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INFN/AE-71/2
10 Febbraio 1971

C. Bernardini: CONTINUOUS CONTRIBUTION FROM RADIATIVE PRODUCTION OF VECTOR MESONS IN e^+e^- REACTIONS. -

1. - The cross section for

$$e^+ e^- \rightarrow V + \gamma \tag{1}$$

where V is a $J^{PL} = 1^{--}$ vector (like ρ^o, ω, ϕ) can be computed in the peaking approximation when the total energy $2E \equiv \sqrt{S}$ of the e^+e^- pair is larger than the central mass m_V:

$$S - m_V^2 \gg \Gamma_V^2 \tag{2}$$

Here Γ_V is the width of the V line.

Here we shall show that cross sections in the range 10^{-33} - $- 10^{-32}$ cm^2 behaving like $1/S$ are obtained for the best known mesons. Also, some peculiarities of the angular distribution of the decay products of the V will be illustrated.

2. - The probability of emission of a photon having energy k is

$$P(k)\, dk = 2 \times \frac{2\alpha}{\pi} L(k, E) \frac{dk}{k}$$

where an extra factor 2 accounts for the possibility that either the e^+ or the e^- radiate before annihilation. The factor L is

$$L(k, E) = \frac{E^2 + (E-k)^2}{2E^2} \ln \frac{E}{m_e}$$

When k satisfies the kinematic relation

$$k = E - \frac{m^2}{4E}$$

a vector meson having the mass m can be produced by annihilation of the surviving pair. The total cross section for

$$e^+ e^- \rightarrow V$$

at mass m can be written (near the peak)

$$\sigma_V(m^2) = \sigma_V(m_V^2) \frac{m_V^2 \Gamma_V^2}{(m^2 - m_V^2)^2 + m_V^2 \Gamma_V^2}$$

where $\sigma_V(m_V^2)$ is the peak cross section. Therefore, the cross section for reaction (1) at a given mass will be

$$\text{(3)} \qquad d\sigma_{\gamma V}(m^2) = \sigma_V(m^2)\, P\left[k(m^2)\right] \frac{dk}{dm^2}\, dm^2$$

We will now integrate over the peak assuming that (2) is valid and thus computing the slow varying P(k) at $k = k_V \equiv k(m_V^2)$ (clearly no serious problem is associated with the infrared catastrophe). Integration gives:

$$\sigma_{\gamma V} = 4\alpha \frac{m_V \Gamma_V}{S} \sigma_V(m_V^2) \frac{L(k_V, E)}{1 - \frac{m_V^2}{S}}$$

When E ranges from .7 to 2.0 GeV the factor $L / 1-(m_V^2/S)$ varies from 5.5 to 4.5 for the most unfavourable case of the ρ meson. We shall fix this factor at 5 since we do not pretend a better than 10% approximation for this naive calculation.

Also, the cross section at peak $\sigma_V(m_V^2)$ is usually written

$$\sigma_V(m_V^2) = \frac{12\pi}{m_V^2} \frac{\Gamma_{V \rightarrow e^+e^-}}{\Gamma_V}$$

corresponding to a definition of the width $\Gamma_{V \rightarrow e^+e^-}$ for lepton-decay of the V. Therefore

$$\sigma_{\gamma V} \approx \frac{240\alpha\pi}{S} \frac{\Gamma_{V\to e^+e^-}}{m_V}$$

or, by introducing the usual γ-V coupling constants g_V (to avoid confusion, use

$$\Gamma_{V\to e^+e^-} = \frac{4\pi\alpha^2}{3} \frac{m_V}{g_V^2}),$$

the $\sigma_{\gamma V}$ can be written

$$\sigma_{\gamma V} \cong 320 \frac{\pi^2\alpha^3}{S} \frac{1}{g_V^2}$$

or, for numerical use:

$$\sigma_{\gamma V} = \frac{4\pi}{g_V^2} \frac{1}{E_{GeV}^2} 10^{-32} \text{cm}^2$$

This formula shows the 1/S dependence of $\sigma_{\gamma V}$ and, by using the Orsay results(1), values in the range $10^{-33} \div 10^{-32}$ are found at E = 1 GeV for the ρ^o, ω, ϕ cases. Detailed results are summarized in Table I.

Note that $\sigma_{\gamma V}$ is nearly independent on V's parameters other than the coupling constant g_V and that the total hadronic cross section via radiative vector meson production is $\sim \sum_V 1/g_V^2$.

