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HADRON DEUTERON INELASTIC SCATTERING

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1. - INTRODUCTION

It is well known that provided the energy is high enough, the scattering of a particle off a nucleus, can be described by the eikonal approximation and the hypothesis of additivity of phase shifts ([1]). The theory should work only when the energy is so high that the geometrical shadow of the single nucleon in the nucleus is a good approximation for the real diffractive shadow and the composite system can be thought as a set of balls fixed in space, but recent theoretical experiments have shown that the theory gives good results even at low energy ([2]). There is some doubt about the theoretical basis of these applications, but one can say at least that Glauber approximation is an order of magnitude estimate of the low energy scattering.

I am here interested in the treatment of the special problem of the deuteron break up. This process enters in the cathegory of the incoherent elastic scattering: actually it is the only incoherent elastic process possible in the case of a deuteron as target nucleus. To begin with, let us define the following notation; the index 1 refers to the target deuteron; 2, 3 respectively to the incoming and the scattered particle; 4, 5 to the unbound nucleon pair in the final state. The hypothesis of addivity of phase shifts reads in this case

$$(1) \qquad \chi(\vec{b},\vec{s}) = \chi_4\ (\vec{b}-\vec{s}/2) + \chi_5\ (\vec{b}+\vec{s}/2)$$

where b is the impact parameter

s is the projection of the relative coordinate on a plane orthogonal to the beam direction.

The recipe is to insert this expression of the phase shift in the eikonal form the scattering amplitude

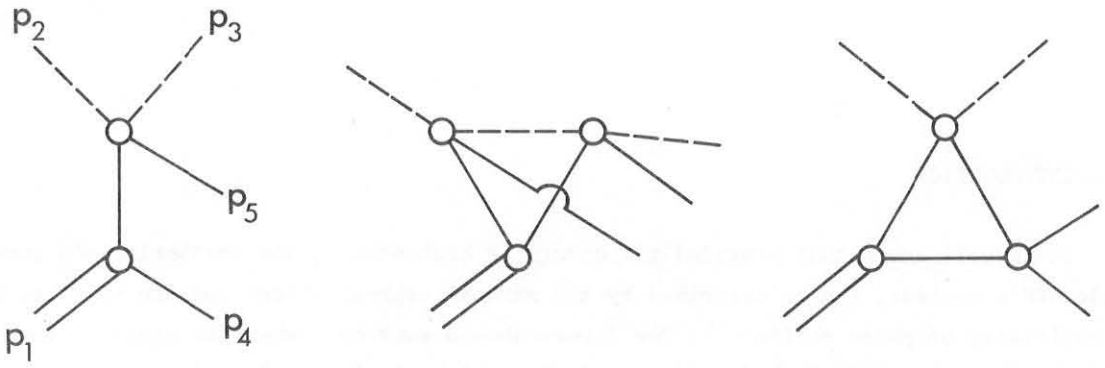
$$(2) \qquad f(\vec{\Delta}) = \frac{ik}{2\pi}\int e^{i\vec{\Delta}\vec{b}}\left[e^{i\chi(\vec{b},\vec{s})}-1\right]\, d^{(2)}\vec{b}$$

where $\vec{\Delta}$ is the momentum transfer

$\vec{k}$ the momentum of the incident particle in the laboratory system

and to sandwich the scattering operator between the initial and final state. In this case the final state is a scattering state of the unbound pair. So in this type of process we have two types of corrections to the so called PWIA: one is the double scattering or shadow correction and the other is the so called final state interaction.

These two corrections can be represented by the two symbolic Feynmann diagrams



For simplicity of treatment I will study separately the two corrections: that is, first of all I will switch off the secondary interaction to study the double scattering corrections and afterwards I will neglect the possibility of second order primary interaction to study the final state interaction.

2. - DOUBLE SCATTERING

In the hypothesis of no influence of the interaction in the final state, we can write the following formula for the break up amplitude

$$(3) \qquad \langle \vec{q}| F|\psi_0\rangle = f_4(\Delta)\psi_0(\vec{q}+\vec{\Delta}/2)+f_5(\Delta)\psi_0(\vec{q}-\vec{\Delta}/2)+\frac{i}{2\pi k}\int \psi_0(\vec{t}+\vec{q}) \times$$

$$\times f_4(\vec{t}+\vec{\Delta}/2) f_5(\vec{t}+\vec{\Delta}/2)\ d^{(2)}t$$

where $\vec{q} = (\vec{p}_4 - \vec{p}_5)/2$ is the C.M. momentum of the two nucleons
$\psi(p)$ is the deuteron wave function in the momentum space.

There is a fundamental difference between the elastic and the quasi elastic scattering on deuteron: while in the first case we have two bodies in the final state and 1 variable is sufficient to fix the final state, in the latter one we need to specify 4 independent quantities to define completely the final state. We could for instance, specify the relative momentum between the outgoing nucleons as for instance in the above formula q and the scattering angle, but any other choice is equally possible. Therefore to have a complete idea of the situation one should be able to draw at fixed scattering angle a quadridimensional plot. But one can see quite well what is going on even with a simple ordinary bidimensional plot: the idea is to take for instance as variable the momentum of the proton p_5 in the lab system and fix the azimuth and colatitude of such momentum with respect to the direction of the incident beam, in such a way to close

the triangle of conservation of momentum for the corresponding two body process on the nucleon of the target, supposed fixed in space and free. If $\varphi_5 = \pi$ and $\vartheta_5 = \vartheta_{el}$ and the scattering angle is fixed, the pattern shown in Fig. 1, 2, 3 show off.

The first peak is called the spectator peak since here the nucleon 4 is interacting with the incident particle while the nucleon 5, on whose momentum we plot the difference cross section, is assumed to assist undisturbed to the process. The second peak is called quasi elastic peak because is centered just on the elastic peak one should see, filling the target with hydrogen.

The physical distinction between the two peaks is clearly justified only if we can distinguish the two nucleons (a neutron-proton pair): otherwise the two peaks will represent exactly the same process seen on two different variables, the spectator momentum or the recoiling nucleon momentum.

At small momentum transfers the two peaks overlap considerably and this overlapping will decrease violently with the momentum transfer. But at high momentum transfer the pattern will change, with the inclusion of an other peak, in the middle of the two representing the double scattering contribution. While the spectator peak appears for all angles almost with the same height, this is not true for the quasi elastic peak which is strongly depressed far from the ϑ_5 of the recoil proton in the elastic process.

The previous considerations are visualized in the Fig. 1, 2, 3. In the first three figures the spectator and the quasi elastic peak are shown at the momentum transfers -.1, -.6, - 1.1 for the quasi elastic scattering of π from deuterium at 9 GeV/c.

