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INFN/AE-71/5
4 Maggio 1971

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ABSTRACT. -

Hadronic configurations and their mixing in the quark parton model are examined and shown to lead naturally to the association of every hadron with a statistical ensemble whole elements are exact copies of the hadron in all its possible configurations. Configuration mixing is achieved by considering every result of scattering as an average over this ensemble. From this point of view it is shown that the usual expressions of the structure functions in the parton model are not statistical averages in any acceptable sense and are therefore incorrect. Previous attempts to incorporate mixing in the parton model have been confused and misleading for this reason. A unified treatment of electroproduction sum rules is given together with a discussion of two of their special solutions in the literature.

1. - INTRODUCTION. -

The concept of configurations and their mixing is not new; it has been discussed by Feynman and Vernon(1) in connection with the theory of influence functionals where configuration mixing occurs be cause the physical situation is unsure either through an incomplete knowledge of the type of interaction system or of the nature of its initial and final states or both. In the quark parton model this problem seems to have been completely misunderstood as evidenced by the erroneous duplication of probabilities(2) and from rather special solutions of some electroproduction sum rules(2, 3). The incorrectness of these solutions have been established either by counter examples which invalidate them or from wrong conclusions derived there from. No proof as yet exists of why they are invalid not of what the correct general solution should be. The purpose of this paper is then two-fold: firstly to discuss as simply as possible the concept of configurations and their mixing within the framework of the quark parton model and secondly to give a unified treatment of electroproduction sum rules, for which experimental numbers exist, and extract therefrom those conclusions that can be retained generally valid. After the ground-work has been laid it will become immediately clear why previous attempts to incorporate mixing in the parton model have been inconsistent hy bridizations leading to incorrect solutions of the electroproduction sum rules. The expressions of the structure functions in the usual form of the parton model(4, 5) are not statistical averages in any acceptable sense and are therefore incorrect.

2. - HADRON CONFIGURATIONS IN THE PARTON MODEL. -

In the parton model the nucleon is regarded as a free gas of non-interacting constituents (partons); to relate the results of such a model with those of current algebra it is necessary to identify partons with quarks. Experiments indicate that for the parton model to survive(2, 3, 6), if only as a useful heuristic device, complicated parton arrangements are necessary. This is the motivation for configuration mixing. The conventional form of the parton model does not incorporate mixing but it is very simple to include it once the basic ideas about configurations are understood.

2.1. - SU(2) Configurations. -

To see how configurations arise assume that the free parton gas belongs to an SU(2) doublet formed by N + 3 proton and neutron type quarks q_1 and q_2 and N antiproton and antineutron type quarks $\overline{q}_1$ and $\overline{q}_2$ respectively. A system soconstituted will be called an N parton assembly even through there are 2N+3 constituents. A proton can be formed from the product state

$$q_1^{\alpha_1} \quad q_1^{\alpha_j} \quad q_2^{\beta_1} \quad q_2^{\beta_k} \quad \bar{q}_{1\sigma_1} \quad \bar{q}_{1\sigma_1} \quad \bar{q}_{2\tau_1} \quad \bar{q}_{2\tau_n}$$

of 2N+3 terms by reduction with the SU(2) invariant tensors $\varepsilon^{\mu\nu}$, $\varepsilon_{\mu\nu}$ and δ^{μ}_{ν} in two ways corresponding to an uncontracted upper and lower index respectively

(C.1.) $\quad p \equiv p \otimes \text{(SU(2) Scalar } S_1)$

$$S_1 = (pn)_-^{n_1} \, (\bar{p}\bar{n})_-^{m_1} \, (\bar{p}p + \bar{n}n)^{k_1}$$

$$(pn)_- = pn - np\,, \qquad (\bar{p}\bar{n})_- = \bar{p}\bar{n} - \bar{n}\bar{p}$$

$$q_1 \; \binom{p}{o}, \quad q_2 \; \binom{o}{n} \qquad \text{etc.}$$

where n_1, m_1 and k_1 are integers satisfying

$$\begin{aligned} n_1 + m_1 + k_1 &= N+1 \\ n_1 - m_1 &= 1 \end{aligned} \tag{1}$$

(C.2.) $\quad p \equiv \bar{n} \otimes \text{(SU(2) Scalar } S_2)$

$$S_2 = (pn)_-^{n_2} \, (\bar{p}\bar{n})_-^{m_2} \, (\bar{p}p + \bar{n}n)^{k_2}$$

and the integers n_2, m_2, k_2 satisfy

$$n_2 + m_2 + k_2 = N+1, \qquad n_2 - m_2 = 2 \tag{2}$$

The two ways in which the product state has been reduced to form a proton will be called configuration classes which because conjugate SU(2) representations are equivalent actually coincide. To better bring out this equivalence rewrite the reduced states, making use of eqs.(1) and (2), as

(C.1.) $$p \equiv p^{N_1+2} n^{N_2+1} \bar{p}^{N_1} \bar{n}^{N_2}, \qquad N_1 + N_2 = N \tag{1'}$$

(C.2.) $$p \equiv p^{N_1'+2} n^{N_2'+1} \bar{p}^{N_1'} \bar{n}^{N_2'}, \qquad N_1' + N_2' = N \tag{2'}$$

in which forms there is indeed no difference between (C.1.) and (C.2.). For any fixed N there are 2N+1 different specifications within a class indexed by the difference $\alpha = N_1 - N_2$ which runs from -N through zero to +N. Each of these 2N+1 specifications will be referred to as a configuration or composite degree of freedom. The two SU(2) classes are thus as follows

(C.1.) $$\alpha = N_1 - N_2, \qquad -N \leq \alpha \leq N, \qquad N_1 + N_2 = N \tag{1''}$$

(C.2.) $$\beta = N_1' - N_2', \qquad -N \leq \beta \leq N, \qquad N_1' + N_2' = N \tag{2''}$$

where by definition $\alpha = (N_1 - N_2)$ is an SU(2) proton configuration of class (C.1.) in an N parton assembly and similarly for β .

