Istituto Nazionale di Fisica Nucleare
Sezione di Padova

INFN/AE-69/9
21 Dicembre 1969

G. Patergnani: MASS DIFFERENCE BETWEEN THE $\pi^{\pm}$ AND THE π^{0}, USING A BETHE-SALPETER EQUATION. -

SUMMARY. -

In a preceding work[1] a bound state of a nucleon-antinucleon pair was studied with a Bethe-Salpeter equation in the ladder approximation.

In this work the electromagnetic interaction is used to produce in the bound state the difference of mass $\Delta\mu$ between the $\pi^{\pm}$ and the π^{0}. It is possible to obtain the right sign of $\Delta\mu$ only if the interaction which binds the states is produced by isoscalar pseudoscalar or isoscalar vector mesons, or with a mixture of isovector pseudoscalar plus isoscalar vector mesons.

A cut off of the order of the mass of the π meson is used in the electromagnetic interaction to obtain the experimental value for $\Delta\mu$.

INTRODUCTION. -

The difference of mass $\Delta\mu$ of the π meson due to the electromagnetic interaction was evaluated using various techniques[2], in good agreement with the experimental value.

The purpose of this work is to calculate $\Delta\mu$, the difference of mass between the π^{0} and $\pi^{\pm}$, in the model in which the π meson is considered a bound nucleon antinucleon state.

We treated this model with a Bethe-Salpeter equation in the ladder approximation in a preceding work(1). Actually this work is intended only to check that model in this respect. We will find that the interaction which seems necessary to bind the $N\bar{N}$ to form the π gives also a $\Delta\mu$ with a correct sign. The numerical value, which can be obtained in agreement with the experimental one, is obtained with a cut off, this time in the electromagnetic interaction, which is of the order of the mass of the π. This cut off seems unavoidable, at least in the ladder approximation.

The graph which we consider in the interaction, which is responsible of the splitting, is the one of Fig. 1. This graph is effec-

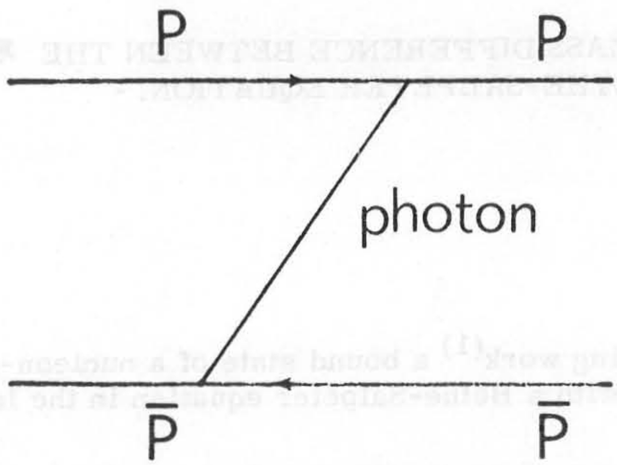

FIG. 1 - This is the unique graph in which the electromagnetic interaction is considered effective.

tive only in forming the π^o. In the ladder approximation with this model, ignoring vertex corrections, there is no other effect of the electromagnetic interaction on the π^o, or $\pi^{\pm}$, but the graph of Fig. 1.

EQUATION AND INTERACTIONS. -

For sake of clearness we report here some formulas and notation from I. In I for taking care of a pseudoscalar plus a vector interaction, we used the equation, in symbolic form:

$$\phi = \tau_\pi g_\pi K^{-1} \left(I_\pi + \frac{\tau g}{\tau_\pi g_\pi} I_v \right) \phi \qquad (1)$$

where I_π, I_v are the propagators of the pseudoscalar and vector mesons, or of the photon, K^{-1} is the kinematical part of the kernel, τ_π,

τ the isotopic spin factor in the interaction. In the computation, the mass 2E of the bound states and $R = \tau_g/(\tau_\pi g_\pi)$ are fixed, and g_π is evaluated as eigenvalue of eq. (1). The method can be extended, and several interactions can act together, adding various terms with different R and I_V.

There is the additional complication that the electromagnetic interaction does not conserve the isotopic spin, and so to use this interaction in (1) it is necessary to introduce two amplitudes ϕ_1 and ϕ_2, relative to the states of isotopic spin T=1 and T=0, both with $T_3=0$. This is accomplished in this way. Indicating with N and P a neutron and a proton, with notation of I one has:

$$\phi_{\frac{1}{2}} = \langle 0 | \psi_P \bar{\psi}_P \pm \psi_N \bar{\psi}_N | B \rangle$$

The electromagnetic interaction, using only the graph of Fig. 1, changes, for what concerns isospin indices, both the ϕ_is in $\langle 0 | \psi_P \bar{\psi}_P | B \rangle$, that is to $(\phi_1 + \phi_2)/2$.