As it is possible to see from formula (2) the double scattering is strongly dependent on the local properties of the wave function, because the argument of the wave function inside the integral contains the external variable $\bar{q}$. One can see immediately that if $\bar{q}$ is not lying in the plane orthogonal to the incident beam, the argument of the wave function can never be zero; therefore the double scattering integral is strongly suppressed unless the longitudinal part of the relative momentum is reduced to zero.

These are the specific prediction of the theory which give the possibility to extract the differential cross section of various particles on neutron targets, taking in proper account the double scattering correction. There is in the literature one specific case in which the prediction of the theory have been checked: that is the scattering of protons of 19.2 GeV/c on deuteron ([5]).

In this experiment the proton is detected at various laboratory angles (40÷65 mr) and its energy spectrum is measured with a spectrometer. This spectrum shows two peaks: the one at lower energies corresponds to the sum of the spectator and the quasi elastic peak integrated an all variables except the energy, the other at higher energies corresponds to the dcuble scattering.

This spectrum has been calculated for the angle by Straumann and Wilkin ([6]) using the analytical expression for the double differential cross section $\frac{d\sigma}{d\Omega_3 ds_{45}}$ which is obtained using the one gaussian parametrization of the wave function. The result is re-

ported in Fig. 4 and the discrepancy in the normalization is probably due to the uncertainty in the experimental determination of the absolute value of the cross section.

The same spectrum at all the available angles was calculated by R.J. Glauber, Kofoed Hansen and Margolis ([7]) using the saddle point method of integration and again the one gaussian approximation of the wave function. The calculation which consist of the integration of the theoretical cross section times the resolution function, shows reasonable agreement with the experimental data. The sensitivity of the theoretical results to the various parameters has been examined: the main results are: the double scattering peak shows high sensitivity to the parameter of the wave function and the minimum between the two peaks is sensitive to the d-wave percentage and to the real part of the amplitude (Fig. 5).

3. - FINAL STATE INTERACTION

The so called final state interaction has a long and venerable history in the field of low energy nuclear physics. It is very well known, for instance, that if the relative energy between the two nucleon is very low, the cross section of the process is enhanced, due to the so called energy resonance in the singlet S state. There is some enhancement due to the low binding energy of the deuteron and therefore to the nearness of the deuteron pole to the physical region. The selection rules for the break up process will determine which one of the two is dominant. The first type of enhancement can be described by the simple formula due to Watson ([8])

$$d\sigma \sim \frac{q^2 dq}{q^2+a^2} \tag{4}$$

where a is the scattering length of the interacting system. This formula is valid if the cross section of the volume of the primary interaction is small with respect to the cross section of the secondary interaction. This is true even for the deuteron, which is quite large (4F), when a is much larger. This enhancement was found experimentally for instance in electron scattering ([9]) (Fig. 6) and has been used in neutron deuteron scattering ([10]) at low energy to measure the scattering length of the neutron - neutron system. But for high energy scattering (above 1 GeV/c) the energy resolution is such that the final state interaction peak is completely distorted by the energy spread and will appear only at very low momentum transfer (for instance for π-mesons of 9 GeV/c at -.1 $(GeV/c)^2$). For higher momentum transfer the final state interaction will play as slowly varying background which has to be subtracted to have a real information on the cross section on neutron. I will recall here the traditional method to take in account this interaction and I will report some results for electron scattering: the case of electron scattering is not substantially different. The cross section will be certainly

lower of five order of magnitudes and the spin structure of the amplitude will be different but the main features of the final state interaction effect are the same as in hadron scattering.

Neglecting as said before the double scattering contribution we find for the break-up amplitude

$$T = f_4(\Delta) \int \psi_f(\vec{r})\, e^{i\vec{\Delta}/2\cdot\vec{r}}\, \psi_0(\vec{r})\, d^3r + f_5(\Delta) \int \psi_f(\vec{r})\, e^{-i\vec{\Delta}/2\cdot\vec{r}}\, \psi_0(r) d^3r \tag{5}$$

where ψ_f if the scattering wave function of the nucleon pair and the particle nucleon amplitudes are

$$f_i = a_f + b_f\, \vec{\tau}\cdot\vec{\tau}_i + i(a_g + b_g\, \vec{\tau}\cdot\vec{\tau}_i)\, \vec{\sigma}_i \cdot \hat{n} \tag{6}$$

and the matrix element for the singlet and the triplet state for the final nucleon pair are

1) for the spin singlet state

$$\langle 0\ 0|T|\nu\ 0\rangle = \frac{1}{q}\, \delta_{\nu 0}\, i\, a_g\, \frac{1}{\pi\sqrt{2}} \sum_{\ell=0}^{\infty} (2\ell+1)[1-(-)^{\ell}] P_\ell(\cos q\hat{\Delta}) \int_0^\infty \psi_\ell^{(0)}(qr)\, j_\ell(\Delta r/2) u(r) dr \tag{7.a}$$

$$\langle T_3\ 0|T|\nu\ 0\rangle = \frac{1}{q}\, \delta_{\nu 0}\, i\, b_g\, \frac{1}{\pi\sqrt{2}} \langle T_z \tau'_z|\vec{\tau}_1\vec{\tau}_4|\tau_z\ 0\rangle \sum_{\ell} (2\ell+1) [1+(-)^{\ell}] P_\ell(\cos q\hat{\Delta}) \int_0^\infty \psi_\ell^{(0)}(qr) j_\ell(\Delta r/2) u(r) dr \tag{7.b}$$

2) for the spin triplet state

$$\langle 0\ \nu'|T|\nu\ 0\rangle = \frac{1}{q}\sqrt{8} \sum_{\substack{\ell\ \ell' \\ m\ m'}} (-i)^{\ell} i^{\ell'} [1+(-)^{\ell'}]\, Y_\ell^m(\hat{q})\, Y_{\ell'}^{*m'}(\hat{\Delta}) \sum_{JM} C_{\ell' s}(JM; m'\nu'') C_{\ell s}(JM; m\nu') \langle \nu''|a_f + i\sigma_{4z}\, a_g|\nu\rangle \int_0^\infty \psi^J_{\ell\ell'}(qr) j_\ell(\Delta r/2) u(r) dr \tag{7.c}$$

$$\langle T_z\ \nu'|T|\nu\ 0\rangle = \frac{1}{q}\sqrt{8}\, \langle T_z \tau'_z|\vec{\tau}\vec{\tau}_4|\tau_z\ 0\rangle \sum_{\substack{\ell\ \ell' \\ m\ m'}} (-i)^{\ell} i^{\ell'} [1-(-)^{\ell'}]\, Y_\ell^m(\hat{q}) Y_{\ell'}^{*m'}(\hat{\Delta}) \sum_{J\,M} C_{\ell' s}(JM; m'\nu'') C_{\ell s}(JM; m\nu') \langle \nu''|b_f + i\sigma_{4z}\, b_g|\nu\rangle \int_0^\infty \psi^J_{\ell\ell'}(qr) j_\ell(\Delta r/2) u(r) dr \tag{7.d}$$

where T_z is the z-component of the isotopic spin of the nucleon pair, τ_z τ'_z are the same for the incident and the scattered particle respectively, and ν, ν' are the z-component of the initial and final spin of the nucleon pair.