2.2.- SU(3) Configurations.-

If the free parton gas of N+3 quarks and N antiquarks belongs to an octet the product state can be reduced with the aid of the SU(3) invariant tensors $\varepsilon^{\mu\nu\lambda}$, $\varepsilon_{\mu\nu\lambda}$ and δ^{μ}_{ν} in three ways to give a proton

(C.1.) $p \equiv ppn \otimes$ (SU(3) Scalar S_1)

(C.2.) $p \equiv p\bar{\lambda} \otimes (pn\lambda)_- \otimes$ (SU(3) Scalar S_2)

(C.3.) $p \equiv \bar{n}\,\bar{\lambda}\bar{\lambda} \otimes (pn\lambda)_- \otimes (pn\lambda)_- \otimes$ (SU(3) Scalar S_3)

$$(pn\lambda)_- = (pn)_-\lambda + (n\lambda)_- p + (\lambda p)_- n, \qquad (pn)_- = pn - np \quad \text{etc.}$$

$$S_1 = (pn\lambda)_-^m \, (\bar{p}\bar{n}\bar{\lambda})_- \, (\bar{p}p + \bar{n}n + \bar{\lambda}\lambda)^k = S_o(m, k)$$

$$S_2 = (pn)_- \, (\bar{p}\bar{n})_- \, S_o(m-1, k)$$

$$S_3 = \bar{p}p \, (pn)_- \, (\bar{p}\bar{n})_- \, S_o(m-2, k)$$

As in eqs. (1') and (2') (C.1.), (C.2.) and (C.3.) can be written as products of the p's, n's and λ's with integer coefficients

$$\text{(C.1.)} \qquad p \equiv p^{N_1+2}\, n^{N_2+1}\, \lambda^{N_3}\, \bar{p}^{N_1}\, \bar{n}^{N_2}\, \bar{\lambda}^{N_3} \qquad N = N_1 + N_2 + N_3 \tag{3.1}$$

$$\text{(C.2.)} \qquad p \equiv p^{N'_1+2}\, n^{N'_2+1}\, \lambda^{N'_3+1}\, \bar{p}^{N'_1}\, \bar{n}^{N'_2}\, \bar{\lambda}^{N'_3+1} \qquad N-1 = N'_1 + N'_2 + N'_3 \tag{3.2}$$

$$\text{(C.3.)} \qquad p \equiv p^{N''_1+1}\, n^{N''_2+2}\, \lambda^{N''_3+2}\, \bar{p}^{N''_1-1}\, \bar{n}^{N'_2+1}\, \bar{\lambda}^{N''_3+2} \qquad N-2 = N''_1 + N''_2 + N''_3 \tag{3.3}$$

Because conjugate SU(3) representations are inequivalent the three SU(3) configuration classes are different as seen from eqs. (3). A specification of the classes equivalent to eqs. (1'') and (2'') takes the form

$$\text{(C.1.)} \qquad \alpha_1 = N_1 + N_2; \quad 0 \leq \alpha_1 \leq N; \qquad \alpha_2 = N_1 - N_2; \quad -\alpha_1 \leq \alpha_2 \leq \alpha_1; \quad N_1 + N_2 + N_3 = N \tag{4.1}$$

$$\text{(C.2.)} \qquad \beta_1 = N'_1 + N'_2; \quad 0 \leq \beta_1 \leq N-1; \qquad \beta_2 = N'_1 - N'_2; \quad -\beta_1 \leq \beta_2 \leq \beta_1; \quad N'_1 + N'_2 + N'_3 = N-1 \tag{4.2}$$

$$\text{(C.3.)} \qquad \gamma_1 = N''_1 + N''_2; \quad 0 \leq \gamma_1 \leq N-2; \quad \gamma_2 = N''_1 - N''_2; \qquad -\gamma_1 \leq \gamma_2 \leq \gamma_1; \quad N''_1 + N''_2 + N''_3 = N-2 \tag{4.3}$$

For notational convenience the pairs of integers (α_1, α_2) etc, specifying a configuration will be written compactly as

$$\begin{aligned}
\alpha &\equiv (\alpha_1, \alpha_2) \equiv (N_1, N_2, N_3) \\
\beta &\equiv (\beta_1, \beta_2) \equiv (N'_1, N'_2, N'_3) \\
\gamma &\equiv (\gamma_1, \gamma_2) \equiv (N''_1, N''_2, N''_3)
\end{aligned} \tag{5}$$

From the above examples it is clear that the concept of hadronic configurations can be easily generalized so that it is neither necessary nor compelling to retain partons less yet limit the consideration to configurations of albegraic origin only. Therefore independently of any model and of the type of configurations one can associate with every hadron the set of all its possible configurations. This set will be referred to as the configuration ensemble.