In this way the eq. (1) becomes simbolically:

$$(2) \quad \begin{vmatrix} \phi_1 \\ \phi_2 \end{vmatrix} = \left\{ (g^2 K \; I \; + g^2 K \; I \;) \begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} + \frac{\alpha^2}{2} K_e I_e \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \right\} \begin{vmatrix} \phi_1 \\ \phi_3 \end{vmatrix}$$

The first term at the right side is the conserving isotopic spin term, the second one is the electromagnetic one: $\alpha = e^2/4\pi$ is the fine structure constant.

The eq. (2) is valid only for $T_3=0$ states; otherways the electro magnetic interaction is not effective in the used approximation. So eq. (2) can give the mass of the π^0, whereas the mass of the $\pi^\pm$ can be related to the g_π as in I.

METHOD OF COMPUTATION. -

The experimental value of $\Delta\mu$ is:

$$\Delta\mu \simeq 4.6 \text{ MeV}.$$

We will try to evaluate

$$\varsigma = \frac{\Delta\mu}{\alpha} \sim 630 \text{ MeV}.$$

Our aim is to solve eq. (2) and to find the mass splitting as consequence of the second term on the right side, that is to evaluate

$\lim_{\alpha \to 0} d(2E)/d\alpha$. In I it was found g_π as function of 2E and g_V. Actually g_π is considered at the value for which the mass of the $\pi^\pm$ is obtained, and g_π is a function of α. So it is required that:

$$dg_\pi = \frac{\partial g_\pi}{\partial (2E)} 2dE + \frac{\partial g_\pi}{\partial \alpha} d\alpha = 0 \qquad \text{that is}$$

$$\frac{2\partial E}{\partial \alpha} = - \frac{\partial g_\pi}{\partial \alpha} \Big/ \frac{\partial g_\pi}{2\partial E} .$$

In order to find numerically $\partial E/\partial \alpha$ it is necessary to solve eq. (2) at least three times, that is for

$$\begin{array}{lll} 2E = m_{\pi^\pm} & 2E \neq m_{\pi^\pm} & 2E = m_{\pi^\pm} \\ \alpha = 0 & \alpha = 0 & \alpha = 0 \end{array}$$

and to construct the g_π's in these situations. We could solve eq. (3), enlarging the matrices used in I, to take in account of the isotopic spin indices. To avoid to diagonalize matrices larger than necessary, we prefer to solve the equation

$$(4) \qquad \phi_1 = \left[(g_\pi^2 K_\pi I_\pi + g_V^2 K_V I_V)(-1) + \frac{\alpha^2}{2} K_e I_e \right] \phi_1$$

The eq. (3) and the eq. (4) have in fact the same eigenvalues for $\alpha \to 0$, if the standard perturbation theory is available. Indicating with ϕ_i^T the solution of the coniugate to the eq. (4), one has, for the splitting Δg_π of g_π due to α^2,

$$(5) \qquad \Delta g_\pi = \overline{\underline{\phi_1^{0T} \quad \phi_2^{0T}}} \; \frac{\alpha}{2} K_e I_e \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} \phi_1^0 \\ \phi_2^0 \end{vmatrix}$$

where ϕ_i^0 are the solutions of (3) for $\alpha \to 0$; referring to the π^0, a T=1 state, $\phi_2^0 = 0$, which makes

$$\Delta g_\pi = \phi_1^{0T} \frac{\alpha}{2} K_e I_e \phi^0 .$$

This is identical to the Δg_π from eq. (4) using the perturbation theory. In this way we can solve eq. (4), avoiding to double the size of the matrices and to solve the coniugate equation.

The same technique is used as in I, only the cut off is introduced in a different way, a la Pauli. This was done the first time in a Bethe-Salpeter equation in ref. (3), and it consists in subtracting in the propagators of the interactions terms like $1/|(p-k)^2 + \bar{\mu}^2|$, where $\bar{\mu}$ is a conveniently large mass, at which the interaction should decrease. In this way, there is the possibility to use different cut offs for the interactions.

An additional difficulty is that the mass μ of the photon is zero, and this makes the functions Δ_n of the appendix of I singular for p=k. In fact one has

$$\lim_{\mu\to 0} \Delta_n(p,k) = \frac{4}{n+1}\left\{\left(\frac{k}{p}\right)^n \frac{\theta(p-k)}{p^2} + (p \rightleftarrows k)\right\}$$

where $\theta(x)$ is the step function. The computation was done giving a small mass μ_e to the photon, $\mu_e \geq \mu_\pi/100$. In this way the step function is not sharp, but the numerical results do not seem sensitive to the value of μ_e.