When the final state is a spin singlet and isospin triplet the Watson theorem can be derived for low energies.

Infact for low energies $q \lesssim \frac{1}{a}$ $a = 15$ F the s-wave Jost function is in good approximation given by

$$f_0(-q) = q - {}^{i}/_{a} \tag{8}$$

therefore the matrix element becomes in the approximation of neglecting all other waves $\ell > 0$

$$T_{ss} = i\, b_g(\Delta)\, \frac{1}{\pi\sqrt{2}} \cdot \frac{1}{f_0(-q)} \int_0^\infty \varphi_0(qr)\, j_0({}^{\Delta r}/_{2})\, u(r)\, dr \tag{9}$$

since $qR \ll 1$ where R is the radius of the deuteron $(4F)\varphi_0(qr)$ can be reasonably approximated by the asymptotic behaviour at $qr = 0$ in all the range of integration

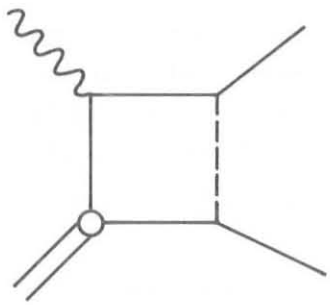
$$T_{ss} \sim i\, b_g(\Delta)\, \frac{\sqrt{2}}{\pi}\, \frac{\langle T_z \tau'_z | \vec{\tau}\ \vec{\tau}_4 | \tau_z\, 0 \rangle}{q - {}^{i}/_{a}} \int_0^\infty j_0({}^{\Delta r}/_{2})\, u(r) r\, dr \tag{10}$$

which determines the proportionality constant of Watson theorem.

When the C.M. momentum is between 100 and 500 MeV/c the form of the scattering wave functions for $\ell, \ell' \leq 6$ is requested for calculating the effect of final state interaction.

This program has been carried out by various authors in the past for electron scattering. To give an idea of the results of this effort I have drawn in Fig. 7 the percentage difference between plane wave and distorted wave calculation for the double differential cross-section $\frac{d\sigma}{dE_e, d\Omega_e}$ on the top of the quasi elastic peak for various values of the momentum transfer. Durand ([11]) and Breitenlohner ([12]) approximated the final state wave function with the solution of the Schrödinger equation with a square well potential, such to fit the scattering phase shifts. The coupling between different angular momenta has been neglected. Kramer ([13]), and Nuttal ([14]) solved numerically the Schrödinger equation for the Hamada Johnston and Gammel and Thaler potentials respectively; they both considered the coupling between angular momenta. Although this method is the most reliable one it is tedious and it has the drawback that it cannot be extended to a relativistic framework. In the literature an other method is reported, the so called dispersive method ([15]): this is based on the very simple observation that the break up amplitude in any partial wave has the same phase of the nucleon-nucleon scattering, a part of an overall phase of the scattering amplitude which is assumed independent of the C.M. momentum of the two nucleons. If one assumes that the amplitude is analytic every where in the relative energy plane and subtracts the behaviour at infinity, one can write a dispersion relation and solve such relation by means of the Ommès

method. Unfortunately this method is not precise enough, since the hypothesis of analyticity is not justified: one can infact show for an exponential potential ([16]) and for a general super position of Yukawa potentials (Bosco ([17])) that the amplitude has together with the unitarity cut and possible poles representing bound states, cuts in the complex plane, which represent anomalous thesholds in the crossed channel of the diagram



Therefore this method suffers of the presence of a free parameter which has to be determined with the experiment ([18]).

When the momentum transfer is higher (from .8 to 6 GeV/c) the C.M. energy of the nucleon pair is too high to allow the determination of phase shifts and the potential concept is no longer appropriate, the nucleon-nucleon interaction is becoming at these energies absorptive and the differential cross section presents a diffraction peak. For the case of pp, for instance this happens quite abruptly at ~1 GeV/c.

At these energies the effect of final state interaction has been calculated by Mc Gee ([19]), using the strong absorption model

$$S_\ell = 1 - \gamma\, e^{-\frac{\nu^2}{q^2}\cdot\ell(\ell+1)}$$

where γ is .6÷1. and $\nu^2 = 2\pi\gamma/\sigma_T$. The percentage effect is reported in Fig. 7.

An other possible treatment has been suggested by R. Smith and C. Wilkin ([20]): in this paper the final state wave function is expressed through its eikonal approximation

$$\psi_f(r) = e^{+i\vec{q}\cdot\vec{r} - \frac{i}{\hbar\sigma}\int_{-\infty}^{z} V(\vec{b},z)dz} \qquad (11)$$

and the "anti-Watson" theorem is postulated

$$e^{-i\vec{q}\cdot\vec{r}}\,\psi_f(r) = e^{2i\delta(\vec{b})}\vartheta(-z) + \vartheta(z) \qquad (12)$$

where z is the relative coordinate along the direction of the C.M. momentum: in words if the two nucleons are meeting on their optical path, there is scattering otherwise not. The idea was applied to electron scattering and the results are included in Fig. 7.

At sufficiently high energy >1 GeV/c there is an other approximate method which

can provide an useful guide for more refined treatments: the closure method. It can be shown that the differential cross section at high energy which is obtained integrating on all other variables except the scattering angle is ([21])

$$\frac{d\sigma}{d\Omega_3} = \int |\langle \psi_f | F | \psi_0 \rangle|^2 d^3q \qquad (13)$$

where the integral is extended in the whole momentum space. For lower energies we should use an Jacobian and we should have a limit on the region of integration given by conservation of energy. For high energies and small angles in the laboratory system the Jacobian can be shown to be in very good approximation 1 and the whole important region of integration is contained in the kinematically allowed zone.

