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INFN/AE-70/5
7 Ottobre 1970

P. Christillin[(x)] and F. Strocchi: CPT INVARIANCE AND FINAL STATE INTERACTIONS. -

ABSTRACT. -

The implications of TCP invariance are critically examined when more than one interaction is responsible for the process under consideration. In particular possible implications of this analysis in CP violating processes and in K-decays are analyzed.

1. - INTRODUCTION. -

It is known that the usual parametrization of the $K_L \rightarrow 2\pi$ decay and of the charge asymmetry in $K_L \rightarrow \pi^{\pm} \ell^{\mp} \nu$ decays[(1)], just to mention two well-known examples, is made with neglect of final state e.m. interactions.

It is reasonable to assume a critical attitude towards this procedure for two reasons. First, it might be possible that CP violation due to H_{γ} would modify the usual results. Second, independently from that, the same might happen due to an incorrect use of TCP since in this case TCP invariance should be valid only to order α with respect to H_W, i.e. the very order of magnitude of the other quantities in play in these decays.

Both possibilities have been exhaustively examined for the above mentioned decays in a previous article[(2)] and have proved to give a small contribution.

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The purpose of this note, along the previous line, is two-fold. We want, first of all to examine critically what is meant by TCP invariance and to which accuracy one is justified in neglecting final state e.m. interactions. Second, by taking the other K decays, forbidded to the lowest order in G by some selection rule, we try to set out whether these final state e.m. effects affect the processes to order, say, αG or to order α times the effective coupling constant. It is clear, that in the first case a relevant contribution, so far neglected, might be expected.

Paragraphs 2 and 3 will be devoted, respectively, to TCP invariance and its tests and to an explicit evaluation of final state e.m. effects, whereas in par. 4 we will discuss briefly the decays $K^{\pm} \to \pi^{\pm}\pi^{o}$, $K_S \to 3\pi^{o}$, $K_{S,L} \to \ell\bar{\ell}$ and the $\Delta S = - \Delta Q$ rule.

2. - TCP INVARIANCE AND PHYSICAL IMPLICATIONS. -

The transformation $TCP \equiv \theta$ is simply defined as the product of T, C and P. Therefore it shares the antiunitary character of T: i.e. it relates a given process with that obtained applying CP, but running backward in time.

More precisely, denoting by $M_{i \to f}$ the transition amplitude from the state $|i\rangle$ to the state $|f\rangle$, one has that TCP invariance implies the following equality

$$\left| M_{i \to f} \right|^2 = \left| M_{\theta f \to \theta i} \right|^2 \tag{2.1}$$

Here θi and θf are the TCP transformed of the states i and f. The invariance of the theory under TCP implies therefore that the T-matrix transforms in the following way

$$\theta T \theta^{-1} = T^{\dagger} \tag{2.2}$$

Direct tests of TCP invariance, through a check of eq. (2.1) are almost impossible to realize for the difficulties connected with the comparison of reversed processes $i \to f$, $\theta f \to \theta i$. For example, phase space reasons and/or additional interactions acting in the final state usually prevent the possibility of reproducing the reversed process of a given one. In particular one cannot hope to reverse a decay process or a reaction induced by weak interactions. Quite generally, when strong interactions act in the final states it is very difficult to compare the direct process $i \to f$ and the TCP-reversed one $\theta f \to \theta i$, because in the former the final states have definite phase relations which is almost impossible to reproduce in the reversed process.

Clearly, the only hope to get some information about TCP invariance lies in the possibility of comparing two direct processes differing by the exchange of particles and antiparticles. In this case one has to analyse which restrictions are imposed by TCP invariance on the differential cross sections.

The simplest case to discuss is when the T-matrix is Hermitian. This situation is realized when the T-matrix may be approximated by an effective Hamiltonian (<u>first order processes</u>)

(2.3) $$T \simeq H_{eff}$$

This is the case, for example, when only electromagnetic or weak interactions are separately responsible for the process $i \to f$. The approximation (2.3) amounts then to neglect second order electromagnetic or weak effects.

<u>Statement 1.</u> - If the T-matrix is hermitian, TCP invariance implies the following equality

(2.4) $$\left| M_{i \to f} \right|^2 = \left| M_{\theta i \to \theta f} \right|^2$$

Proof. One has in fact

$$M_{i \to f} \equiv \langle i | T | f \rangle = \langle f | T^\dagger | i \rangle^* = \langle f | T | i \rangle^* = M^*_{f \to i}$$

and eq. (2.1) reduces to eq. (2.4).

