 \mathrm{c}$ | 27 | 6 | 18 | 189 |
| $4 \gamma 0 \mathrm{c}$ | 7 | 1 | 20 | 108 |
| $5 \gamma 0{ }^{\prime}$ | 4 | 1 | 4 | 58 |
| $6 \gamma$ OC | 1 | 0 | 0 | 22 |
| ${ }^{0} \gamma_{12}$ | 56 | 13 | 54 | 460 |
| $1 \gamma 1 \mathrm{C}$ | 14 | 1 | 15 | 137 |
| 28 IC | 9 | 1 | 11 | 63 |
| $0 \gamma 2 \mathrm{C}$ | 75 | 13 | 74 | 525 |
| $1 \gamma 2 \mathrm{C}$ | 5 | 2 | 14 | 132 |
| 232 C | 2 | 1 | 6 | 58 |
| Total | 273 | , 63 | 311 | 2651 |

The invariant mass distributions for signal and background events have been taken into account, to minimize subtraction uncer tainties.

As discussed in par. 1 if only events corresponding to an invariant mass greater than 540 MeV are considered, the signal should be reduced by $10 \%$, while almost two thirds of the background is removed (see Fig. 10).

Fig. 9 shows the invariant mass distribution for the background events at $721.5 \mathrm{MeV} / \mathrm{c}$ evaluated from neutron angle and time of flight.

Fig. 5 shows the invariant mass distributions predicted for $n \pi^{0} \pi^{0}$ final states produced according to phase space. It can be seen that the kinematic limit varies appreciably with the incoming momentum.

In order to subtract the background events channel by channel, equivalent cuts have to be applied to the different samples. These cuts correspond to appropriate invariant mass values which can be evaluated requiring at each momentum the inclusion of the same frac tion of the invariant mass distribution for phase space generated events.

Equivalent cut values are given in Table IV.

TABLE IV

| INCOMING MOMENTUM | 752.5 | 737.5 | 729.5 | 721.5 |
| :---: | :---: | :---: | :---: | :--- |
|  | 540 | 535 | 530 | 525 |
| INVARIANT <br> MASS (MeV) <br> CUTS | 530 | 526 | 521 | 517 |
|  |  | 520 | 519 | 515 |

With these cuts at the different beam momenta, each incoming pion has the same probability to produce a background event in the accepted region of invariant mass, and normalizations of different samples can be done simply on the numbers of incoming pions.

Table V shows the events observed with different topology at $752.5 \mathrm{MeV} / \mathrm{c}$, at different invariant mass cuts, together with the back grounds.

Table VI shows the (Signal + Background)/Background ratios, for each observed topology and for different invariant mass cuts.


FIG. 9

Invarlant Mass Distribution<br>$P_{\pi}=752,5 \mathrm{MEV} / \mathrm{C}$<br>....- Not normallzed Phase space contrlbutlon<br>........ Not normallzed $\eta^{\circ}$ contrlbution<br>- Expurimental Data.