3. - STATISTICAL WEIGHT OF A CONFIGURATION AND THE MIXING PROBLEM. -

The task in this section is to define a unique probability function for each of the configurations introduced in the last section. To this end consider an SU(3) configuration of any class in an N parton assembly of the proton. The total energy of the proton as a non-interacting gas of partons is given by

$$E = \sum_{q,k} N_k(q)\, \varepsilon_k(q) \tag{6}$$

where q runs over all quarks and antiquarks, $\varepsilon_k(q)$ the energy of the k-th level of the parton q and $N_k(q)$ the occupation number of the level in question. The occupation numbers satisfy

$$\begin{aligned}
&N_1 + 2 = \sum_i N_i(q_1); \quad N_1 = \sum_j N_j(\bar{q}_1); \quad N_2 + 1 = \sum_k N_k(q_2); \\
&N_2 = \sum_l N_l(\bar{q}_2); \quad N_3 = \sum_r N_r(q_3) = \sum_s N_s(\bar{q}_3)
\end{aligned} \tag{7}$$

The specification of the occupation numbers $N_k(q)$ for all energy levels and all partons consistent with eqs. (6) and (7) defines a dynamical state of the configuration $\alpha \equiv (N_1, N_2, N_3)$. A particular dynamical state can be obtained in many ways since it is determined by the number of partons of each kind in an allowed energy level without stating which ones[7]. The total number of ways of obtaining a particular state will be referred to as usual as the number of complexions (not to be confused with configurations). If $g_k(q)$ is the statistical weight of the k-th energy level of the parton q the total number of complexions of the state with occupation numbers $N_k(q)$ subject to eqs. (6) and (7) is in standard form

$$\overline{W_\alpha}\,(N,j) = \prod_q N(q)\,!\,\prod_k \frac{g_k(q)^{N_k(q)}}{N_k(q)\,!} \tag{8}$$

where j (j=1, 2, 3) stands for the configuration class. We now define the probability $p_\alpha\,(N,j)$ of the configuration $\alpha \equiv (N_1, N_2, N_3)$ of the j-th class as proportional to the total number of complexions obtained from $\overline{W_\alpha}\,(N,j)$ by summing over all values of the occupation numbers $N_k(q)$ consistent with the constraints (6) and (7). Calling this number $W_\alpha\,(N,j)$ we get

$$p_\alpha\,(N,j) = \frac{W_\alpha\,(N,j)}{\sum_{M,k,\beta} W_\beta\,(M,k)}\,; \qquad \sum_{N,j,} p_\alpha\,(N,j) = 1 \qquad j = 1,2,3 \tag{9}$$

The probabilities of the j-th configuration class $p(N,j)$ and of the N-th assembly $p(N)$ follow easily from eq. (9)

$$p(N,j) = \frac{\sum_\alpha W_\alpha\,(N,j)}{\sum_{M,k,\beta} W_\beta\,(M,k)} = \sum_\alpha p_\alpha\,(N,j); \qquad \sum_{N,j} p(N,j) = 1 \tag{10}$$

$$p(N) = \frac{\sum_{j,\alpha} W_\alpha\,(N,j)}{\sum_{M,k,\beta} W_\beta\,(M,k)} = \sum_{j,\alpha} p_\alpha\,(N,j); \qquad \sum_N p(N) = 1 \tag{11}$$

The probability $p_j(N)$ of the j-th class in an N parton assembly for any fixed N is given by

$$p_j(N) = \frac{\sum_\alpha W_\alpha\,(N,j)}{\sum_{k,\beta} W_\beta\,(N,k)} = \frac{p(N\,j)}{\sum_k p(N\,k)}\,; \qquad \sum_j p_j(N) = 1 \tag{12}$$

The value of any function F of scattering, such as structure functions etc., which one would expect to measure is therefore given by

$$\langle F \rangle = \sum_{N,j,\alpha} p_\alpha (N, j) F_\alpha (N, j) = \sum_\alpha p_\alpha F_\alpha \tag{13}$$

where F_α is the value of F in the configuration α and in the last step the configurations have been reordered so that α runs over the entire configuration ensemble so that explicit indication of the configuration class and assembly becomes unnecessary. Eq. (13) agrees with a similar one given by Feynman and Vernon. The measured value of F is thus an average over the configuration ensemble; this averaging will be referred to as configuration mixing. Using eqs. (10) - (12) $\langle F \rangle$ can be written in other useful forms

$$\langle F \rangle = \sum_N p(N) F(N); \qquad p(N) F(N) = \sum_{j,\alpha} p_\alpha (N, j) F_\alpha (N, j) \tag{13.1}$$

$$\langle F \rangle = \sum_{N,j} p(N, j) F(N, j); \qquad p(N, j) F(N, j) = \sum_\alpha p_\alpha (N, j) F_\alpha (N, j) \tag{13.2}$$

$$\langle F \rangle = \sum_{N,j} p_j(N) F_j(N); \qquad p_j(N) F_j(N) = \sum_\alpha p_\alpha (N, j) F_\alpha (N, j) = p(N, j) F(N, j)$$

$$F_j(N) = F(N, j) \sum_k p(N, k) \tag{13.3}$$

From these equations it follows at once that it is wrong to write

$$\langle F \rangle = \sum_N p(N) \sum_j p_j(N) F_j(N) \tag{14}$$

because it involves an incorrect duplication of probabilities since inserting into (14) from (11) and (13.3) gives