The integration can be formally written in the following way ([3])

$$\frac{d\sigma}{d\Omega_3} = \sum_f \langle \psi_0 | F^+ | \psi_f \rangle \langle \psi_f | F | \psi_0 \rangle \qquad (14)$$

If the final state is a n-p system

$$\sum_f |\psi_f\rangle \langle \psi_f| = 1 - |\psi_0\rangle \langle \psi_0|$$

therefore

$$\frac{d\sigma}{d\Omega_3} = \langle \psi_0 | F^+ F | \psi_0 \rangle - |\langle \psi_0 | F | \psi_0 \rangle|^2 \qquad (15)$$

the first term in the so called plane-wave approximation and the second term is the elastic differential cross-section and represent the final state interaction. We can therefore see that

$$\left.\frac{d\sigma}{d\Omega_3}\right|_{F.S.} = \left.\frac{d\sigma}{d\Omega_3}\right|_{P.W.} - \left.\frac{d\sigma}{d\Omega_3}\right|_{el} , \qquad (16)$$

therefore we can write the percentage difference

$$\Delta J = \frac{J - J_0}{J_0} = - \frac{\left.\frac{d\sigma}{d\Omega_3}\right|_{el}}{\left.\frac{d\sigma}{d\Omega}\right|_{P.W.}} \qquad (17)$$

In the case of a spin and i-spin independent amplitude we can write:

$$\left.\frac{d\sigma}{d\Omega_3}\right|_{P.W.} = 2|f(\Delta)|^2 (1+S(\Delta))$$

$$\left.\frac{d\sigma}{d\Omega_3}\right|_{el} = 4|f(\Delta)|^2 S^2(\Delta/2)$$

where $S(\Delta) = \int e^{i\vec{\Delta}\cdot\vec{r}} |\psi(r)|^2 d^3$

therefore the percentage difference is

$$\Delta J = -2 \frac{S^2(\Delta/2)}{1+S(\Delta)} \tag{18}$$

If the final state is a pure T = 1 state (pp, nn), the completeness relation is:

$$\sum_f |\psi_f><\psi_f| = 1 \quad , \tag{19}$$

and therefore

$$\left.\frac{d\sigma}{d\Omega_3}\right|_{F.S.} = \left.\frac{d\sigma}{d\Omega_3}\right|_{P.W.} \quad . \tag{20}$$

These are the predictions of the closure approximation and we can compare them with the "exact" calculation in the case of n-p in the final state. We see in fact that the main features of the ΔJ are the same as ΔI, that is:

1) it is very large for $\Delta \sim 0$
2) is always negative.

There is, on the other hand, one difference, that is while $\Delta J \to 0$ already at .2 GeV/c)^2, ΔI is constant and around -2% and then increases again in absolute value with the momentum transfer. It is thought that at high momentum transfer, while the maximum of the peak in $\frac{d\sigma}{dE_3\, d\Omega_3}$ is lowering, its width is increasing such that the aerea is constant and equal to the aerea without final state interaction.

As in the low energy scattering from nuclei, the Glauber theory is applied with success and provides an order of magnitude estimate, the high energy closure approximation can be used even at low energy to have an explanation of the general trends and a guide for the elaborate calculations.

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FIGURE CAPTIONS

Fig. 1 - The predicted differential cross section $\frac{d\sigma}{dp_5\, d\Omega_3\, d\Omega_5}$ for inelastic scattering of π^- on deuterium at 9 GeV/c, against proton momentum p_5 for fixed momentum transfer and fixed proton angle and azimuth $\vartheta_5 = \vartheta_{el}$, $\varphi_5 = \pi$, where the (XZ) plane is defined to be the pion scattering plane. The momentum transfer is $-.1\ (GeV/c)^2$. The dashed and solid line correspond respectively to the calculation without and with D-state for the deuteron.

Fig. 2 - The same where $t = -.6\ (GeV/c)^2$. The double scattering enhancement is appearing.

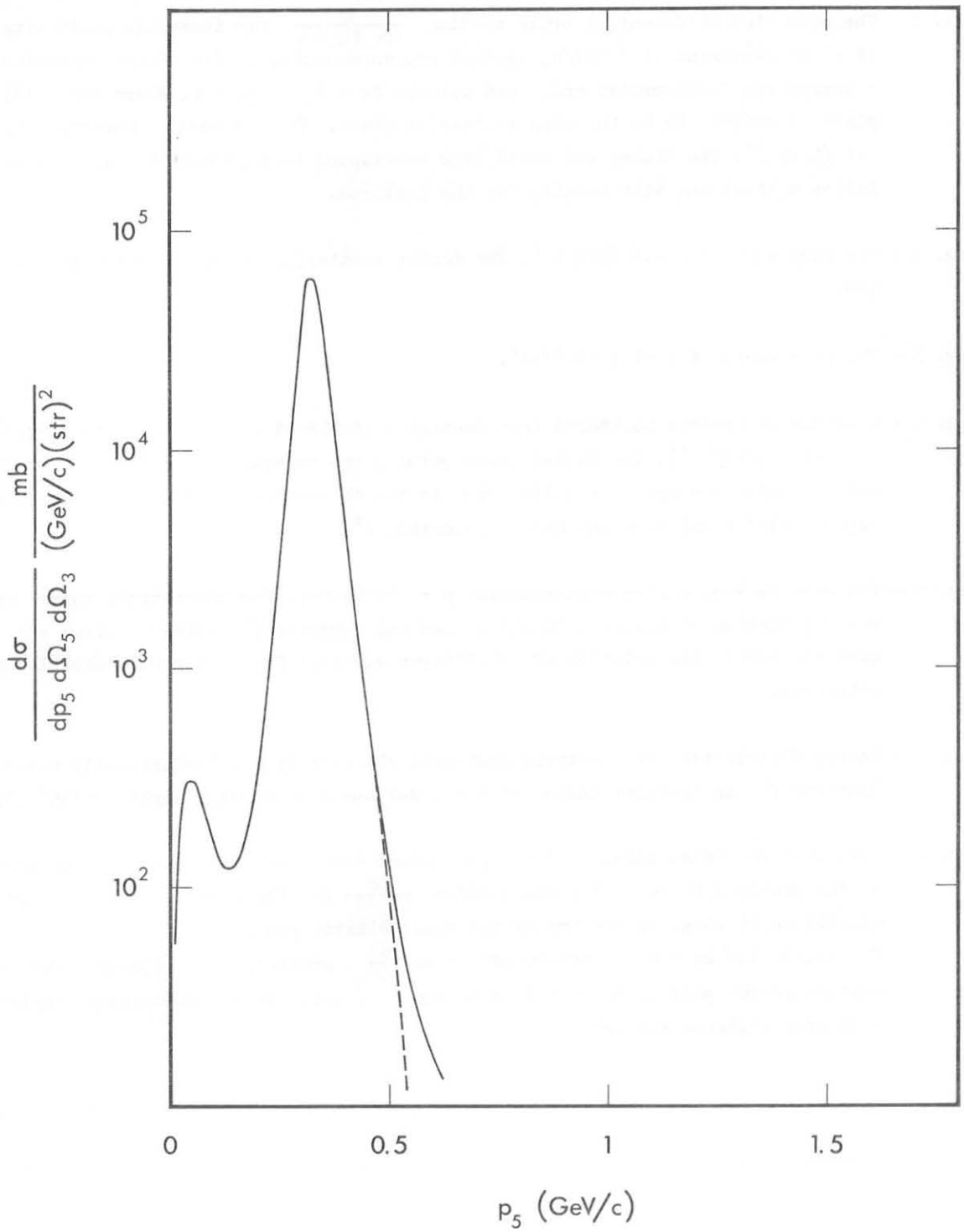
Fig. 3 - The same where $t = -1.2\ (GeV/c)^2$.