Under the assumptions of Statement 1 one may easily prove the equality of mean lives of particles and antiparticles in TCP-conjugate channels[3]. We would like to stress, however, that even for pure weak or electromagnetic processes the T-matrix can be replaced by an hermitian Hamiltonian only to order G and α respectively

$$T = H + H \frac{1}{E - H_o + i\mathcal{E}} H + \ldots \quad \neq T^\dagger = H + H \frac{1}{E - H_o - i\mathcal{E}} H + \ldots$$

Moreover, in the general case the non-hermiticity of the T-matrix may arise not only from second order effects but also because an additional interaction is acting in the outcoming channel. For example, in weak processes electromagnetic interactions may be effective in the final state.

On the other hand in electromagnetic processes, like electro and photo production, strong interactions are present in the final states.

These effects have been extensively studied in the literature in the case of strong interactions[4]. The case of final state electromagnetic effects will be discussed in sect. 4.

A simplification of the problem occurs when a sum is made over the final states.

Statement 2.- Equation (2.4) remains true also in the presence of final state strong interactions provided a sum is performed over the final states f so to get a subspace $\mathcal{H}'$ invariant under TCP and closed under strong interactions:

$$\sum_{f \in \mathcal{H}'} \left| M_{i \to f} \right|^2 = \sum_{f \in \mathcal{H}'} \left| M_{\theta i \to \theta f} \right|^2$$

where $\theta \mathcal{H}' = \mathcal{H}'$ and $\Omega^{\pm}_{st} \mathcal{H}' = \mathcal{H}'$, $\Omega^{\pm}_{st}$ being the Møller operators for strong interactions.

Proof. As a matter of fact when $|i\rangle$ is a decaying state, or a scattering state below threshold, stable with respect to strong interactions, (by assumption strong interactions are effective only in the final state) one may get

$$\Omega^{+}_{st} |i\rangle = |i\rangle$$

with a suitable redefinition of phases. Hence

$$\sum_{f \in \mathcal{H}'} \left| M_{\theta i \to \theta f} \right|^2 = \sum_{f \in \mathcal{H}'} \left| \langle \theta i | \Omega^{-\dagger}_{st} H \Omega^{+}_{st} | \theta f \rangle \right|^2 =$$

$$= \sum_{f \in \mathcal{H}'} \left| \langle i | \Omega^{+\dagger}_{st} H \Omega^{-}_{st} | f \rangle^{*} \right|^2 = \sum_{f \in \mathcal{H}'} \left| \langle i | H | f \rangle \right|^2 = \sum_{f \in \mathcal{H}'} \left| M_{i \to f} \right|^2$$

The conditions of the above Statement are obviously satisfied when $\mathcal{H}'$ consists of a single eigenstate of the strong interactions. In this case TCP invariance implies the usual relation between matrix elements

(2.5) $$\langle i | T | f \rangle = \langle \theta i | T | \theta f \rangle e^{2i \delta_f}$$

where δ_f is the strong interaction scattering phase for the state f.

The above remarks cannot be applied straightforwardly when electromagnetic interactions act as final state interactions.

For example, in the case of a process induced by H_W in the presence of e.m. interactions, $T = \Omega_\gamma^{+\dagger} H_W \Omega_\gamma^+$ and, formally, one could repeat the arguments used above for Ω_{st} in order to relate $|\theta f\rangle$ with $|f\rangle$ (i.e. closure of the subspace etc.). However the equation $\Omega_\gamma^{\pm} |i\rangle = |i\rangle$ can no longer be obtained due to the massless character of the photon (the threshold starts at zero energy). Since the emission of as many soft photons as possible may take place one cannot any longer find the eigenstates of H_γ. So, for instance, charged particles are stable, in the presence of Ω_γ, only to order $\sqrt{\alpha}$ (bremsstrahlung).

Thus, final state electromagnetic effects make the T-matrix non hermitian and the above equations (2.4), (2.5) do no longer hold in general. Clearly the non-hermiticity of T, due to electromagnetic effects, is of order α the hermitian part of T and in most cases can be neglected. These effects might however come into play in the analysis of CP-violating processes, which are in fact of the order αG. For these reasons it seems worthwhile to evaluate the non hermitian contribution due to H_γ expecially in K-decays (sect. 4)[x].