$$F = \sum_{N,k,j,\beta,\alpha} p_\beta (N, k) p_\alpha (N, j) F_\alpha (N, j) \tag{15}$$

which has absolutely nothing to do with eq. (13). Eq. (14) was first written down by Llewellyn-Smith[2] in the hope of introducing mixing through the $p_j(N)$. This is unnecessary as mixing is already contained in p(N). Before closing this section we sketch briefly the derivation of some results which will prove useful in the next section. To every configuration $\alpha \equiv (N_1, N_2, N_3)$ there correspond unique mean values $\overline{N}_k(q)$ of the occupation numbers $N_k(q)$ calculable from the method of steepest descents;

$$\overline{N}_k(q) = N(q) \frac{g_k(q)\, e^{-\beta \mathcal{E}_k(q)}}{Z(q)} ; \qquad Z(q) = \sum_k g_k(q)\, e^{-\beta \mathcal{E}_k(q)} \tag{16}$$

where β is related to the temperature T of the assembly and to the saddle point ζ in the Darwin-Fowler method by

$$\beta = \frac{1}{k_B T} = -\log \zeta \tag{17}$$

with k_B the Boltzmann constant. Thus an equivalent specification of a configuration is in terms of the mean values $\overline{N}_k(q)$ which because of eqs. (4) and (7) satisfy

$$\sum_{q,k} \overline{N}_k(q) = \sum_q N(q) = 2N + 4 - j, \qquad j = 1, 2, 3 \tag{18}$$

where q runs over all quarks and antiquarks and j is the class to which the configuration specified by the $\overline{N}_k(q)$ belongs. If the only assemblies realised in the nucleon are those for which $N \to \infty$ it follows at once that the three SU(3) configuration classes coincide asymptotically and we have from eqs. (9)-(12)

$$p(N, j=1) = p(N, j=2) = p(N, j=3); \qquad p_1(N) = p_2(N) = p_3(N) = \frac{1}{3} \tag{19}$$

If the only configurations realizable are those for which $\alpha \equiv (N_1, N_2, N_3) \to \infty$ eqs. (19) remain valid but in addition we have

$$p_\alpha(N, j=1) = p_\alpha(N, j=2) = p_\alpha(N, j=3) \tag{19'}$$

$$p_\alpha(N, j) = p_\beta(N, j) \tag{20}$$

Eqs.(19) and (19') but not (20) are always true for the two SU(2) configuration classes for all N.

4. - ELECTROPRODUCTION SUM RULES AND THEIR SOLUTIONS. -

In the familiar form of the parton model the electroproduction structure function $W_2(\nu, q^2)$ is given by[4, 5]

$$\nu W_2(\nu, q^2) = F_2(\omega) = \sum_N p(N) \sum_i Q_i^2 \omega f_i^N(\omega) \tag{21}$$

where $\omega = q^2/2M\nu$ is the Bjorken variable; Q_i the charge of the i-th parton, $f_i^N(x)$ the probability density that the i-th parton has a fraction x of the nucleon's longitudinal momentum and p(N) the probability of the N-th parton assembly. $f_i^N(x)$ is normalised

$$\int_0^1 dx\, f_i^N(x) = 1 \tag{22}$$

We shall first show that eq. (21), as it stands, is incomplete and incorrect. To this end consider the quark parton model and let the nucleon belong to an octet. Define the functions $f_q^N(x)$, for q running over all quarks and antiquarks, by

$$f_q^N(x) = \frac{1}{N(q)} \sum_1^{N(q)} f_1^N(x); \qquad \int_0^1 dx\, f_q^N(x) = 1 \tag{23}$$

where the summation index (1) covers all partons of type q and the total number N(q) of such partons depends on the configuration and the SU(3) class. The N(q) satisfy

$$\sum_q N(q) = 2N + 4 - j, \qquad j = 1, 2, 3 \tag{18}$$

where j stands for the configuration class. Substituting from (23) into (21) yields

$$F_2(\omega) = \sum_N p(N)\, F_{2\alpha}((N, j); \omega) \tag{24}$$

where

$$F_{2\alpha}((N,j);\omega) = \sum_q Q_q^2 N(q)\, f_q^N(\omega) \tag{25}$$

is the structure function in the configuration α. Eq. (24) or its equivalent eq. (21) is incomplete because there is no summation in it over all distributions of the N(q) consistent with the constraint (18) for fixed N and j = 1, 2, 3[8]. It is also incorrect because it and the equation obtained from it by summing over all values of the N(q) subject to (18)

$$F_2(\omega) = \sum_N p(N) \sum_{j,\alpha} F_{2\alpha}((N\ j);\omega) \tag{26}$$

do not agree with eqs. (13) and with any acceptable method of averaging. To see clearly the error in eqs. (24) and (26) introduce into them the expression of p(N) in eq. (11) to obtain

$$F_2(\omega) = \sum_{N,k,\beta} p_\beta(N,k)\ F_{2\alpha}((N,j);\omega) \tag{24'}$$

$$F_2(\omega) = \sum_{N,k,\beta} p_\beta(N,k) \sum_{j,\alpha} F_{2\alpha}((N,j);\omega) \tag{26'}$$

which are anything but statistical averages. Little doubt then that previous attempts to incorporate mixing in the parton model starting from eq. (21) have been confused and misleading[2,3]. Making use of eqs. (25) and (13) the correct expression for the structure function is

$$F_2(\omega) = \sum_{N,j,\alpha} p_\alpha(N,j)\, F_{2\alpha}((N,j);\omega) = \sum_\alpha p_\alpha\, F_{2\alpha}(\omega) = \langle F_2(\omega)\rangle \tag{27}$$

where in the last expression but one the index α runs over the entire configuration ensemble Eq. (27) is the correct statistical average which accounts for configuration mixing[1].