Fig. 4 - Spectrum of protons scattered from deuterium (incident momentum p = 18.29 GeV/c, $t = -1.2\ (GeV/c)^2$). The dashed curve joining the experimental points is meant only to guide the eye. The solid curve is the theoretical prediction neglecting experimental resolution and meson production ([6]).

Fig. 5 - The same as Fig. 4 (incident momentum p = 19. GeV/c). The theoretical curve is the calculation of Glauber, Kofoed Hansen and Margolis ([7]) using as final state wave functions, the solution of Schrödinger equation for an harmonic oscillator potential.

Fig. 6 - Energy distribution of electrons scattered elastically and inelastically from the deuteron for an incident energy of 146.9 MeV and a scattering angle of 135° ([9]).

Fig. 7 - Review of the calculations for the percentage effect of final state interaction on the double differential cross section $\frac{d\sigma}{dE_e\, d\Omega_e}$ for the electron-deuteron inelastic scattering, on the top of the quasi elastic peak.
The solid line is the percentage effect on $\frac{d\sigma}{d\Omega_e}$, predicted by closure for a neutron proton pair in the final state and a structureless elementary amplitude (for instance scalar).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10^4
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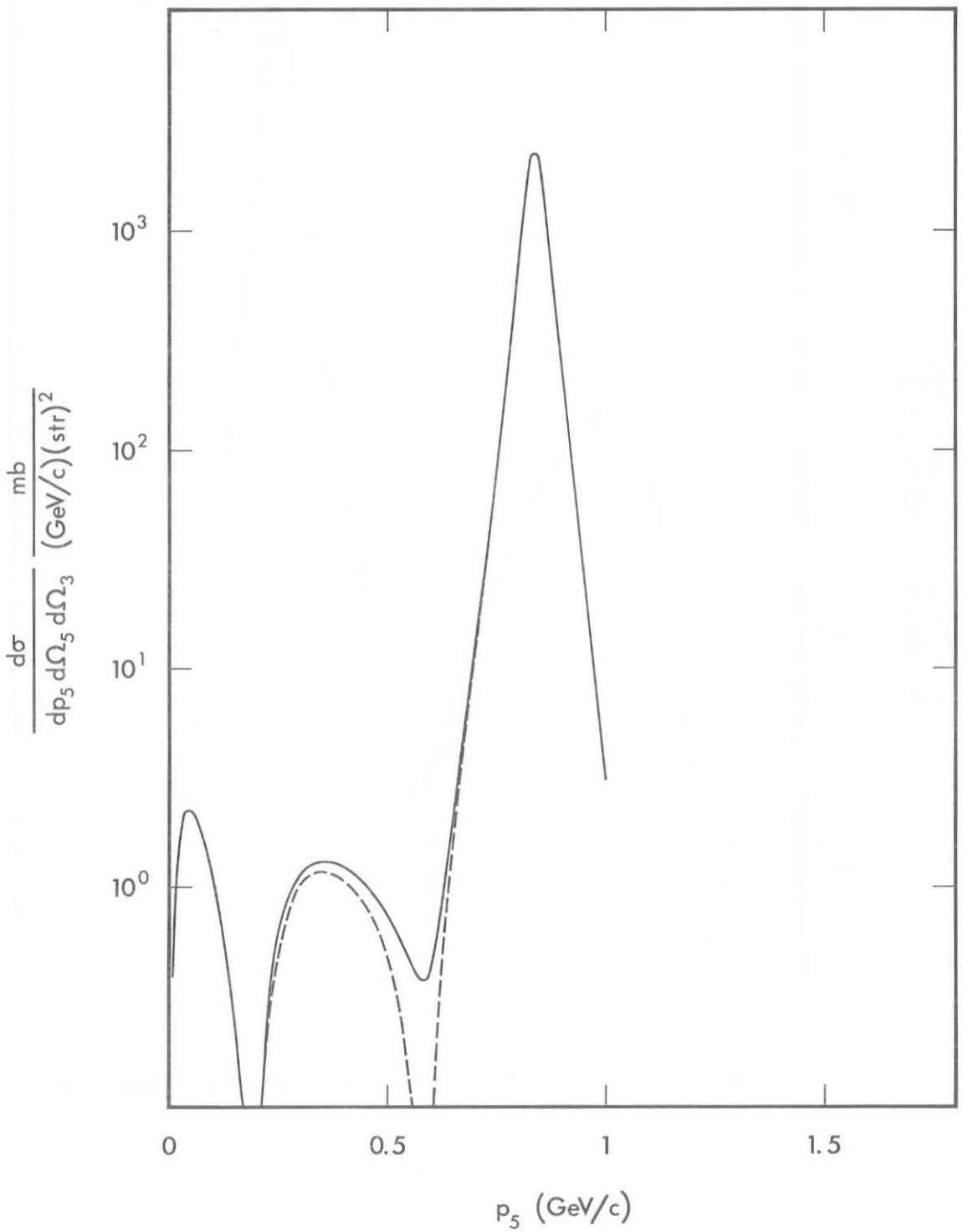
0
0.5
1
1.5
p_5 (GeV/c)