To this purpose we need an expression for the T matrix when two interactions are present in the final state. For concreteness, we will discuss the case of a weak process in the presence of strong and electromagnetic final state interactions:

$$H = H_o + H_{st} + H_\gamma + H_W \tag{2.6}$$

According to the formal theory of scattering one has

$$T = \Omega_{st}^{-\dagger} (H_\gamma + H_W) \Omega^+_{st+em+W} + T_{st} \tag{2.7}$$

where $\Omega^{\pm}$ are the Møller operators and T_{st} is the T matrix due to strong interactions. By expanding T with respect to H_W one easily gets

$$T = \Omega_{st}^{-\dagger} (\Omega_\gamma^{-\dagger} H_W \Omega_\gamma^+ + H_\gamma \Omega_\gamma^+) \Omega_{st}^+ + T_{st} + O(H_W^2) \tag{2.8}$$

(x) - For $K_L \to 2\pi$ and $K_L \to \pi \ell \nu$ decays see ref. 2.

where

$$\Omega_{\gamma}^{\pm} = (E - H_o - H_{st} - H_{\gamma} \pm i\varepsilon)^{-1} H_{\gamma} + 1 \tag{2.9}$$

Thus, due to electromagnetic final state interactions the weak Hamiltonian is replaced by a non-hermitian effective Hamiltonian

$$H_{eff} = \Omega_{\gamma}^{-\dagger} H_W \Omega_{\gamma}^{+} \tag{2.10}$$

To summarize, we recall that whenever a process happens to the second order, which may be due either to an interplay of two different Hamiltonians or even to one Hamiltonian alone, the T matrix is no longer hermitian. Then, in the presence of H_{γ} or H_W, it is necessary to estimate the neglected nonhermitian part, before drawing conclusions from TCP invariance of the single Hamiltonians.

3.- EVALUATION OF FINAL STATE e.m. EFFECTS.-

In order to evaluate final state electromagnetic effects we define

$$\mathcal{T} = \Omega_{\gamma}^{-\dagger} H_W \Omega_{\gamma}^{+} + H_{\gamma} \Omega_{\gamma}^{+} \equiv H_{eff} + H_{\gamma} \Omega_{\gamma}^{+} \tag{3.1}$$

and split H_{eff} into its TCP even and odd parts

$$H_{eff} = H_+ + H_- ; \quad 2H_{\pm} = \Omega_{\gamma}^{-\dagger} H_W \Omega_{\gamma}^{+} \pm \Omega_{\gamma}^{+\dagger} H_W \Omega_{\gamma}^{-} ; \quad \theta H_{\pm} \theta^{-1} = \pm H_{\pm} \tag{3.2}$$

with

$$H_+ \sim H_W + O(\alpha); \qquad H_- \simeq O(\alpha)$$

Then we evaluate H_- by means of a unitarity sum rule. Below threshold $\Omega_{st}^{\pm}$ are unitary operators and the S matrix may be written in the following form

$$\begin{aligned} S = 1 + iT = 1 + iT_{st} + \Omega_{st}^{-\dagger} \Omega_{st}^{+} \Omega_{st}^{+\dagger} \mathcal{T} \Omega_{st}^{+} = S_{st} + S_{st} \Omega_{st}^{+\dagger} \mathcal{T} \Omega_{st}^{+} = \\ = S_{st} \Omega_{st}^{+\dagger} (1 + \mathcal{T}) \Omega_{st}^{+} \end{aligned} \tag{3.3}$$

The unitarity of S, implies that of $1+\mathcal{T}$. One gets immediately

$$i(\mathcal{T}-\mathcal{T}^{\dagger}) = \mathcal{T}\mathcal{T}^{\dagger} \tag{3.4}$$

Now, from eq. (3.1) one has

$$\mathcal{T}-\mathcal{T}^{\dagger} = H_{-} + \mathbb{T}_{\gamma} - \mathbb{T}_{\gamma}^{\dagger} \tag{3.5}$$

and finally

$$iH_{-} \simeq \sum_{n} \mathcal{T}|n\rangle\, \rho(n) \langle n|\mathcal{T}^{\dagger} \tag{3.6}$$

where it has been explicitely exhibited the dependence of H_{-} on the phase space of the intermediate states.

In the preceding sum rule, the largest contributions, provided the matrix elements have the same strenght, will come of course from the intermediate states with the lowest phase space.