The structure functions of the proton and neutron may be different for a host of reasons which we need not go into. Denoting with bars all quantities referring to the neutron we have for the proton and neutron structure functions $F_{2p}(\omega)$ and $F_{2n}(\omega)$ respectively

$$F_{2p}(\omega) = \sum_{\alpha} p_{\alpha} F_{2p\alpha}(\omega) = \sum_{N,j,\alpha} p_{\alpha}(N,j) F_{2p\alpha}((N,j);\omega) \tag{28.1}$$

$$F_{2n}(\omega) = \sum_{\alpha} \bar{p}_{\alpha} F_{2n\alpha}(\omega) = \sum_{N,j,\alpha} \bar{p}_{\alpha}(N,j) F_{2n\alpha}((N,j);\omega) \tag{28.2}$$

Consider the electroproduction sum rules[2, 3, 6]

$$I_p = \int_0^1 \frac{d\omega}{\omega} F_{2p}(\omega) = \sum_{N,j,\alpha} p_{\alpha}(N,j) \sum_q Q_q^2 N(q) \tag{29.1}$$

$$I_n = \int_0^1 \frac{d\omega}{\omega} F_{2n}(\omega) = \sum_{N,j,\alpha} \bar{p}_{\alpha}(N,j) \sum_q Q_q^2 \bar{N}(q) \tag{29.2}$$

$$J_p = \int_0^1 d\omega\, F_{2p}(\omega) = \sum_{N,j,\alpha} p_{\alpha}(N,j)\,\omega(N) \sum_q Q_q^2 N(q) \tag{30.1}$$

$$J_n = \int_0^1 d\omega\, F_{2n}(\omega) = \sum_{N,j,\alpha} \bar{p}_{\alpha}(N,j)\,\omega(N) \sum_q Q_q^2 \bar{N}(q) \tag{30.2}$$

where

$$\omega(N) = \int_0^1 \omega\, f_q^N(\omega)\, d\omega = \int_0^1 \omega\, \bar{f}_q^N(\omega)\, d\omega \tag{31}$$

is the average value of the Bjorken variable in an N-parton assembly of the nucleon. Making use of the charge symmetry relations

$$N_1(p) = \bar{N}_2(n); \qquad N_2(p) = N_1(n); \qquad N_3(p) = \bar{N}_3(n) \tag{32}$$

where the subscripts stand for the three kinds of quarks $q_1 \equiv p$, $q_2 \equiv n$, $q_3 \equiv \lambda$, (p) and (n) indicating the number of these quarks in the proton and neutron respectively, and the canonical values of quark charges we get from (29) and (30)

$$\Delta I = I_p - I_n = \frac{1}{3} + \sum_{N,j,\alpha} \frac{8}{9}(N_1(p)\, p_\alpha\ (N,j) - N_2(p)\, \bar{p}_\alpha\ (N,j)) +$$

$$+ \sum_{N,j,\alpha} \frac{2}{9}(N_2(p)\, p_\alpha\ (N,j) - N_1(p)\, \bar{p}_\alpha\ (N,j)) + \tag{33}$$

$$+ \sum_{N,j,\alpha} \frac{2}{9} N_3(p)\,(p_\alpha (N,j) - \bar{p}_\alpha (N,j))$$

$$\Delta J = J_p - J_n = \frac{1}{3}\langle\omega\rangle + \sum_{N,j,\alpha} \frac{8}{9}(N_1(p)\, p_\alpha\ (N,j) - N_2(p)\, \bar{p}_\alpha\ (N,j))\omega(N) +$$

$$+ \sum_{N,j,\alpha} \frac{2}{9}(N_2(p)\, p_\alpha\ (N,j) - N_1(p)\, \bar{p}_\alpha\ (N,j))\omega(N) + \tag{34}$$

$$+ \sum_{N,j,\alpha} \frac{2}{9} N_3(p)\,(p_\alpha\ (N,j) - \bar{p}_\alpha\ (N,j)\omega\ (N)$$

The above equations are greatly simplified if

$$p_\alpha (N,j) = \bar{p}_\alpha (N,j) \tag{35}$$

that is if the probability of a configuration is a property of the nucleon. In this case eqs. (33) and (34) become

$$\Delta I = \frac{1}{3} + \sum_{N,j,\alpha} p_\alpha (N,j)\ D_\alpha\ (N,j) \tag{33'}$$

$$\Delta J = \frac{\langle\omega\rangle}{3} + \sum_{N,j,\alpha} p_\alpha (N,j)\ D_\alpha\ (N,j)\omega(N) \tag{34'}$$

with

$$\langle \omega \rangle = \sum_{N,j,\alpha} p_\alpha (N,j)\, \omega(N) \tag{36}$$

the overall average of the Bjorken variable and

$$D_\alpha (N,j) = \frac{2}{3} (N_1(p) - N_2(p) \tag{37}$$

4.1.- Llewellyn-Smith's Inequality for ΔI.-

Llewellyn-Smith[2] has derived the following inequality for ΔI

$$|\Delta I| \leqslant 1/3 \tag{38}$$

Later shown to be invalid. It is interesting to go through the derivation of (38) to better understand the assumptions involved. Smith considers SU(2) configurations and writes