The numerical part of the computation is essentially as in I, but the computation is more critical than in I, because it is necessary to consider differences between values of g_π for different 2E and α, in which three figures are often lost. We hope that this loss of relative precision is not too critical, because the values obtained for g_π with the two methods of ref. s (11) and (12) of I are often coincident for eight figures for the lowest eigenvalues. Beside this we hope that the systematic error due to the limited number of gaussian integration points is equal in both term of the difference and so eliminated. The computation is made for the following values

$$2E = 139 \text{ MeV}, \qquad \alpha = 0$$
$$2E = 139 \text{ MeV}, \qquad \alpha/2\tau_\pi g_\pi = 0.1$$
$$2E = 300 \text{ MeV}, \qquad \alpha = 0.$$

The values of 2E are so different because g_π is very weakly dependent on them.

The masses of the mesons of the interaction are, as in I, in nucleonic mass m = 1

$$\frac{\mu}{m} = 0.1485 \text{ MeV} \qquad \text{for the pseudoscalar interactions}$$

$$\frac{\mu}{m} = 0.850 \qquad \text{for the vector interactions.}$$

CONCLUSIONS AND RESULTS. -

The results are given in the graphs of Fig. s 2, 3, 4 and in Table I. The abscissa contains the cut off's of the electromagnetic interaction in unit nucleonic mass = 1. Near each curve there is, in the same units, the value of the cut off of the strong interaction which binds the $N\bar{N}$ to form the π. In Table I there is, for each cut off, the values of the coupling constants of the strong interaction which binds the state.