FIG. 10

| Topologies | Invariant Mass cut 500 MeV |  |  |  | Invariant Mass cut 520 MeV |  |  |  | Invariant Mass cut 530 MeV |  |  |  | Invariant Mass cut 540 MeV |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Momentum } \\ & \mathrm{MeV} / \mathrm{c} \end{aligned}$ | 721.5 | 729.5 | 737.5 | 752.5 | 721.5 | 729.5 | 737.5 | 752.5 | 721.5 | 729.5 | 737.5 | 752.5 | 721.5 | 729.5 | 737.5 | 752.5 |
| $0 \gamma 0 \mathrm{C}$ | 4 | 1 | 4 | 69 | 1 | 1 | 4 | 55 | 1 | 1 | 4 | 48 | 1 | 0 | 2 | 38 |
| $1 \gamma 0 \mathrm{C}$ | 12 | 5 | 21 | 187 | 11 | 4 | 17 | 158 | 7 | 3 | 13 | 131 | 6 | 0 | 9 | 102 |
| $2 \gamma$ OC | 22 | 9 | 42 | 345 | 21 | 6 | 35 | 289 | 18 | 4 | 29 | 260 | 16 | 3 | 21 | 210 |
| $3 \gamma 0 \mathrm{C}$ | 19 | 3 | 15 | 143 | 15 | 3 | 12 | 121 | 13 | 3 | 10 | 108 | 10 | 2 | 7 | 81 |
| $4 \gamma 0 \mathrm{C}$ | 6 | 1 | 18 | 91 | 5 | 1 | 15 | 87 | 5 | 1 | 10 | 79 | 4 | 0 | 7 | 75 |
| 57 OC | 3 | 1 | 4 | 53 | 2 | 1 | 3 | 50 | 2 | 1 | 3 | 47 | 2 | 1 | 2 | 36 |
| $6 \gamma$ OC | 1 | 0 | 0 | 21 | 1 | 0 | 0 | 21 | 1 | 0 | 0 | 20 | 1 | 0 | 0 | 18 |
| $0 \gamma 1 \mathrm{C}$ | 44 | 5 | 38 | 327 | 33 | 3 | 31 | 274 | 27 | 3 | 27 | 234 | 23 | 3 | 18 | 175 |
| $1 \gamma 1 \mathrm{C}$ | 8 | 1 | 12 | 102 | 6 | 0 | 10 | 86 | 5 | 0 | 9 | 75 | 5 | 0 | 6 | 62 |
| $2 \gamma 1 \mathrm{C}$ | 6 | 0 | 9 | 46 | 5 | 0 | 8 | 42 | 4 | 0 | 4 | 34 | 2 | 0 | 2 | 27 |
| $0 \gamma 2 \mathrm{C}$ | 65 | 9 | 60 | 405 | 48 | 8 | 50 | 341 | 41 | 7 | 43 | 302 | 31 | 4 | 33 | 217 |
| $1 \gamma 2 \mathrm{C}$ | 5 | 2 | 11 | 110 | 5 | 2 | 8 | 9.5 | 4 | 2 | 6 | 75 | 3 | 1 | 5 | 58 |
| $2 \gamma 2 \mathrm{C}$ | 2 | 0 | 6 | 44 | 1 | 0 | 5 | 39 | 1 | 0 | 4 | 35 | 1 | 0 | 3 | 33 |
| Total | 197 | 37 | 242 | 1943 | 154 | 29 | 198 | 1658 | 129 | 25 | 162 | 1448 | 106 | 14 | 115 | 1132 |

## TABLE VI

| Cuts | $\begin{aligned} & \text { total } \\ & \text { S/B } \end{aligned}$ | $\begin{aligned} & 0 \gamma \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 1 \gamma \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 2 \gamma \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 3 \gamma \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 4 \gamma \\ & 0 C \end{aligned}$ | $\begin{aligned} & 5 \gamma \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 6 \gamma \\ & 0 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 0 \gamma \\ & 1 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 1 \% \\ & 1 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 2 \gamma \\ & 1 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 0 \gamma \\ & 2 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 1 \gamma \\ & 2 \mathrm{C} \end{aligned}$ | $\begin{aligned} & 2 \gamma \\ & 2 \mathrm{C} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 110 \mathrm{MeV} / \mathrm{c}^{2} \\ & \theta_{\mathrm{N}}<22^{\circ} \end{aligned}$ | 1:52 | 1.85 | 1.83 | 1:67 | 1:37 | 1: 44 | 2:38 | 8,1 | 1. 40 | 1. 65 | 1. 11 | 1. 2 | 2. 32 | 2.39 |
| $\begin{aligned} & 500 \mathrm{MeV} / \mathrm{c}^{2} \\ & { }^{9} \mathrm{~N}<22^{\circ} \end{aligned}$ | 1.52 | 2.84 | 1. 82 | 1. 75 | 1.44 | 1.35 | 2.45 | 7.8 | 1.40 | 1.80 | 1.14 | 1.12 | 2.26 | 2.04 |
| $\begin{aligned} & 520 \mathrm{MeV} / \mathrm{c}^{2} \\ & { }^{\theta_{\mathrm{N}}}<22^{\circ} \end{aligned}$ | 1.64 | 3.39 | 1.83 | 1.73 | 1.49 | 1.47 | 3.08 | 7.8 | 1.52 | 1.98 | 1. 20 | 1.21 | 2.34 | 3.41 |
| $\begin{aligned} & 530 \mathrm{MeV} / \mathrm{c}^{2} \\ & { }^{\theta_{\mathrm{N}}<22^{\circ}} \end{aligned}$ | 1. 70 | 2. 96 | 2.11 | 1.89 | 1. 54 | 1. 79 | 2.89 | 7.4 | 1.52 | 1.97 | 1.57 | 1.23 | 2.32 | 2.6 |
| $\begin{aligned} & 540 \mathrm{MeV} / \mathrm{c}^{2} \\ & \theta_{\mathrm{N}}<22^{\circ} \end{aligned}$ | 1.85 | 4.7 | 2. 52 | 1.95 | 1.58 | 2.52 | 2.67 | 6.7 | 1.48 | 2.09 | 2.5 | 1.18 | 2.39 | 3.06 |