$$\Delta I = \sum_N p(N) \sum_{J=1}^{2} p_j(N) \Delta I_j(N) \tag{39}$$

with

$$\Delta I_j(N) = \sum_q Q_q^2 N(q) - \sum_q Q_q^2 \bar{N}(q) = \frac{1}{3} + \frac{2}{3} (N_1(p) - N_2(p)) \tag{40}$$

and for the two configuration classes j = 1, 2 sets

$$N_1(p) - N_2(p) = 0 \quad j = 1; \qquad N_1(p) - N_2(p) = -1 \quad j = 2 \tag{41}$$

so that (40) yields

$$\Delta I_1(N) = 1/3; \qquad \Delta I_2(N) = -1/3 \tag{42}$$

Substituting from (42) into (39) and defining

$$\mathcal{E} = \sum_N p(N)\, p_2(N) \qquad 0 \leqslant \mathcal{E} \leqslant 1 \tag{43}$$

yields

$$\Delta I = \frac{1}{3} - \frac{2}{3}\varepsilon \tag{44}$$

which together with (43) gives the inequality (38). To begin with eq. (39) is wrong because it is not a meaningful statistical average as it duplicates probabilities (c.f. eqs. (14) and (15)); the values of $N_1(p)-N_2(p)$ in eqs. (41) are completely arbitrary and since SU(2) configuration classes are equivalent

$$p_1(N) = p_2(N) = 1/2 \qquad \text{(c.f. eqs. (19) and (20)).}$$

so that eq. (44) should give $\Delta I = 0$ and the inequality is trivially satisfied.

4.2.- Gourdin's Solutions.-

Since experiments exclude simple parton assemblies it has been argued that neutral vector mesons (gluons) which could provide binding of the charged partons (quarks) are necessary. Treating the partons (both charged and neutral) as free is then a first approximation. Gourdin[3] has considered this possibility and tries to extract information on the average values of the Bjorken variable $\omega = q^2/2M\nu$ and the ratio of the total number of neutral partons (gluons) to charged ones from the sum rules in eqs. (30). These averages are understood to be taken over all parton assemblies; however values for them are given for the three SU(3) classes (C.1.), (C.2.) and (C.3.). It is theoretically possible to define such averages but they cannot be related to or extracted from experimental data because these latter, by their very nature, are averages over all parton configurations, classes and parton assemblies etc. To see the assumptions involved we now consider the derivation of Gourdin's solutions for these averages.. Let N_j ($j = 1, 2, 3$) be the total number of partons of all kinds in a parton assembly of configuration class j; the parameter η_j defined by,

$$1 - \eta_j = \frac{1}{N_j} \sum_q N(q) = \frac{1}{N_j}(2N+3) \tag{45}$$

where q runs over all charged partons, is the fraction of gluons in the nucleon when it is in the configuration class j. It is also assumed that the nucleon's longitudinal momentum is equally distributed among the partons so that the average longitudinal momentum per parton in the configuration class j is

$$\omega_j(N) = \int_0^1 dx\, f_q^N(x)\; x = \frac{1}{N_j} \qquad (46)$$

where $f_q^N(x)$ is the density function in eq. (23). Using eq. (24), which as shown previously is not a meaningful statistical average, the following averages

$$\left\langle \frac{N(q)}{N_j} \right\rangle = \sum_N p(N)\, \frac{N(q)}{N_j} \qquad (47)$$

are defined for all charged partons in the configuration class j. Making use now of eqs. (47), (46), (45), (32), (25) and (24) yields for the sum rules J_p and J_n.

$$J_p = \frac{1}{9}\left(1 - \langle \eta_j \rangle\right) + \frac{2}{3}\langle \omega_j \rangle + \frac{2}{3}\left\langle \frac{N_1(p)}{N_j} \right\rangle \qquad (48.1)$$

$$J_n = \frac{1}{9}\left(1 - \langle \eta_j \rangle\right) + \frac{1}{3}\langle \omega_j \rangle + \frac{2}{3}\left\langle \frac{N_2(p)}{N_j} \right\rangle \qquad (48.2)$$

where all angular brackets are averages with the weight p(N) over all N as in eq. (47). From eqs. (48) one gets

$$\Delta J = J_p - J_n = \frac{1}{3}\langle \omega_j \rangle + \frac{2}{3}\left\langle \frac{N_1(p) - N_2(p)}{N_j} \right\rangle \qquad (49)$$

On the left hand sides of these equations are experimental quantities. J_p, J_n and ΔJ while the right hand sides contain parameters which depend on the SU(3) configuration class. The two sides cannot therefore be compared with each other because an experimental quantity is already an average over all SU(3) configurations and classes thereof. Granted one can carry through with the derivation the following values are found for $\langle \eta_j \rangle$ and $\langle \omega_j \rangle$ by making specific choices for the parameter $\langle (N_1(p) - N_2(p))/N_j \rangle$ in the three configuration classes (C.1.), (C.2.) and (C.3.)

$$\text{(C.1.)} \quad j = 1. \qquad \left\langle \frac{N_1(p)}{N_j} \right\rangle = \left\langle \frac{N_2(p)}{N_j} \right\rangle \neq \left\langle \frac{N_3(p)}{N_j} \right\rangle \qquad (50.1)$$