It is known, for instance, that the phase-space of $2\pi\gamma$ (soft γ) is practically the same as the 2π and that the ratio of the adimensional phase space[5] of 3π to 2π is $0(10^{-3})$. By means of the previous parametrization, the asymmetry of particle antiparticle in CPT conjugate channels can be easily found to be

$$\delta\Gamma = \frac{\Gamma(A\to f)-\Gamma(\bar{A}\to\bar{f})}{\Gamma(A\to f)+\Gamma(\bar{A}\to\bar{f})} = \frac{2\,\mathrm{Im}(A_{+}A_{-}^{*})}{|A_{+}|^{2}+|A_{-}|^{2}} \simeq \frac{2\,\mathrm{Im}\,A_{-}}{A_{+}} \tag{3.7}$$

where

$$A_{+} = \langle A|H_{+}|f\rangle \;; \qquad A_{-} = i\langle A|H_{-}|f\rangle$$

Now, the first interesting point which can be easily derived from (3.6) is that for a process whatsoever

$$|A_{-}| \leq G\,\rho(2\pi)\alpha \simeq G\times 10^{-4}$$

So, if $A\to f$ can proceed via H_{+} to the first order without being suppressed by some selection rules, CPT invariance can be safely tested to 10^{-4}, without worrying for final state e.m. effects.

Second, for processes forbidden in H_+ to the lowest order, one might expect an appreciable contribution from $\mathrm{Im}\,H_-$ (if, as reasonable, it is of the same order as H_-).

That is what we will examine in the following paragraph.

4. - K DECAYS AND FINAL STATE ELECTROMAGNETIC EFFECTS. -

We now apply the previous formalism to some cases where, on the basis of the above arguments, the anti-hermitian part might give a sizable contribution to the asymmetry. For the discussion of $K_L \rightarrow 2\pi$ and $K_L \rightarrow \pi \ell \nu$ decays we refer to (2).

When $T = T^\dagger$, TCP really forbids any total decay rate asymmetry and no sign of CP non-invariance can appear. As a matter of fact TCP invariance implies

$$\langle A | T | f \rangle = \langle \bar{A} | T^\dagger | \bar{f} \rangle^* = \langle \bar{A} | T | \bar{f} \rangle^* \tag{4.1}$$

whereas CP invariance gives

$$\langle A | T | f \rangle = \langle \bar{A} | T | \bar{f} \rangle \tag{4.2}$$

Thus, one cannot test eq. (4.2) if eq. (4.1) has to hold. However when $T \neq T^\dagger$, the two equations are independent. An asymmetry of the order A_- / A_+ is then allowed by TCP invariance, so that its presence is a direct test of eq. (4.2).

We will examine the $K^\pm \rightarrow \pi^\pm \pi^0$, $K_S \rightarrow 3\pi^0$, $K_{L,S} \rightarrow \ell \bar{\ell}$ decays and the $\Delta S = -\Delta Q$ rule.

i) We take as a first example $K^\pm \rightarrow \pi^\pm \pi^0$, which is the only $K^\pm$ decay interesting for our purpose since it is depressed by the $|\Delta I| = 1/2$ selection rule.

It is known that it occurs to order $\sqrt{\alpha}\, G$.

The contribution to A_- from the intermediate states with the lowest phase space (i.e. $\pi\pi$, $\pi\pi\gamma$, $\pi\pi\pi$) are of the following order

$$\langle K^+ | \mathcal{T} | \pi^+ \pi^0 \rangle \, \rho(2\pi) \, \langle \pi^+ \pi^0 | \mathcal{T}^\dagger | \pi^+ \pi^0 \rangle \simeq \sqrt{\alpha}\, G \, \rho(2\pi) \alpha \tag{4.3}$$

$$\langle K^+ | \mathcal{T} | \pi^+ \pi^0 \gamma \rangle \, \rho(2\pi\gamma) \, \langle \pi^+ \pi^0 \gamma | \mathcal{T}^\dagger | \pi^+ \pi^0 \rangle \simeq \sqrt{\alpha}\, G \, \rho(2\pi\gamma) \sqrt{\alpha} \tag{4.4}$$

(4.5) $\langle K^+|\mathcal{T}|\pi^+\pi\pi\rangle\,\rho(3\pi)\,\langle\pi^+\pi\pi|\mathcal{T}^\dagger|\pi^+\pi^o\rangle \simeq G\,\rho\,(3\pi)\alpha$

The above estimations easily follow from $\mathcal{T} = H_{eff} + \mathcal{T}_\gamma$, where H_{eff} has to the lowest order the same selection rules as H_+.

Then A_-, and consequently its imaginary part, is at most, of the order $\alpha G\,\rho\,(2\pi\gamma)$. Although these order of magnitude estimations are very crude, since no energy dependence of the matrix elements is taken into account, they indicate that the ratio A_-/A_+ cannot be greater than $0(10^{-3})$.

Hence CP violation effects in A_- can at most be of order α with respect to the effective Hamiltonian.

In this connection it is perhaps interesting to remark that if $[H_\gamma, CP] = 0$, then $\mathrm{Im}\,A_- = 0$ to $0(\alpha\epsilon)$.