The ratios show significant improvements for increasing missing mass cuts as expected from kinematic and geometric consi deration (see par. 1). A separate analysis on single channels shows a reasonable agreement with Monte Carlo prediction, also for the topo logies $0 \gamma 1 \mathrm{C}, 0 \gamma 2 \mathrm{C}$ and $3 \gamma 0 \mathrm{C}$. In these channels two simultaneous effect contribute to keep low signal/noise ratio, even increasing the missing mass cuts: the large contributions of background reactions as

$$
\pi^{-} \mathrm{p} \rightarrow \mathrm{n} \pi^{+} \pi^{-} ; \quad \pi^{-} \mathrm{p} \rightarrow \mathrm{n} \pi^{0} \pi^{0}
$$

and the small contribution of the $\eta_{0}$ decays to these topologies. The numbers of events at 540 MeV cut, after subtraction of the norma lized background, have been fitted to obtain the $\eta^{\circ}$ branching ratios.
7. - FIT. -

The final event sample after background subtraction is given in Table VII. For each topology the corrected number of $\eta^{\circ}$ events is given, together with the statistical error.

With the experimental data and the detection efficiency obtained from the simulation, the following equation can be written:

$$
\begin{equation*}
\sum_{1}^{5} a_{i k} N_{k}=n_{i} \tag{i=1,13}
\end{equation*}
$$

where
18.
$n_{i}=$ number of events for the i-topology
$\mathrm{N}_{\mathrm{K}}=$ number of $\eta_{\mathrm{O}}$ parents decaying in the K mode
$\mathrm{a}_{\mathrm{ik}}=$ M.C. probability.

TABLE VII

| Topologies | Signal+ <br> Background | Normalized <br> Background | Subtracted <br> Sample |
| :--- | ---: | ---: | ---: |
| $0 \gamma 0 \mathrm{C}$ | $38 \pm 6.1$ | $8.1 \pm 4.6$ | $29.9 \pm 7.6$ |
| $1 \gamma 0 \mathrm{C}$ | $102 \pm 10.1$ | $40.5 \pm 10.5$ | $61.5 \pm 14.5$ |
| $2 \gamma 0 \mathrm{C}$ | $210 \pm 14.5$ | $108 . . \pm 17.0$ | $102 . \pm 22.3$ |
| $3 \gamma 0 \mathrm{C}$ | $81 \pm 9.0$ | $51.3 \pm 11.7$ | $29.7 \pm 14.8$ |
| $4 \gamma 0 \mathrm{C}$ | $75 \pm 8.6$ | $29.7 \pm 8.9$ | $35.3 \pm 12.4$ |
| $5 \gamma 0 \mathrm{C}$ | $36 \pm 6.0$ | $13.5 \pm 6.0$ | $22.5 \pm 8.4$ |
| $6 \gamma 0 \mathrm{C}$ | $18 \pm 4.2$ | $2.7 \pm 2.7$ | $15.3 \pm 5.0$ |
| $0 \gamma 1 \mathrm{C}$ | $175 \pm 13.2$ | $118.8 \pm 18$. | $56.2 \pm 22.3$ |
| $1 \gamma 1 \mathrm{C}$ | $62 \pm 6.3$ | $29.7 \pm 8.9$ | $32.3 \pm 11.9$ |
| $2 \gamma 1 \mathrm{C}$ | $27 \pm 5.2$ | $10.8 \pm 5.4$ | $16.2 \pm 7.5$ |
| $0 \gamma 2 \mathrm{C}$ | $217 \pm 14.7$ | $183.6 \pm 22.2$ | $33.4 \pm 26.7$ |
| $1 \gamma 2 \mathrm{C}$ | $58 \pm .7 .6$ | $24.3 \pm 8.1$ | $33.7 \pm 11.0$ |
| $2 \gamma 2 \mathrm{C}$ | $33 \pm 5.7$ | $10.8 \pm 5.4$ | $22.2 \pm 7.8$ |
| 1 |  |  |  |