Substituting from (50) into (45) yields

$$\left\langle \frac{N_2(p)}{N_j} \right\rangle = \frac{1}{6}\,(1 - \langle \eta_j \rangle) - \frac{1}{2}\,\langle \omega_j \rangle$$

making use of which together with (50) in eqs. (48) and (49) gives

$$\left\langle \frac{1}{N_j} \right\rangle = \langle \omega_j \rangle = 3\Delta J; \qquad \langle \eta_j \rangle = 1 - \frac{9}{2} J_n \tag{51.1}$$

(C.2.) $\quad j = 2 \qquad \left\langle \frac{N_1(p)}{N_j} \right\rangle = \left\langle \frac{N_2(p)}{N_j} \right\rangle = \left\langle \frac{N_3(p)-1}{N_j} \right\rangle$ (50.2)

Going through the same steps as those under (C.1.) one finds

$$\left\langle \frac{1}{N_j} \right\rangle = \langle \omega_j \rangle = 3\Delta J; \qquad \langle \eta_j \rangle = 1 - \frac{9}{2} J_n - 3\Delta J \tag{51.2}$$

(C.3.) $\quad j = 3 \qquad \left\langle \frac{N_1(p)}{N_j} \right\rangle = \left\langle \frac{N_2(p)-1}{N_j} \right\rangle = \left\langle \frac{N_3(p)-2}{N_j} \right\rangle$ (50.3)

Using (50.3) in eqs. (48) and (49) as before gives

$$\left\langle \frac{1}{N_j} \right\rangle = \langle \omega_j \rangle = -\,3\Delta J; \qquad \langle \eta_j \rangle = 1 - \frac{9}{2} J_n \tag{51.3}$$

Because experimentally $\Delta J > 0$, the negative value of $\langle 1/N_j \rangle$ in eq. (51.3) led to the conclusion that the configuration class (C.3.) is excluded experimentally[3]. Actually this erroneous conclusion or any other of its kind was unavoidable since eqs. (51.1) - (51.3) originated from wrong premises. The choices made in eqs. (50.1) - (50.3) are completely arbitrary and not physically motivated although they could be suggested for a particular configuration by the form of eqs. (3.1) - (3.3). If gluons were really necessary they could be introduced as follows:

Let $\eta_\alpha(N, j)$ be the fraction of gluons present in an SU(3) configuration α; in place of eq. (45) we have:

$$1 - \eta_\alpha(N,j) = \frac{1}{N_t}\sum_q N(q) = \frac{1}{N_t}(2N+3) \tag{45'}$$

where N_t is the total number of partons of all kinds, both charged and neutral, and in place of (46)

$$\omega(N) = \int_0^1 dx\, f_q^N(x)\, x = \frac{1}{N_t} \tag{46'}$$

Eq. (47) now becomes

$$\left\langle \frac{N(q)}{N_t} \right\rangle = \sum_{N,j,\alpha} p_\alpha(N,j)\frac{N(q)}{N_t} \tag{47'}$$

This and other ensemble averages are not configuration class parameters. Substituting from (47') , (46'), (45') and (32) into (30) gives (35)

$$I_p = \frac{1}{9}(1 - \langle\eta\rangle) + \frac{2}{3}\langle\omega\rangle + \frac{2}{3}\left\langle \frac{N_1(p)}{N_t} \right\rangle \tag{48.1'}$$

$$J_n = \frac{1}{9}(1 - \langle\eta\rangle) + \frac{1}{3}\langle\omega\rangle + \frac{2}{3}\left\langle \frac{N_2(p)}{N_t} \right\rangle \tag{48.2'}$$

and for ΔJ,

$$\Delta J = J_p - J_n = \frac{1}{3}\langle\omega\rangle + \frac{2}{3}\left\langle \frac{N_1(p) - N_2(p)}{N_t} \right\rangle; \qquad \langle\omega\rangle = \left\langle \frac{1}{N_t} \right\rangle \tag{49'}$$

or in an equivalent form

$$\Delta J = J_p - J_n = \frac{1}{3}\langle\omega\rangle + \langle D\omega\rangle$$

$$\langle D\omega\rangle = \sum_{N,j,\alpha} p_\alpha(N,j)\, D_\alpha(N,j)\,\omega(N) \tag{34''}$$

$$D_\alpha(N,j) = \frac{2}{3}(N_1(p) - N_2(p))$$

In eqs. (48') and (49') there is no reference to configuration classes. All of the quantities in the right hand sides of these equations are parameters of the model to be determined by further inputs. Without these inputs nothing can be said about them and hence must be regarded as unknown. Configuration mixing has therefore considerably reduced the predictive power of the parton model and the inclusion of gluons adds one more parameter to the unknowns.