Fig. 1


$\frac{d\sigma}{dp_5\, d\Omega_5\, d\Omega_3}$ $\frac{mb}{(GeV/c)(str)^2}$
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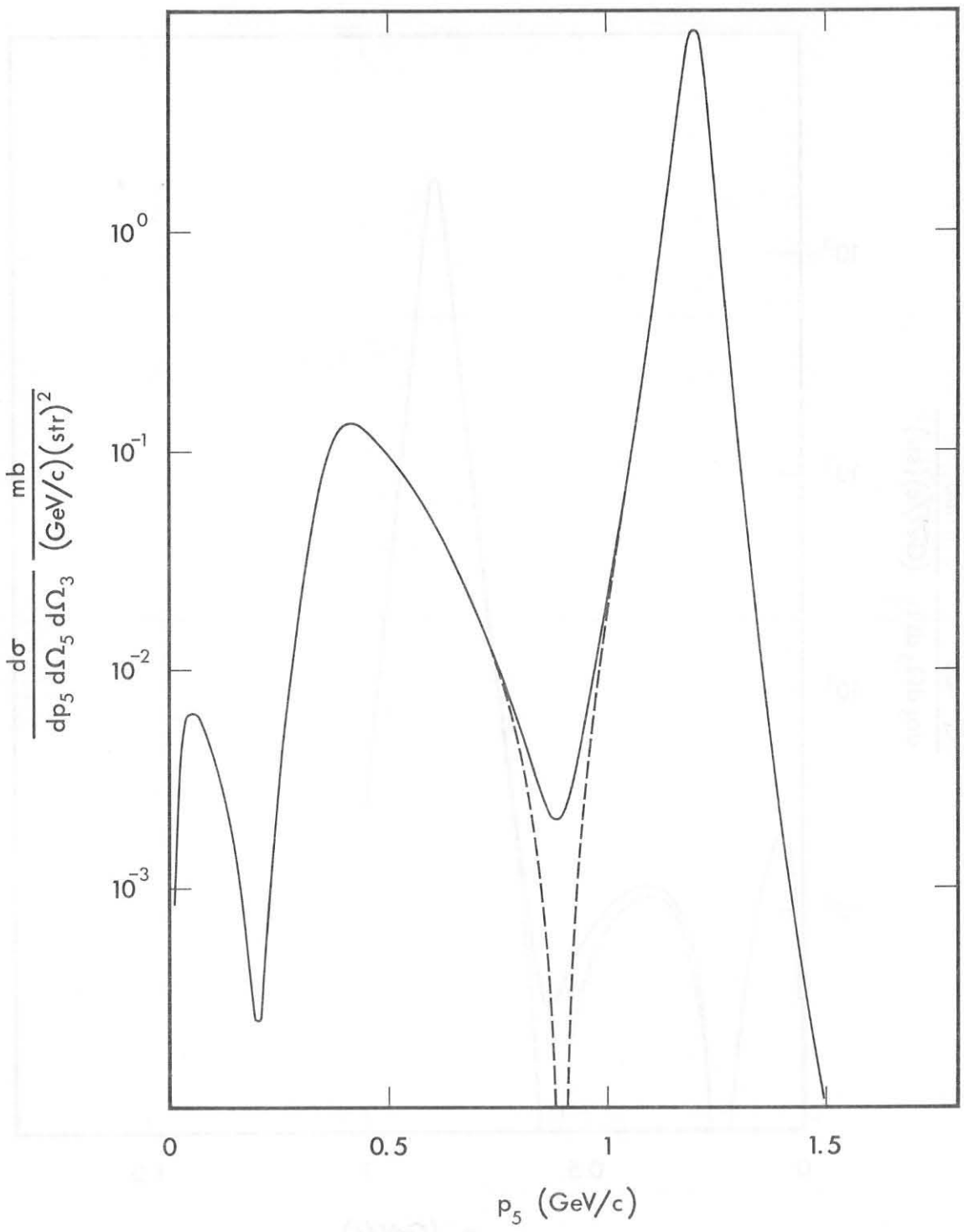
p_5 (GeV/c)

Fig. 2


$\frac{d\sigma}{dp_5\, d\Omega_5\, d\Omega_3}\ \frac{mb}{(GeV/c)(str)^2}$
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0
0.5
1
1.5
p_5 (GeV/c)

Fig. 3

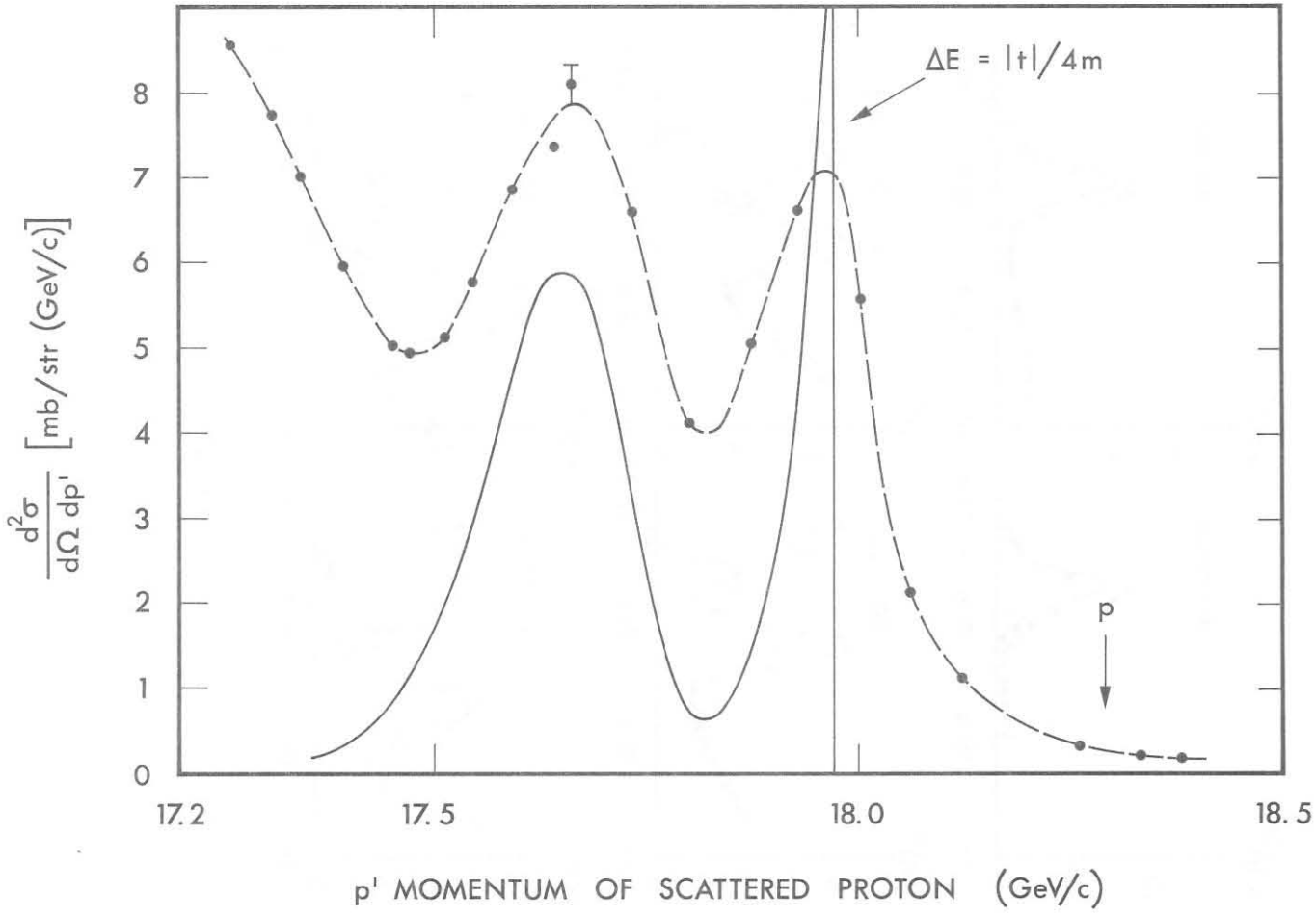
ΔE = |t|/4m
p
$\frac{d^2\sigma}{d\Omega\, dp'}$ [mb/str (GeV/c)]
0
1
2
3
4
5
6
7
8
17.2
17.5
18.0
18.5
p' MOMENTUM OF SCATTERED PROTON (GeV/c)