To find the best fitted solution for the $\mathrm{N}_{\mathrm{K}}^{\prime} \mathrm{s}$, we have to minimize the function

$$
\chi^{2}=\sum_{1}^{13} \frac{\left(\sum_{1}^{5} \mathrm{a}_{\mathrm{ik}} \mathrm{~N}_{\mathrm{k}}-\mathrm{n}_{\mathrm{i}}\right)^{2}}{\sigma_{\mathrm{EXP}}{ }^{2}+\left(\sum_{1}^{5} \mathrm{k}^{5} \Delta \mathrm{a}_{\mathrm{ik}} \mathrm{~N}_{\mathrm{k}}\right)^{2}}
$$

where

$$
\begin{aligned}
& \Delta a_{i k}=M . C \text {. uncertainties on } a_{i k} \\
& \left.\sigma_{\text {EXP }}^{2}=n_{i(\text { SIGNAL })}+f n_{i(\text { NORMAL BACKGR. }}\right) \\
& \\
& f=\text { normalizing factor. }
\end{aligned}
$$

The MINFUN program ${ }^{(5)}$ has been used to search the best solu tion. Parameter space has been first explored in searching mode
and final fitted values have been obtained running the program in converging mode.

## 8. - CONCLUSIONS. -

a) All topologies included

13 equations with 5 unknown

$$
\chi^{2}=13.4
$$

| Decay mode | Fitted number of parents |
| :--- | :---: |
| $2 \gamma$ | $201.6 \pm 31.6$ |
| $\pi^{\circ} \gamma \gamma$ | $-37.6 \pm 41.2$ |
| $3 \pi^{\circ}$ | $173.1 \pm 47.5$ |
| $\pi^{+} \pi^{-} \pi^{\circ}$ | $127.6 \pm 39.0$ |
| $\pi^{+} \pi^{-} \gamma$ | $35.8 \pm 42.8$ |

## VARIANCE MATRIX

| 0.1001D | 04 | -0.5022D | 03 | 0.3881 D | 03 | 0.8690D | 01 | -0.3658D | 02 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.5022D | 03 | 0.1702 D | 04 | -0.1574D | 04 | -0.6891D | 01 | 0.9897 D | 01 |
| 0.3881 D | 03 | -0.1574D | 04 | 0.2251 D | 04 | 0.4127 D | 01 | -0.5827D | 01 |
| 0.8690D | 01 | -0.6891D | 01 | 0.4127 D | 01 | 0.1521 D | 04 | -0.1191D | 04 |
| -0.3658D | 02 | 0.9897 D | 01 | -0.5827D | 01 | -0.1191D | 04 | 0.1838 D | 04 |