The assumption of equal distribution of the nucleon's longitudinal momentum among the partons reduces the total number of unknowns by one as it establishes a relation between $\langle \eta \rangle$, $\langle N/N_t \rangle$ and $\langle \omega \rangle =$ $= \langle 1/N_t \rangle$. To see this take the ensemble average of $(2N+3)\,\omega(N)$ and $1-\ \eta_\alpha(N,j)$ and obtain from (45') and (46')

$$\langle \eta \rangle + 2 \left\langle \frac{N}{N_t} \right\rangle = 1 - 3\langle \omega \rangle = 1 - 3 \left\langle \frac{1}{N_t} \right\rangle \tag{52}$$

where $N = N_1+N_2+N_3$ is the total number of antiquarks. Since $N \geqslant 0$ and $\eta_\alpha(N,j) \geqslant 0$ their average values are constrained thus

$$\langle \eta \rangle \leqslant 1-3\langle \omega \rangle ; \qquad \left\langle \frac{N}{N_t} \right\rangle \leqslant 1/2\,(1-3\langle \omega \rangle) \tag{53}$$

To have an idea of these upper limits let us compute $\langle \omega \rangle$. Multiply ΔI in (33') by $\langle \omega \rangle$ and subtract from (34') to get

$$\Delta J = \Delta I \cdot \langle \omega \rangle + \sum_{N,j,\alpha} p_\alpha(N,j)\,(\omega(N) - \langle \omega \rangle)\, D_\alpha(N,j) \tag{54}$$

$\langle \omega \rangle$ can be calculated from this last equation in two ways. In a first approximation identify the terms in eq. (54) as follows

$$\Delta I . \langle \omega \rangle = \int_{1/12}^{1} d\omega\, (F_{2p}(\omega) - F_{2n}(\omega)) \tag{55.1}$$

$$\sum_{N,j,\alpha} p_\alpha(N,j)\,(\omega(N) - \langle \omega \rangle)\, D_\alpha(N,j) = \int_{0}^{1/12} d\omega\, (F_{2p}(\omega) - F_{2n}(\omega)) \tag{55.2}$$

where the cutt-off $\omega = 1/12$ corresponds to the lowest value of ω for which the neutron data have been analysed[6]. Inserting the experimental numbers in (55.1) gives

$$\langle\omega\rangle = \Delta J/\Delta I = \begin{matrix} 0.3: & I = 0.13 \\ 0.2: & I = 0.19 \end{matrix} \tag{56}$$

The inequalities (53) then become

$$\langle\eta\rangle \lessgtr \begin{matrix} 0.1: & I = 0.13 \\ 0.4: & I = 0.19 \end{matrix}$$

$$\left\langle\frac{N}{N_t}\right\rangle \lessgtr \begin{matrix} 0.05: & I = 0.13 \\ 0.2\ : & I = 0.19 \end{matrix} \tag{57}$$

Next note that the integral in (55.2) is over the diffractive sector of the scaling curves $F_{2p}(\omega)$ and $F_{2n}(\omega)$ where the proton and neutron look very much alike; differences between the two particles certainly tend to disappear for large N parton assemblies[5]. As a result large N assemblies dominate the ensemble sum in the left hand side of (55.2). We can thus use the results of eqs. (19), (19') and (20), valid for $N \to \infty$, and define through (33') and (34') asymptotic values of ΔI and $\langle\omega\rangle$, call them ΔI_a and $\langle\omega\rangle_a$, in this limit.

$$\Delta I_a = 1/3; \qquad \Delta J = \Delta I_a \langle\omega\rangle_a; \qquad \omega\,(N \to \infty) = \langle\omega\rangle_a \tag{58}$$

Substituting from (58) into (54) and making use of (33') gives

$$\langle\omega\rangle = \frac{\Delta J(2\Delta I_a - \Delta I)}{(\Delta I_a)^2} = \begin{matrix} 0.19: & I = 0.13 \\ 0.17: & I = 0.19 \end{matrix} \tag{56'}$$

The inequalities (53) now become

$$\langle \eta \rangle \leq \begin{array}{ll} 0.42 & I = 0.13 \\ 0.49 & I = 0.19 \end{array}$$

$$\left\langle \frac{N}{N_t} \right\rangle \leq \begin{array}{ll} 0.21 & I = 0.13 \\ 0.25 & I = 0.19 \end{array} \qquad (57')$$

From eqs. (56) and (56') it follows at once that the assumption of equal distribution of the longitudinal momentum of the nucleon among the partons, that is $\langle \omega \rangle = \langle 1/N_t \rangle$, implies that the fraction of the three valence quarks to the total number of partons within the nucleon is about 50%

$$\left\langle \frac{3}{N_t} \right\rangle = 3 \left\langle \frac{1}{N_t} \right\rangle = 3 \langle \omega \rangle \simeq 0.54 \qquad (59)$$

The background sea of quarks and antiquarks[5] is thus simply inexistent, the maximum fraction of antiquarks of this sea from eqs. (57) and (57') being

$$\left\langle \frac{N}{N_t} \right\rangle \simeq \left\langle \frac{1}{N_t} \right\rangle \qquad (60)$$

If a quark-antiquark sea is excluded by the simplicity assumption $\langle \omega \rangle = \langle 1/N_t \rangle$, an interesting structure emerges if gluons are allowed to take its place. According to eqs. (57), (57') and (59) it is possible to accommodate an equal number of valence quarks and gluons within the nucleon. Disposing the former at the vertices of a triangle and the latter as the sides gives rise to a rubber band structure of the nucleon. Such a structure is widely used in the dual resonance model although what has just been said cannot pass as an indication of a necessity for this structure of the hadron.

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(7) - We shall not worry about the appropriate statistics of the partons and for simplicity take them to the bosons.

(8) - By definition p(N) is the probability of the N parton assembly so that the summation over N in eq. (24) does not implicitly include those over j and α because conventional parton models do not know of, or at least ignore, the existence of these.