Fig. 4

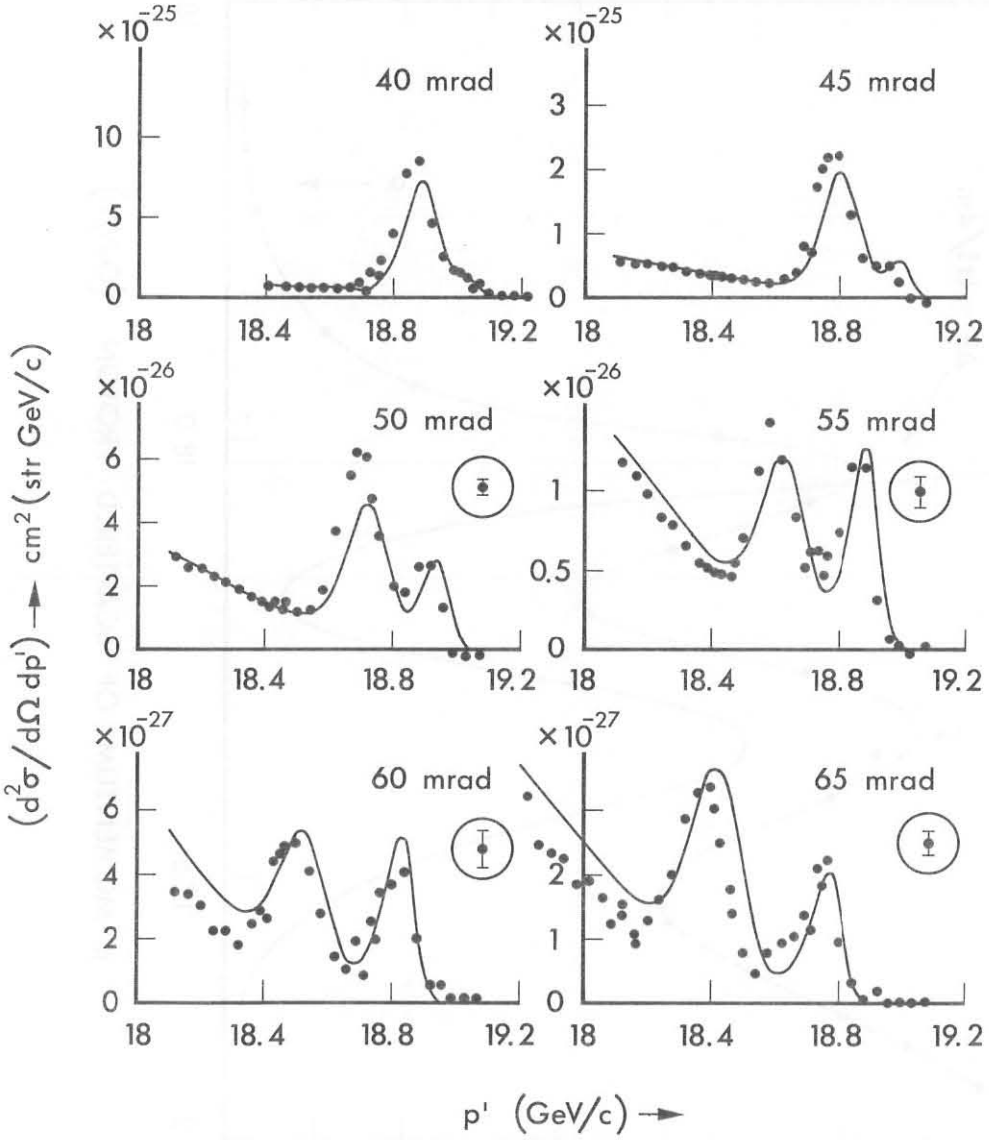
×10⁻²⁵
40 mrad
×10⁻²⁵
45 mrad
×10⁻²⁶
50 mrad
×10⁻²⁶
55 mrad
×10⁻²⁷
60 mrad
×10⁻²⁷
65 mrad
18
18.4
18.8
19.2
(d²σ/dΩ dp') → cm² (str GeV/c)
p' (GeV/c) →