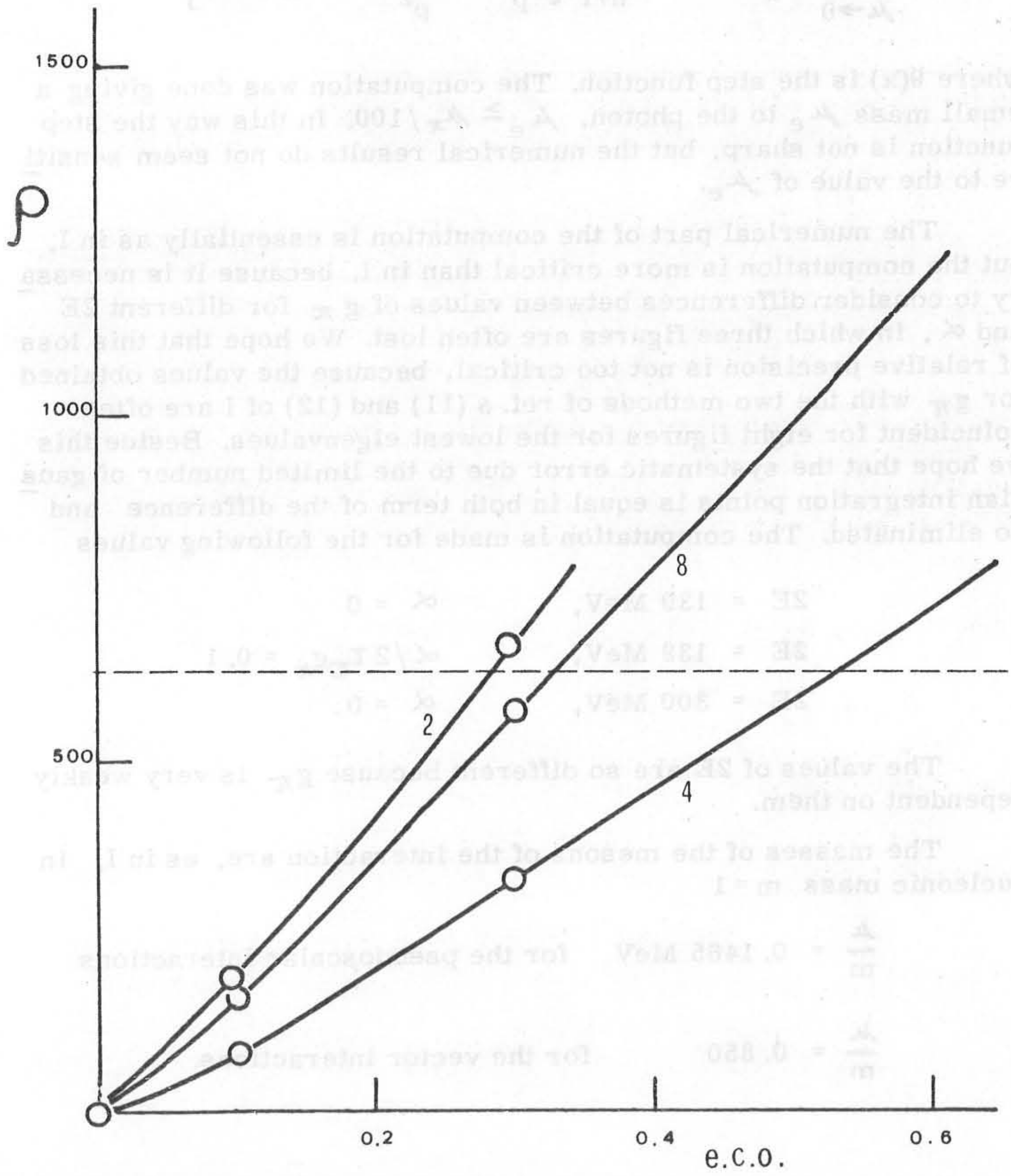

FIG. 2 - ρ as function of the electromagneti c cut off (e. c. o.). The strong interaction is isovector pseudoscalar plus isoscalar vector. (The dashed line is at the experimental value of ρ).

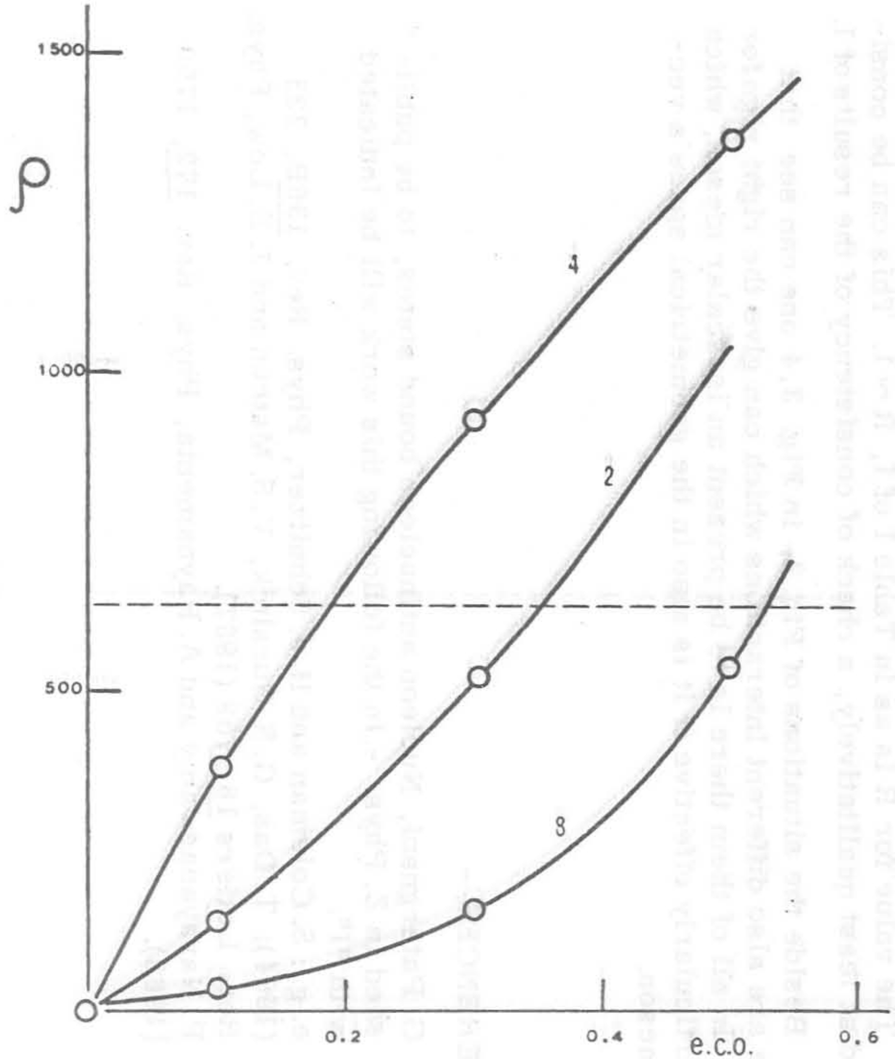

FIG. 3 - ρ as function of the electromagnetic cut off (e. c. o.). The strong interaction is isoscalar pseudoscalar. (The dashed line is at the experimental value of ρ).