As a matter of fact

$$\text{(4.6)}\quad \begin{aligned} iA_- &= \langle K^+|H_-|\pi^+\pi^o\rangle = \langle K^+|CP^{-1}\,CPH_-CP^{-1}\,CP|\pi^+\pi^o\rangle = \\ &= iA_-^x + 0(\alpha\epsilon) \end{aligned}$$

ii) Next we consider the $K_S \to 3\pi^o$ decay, which is forbidden to order ϵ, since the $3\pi^o$ are in a CP = -1 state. In this case the contribution of $\langle K_S|H_-|3\pi^o\rangle$, in order to be significant should be greater than $\epsilon\langle K_2^o|H_+|3\pi^o\rangle$ i.e. ϵG since this decay should be detected against the predominant $K_S \to 2\pi$ mode.

Now

(4.7) $\langle K_S|H_-|3\pi^o\rangle \simeq \langle K_1^o|H_-|3\pi^o\rangle = \sum_n \langle K_1^o|\mathcal{T}|n\rangle\,\rho\,(n)\,\langle n|\mathcal{T}^\dagger|3\pi^o\rangle$

The only significant contribution is due to the 2π state, so that

$$\text{(4.8)}\quad \begin{aligned} \langle K_S|H_-|3\pi^o\rangle &\simeq \langle K_1^o|\mathcal{T}|2\pi\rangle\,\rho\,(2\pi)\,\langle 2\pi|\mathcal{T}^\dagger|3\pi^o\rangle \sim \\ &\sim G\,\rho\,(2\pi)\alpha \end{aligned}$$

We note that in the evaluation of the above matrix element an explicit CP violation has been attributed to H_γ, for instance through a diagram of the type (see Fig. 1).

Nevertheless the contribution of the antihermitian part of H_{eff} except for same dinamical enhancement, is smaller than the mixing of CP eigenstates in the K_S.

A fortiori, this contribution is negligible for all K_L decays, which are not hindered by selection rules.

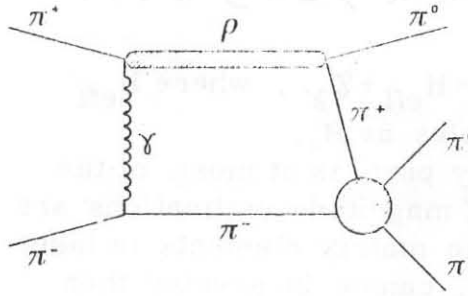

FIG. 1 - A possible diagram for CP violation through electromagnetic interaction.

iii) Finally, the application of this technique shows at once that a possible simulation of TCP non-invariance may arise in $K_{SL} \to \ell\bar{\ell}$ (7).

As a matter of fact the parameters a and b appearing in the invariant amplitude $M = \bar{u}(\ell)[a + ib\,\gamma_5]\,v(\bar{\ell})$ are relatively real, if one takes into account only H_+ (8).

Their imaginary parts are compactly evaluated by means of our unitarity sum rule for H_-.

$$\text{(4.9)}\quad \text{Im}\binom{a}{b} = \langle K_2^o|H_-|\ell\bar{\ell}, \pm\rangle = \sum_n \langle K_2^o|\mathcal{T}|n\rangle\,\rho(n)\,\langle n|\mathcal{T}^\dagger|\ell\bar{\ell}, \pm\rangle \simeq$$
$$\simeq \langle K_2^o|\mathcal{T}|\gamma\gamma\rangle\rho(\gamma\gamma)\langle\gamma\gamma|\mathcal{T}^\dagger|\ell\bar{\ell}, \pm\rangle$$

where $|\pm\rangle$ are the CP eigenstates of the $\ell\bar{\ell}$ pair.

However nothing definite can be said about their relative phases (and therefore about a pseudo TCP violation).

iv) Last, a word about the $\Delta S = -\Delta Q$ rule.

The $K \to \pi\ell\nu$ decays, in which CP violation and the $\Delta S = -\Delta Q$ rule may be simultaneously analized, are usually discussed by introducing the following parameters

$$x = \frac{g}{f} = \frac{\langle\pi\ell^+\nu|H|\bar{K}_o\rangle}{\langle\pi\ell^+\nu|H|K_o\rangle} \qquad x' = \frac{\langle\pi\ell^-\nu|H|K_o\rangle}{\langle\pi\ell^-\nu|H|\bar{K}_o\rangle} = \frac{g^*}{f^*} = x^*$$

Now, with H replaced by T, it is no longer true that $x' = x^*$.

They differ, however, only for terms of $0(\alpha)$, so that the usual formulae are, at present, completely justified.

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