Fig. 5

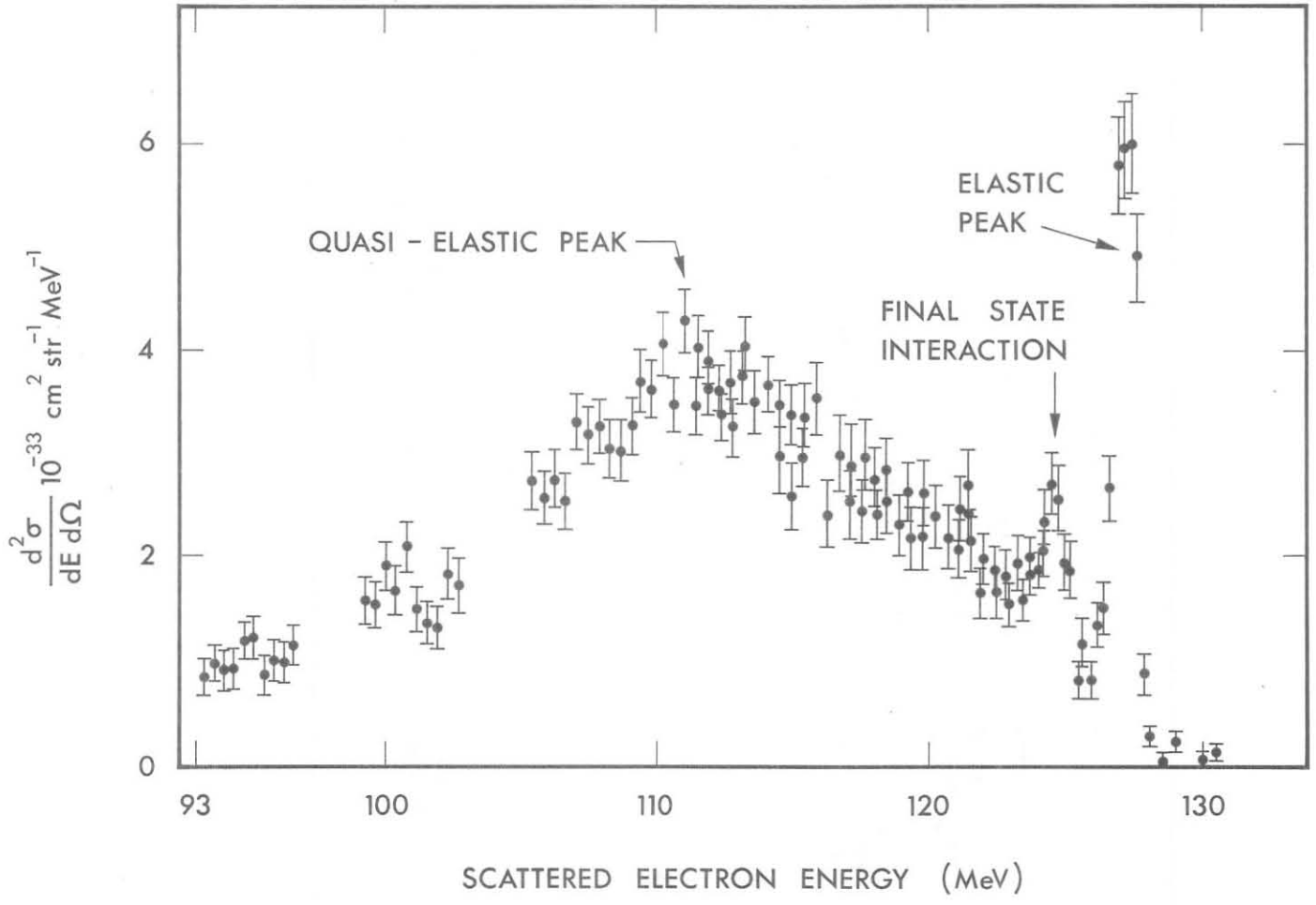
QUASI - ELASTIC PEAK
ELASTIC PEAK
FINAL STATE INTERACTION
$\frac{d^2\sigma}{dE\,d\Omega}\,10^{-33}$ cm^2 str^{-1} MeV^{-1}
0
2
4
6
93
100
110
120
130
SCATTERED ELECTRON ENERGY (MeV)

Fig. 6

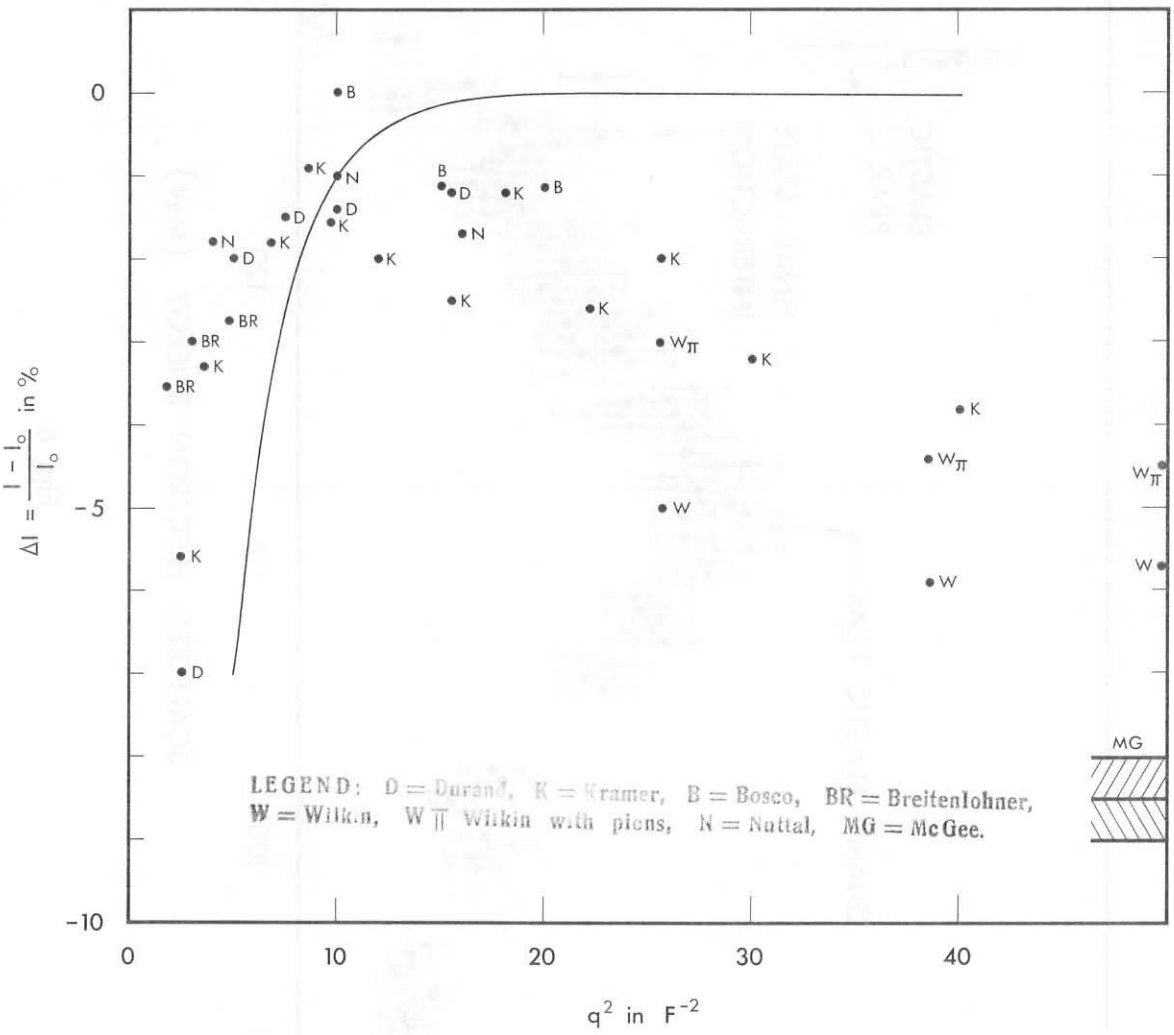
0
-5
-10
0
10
20
30
40
$\Delta I = \frac{I - I_o}{I_o}$ in %
q^2 in F^{-2}
LEGEND: D = Durand, K = Kramer, B = Bosco, BR = Breitenlohner, W = Wilkin, W_π Wilkin with pions, N = Nuttal, MG = McGee.
MG

Fig. 7