The resulting branching ratios are:

| BRANCHING RATIOS |  |
| :---: | :---: |
| $37.4 \pm 5.8 \%$ | $2 \gamma$ |
| $-7.0 \pm 7.6 \%$ | $\pi^{\circ} \gamma \gamma$ |
| $32.1 . \pm 8.8 \%$ | $3 \pi^{\circ}$ |
| $23.85 \pm 7.2 \%$ | $\pi^{+} \pi-\pi 0$ |
| $6.65 \pm 7.9 \%$ | $\pi+\pi-\gamma$ |

The $\pi^{0} \not \partial \gamma$ mode seems consistent with 0 .
20.
b) Since the $\pi^{\mathrm{o} \gamma \gamma}$ decay mode of the $\eta^{0}$ seems consistent with zero, a separate fit has been done without the $\eta^{0} \rightarrow \pi^{0} \gamma \gamma$ channel: 13 equations with 4 unknowns $\chi^{2}=14.3$.

| Decay mode | Fitted number of parents |
| :--- | :---: |
| $2 \gamma$ | $190.6+29.2$ |
| $3 \pi^{\circ}$ | $138.6 \pm 28.2$ |
| $\pi^{+} \pi-\pi \circ$ | $127.4 \pm 39.0$ |
| $\pi^{+} \pi-\gamma$ | $36.07 \pm 42.8$ |

VARIANCE MATRIX

| 0.8532 D | 03 | -0.7634 D | 02 | 0.6660 D | 01 | -0.3367 D | 02 |
| ---: | :--- | ---: | :--- | ---: | :--- | ---: | :--- |
| -0.7634 D | 02 | 0.7958 D | 03 | -0.2245 D | 01 | 0.3326 D | 01 |
| 0.6660 D | 01 | -0.7958 D | 03 | 0.1521 D | 04 | -0.1191 D | 04 |
| -0.3367 D | 02 | 0.3326 D | 01 | -0.1191 D | 04 | 0.1838 D | 04 |

The resulting branching ratios are:

## BRANCHING RATIOS

| $38.6+5.9 \%$ | $2 \gamma$ |
| :---: | :--- |
| $28.1 \pm 5.7 \%$ | $3 \pi \circ$ |
| $25.8 \pm 7.9 \%$ | $\pi+\pi-\pi \circ$ |
| $7.4 \pm 8.5 \%$ | $\pi+\pi-\gamma$ |

The fraction of neutral decays of the $\eta^{\circ}$ results $\Gamma\left(\eta^{\circ} \rightarrow\right.$ neutrals $) /$ $/ \Gamma\left(\eta^{\circ} \rightarrow\right.$ total $)=0.08$. These ratios are in good agreement with the known world average.

## APPENDIX I. -

An integral consistency check can be performed considering the CM opening angle distributions for the $2 \gamma$ events. These distributions allow a clean separation of events produced by $\eta^{\circ}$ decay.

On the background samples, we can therefore determine the number of events deriving from $2 \pi^{\circ}$ and $3 \pi^{\circ}$ channels and appearing as $2 \gamma$ in the opening angle region typical of the $~ \geqslant 0$ 。

With the invariant mass cut at 540 MeV and $142<\theta_{0 \mathrm{P}}<180$, the normalized number is:

$$
\mathrm{N}_{2 \pi} \mathrm{o}+3 \pi \mathrm{o}=21 \pm 6.7
$$

In the sample at $752.5 \mathrm{MeV} / \mathrm{c}$, with the same cut in the invariant mass, the number of $2 \gamma$ events with $142<\theta_{0 \mathrm{P}}<180$ is:

$$
\mathrm{N}_{\eta \mathrm{o}+2 \pi \mathrm{o}+3 \pi \mathrm{o}}=114 \pm 10.7
$$

The detector efficiency for both gammas of the $\eta$ is $0.462 \pm$ $\pm 0.014$ (see Table II).

We can therefore evaluate the number of $\eta^{\circ} \rightarrow 2 \gamma$ in our sample

$$
(\eta \rightarrow 2 \gamma)=\frac{\left(114+\frac{10.7)-(21+6.7)}{0.462 \pm 0.014}\right.}{}=201 \pm 28
$$

in good agreement with the previous estimate.

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