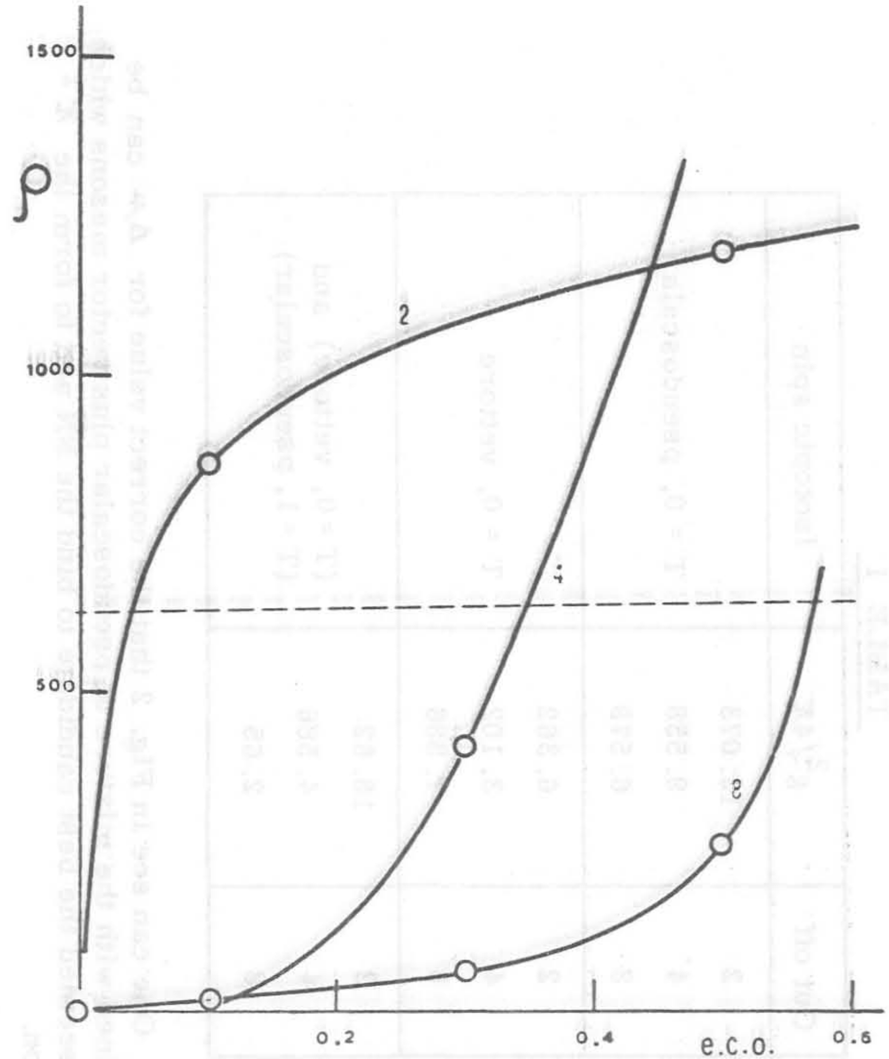

FIG. 4 - ρ as function of the electromagnetic cut off (e. c. o.). The strong interaction is isoscalar vector. (The dashed line is at the experimental value of ρ).

TABLE I

Cut off	$g^2/4\pi$	Isotopic spin
2	15.073	
4	9.558	T = 0, pseudoscalar
8	6.579	
2	6.362	
4	3.102	T = 0, vettore
8	1.886	
2	18.62	
4	4.566	(T = 0, vettore) and (T = 1, pseudoscalar)
8	2.65	

One can see in Fig. 2 that the correct value for $\Delta\mu$ can be obtained with the mixture of pseudoscalar plus vector mesons which in I seemed the best candidate to bind the $N\bar{N}$ and to form the $\pi^{\pm}$ meson.

The value for R is as in Table I of I, $R \sim 1$. This can be considered at least qualitatively, a check of consistency of the results of I.

Beside the situations of Fig. 2, in Fig. 3, 4 one can see that there are also different interactions which can give the right sign for $\Delta\mu$. In all of them there is to be present an isoscalar meson, which is particularly effective if it is also in the geometrical space a vector meson.

REFERENCES. -

(1) - G. Patergnani, Nucleon antinucleon bound states, to be published in Z. Phys. - In the following this work will be indicated with "I".

(2) - e. g. : S. Coleman and H. J. Schnitzer, Phys. Rev. 136B, 223 (1964); T. Das, G. S. Guralnik, V. S. Mathun and J. E. Low, Phys. Rev. Letters 18, 759 (1967).

(3) - P. Narayanaswamy and A. Paynamenta, Phys. Rev. 172, 1750 (1968).