TABLE I

V	$g_V^2/4\pi$	$\sigma_{\gamma V}$(E = 1 GeV)
ρ^o	2.0 ± 0.1	5×10^{-33} cm^2
ω	14.8 ± 2.8	7×10^{-34} cm^2
ϕ	11.0 ± 1.6	1×10^{-33} cm^2
$\rho^o + \omega + \phi$		6.7×10^{-33} cm^2

3.- The most important final states from the decay of known V mesons will be

$$\rho^0 \to \pi^+\pi^-$$
$$\omega \to \pi^+\pi^-\pi^0,\ \pi^0\gamma$$
$$\phi \to K^+K^-,\ K_1^0K_2^0,\ \pi^+\pi^-\pi^0$$

Using the known branching ratios[2] we compute the cross sections

$$\sigma_{\gamma\pi^+\pi^-} = \sigma_{\gamma\rho}$$
$$\sigma_{\gamma\pi^+\pi^-\pi^0} = 0.9\,\sigma_{\gamma\omega} + 0.2\,\sigma_{\gamma\phi}$$
$$\sigma_{\gamma\pi^0} = 0.1\,\sigma_{\gamma\omega}$$
$$\sigma_{\gamma K^+K^-} = 0.5\,\sigma_{\gamma\phi}$$
$$\sigma_{\gamma K_1^0K_2^0} = 0.3\,\sigma_{\gamma\phi}\ (\approx \sigma_{\gamma\,\pi^+\pi^-\pi^+\pi^-\pi^0})$$

The detection efficiency problem depends very much on the apparatus and we will only consider now the simple case of the two body decay

$$V \to P + P$$

where P is a charged pseudoscalar. Since V moves along the beams with velocity

$$\beta(m^2) = \frac{S - m^2}{S + m^2}$$

the two P-particles will be coplanar with the beams but non-collinear. The angular distribution in the V center of mass is $\sim \sin^2\theta_{CM}$ where θ_{CM} is the angle from the e^+e^- line.

The general formula for relating θ_i, the angle of one of the P in the lab with respect to the e^+e^- line, to θ_{CM}, the V mass m and S is

$$\operatorname{tg}\theta_i = \frac{2m\sqrt{S}\,\lambda\sin\theta_{CM}}{\pm\lambda(S+m^2)\cos\theta_{CM} + (S-m^2)}$$

with

$$\lambda = \left(1 - \frac{4m_p^2}{m^2}\right)^{1/2}$$

The $\pm$ sign in the denominator refers to the two cases of CM momentum having a beam-component antiparallel or parallel to the CM velocity. When $m_p \ll m$, $S \gg m^2$ (like for the ρ^0 at 1 GeV)(3)

$$\mathrm{tg}\,\theta_i \approx \frac{2m}{\sqrt{S}} \begin{cases} \mathrm{tg}\,\theta/2 \\ \\ +\mathrm{cotg}\,\theta/2 \end{cases} \qquad (\theta \text{ for } \theta_{CM})$$

On the other hand, when $\lambda \to 0$ (like in the $\phi \to K^+K^-$ case)

$$\mathrm{tg}\,\theta_i \approx \frac{2m\sqrt{S}\,\lambda}{S-m^2}\sin\theta_{CM}$$

The efficiency of the apparatus has to be computed for a given mass m. Call it $\eta(m^2)$, then the effective measured cross section will be

$$\sigma_{\gamma PP} = \int \eta(m^2)\, d\sigma_{\gamma PP}(m^2)$$

where

$$d\sigma_{\gamma PP}(m^2) = d\sigma_{\gamma V}(m^2)\,\frac{\Gamma_{V \to PP}}{\Gamma_V}$$

and $d\sigma_{\gamma V}$ is given by formula (3).

Also, threshold problems for particle detection could be important and the angle dependence of the P's momenta can be learned from

$$|\vec{P}_i| = \frac{m\lambda \sin\theta}{2\sin\theta_i}$$

Eventually, the problem of detecting a forward or backward γ in coincidence should be considered. This problem is presumably difficult because of beam-gas or beam-beam bremsstrahlung but the requirement that the γ energy should be around

$$k_V = E - \frac{m_V^2}{4E}$$

should help.

I warmly thank the experimental groups at Adone and Bruno Touschek for many stimulating conversations.

REFERENCES AND FOOTNOTES. -

(1) - See for example the recollection by J. Haissinski, Production of V-mesons in e^+e^- colliding beams, Argonne Conference May 1969. Any other Orsay data recollection will do.

(2) - Particle Data Group, Phys. Letters 33 B, (1970).

(3) - The minimum angle Δ between the two charged tracks is readily obtained corresponding to $\theta_{CM} = \pi/2$. In the approximation for the ρ case

$$tg\frac{\Delta}{2} \approx \frac{2m}{\sqrt{S}} = \frac{m}{E}$$

and, at $E = 1\,GeV$, $\Delta \approx 75^o$. In general

$$tg\frac{\Delta}{2} = \frac{2m\sqrt{S}\,\lambda}{S-m^2}$$

The case $\phi \to K^+K^-$ corresponds to small angle K pairs, hardly detectable with large angle telescopes.